Question 3 Solutions

For the production of a local play, 19 people auditioned and 7 of them joined the cast. Of these, 5 had named roles.

In how many ways could this have happened?

Auditioning done by $19 \to 7$, $\therefore$

$$\binom{19}{7}.$$

5 "named roles" $\therefore$ ${}^7P_5$.

$\therefore$ Final answer: $\binom{19}{7}\,{}^7P_5$

In how many ways can 5 boys and 6 girls be arranged in a line so that the boys and girls are in separate groups?

(BBBBB)(GGGGGG)

$2 \times 5! \times 6!$ ($5!$ & $6!$ different ways to order the groups).

How many 4-digit numbers have the property that the first and last digits are different, and all the digits are odd?

0 1 2 3 4 5 6 7 8 9

5×4, first & last digit.
5^2 (two other digits).
$\therefore 5^3 \times 4$.

b)

In how many ways can 4 boys and 8 girls be arranged in a line so that two particular students are not next to each other?

Consider case when they ARE together:

(P_1P_2)RRRRRRRRRR

$= 11! \times 2$

$\therefore |U| - |\text{Together}| = |\sim\text{Together}|$

$= 12! - 11! \times 2$

How many 5-digit numbers have the property that the first and last digits are different, and all the digits are even?

0 1 2 3 4 5 6 7 8 9

$4 \times 4 \times 5^3$ (first term can't be zero).

In how many ways can 7 boys and 7 girls be arranged in a line so that the boys and girls are alternating?

(BG)(BG)(BG)(BG)(BG)(BG)(BG)

Line up 7 boys : 7!

Line up 7 girls : 7!

Either boys or girls first, ∴

$\underline{2 \times (7!)^2}$.

In how many ways can 6 boys and 7 girls be arranged in a circle so that two particular students are next to each other?

(BG) PPPPPPPPPPP

(n-1)!

12 groups.

11! ways in a circle.

Two ways to change BG group.

∴ $2 \times 11!$

In how many ways can 6 boys and 3 girls be arranged in a circle so that the boys and girls are in separate groups?

(BBBBBB)(GGG)

$= 1! \times 6! \times 3!$

One way on circle; 6 ways to order boys, 3 ways to order girls.

In how many ways can 8 boys and 4 girls be arranged in a circle so that two particular students are not next to each other?

Consider the two students sitting together.

$$(P_1 P_2)\,RRRRRRRRRR$$

$$= (10!) \times 2.$$

Deduct from universal case:

$$11! - 2 \times 10!$$