Network Dynamics and Graph - HW3 - Politecnico di Torino - Hafez Ghaemi - S289963

```
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt

import networkx.generators.random_graphs as rg
import math

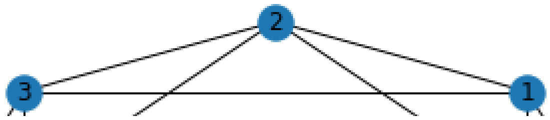
import collections
from networkx.generators.random_graphs import watts_strogatz_graph as ws
import itertools

```

Preliminary parts

```
G = nx.Graph()
k = 4
n = 8
for i in range(n-2):
    G.add_edge(i, i+1)
    G.add_edge(i, i+2)
G.add_edge(n-2, n-1)
G.add_edge(n-2, 0)
G.add_edge(n-1, 0)
G.add_edge(n-1, 1)

nx.draw_circular(G, with_labels = True)
plt.savefig("k_reg.eps",format='eps')
```




```
def sim_SIR_epidemic(G, n_weeks = 5, initial_states = 'random', random_sick = 3, beta = 0.
    '''
    G: networkx.classes.graph.Graph, graph
    n_weeks: int, number of simulation weeks
    initial_states: list, initial state of agents or default 'random' initialize sick peop
    random_sick: int, how many sick agents if initial state is randomly generated, default
    beta: float, the probability that the infection is spread from an infected individual
    rho: float, the probability that an infected individual will recover, default 0.7
    until_end: boolean, indicates whether the simulation continues until the end or finish
    '''

    # Number of people
    n_agents = len(G)

    A_ = [0,1,2] # 0-S, 1-I, 2-R

    states = [0 for _ in range(n_agents)] # 0-S, 1-I, 2-R

    if initial_states == 'random':
        choices = np.random.choice(G.nodes(),size = random_sick, replace= False)
        for i in choices:
            states[i] = 1
        initial_states = states.copy()

    old_states = initial_states.copy()
#     print('Initial states of agents: {}'.format(initial_states))

    W = nx.convert_matrix.to_numpy_matrix(G)

    weeks = []
    weeks.append(initial_states.copy())

    newly_infected_list = [1]
    for week in range(n_weeks):
        if not until_end:
            if not 1 in states:
                #print('No one is sick! \nGREAT SUCCESS!')
                break

#         print("States: {}, \nWeek: {}".format(states, week+1))
        newly_infected = 0
        for agent in range(n_agents):
            # if S:
            if old_states[agent] == 0:
                ngh_states = np.squeeze(np.asarray(W[agent]))*old_states # get neighbors'
                m = collections.Counter(ngh_states)[1] # count sick neighbors
```

```
                if np.random.rand() < (1-(1-beta)**m):
                    states[agent] = 1 # get sick
                    newly_infected += 1
            # if I:
            elif old_states[agent] == 1:
                if np.random.rand() < rho:
                    states[agent] = 2 # get recovered

        weeks.append(states.copy())
        newly_infected_list.append(newly_infected)

        old_states = states.copy()
#         print(old_states)
    return weeks, newly_infected_list
        # weeks = weeks[:5]
```

