Question 7.

a)

Find the solution to the recurrence relation

$a_n = 14a_{n-1} - 45a_{n-2}$ for all $n \geq 2$

which satisfies the initial conditions $a_0 = 11$ and $a_1 = 83$.

$a_n - 14a_{n-1} + 45a_{n-2} = 0$

$r^2 - 14r + 45 = 0$

$r \quad X \quad -9 \quad -5$

$(r-9)(r-5) = 0$

$r = 9, 5.$

$\therefore a_n = A(9)^n + B(5)^n$

$11 = A + B \Rightarrow B = 11 - A$

$83 = 9A + 5B \Rightarrow 83 = 9A + 5(11 - A)$

$\Rightarrow 83 = 9A + 55 - 5A$

$\Rightarrow 83 = 4A + 55$

$\Rightarrow 4A = 28$

$A = 7 \quad B = 4 \quad \therefore a_n = 7(9)^n + 4(5)^n$

b)

Find the general solution to the recurrence relation

$b_n = 7b_{n-1} + 8b_{n-2} - 14n + 37$ for all $n \geq 2$

which satisfies the initial conditions $b_0 = -5$ and $b_1 = 13$.

h_s:

$b_n - 7b_{n-1} - 8b_{n-2} = 0$

$r^2 - 7r - 8 = 0$

$r \quad X \quad -8 \quad 1$

$(r-8)(r+1) = 0$

$r = 8, -1$

$an+b$

$b_n = A(8)^n + B(-1)^n + p_s.$

$p_s = an + b - 7(a(n-1) + b) - 8(a(n-2) + b) = -14n + 37$

$\Rightarrow an + b - 7an + 7a - 7b - 8an + 16a - 8b = -14n + 37$

$\Rightarrow -14an + 23a - 14b = -14n + 37$

$\Rightarrow a = 1$

$23 - 14b = 37$

$-14b = 14$

$b = -1$ $\therefore p_s = n - 1$

$b_n = A(8)^n + B(-1)^n + n - 1$

$-5 = A + B - 1$

$A + B = -4 \Rightarrow B = -4 - A$

$13 = 8A - B$

$13 = 8A + 4 + A$

$9A = 9 \quad A = 1. \quad B = -5$

$b_n = 8^n - 5(-1)^n + n - 1$

c)

Find the general solution to the recurrence relation

$c_n = 9c_{n-1} - 20c_{n-2} + 2 \times 4^n$ for all $n \geq 2$

which satisfies the initial conditions $c_0 = 2$ and $c_1 = -15$.

h_s:

$c_n - 9c_{n-1} + 20c_{n-2} = 0$

$r^2 - 9r + 20 = 0$

$(r-5)(r-4) = 0$

$r = 5, 4$

$c_n = A(5)^n + B(4)^n + p_s$

$p_s = Cn4^n$, as 4^n is alr. sol.

$Cn4^n - 9C(n-1)4^{n-1} + 20C(n-2)4^{n-2}$

$= Cn4^n - 9Cn4^{n-1} + 9C4^{n-1} + 20Cn4^{n-2} - 40C4^{n-2}$

$= 9C4^{n-1} - 40C4^{n-2} = 2 \times 4^n$

$C = -8$

$c_n = A(5)^n + B(4)^n - 3n(4)^n$

$2 = A + B$

$-15 = 5A + 4B - 32$

$B = 2 - A$

$5A + 4B = 17$

$5A + 8 - 4A = 17$

$A = 9 \qquad B = -7$

$\therefore c_n = 9(5)^n - 7(4)^n - 8n(4)^n$

Note:

Be careful with Ps. Make sure your solution is unique.