Chapter 10

Sensory Delay

Various natural and artificial systems consist of multiple autonomous agents that communicate with each other and work collaboratively. Examples include rescue robots [1], bacterial colonies [2, 3], warehouse trucks [4], and human crowds [5]. The interaction between these agents often creates complex behaviors. An important parameter of such systems is the *sensory delay* between when an agent receives a signal and when it reacts to it. Depending on the system, the reason for this delay can be optical, chemical, or biological. It is also possible that the delay is intentionally added to engineer a desired system behavior.

Autonomous robots represent typical examples of autonomous agents (Fig. 10.1). They are widely employed in industry and operate at different scales. A robot fundamentally has two sets of elements: *sensors* and *motors*. While sensors provide the robots with information about their surroundings, motors enable them to move or do work in their environment. The introduction of a sensory delay can alter the behavior of the robots, both at the individual

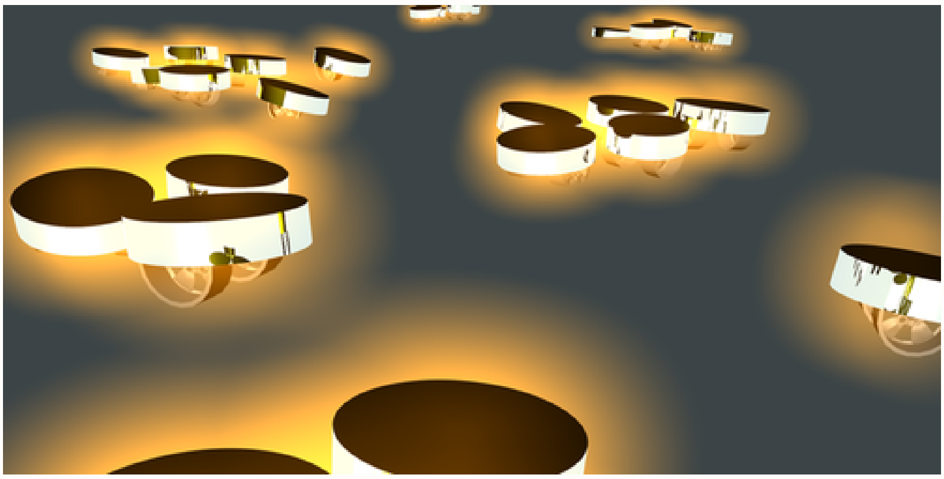

Figure 10.1: **Swarming of robots.** Each robot emits light and adjusts its speed depending on the total light intensity it measures. By controlling the sensory delay, it is possible to engineer the behavior of a group of robots, getting them to cluster together or to disperse away from each other — a feature that can be interesting for many applications, e.g., in delivery and search-and-rescue operations. *Picture by Mite Mijalkov.*

and group level [6].

In this chapter, we simulate the behavior of robots with sensory delay. First, we analyze the response of a single robot to a light field as a function of sensory delay. Then, we simulate a group of robots and show that the clustering behavior of these robots depends strongly on their sensory delay. Specifically, we show that the robots' sensory delay can be tuned to make them cluster together or explore their environment by dispersing away from each other.

Example codes: Example Python scripts related to this chapter can be found on:
https://github.com/softmatterlab/SOCS/tree/main/Chapter%5F10%5FSensory%5FDelay
Readers are welcome to participate in the discussions related to this chapter on:
https://github.com/softmatterlab/SOCS/discussions/19

10.1 A light-sensitive robot

First, we consider a single robot that moves in a two-dimensional plane while subject to rotational diffusion. The equations of motion for this autonomous robot can be written as:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} & = & v\cos\phi(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} & = & v\sin\phi(t) \\ \dfrac{\mathrm{d}\phi(t)}{\mathrm{d}t} & = & \sqrt{\dfrac{2}{\tau}}w_\phi \end{cases} \tag{10.1}$$