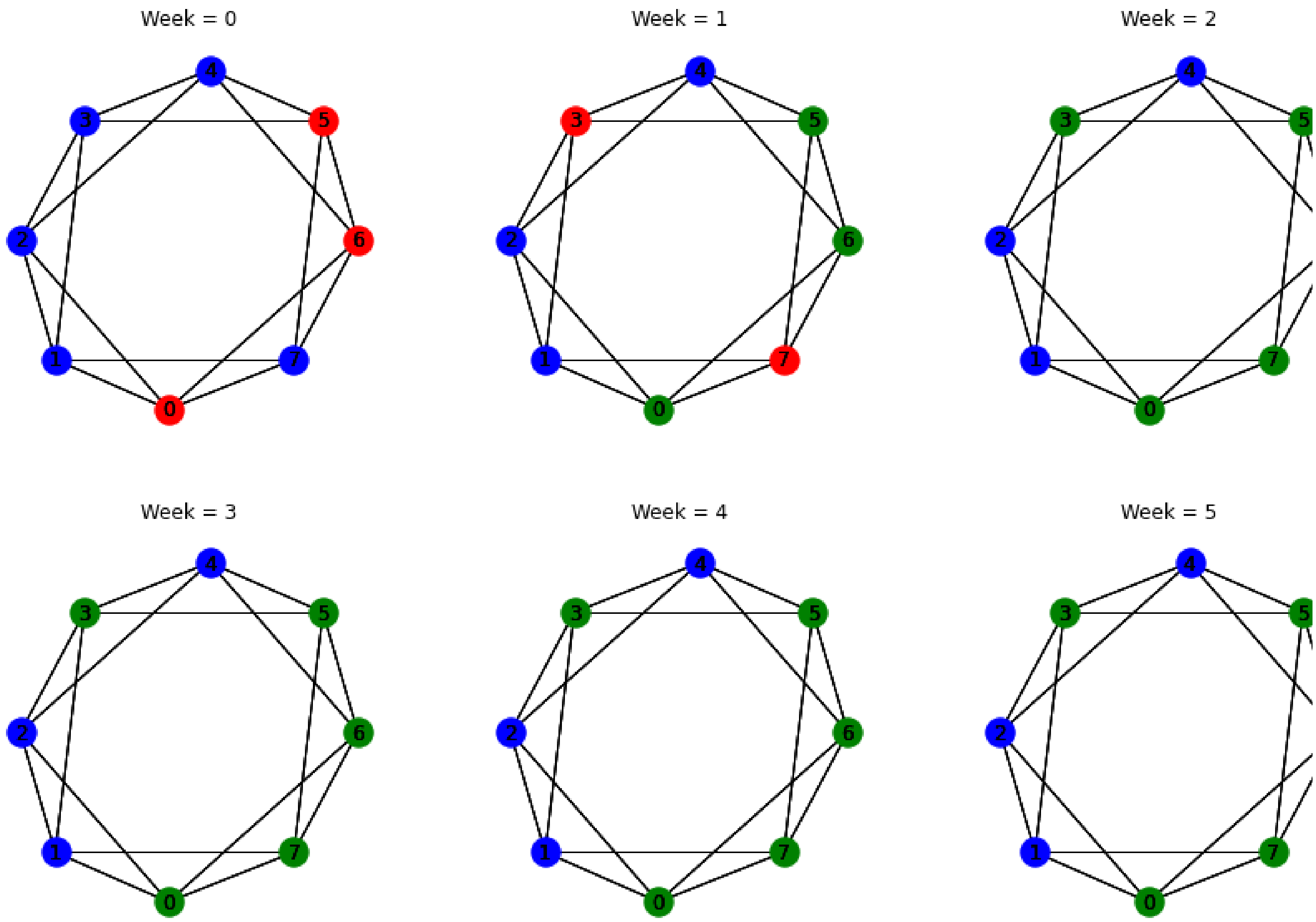
```
def get_sir_count(weeks):
    '''
    weeks: list, list of states through weeks
    '''
    susceptibles = [collections.Counter(np.asarray(w))[0] for w in weeks]
    infected = [collections.Counter(np.asarray(w))[1] for w in weeks]
    recovered = [collections.Counter(np.asarray(w))[2] for w in weeks]
    #print('susceptibles: {}, \ninfected: {}, \nrecovered: {}'.format(susceptibles, infect
    return susceptibles, infected, recovered
```

```
weeks, newly_infected_list = sim_SIR_epidemic(G, n_weeks=5, initial_states='random', rando
# Plot the infection spread
pos = nx.spectral_layout(G)
fig = plt.figure(figsize=(15,10))
for t in range(0,len(weeks)):
    plt.subplot(2,3,t+1)
    week = weeks[t]

    x = np.array(week.copy())
    nx.draw_spectral(G,
        with_labels=True,
        nodelist=np.argwhere(x==0).T[0].tolist(),
        node_color = 'b')
    nx.draw_spectral(G,
        with_labels=True,
        nodelist=np.argwhere(x==1).T[0].tolist(),
        node_color = 'r')
    nx.draw_spectral(G,
        with_labels=True,
        nodelist=np.argwhere(x==2).T[0].tolist(),
        node_color = 'g')
    plt.title('Week = {0}'.format(t))
```

```
susceptibles, infected, recovered = get_sir_count(weeks)

