2020 T1

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Most of these solutions have been derived entirely by me, so there may
be imperfections and wrong answers/reasonings. Please contact me at my
student email for any corrections.

These solutions were made to aid the preparation of MATH1231 students,
not replace it. Good luck :)
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Q3) c) Find an equation for the tangent plane to S at $P(\sqrt{5}, -\sqrt{5}, 20)$ and enter it in the box below:
Solution:

$$\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x - f(x) \\ y - f(y) \\ z - f(z) \end{pmatrix}$$

Q3) d) We can observe that the tangent plane at the point P passes through the origin. Write down the coordinates of another point Q, on the surface S, different from P, which also has the property that the tangent plane to S at Q passes through the origin.
Solution:
All we have to do is take a point on the same set as the original solution.
So here, we can choose $(\sqrt{5}, \sqrt{5}, 20), (-\sqrt{5}, -\sqrt{5}, 20)$, etc. This is due to the symmetry of the function.

Q5)
c)

Q7)
a) By implementing the initial conditions determine values for A_0 and A_1.
Solution:
A_0 can be found my considering that $x(0) = A_0$. Hence, $A_0 = 4$.
A_1 can be further found by using the fact that y(0) = 0, and hence, $y'(0) = 4 \times x(0) = 16$.

b) By substituting the two series (3) and (4) into the differential equation (1) we can show that we have a relationship of the form:

$$A_{n+1} = c(n)B_n$$

Solution:

$$\begin{aligned} x'(t) &= A_1 + 2 \cdot A_2 t + \ldots + (n+1)A_{n+1}t^n \\ y(t) &= 4B_0 + 4B_1 t + \ldots + 4B_n t^n + 4B_{n+1}t^{n+1} \\ \therefore \quad A_{n+1} &= \frac{4}{n+1} \end{aligned}$$

c) Using the results above we can calculate that the 4th degree Maclaurin series of the form:

$$x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + \ldots + c_n t^n$$

Note that $c_0 = A_0$, and therefore, we can often find c_0 using $A_{n+1} = c(n)B_n$ and symmetrically $B_{n+1} = c(n)A_n$.

d)

By combining equations (1) and (2) into a single differential equation we show that

$$\frac{d^2x}{dt^2} - 16x = 0$$

By solving the second order differential equation in part d) together with the initial conditions provided, express x(t) as a hyperbolic trigonometric functions.

Solution:

Consider the homogenous equation $\lambda^2 - 16 = 0$; where we find the values $\lambda = 4$ and $\lambda = -4$.

Now, the equation $A \exp 4t + B \exp -4t$ can be created.

Consider that x(0) = 4, and x'(0) = 4y(0) = 0.

Therefore:

$$\begin{aligned} 4 &= A + B \\ 0 &= 4A - 4B \\ A &= 2 \\ B &= 2 \end{aligned}$$

Then, the function $2e^{4t} + 2e^{-4t}$, can be made into a hyperbolic trig function.

Q8)

Two bank accounts are opened simultaenously. We call them account X and account Y.

- Account X starts with 100,000 dollars and earn 5% interest per annum.
- At the end of each year the interest is paid into account X.
- Account Y starts with 300,000 dollars and earn 7% interest per annum.
- At the end of each year, half the interest is paid into account X and half into account Y.

  Let x_n and y_n be the amount of money (in dollars) in X and Y respectively at the end of n years, including the nth interest payment. Write:

  $$v_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

  a) Find a matrix A such that $v_{n+1} = Av_n$ for all non-negative integers n. Enter the matrix A in the box below.

  Solution:

  First figure out in linear equations the amount of money in each account.

  $X = 100000 \cdot 1.05 + 300000 \cdot 0.035$

  $Y = 300000 \cdot 1.035$

  Therefore, the matrix is constructed as such:

  $$\begin{pmatrix} 1.05 & 0.035 \\ 0 & 1.035 \end{pmatrix}$$

  d) After finding the eigenvectors and eigenvalues, we can use the diagonalisation of the matrix to find powers of A, and hence, find the value of each account in 9 years. Consider:

  $$\begin{aligned} v_{n+1} &= Av_n \\ v_{11} &= A^{11}v_0 \\ &= MD^{11}M^{-1}v_0 \end{aligned}$$

  You can do this on maple fairly easily:

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A := <<1.05, 0>|<0, 1.035>>
B := <<1, 0>|<-0.919145030018057, 0.393919298579170>>
B.(A^9).MatrixInverse(B).<150000, 250000>
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Q9)

b)

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For a set S to be considered linearly independent, it must be shown such
that:

μ1x1 + μ2x2 + μ3x3 + ... = 0, such that x1, x2, x3, ... in S, and μ1,
μ2, μ3 = 0.

Visually, when the set S's matrix representation is row reduced, this
means that each column is leading, as it can be seen:
μ1p1(x) + μ2p2(x) = 0

and therefore, μ1 = 0, μ2 = 0. Therefore, linearly indepenent.
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c)

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The dimension can either be 3, or 2, since that we've derived that the
set is linearly independent. However, part a) has shown that there are
polynomials that don't exist within the set, and hence, the dimension
must be 2.
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d)

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We are given that B is in the set S, and we have also derived that the
dimension of S is two. **Basis** vectors are defined as vectors that are
linearly independent within a specific set S, and to be considered a
**basis**, there must be atleast n vectors within an nth-dimensional
vector space.

Hence, there are two _linearly independent_ vectors in B, and S is two
dimensional, therefore, B forms a basis for S.
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Q11)

a)

$$\begin{aligned} P(W) &= P(RW) + P(BW) \\ &= \frac{1}{6} \cdot \frac{1}{20} + \frac{5}{6} \cdot \frac{1}{6} \end{aligned}$$

b)

$$P(RW|W) = \frac{P(RW \cap W)}{P(W)} = \frac{\left(\frac{1}{6} \cdot \frac{1}{20}\right)}{P(W)}$$

c)

$$\begin{aligned} P(\text{LOSS} \mid \text{WIN} >= 1) &= \frac{P(LOSS \cap WIN \geq 1)}{P(WIN \geq 1)} \\ &= \frac{\left(\frac{1}{6} \cdot \frac{19}{20} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{20} \cdot \frac{5}{6}\right)}{\frac{1}{20} \cdot \frac{1}{6} + \frac{1}{20} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{19}{20}} \end{aligned}$$

Look above for reasonings :(