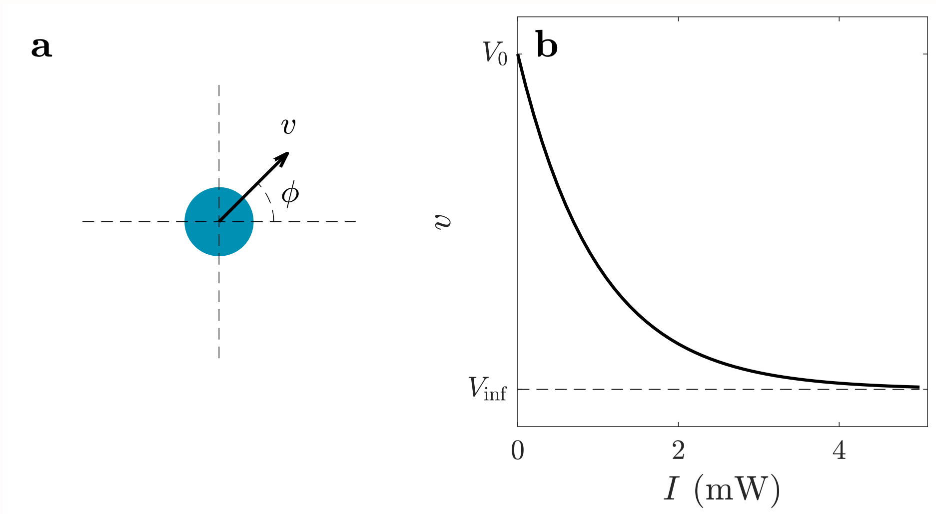

Figure 10.2: **Working principle of a light-sensitive robot. a** The motion of the robot is described by Eq. 10.1. It moves with a speed v that is a function of the optical intensity it measures, as shown in **b**, in a direction ϕ that randomly changes over time. **b** Robot's speed as a function of the measured light intensity (Eq. 10.2).

where (x, y) are the position of the robot, ϕ is its orientation, v is its speed, τ represents its rotational diffusion timescale, and w_ϕ is a white noise. The schematic representation is shown in Fig. 10.2a. The robot adjusts its speed v according to the light intensity that it measures: It changes its speed from V_0 to $V_{\rm inf}$ with an exponential decay function:

$$v = V_{\rm inf} + (V_0 - V_{\rm inf})e^{-I}, \tag{10.2}$$

where I is the measured light intensity. The robot is going to move at its maximum speed (V_0) if the measured light intensity is zero and exponentially slow down to the minimum speed ($V_{\rm inf}$) as the measured light intensity gets higher, as shown in Fig. 10.2b.

Although here we analyze the case of an autonomous robot, there is a variety of real-world agents that perform similar motion. Examples include microswimmers, motile bacteria and animals. All these agents adjust their speed according to the sensory inputs they receive from their environment. After all, with a constant speed, the robot would practically perform an active Brownian motion (see the equations for active Brownian motion in Chapter 9).

Exercise 10.1: Simulation of a light-sensitive robot. Simulate the motion of a robot (Eq. 10.1) with light-sensitive speed (Eq. 10.2).

a. Simulate the motion of the robot in the absence of light (i.e., $I \equiv 0$ and $v \equiv V_0$). Show that this is a standard active Brownian motion with a persistence length $L = V_0\tau$.

b. Simulate the motion of a robot in a periodic light pattern given by

$$I(x) = \left[\sin\left(2\pi\frac{x}{\Lambda}\right)\right]^2.$$

Study the robot's motion as a function of Λ/L. Where does the robot spends most of its time? In the bright or in the dark fringes?

c. Consider now a time-varying light pattern given by

$$I(x,t) = \left[\sin\left(2\pi\frac{x-ct}{\Lambda}\right)\right]^2.$$

Show that this light pattern induces an average motion of the robot along the x-direction. Study the robot motion as a function of the parameter c (it can be useful to consider the relation between the characteristic length scales $c\tau$, Λ, and L). What is the value of c that maximizes the robot's motion? Does the average robot motion change direction as a function of the parameters?

10.2 Single robot with sensory delay

So far, we considered the case where the measured light intensity affects the robots speed immediately with no delay. In this section, we consider that the propulsion speed of the robot v is a function of the measured light intensity with a delay:

