2020 T1

```
Written by Haeohreum Kim (z5480978)

Most of these solutions have been derived entirely by me, so
there may be imperfections and wrong answers/reasonings. Please
contact me at my student email for any corrections.

These solutions were made to aid the preparation of MATH1231
students, not replace it. Good luck :)
```

Q3) c) Find an equation for the tangent plane to S at $P(\sqrt{5}, -\sqrt{5}, 20)$ and enter it in the box below:
Solution:

$$\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x - f(x) \\ y - f(y) \\ z - f(z) \end{pmatrix}$$

Q3) d) We can observe that the tangent plane at the point P passes through the origin. Write down the coordinates of another point Q, on the surface S, different from P, which also has the property that the tangent plane to S at Q passes through the origin.
Solution:
All we have to do is take a point on the same set as the original solution.
So here, we can choose $(\sqrt{5}, \sqrt{5}, 20), (-\sqrt{5}, -\sqrt{5}, 20)$, etc. This is due to the symmetry of the function.

Q7)
a) By implementing the initial conditions determine values for A_0 and A_1.
Solution:
A_0 can be found my considering that $x(0) = A_0$. Hence, $A_0 = 4$.
A_1 can be further found by using the fact that y(0) = 0, and hence,
$y'(0) = 4 \times x(0) = 16$.

b) By substituting the two series (3) and (4) into the differential equation (1) we can show that we have a relationship of the form:

$$A_{n+1} = c(n)B_n$$

Solution:

$$\begin{aligned} x'(t) &= A_1 + 2 \cdot A_2 t + \ldots + (n+1)A_{n+1}t^n \\ y(t) &= 4B_0 + 4B_1 t + \ldots + 4B_n t^n + 4B_{n+1}t^{n+1} \\ \therefore \quad A_{n+1} &= \frac{4}{n+1} \end{aligned}$$

c) Using the results above we can calculate that the 4th degree Maclaurin series of the form:

$$x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + \ldots + c_n t^n$$

Note that $c_0 = A_0$, and therefore, we can often find c_0 using $A_{n+1} = c(n)B_n$ and symmetrically $B_{n+1} = c(n)A_n$.

d)

By combining equations (1) and (2) into a single differential equation we show that

$$\frac{d^2x}{dt^2} - 16x = 0$$

By solving the second order differential equation in part d) together with the initial conditions provided, express x(t) as a hyperbolic trigonometric functions.

Solution:

Consider the homogenous equation $\lambda^2 - 16 = 0$; where we find the values $\lambda = 4$ and $\lambda = -4$.

Now, the equation $A \exp 4t + B \exp -4t$ can be created.

Consider that x(0) = 4, and x'(0) = 4y(0) = 0.

Therefore:

$$\begin{aligned} 4 &= A + B \\ 0 &= 4A - 4B \\ A &= 2 \\ B &= 2 \end{aligned}$$

Then, the function $2e^{4t} + 2e^{-4t}$, can be made into a hyperbolic trig function.

Q8)

Two bank accounts are opened simultaenously. We call them account X and account Y.

- Account X starts with 100,000 dollars and earn 5% interest per annum.
- At the end of each year the interest is paid into account X.
- Account Y starts with 300,000 dollars and earn 7% interest per annum.
- At the end of each year, half the interest is paid into account X and half into account Y.

  Let x_n and y_n be the amount of money (in dollars) in X and Y respectively at the end of n years, including the nth interest payment. Write:

  $$v_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

  a) Find a matrix A such that $v_{n+1} = Av_n$ for all non-negative integers n.
  Enter the matrix A in the box below.
  Solution:
  First figure out in linear equations the amount of money in each account.
  $X = 100000 \cdot 1.05 + 300000 \cdot 0.035$
  $Y = 300000 \cdot 1.035$
  Therefore, the matrix is constructed as such:

  $$\begin{pmatrix} 1.05 & 0.035 \\ 0 & 1.035 \end{pmatrix}$$