```



Problem 1.1

Random Initial Condition

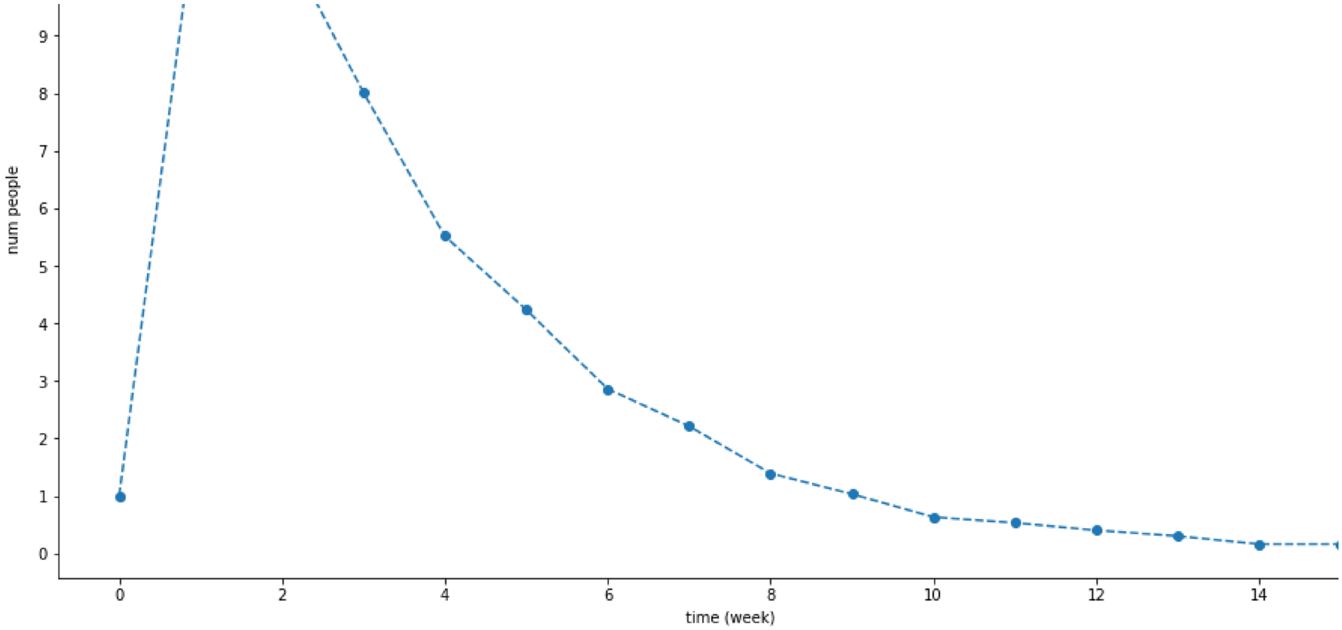
```
G = nx.Graph()
k = 4
n = 500
for i in range(n-2):
    G.add_edge(i, i+1)
    G.add_edge(i, i+2)
G.add_edge(n-2, n-1)
G.add_edge(n-2, 0)
G.add_edge(n-1, 0)
G.add_edge(n-1, 1)
```

```

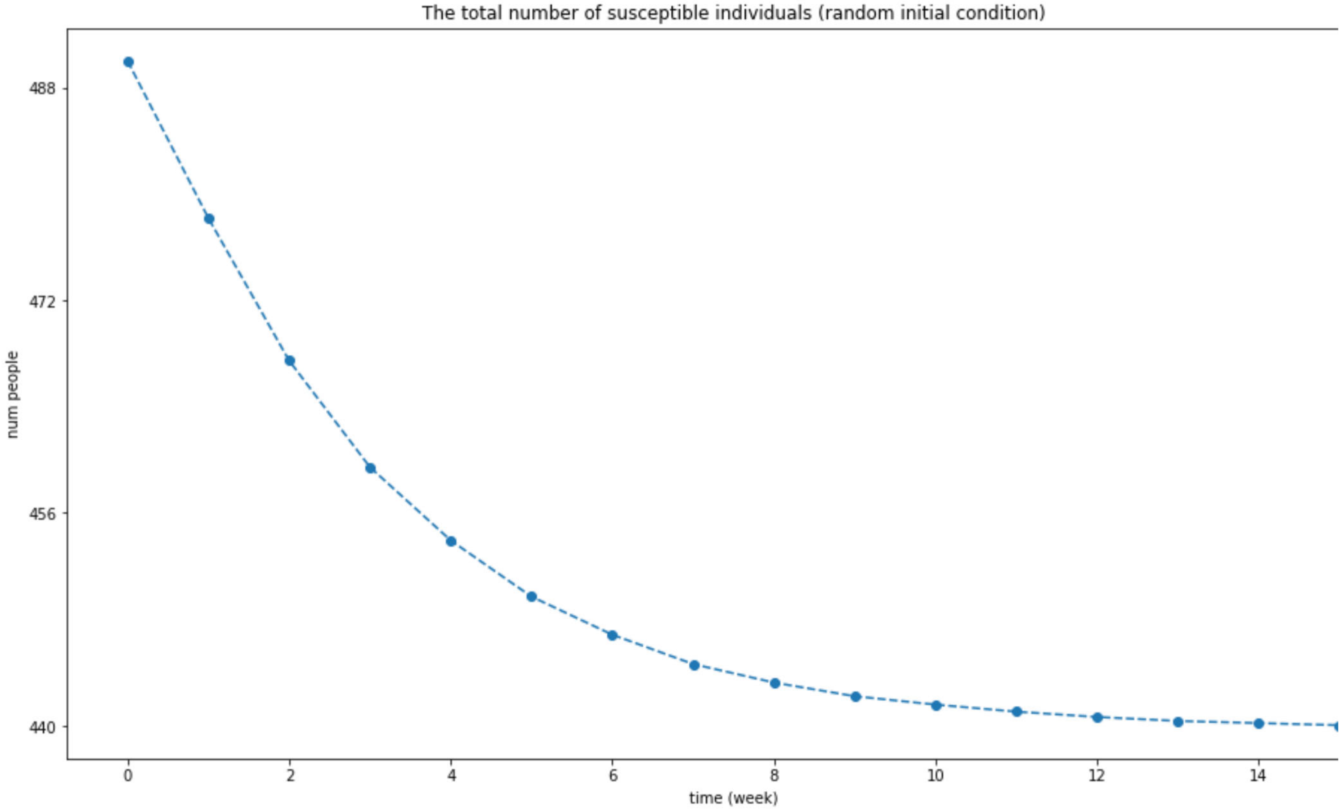

N = 100
sus_list = []
inf_list = []
rec_list = []
new_inf_list = []
for n in range(N):
    # print("Simulation #{}".format(n+1))
    weeks, newly_infected_list = sim_SIR_epidemic(G, n_weeks=15, initial_states='random',
    susceptibles, infected, recovered = get_sir_count(weeks)
    # print('newly infected: {}\n'.format(newly_infected_list))
    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    new_inf_list.append(newly_infected_list)
new_inf_list = np.array(new_inf_list)