$$v = v(I(t-\delta)), \tag{10.3}$$

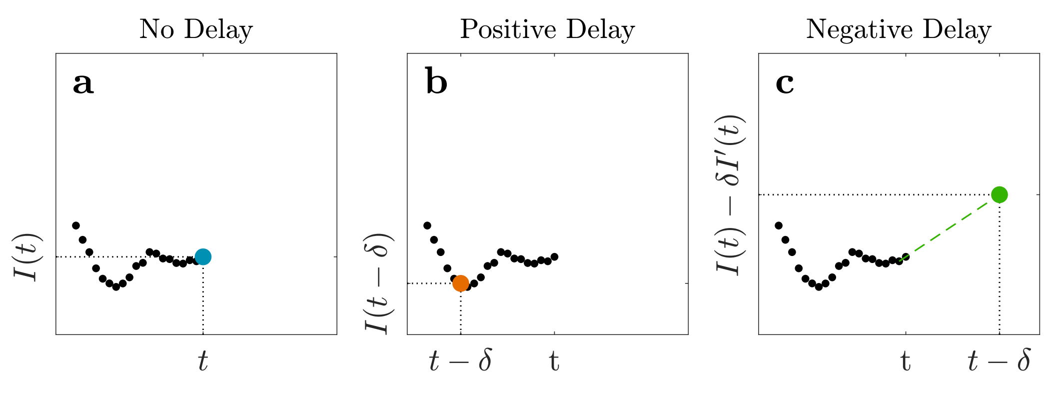

Figure 10.3: **Null, positive, and negative sensory delay.** **a** When the robot reacts to the measured light intensity (black dots) with *no delay* ($\delta = 0$), it uses the currently measured value $I(t)$ (blue dot) to adjust its speed. **b** When the robot reacts to the measured light intensity with *positive delay* ($\delta > 0$), it uses the past value measured δ time before $I(t-\delta)$ (orange dot). **c** When the robot reacts with *negative delay* ($\delta < 0$), it uses the extrapolated future value of the light intensity $I(t-\delta) \approx I(t) - \delta I'(t))$ (green dot).

where $I(t-\delta)$ refers to the light intensity that the robot measured δ time before. We will observe that the robot's qualitative response to a light field can be tuned by changing only this delay parameter.

If we have *no sensory delay* ($\delta = 0$), the robot uses the current value of the light intensity it measures ($I(t)$) instantly, as shown in Fig. 10.3a. If we have a *positive delay* ($\delta > 0$), the robot reacts to the value of the past intensity that is measured δ time before, as shown in Fig. 10.3b. Although less intuitive, it is also possible to have a *negative delay* ($\delta < 0$): The robot can use the current intensity and its trend (time derivative) in order to approximate a future state of the light intensity to adjust its speed, i.e., $I(t-\delta) = I(t) - \delta I'(t)$ for $\delta < 0$, as shown in Fig. 10.3c. This can be interpreted as the robot acting in response to the predicted future state of the system.

In this section, we will focus on how the delay parameter δ can be engineered to control the behavior of an autonomous robot. Specifically, a positive sensory delay ($\delta > 0$) is going to make the robot spend more time in higher-light-intensity (i.e., lower-speed) regions and a negative sensory delay ($\delta < 0$) will make the robot escape from high-intensity (i.e., low-speed) regions.

Exercise 10.2: Robot in a Gaussian light intensity. Simulate the motion of a light-sensitive robot (Eq. 10.1) in a Gaussian light intensity profile (as shown in Fig. 10.4a)

$$I(x,y) = I_0 e^{-(x^2+y^2)/r_0^2},$$

where $I_0 = 1\,\mathrm{W}$ and $r_0 = 1\,\mathrm{m}$ are constants. Use the exponential decay function in

