

Question 5.

a)

A 12-sided die has each of the integers 1 through to 12 written on its faces.

If 7 different people each roll their own 12-sided die, in how many ways can exactly 5 people roll a 9?

$$\binom{7}{5} \times 11^2$$

$7C5 \times 1 \rightarrow$ Choose 5 players out of 7 to receive a 9.

$11^2 \Rightarrow$ other players receive any other value.

b)

Suppose 30 different people each roll their own 10-sided die.

In how many ways can exactly 6 people roll a 6, exactly 7 people roll a 7, ..., and exactly 9 people roll a 9?

$$\binom{30}{6} \binom{24}{7} \binom{17}{8}$$

6 people out of 30

\Rightarrow 7 people out of 24

\Rightarrow 17 people out of 8.

c)

How many 6-digit numbers less than 361146 do not contain any digits greater than 7?

First digit:

0, 1, 2.

Remaining Digits:

0, 1, 2, 3, 4, 5, 6, 7.

$$\therefore 2 \times 8^5 + 6 \times 8^4 + 1 \times 8^3 + 1 \times 8^2 + 4 \times 8^1 + 6 \times 8^0.$$

How many 5-digit numbers less than 43784 do not contain any digits greater than 8?

First digit:

1, 2, 3

if equal

$$\rightarrow 3 \times 9^4 + 3 \times 9^3 + 7 \times 9^2 + 8 \times 9 + 4 \times 9^0$$

cannot
be
zero

if equal

How many 6-digit numbers less than 562244 do not contain any digits greater than 7?

First Digit:

1, 2, 3, 4

$$\therefore 4 \times 8^5 + 6 \times 8^4 + 2 \times 8^3 + 2 \times 8^2 + 4 \times 8^1 + 4$$