```

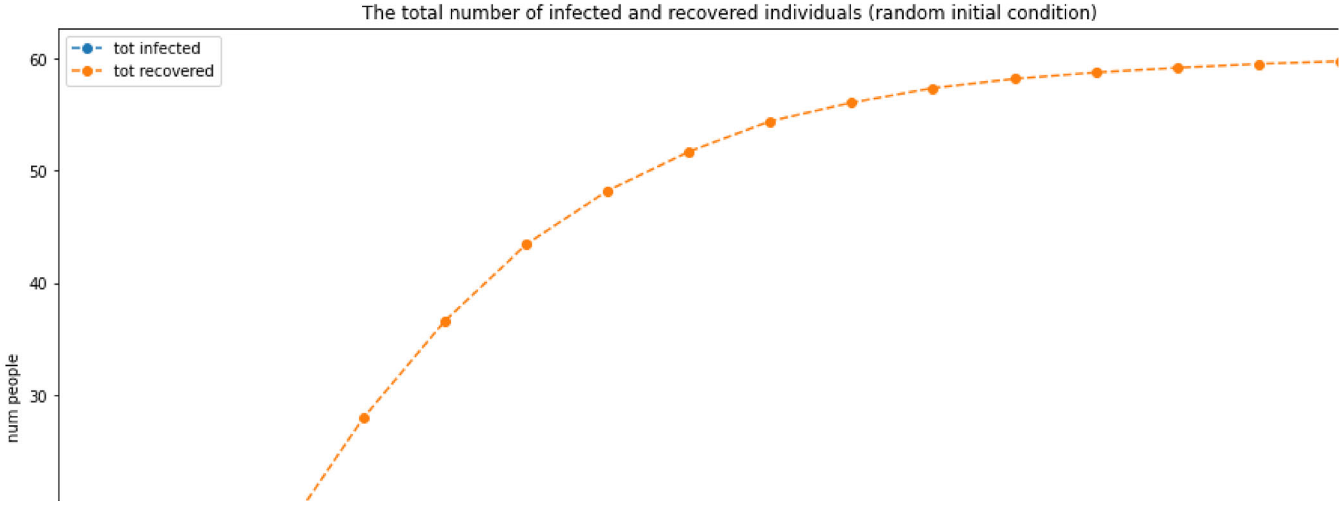
```
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
ax.set_yticks(yint)
ax.set_title("The number of newly infected individuals (random initial condition)")
plt.savefig("new_inf.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot avg sus',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1, len(avg_sus))
ax.set_yticks(yint)
ax.set_title("The total number of susceptible individuals (random initial condition)")
plt.savefig("tot_sus.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_inf, label='tot infected',linestyle='--', marker='o')
ax.plot(avg_rec, label='tot recovered',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (random initial condi
ax.legend(loc='best')
plt.savefig("tot_rec_inf.eps",format='eps')
```

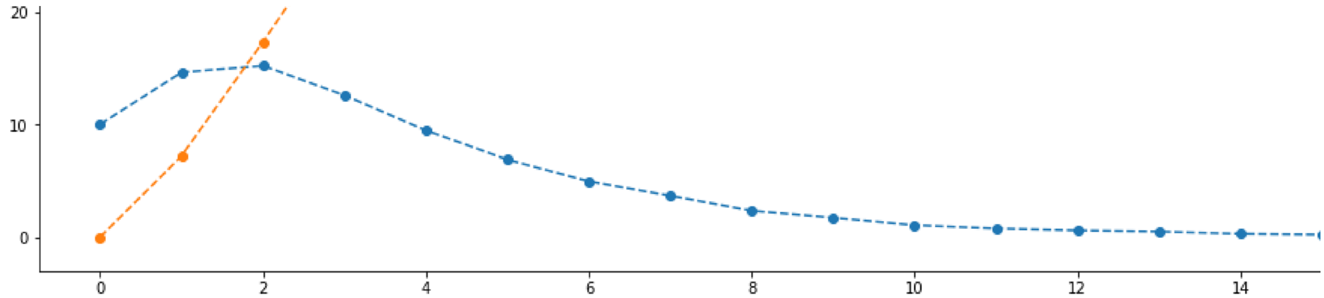


The total number of susceptible individuals (random initial condition)