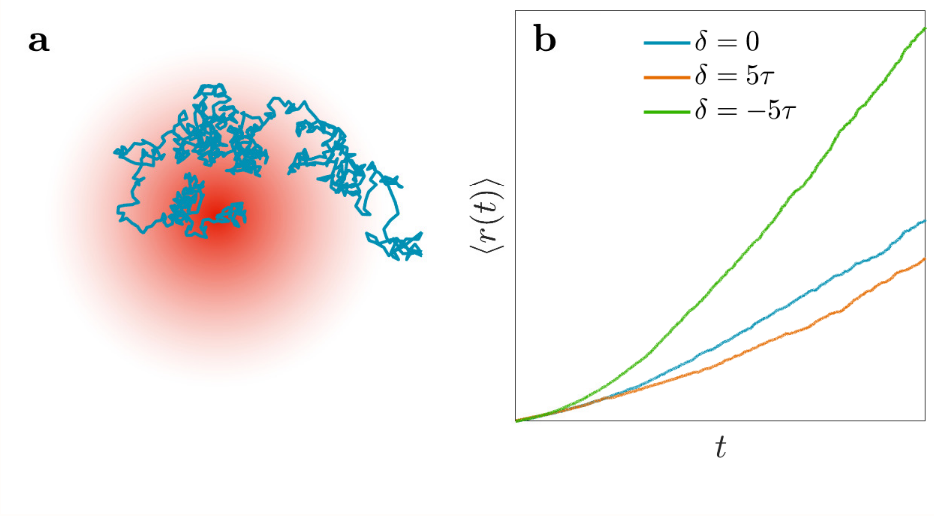

Figure 10.4: **Robot in Gaussian light field with sensory delay. a** An autonomous robot that moves according to Eq. 10.1 is placed in a Gaussian light field starting from the center. The statistical properties of the robot's motion can be tuned by controlling the delay δ. A sample trajectory with no sensory delay ($\delta = 0$) is shown (blue line). **b** The average distance of the robot from the center as a function of time for no delay (blue line, $\delta = 0$), for positive delay (red line, $\delta = 5\tau$), and for negative delay (green line, $\delta = -5\tau$). The robot with a positive sensory delay tends to stay closer to the high-intensity region, whereas the robot with a negative sensory delay escapes the high-intensity region quickly. The parameters used for this simulation are $I_0 = 1\,\mathrm{W}$, $r_0 = 1\,\mathrm{m}$, $V_0 = 1\,\mathrm{m\,s^{-1}}$, $V_\infty = 0.1\,\mathrm{m\,s^{-1}}$.

Eq. 10.2 (Fig. 10.2b) with delay for the particle's response to the light field, i.e.,

$$V(I,\delta) = V_{\mathrm{inf}} + (V_0 - V_{\mathrm{inf}})e^{-I(t-\delta)},$$

where $I(t-\delta)$ is the light intensity measured by the robot with sensory delay δ.

a. Simulate the motion of a robot with no sensory delay ($\delta = 0$). Start with the robot in the middle of the Gaussian light field ($x(0) = 0$, $y(0) = 0$). Plot some example trajectories as shown in Fig. 10.4a.

b. Play with the parameter δ and visualize the differences in robot's behavior. Observe that the robot is more likely to stay longer in the higher-light-intensity region for positive delay and escapes quicker for negative delays.

c. Measure and plot the average radial distance from the center $\langle r(t) \rangle$ with $r(t) = \sqrt{x(t)^2 + y(t)^2}$ for different delays ($\delta = 0$, $\delta = 5\tau$ and $\delta = +5\tau$). Show the quantitative difference, as illustrated in Fig. 10.4b.

So far, we have studied the behavior of the robot in an unbounded environment. However, boundaries are often present. In the next exercise, we are going to examine the position distributions of the robot as a function of sensory delay in a bounded environment.

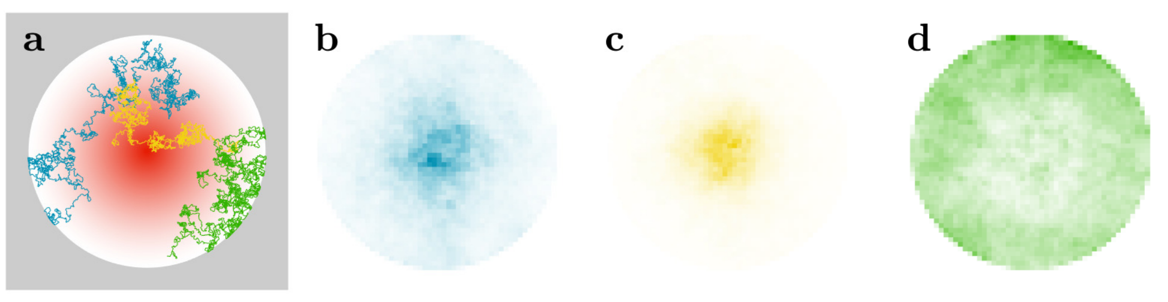

Figure 10.5: **Robot in a circular well.** **a** An autonomous robot that moves according to Eq. 10.1 is placed in a Gaussian light field confined within a circular well with solid boundaries. The distribution of the robot's trajectory can be tuned by controlling δ. Sample trajectories with no sensory delay (blue line, $\delta = 0$), positive sensory delay (yellow line, $\delta = 5\tau$), and negative sensory delay (green line, $\delta = 5\tau$) are shown. **b** With $\delta = 0$, the robot spends most of its time at intermediate radius values. **c** With a positive sensory delay ($\delta = 5\tau$), the robot spends most of its time in the highest-intensity (i.e., lowest-speed) region. **d** With a negative sensory delay ($\delta = -5\tau$), the robot spends most of its time in the lowest-intensity (highest-speed) region. The parameters used for this simulation are $R = 2\,\mathrm{m}$, $I_0 = 1\,\mathrm{W}$, $r_0 = 1\,\mathrm{m}$, $V_0 = 1\,\mathrm{m\,s^{-1}}$, $V_\infty = 0.1\,\mathrm{m\,s^{-1}}$.

