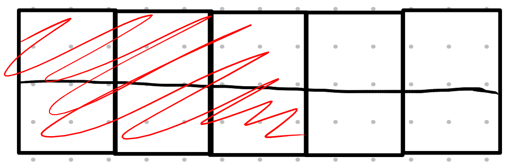
Question 4.

A firm works for the same 5 days each week.

Every employee must work exactly 3 full days and 2 half-days each week.

A half-day can be either morning or afternoon, and two half-days cannot be held on the same day.

a) How many possible different weekly schedules are there?



Choose 3 full days to work.
$^5C_3$

Now, four different ways to choose half days.
(mm, mn, nn, nm).

$\therefore$ $^5C_3 \times 4 = 210$.

b) If the firm has 186 employees, how many people must have the same work schedule for a particular week?

By PHP, $\text{ceil}\left(\frac{186}{40}\right) = 5$

c) What is the smallest number of employees needed to guarantee at least 4 workers have exactly the same schedule?

Find smallest number of employees x: $\frac{x}{40} > 3$ $\quad \underline{x = 121}$

Another example:

A firm works for the same 6 days each week.

Every employee must work exactly 2 full days and 4 half-days each week.

A half-day can be either morning or afternoon, and two half-days cannot be held on the same day.

a) How many possible different weekly schedules are there?

To find the general formula:

$\binom{n}{n-k} 2^k$. $\quad$ k: half days
$\quad$ n: days

$\binom{n}{n-k}$ = full days

2^k = two sessions (morning / arvo).

2 gives a "decision tree" type division of "choices".

$\therefore$ $\binom{6}{2} 2^4 = 240$