The total number of infected and recovered individuals (random initial condition)





Realistic Initial Condition

```
G = nx.Graph()
k = 4
n = 500
for i in range(n-2):
    G.add_edge(i, i+1)
    G.add_edge(i, i+2)
G.add_edge(n-2, n-1)
G.add_edge(n-2, 0)
G.add_edge(n-1, 0)
G.add_edge(n-1, 1)

sus_list = []
inf_list = []
rec_list = []
```

```
new_inf_list = []

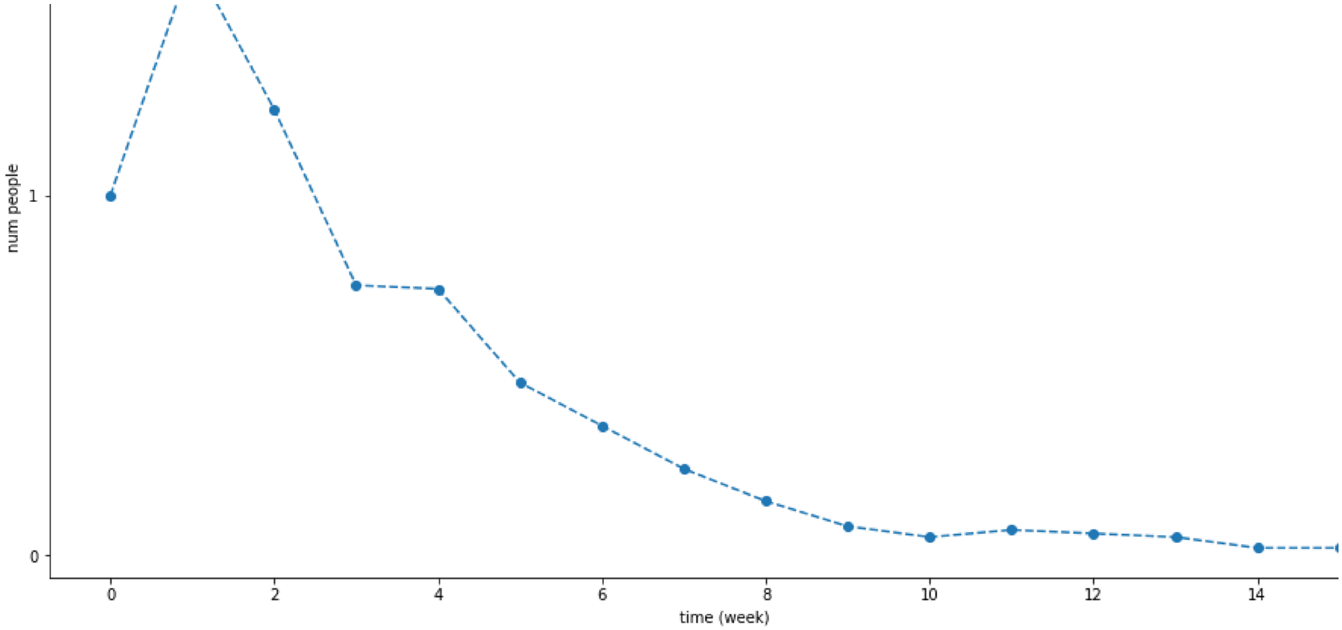
init_inf = [0]*n
for i in range(8):
    init_inf[i] = 1

N = 100

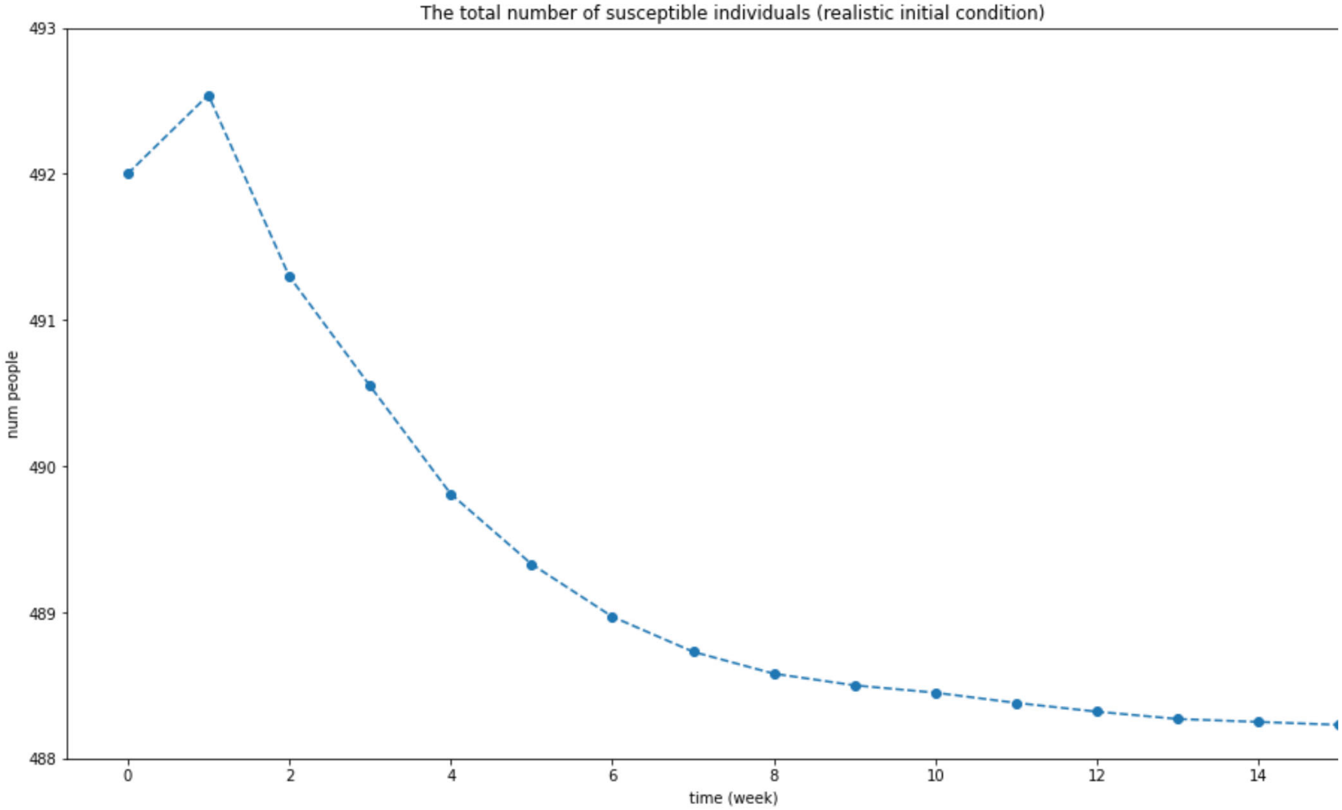
for i in range(N):
    # print("Simulation #{}".format(i+1))
    weeks, newly_infected_list = sim_SIR_epidemic(G, n_weeks=15, initial_states=init_inf,
    susceptibles, infected, recovered = get_sir_count(weeks)
    # print('newly infected: {}\n'.format(newly_infected_list))
    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    new_inf_list.append(newly_infected_list)