Exercise 10.3: Robot in a circular well. Repeat the simulation in Exercise 10.2 ($I_0 = 1\,\mathrm{W}$, $r_0 = 1\,\mathrm{m}$, $V_0 = 1\,\mathrm{m\,s^{-1}}$, $V_\infty = 0.1\,\mathrm{m\,s^{-1}}$) but with a solid circular boundary of radius $R = 2$, as shown in Fig. 10.5a. *[Hint: The boundary can be implemented by, at each time step, checking whether the robot is outside the well (i.e., $x(t)^2 + y(t)^2 > R^2$) and, if so, reflecting back its position into the well (i.e., $x(t) \to x(t)R/\sqrt{x(t)^2+y(t)^2}$, $y(t) \to y(t)R/\sqrt{x(t)^2+y(t)^2}$).]*

a. Calculate the spatial probability distribution of the robot as a function of delay. Show that the robot tends to spend more time near the center (borders) when the delay is positive (negative) as shown in Figs. 10.5b-d. Can you find a value of the delay for which the probability distribution is uniform? *[Hint: Check Ref. [6].]*

b. Compute and plot the radial drift of the particle (average radial displacement in one time step) as a function of the radius ($x^2 + y^2$). Can you find a value of the delay for which the radial drift vanishes? *[Hint: Check Ref. [6].]*

10.3 Multiple robots with sensory delay

We will now extend our observations for a single robot to multiple robots. In this case, instead of having an externally-applied light field, each robot will emit its own light field, which is then measured by the other robots. Each robot will measure the cumulative light intensity that is created by the other robots while disregarding its own light. Like for the case of a single robots, multiple robots are also going to adjust their speed based on their measured light intensity with a delay δ.

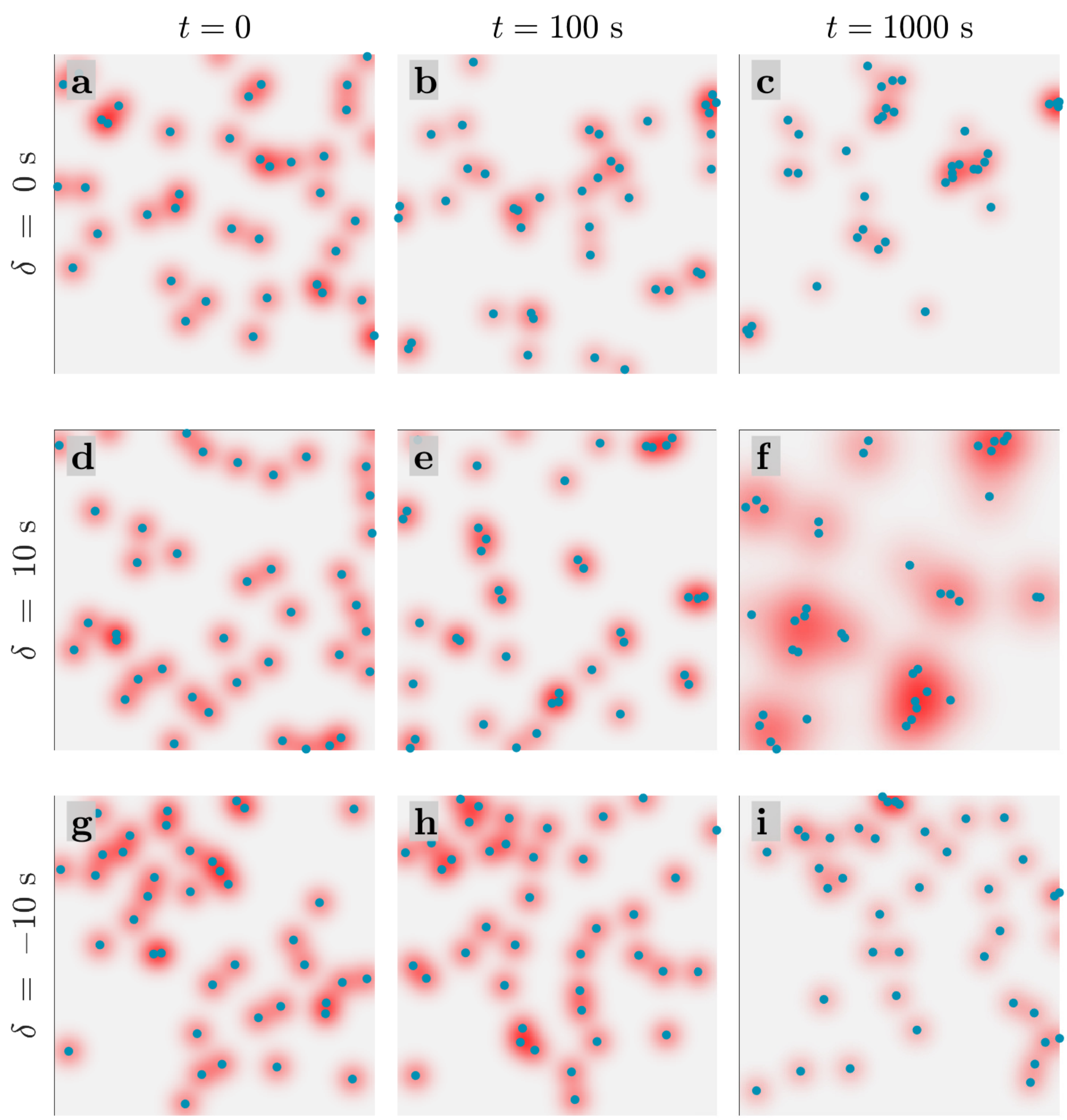

Figure 10.6: **Tuning the collective behavior of robots with sensory delay. a-c** Snapshots of 40 autonomous light-sensitive robots moving in an arena with no sensory delay at **a** $t = 0$ s, **b** 100 s, and **c** $t = 1000$ s. The robots tend to stay close to each other. **d-f** With positive sensory delay $\delta = +10$ s, the robots get closer over time and form clusters. **g-i** With negative sensory delay $\delta = -10$ s, the robots move away from each other and spread around the arena. The parameters used for this simulation are $R = 2$ m, $I_0 = 5$ W, $r_0 = 1$ m, $V_0 = 1\,\mathrm{m\,s^{-1}}$, $V_\infty = 0.1\,\mathrm{m\,s^{-1}}$.

We consider n robots with positions (x_i, y_i), $i = 1, \ldots, n$. Assuming that each robot emits a light field with a Gaussian profile, the total light intensity on each robot can be calculated as

$$I_i(t) = \sum_{j \neq i} I_0 \exp\left[-\frac{(x_j - x_i)^2 + (y_j - y_i)^2}{r_0^2}\right], \tag{10.4}$$

where r_0 is the decaying length-scale of the signal and I_0 is the intensity of the light emitted from each robot. Since individual robots are attracted to high (low) intensity region with positive (negative) sensory delay, multiple light emitting robots cluster together with positive delay (as shown in Figs. 10.6d-f) while they diverge away from each other with negative delay (as shown in Figs. 10.6g-i).

Exercise 10.4: Multiple robots with sensory delay. Simulate $n = 50$ robots in a square box of length $L = 10$ with periodic boundary conditions. Calculate the light field measured by each robot using Eq. 10.4 and adjust the robot speed according to Eq. 10.2. Use $\tau = 2\,\mathrm{s}$ $r_0 = 1\,\mathrm{m}$, and $I_0 = 5\,\mathrm{W}$. Place the robots randomly and start the simulation.

a. Simulate this multiple-robot system and observe its behavior. Show that the robots tend to stay close to each other when there is no delay, as shown in Figs. 10.6a-c.

b Show that the robots come closer together when there is a positive delay and form clusters, as shown in Figs. 10.6d-f.

c Show that the robots move away from each other and explore the arena when there is a negative delay, as shown in Figs. 10.6g-i.

10.4 Further readings

The simulations in this chapter are largely inspired by Ref. [6], where you can find the details of the simulations with single and multiple robots as well as the theory behind these results and a set of experiments with real robots.

While this chapter has focused on sensory delay affecting the speed of the autonomous robots, Ref. [7] generalizes these results to the case where the robot's rotational diffusion coefficient also depends on the measured light intensity and is subject to sensory delay. It provides an extensive study of how the distribution and radial drift of the light-sensitive robots depend on the interaction between the two delay parameters.

Ref. [8] provides a review where these results are placed in a more general context and related to stochastic integrals and to the so-called Itô-Stratonovich dilemma (see also Chapter 7).

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