new_inf_list = np.array(new_inf_list)
```

```
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
ax.set_yticks(yint)
ax.set_title("The number of newly infected individuals (realistic initial condition)")
plt.savefig("new_inf_real.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot avg sus',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1)
ax.set_yticks(yint)
ax.set_title("The total number of susceptible individuals (realistic initial condition)")
plt.savefig("tot_sus_real.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_inf, label='tot infected',linestyle='--', marker='o')
ax.plot(avg_rec, label='tot recovered',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (realistic initial co
ax.legend(loc='best')
plt.savefig("tot_rec_inf_real.eps",format='eps')

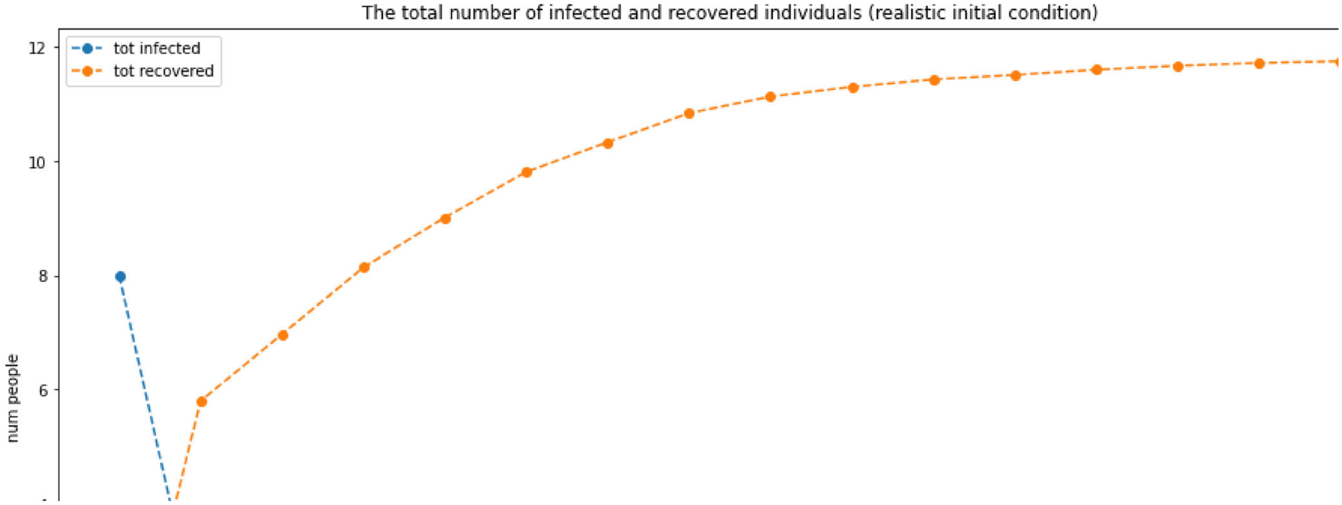
```

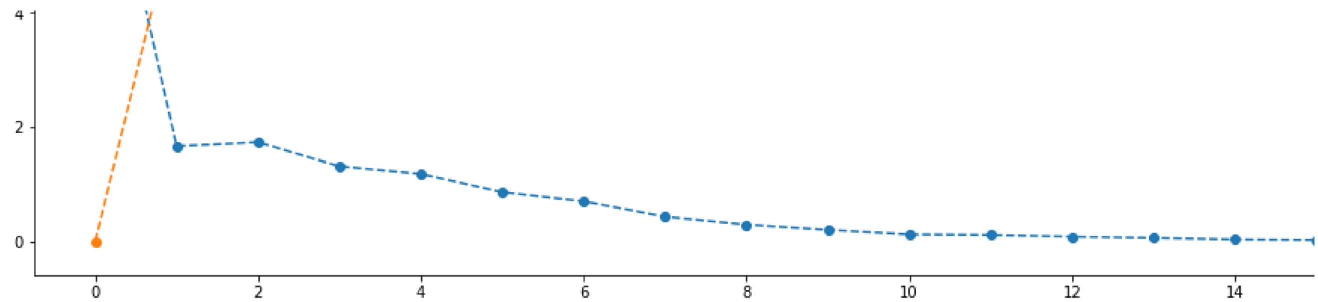


The total number of susceptible individuals (realistic initial condition)



The total number of infected and recovered individuals (realistic initial condition)





Problem 1.2

```
def generate_graph(k, n_nodes, verbose = False):
    '''
    k: int, average degree
    n_nodes: int, number of nodes
    verbose: boolean, for printing information
    '''
    t = 1
    G = nx.random_graphs.complete_graph(k+1) # generate an initial complete graph with k+1
#     nx.draw_circular(G)
    while 1:
        degrees = [deg for (node, deg) in G.degree()] # degree of nodes
        prob = [d/sum(degrees) for d in degrees]
        sel_nodes = np.random.choice(list(G.nodes), size=int((k+(-1)**(t*k))/2), p=prob, r
```

```
    G.add_edges_from([(G.number_of_nodes(), n) for n in sel_nodes]) # add edge from n+
#     G.add_edge(len(G.nodes()), sel_nodes)
    avg_deg = 2*G.number_of_edges() / float(G.number_of_nodes())
#     print("Time:", t,"\nAvg degree:", avg_deg)

    t += 1
    if G.number_of_nodes() == n_nodes:
      break
#  nx.draw_circular(G)
  if verbose:
    print('Graph with {} nodes and {} degree is created using preferential attachment.
  return G

```

```
k = 6
N = 500

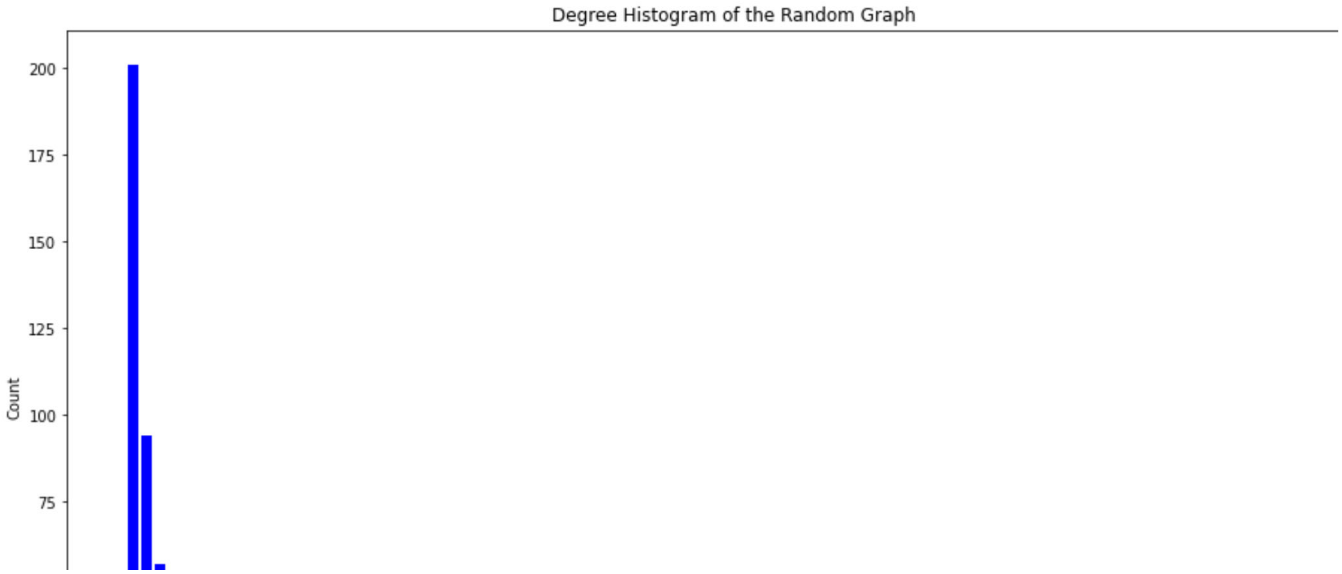
G = generate_graph(k,N, verbose=False)
degree_sequence = sorted([d for n, d in G.degree()], reverse=True)  # degree sequence
# print "Degree sequence", degree_sequence
degreeCount = collections.Counter(degree_sequence)
deg, cnt = zip(*degreeCount.items())

fig, ax = plt.subplots(figsize=(16,9))
ax.bar(deg, cnt, width=0.80, color='b')

ax.set_title("Degree Histogram of the Random Graph")
ax.set_ylabel("Count")
ax.set_xlabel("Degree")
ax.set_xticks([d for d in deg])
ax.set_xticklabels(deg)
degs = list(dict(G.degree()).values())
print("The average degree of nodes:", np.mean(np.array(degs)))
plt.savefig("deg_hist.eps",format='eps')

```

```
The average degree of nodes: 6.0
```



Problem 2

Random Initial Condition

```
sus_list = []
inf_list = []
rec_list = []
new_inf_list = []

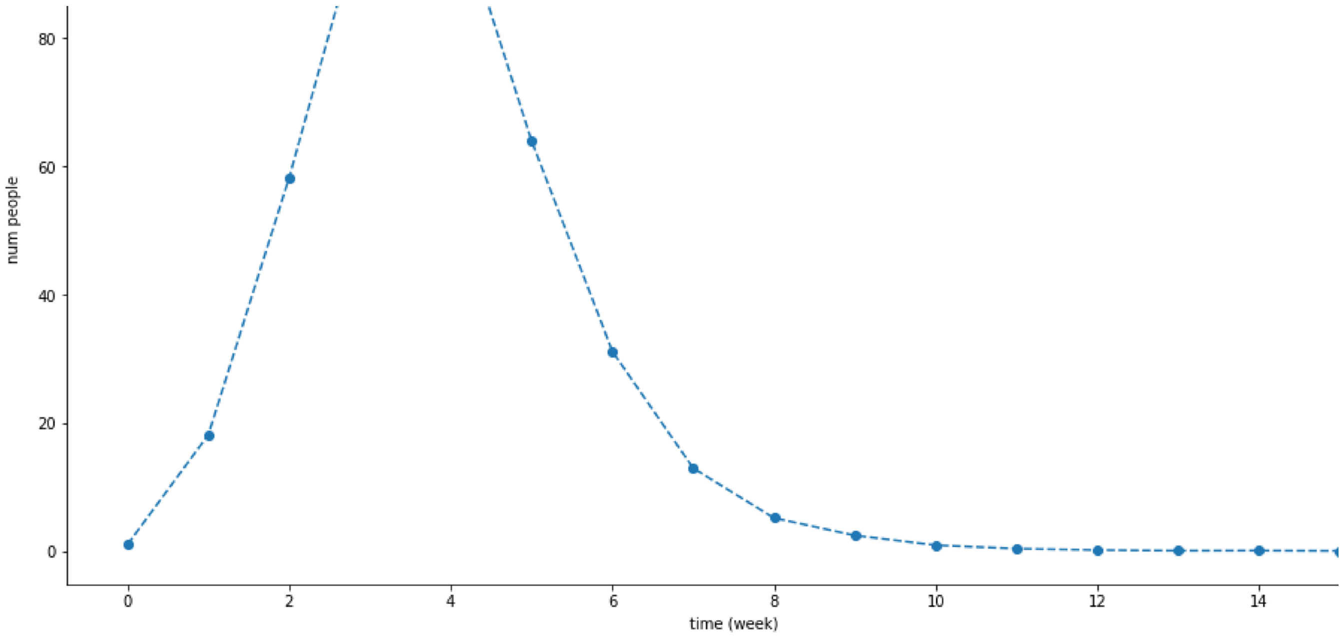
k = 6
beta = 0.3
rho = 0.7
n=500
G2 = generate_graph(k,n, verbose=False)

N = 100
for i in range(N):
    #print("Simulation #{}".format(i+1))
    weeks, newly_infected_list = sim_SIR_epidemic(G2, n_weeks=15, initial_states='random',
    susceptibles, infected, recovered = get_sir_count(weeks)
    #print('newly infected: {}\n'.format(newly_infected_list))
    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    new_inf_list.append(newly_infected_list)
```

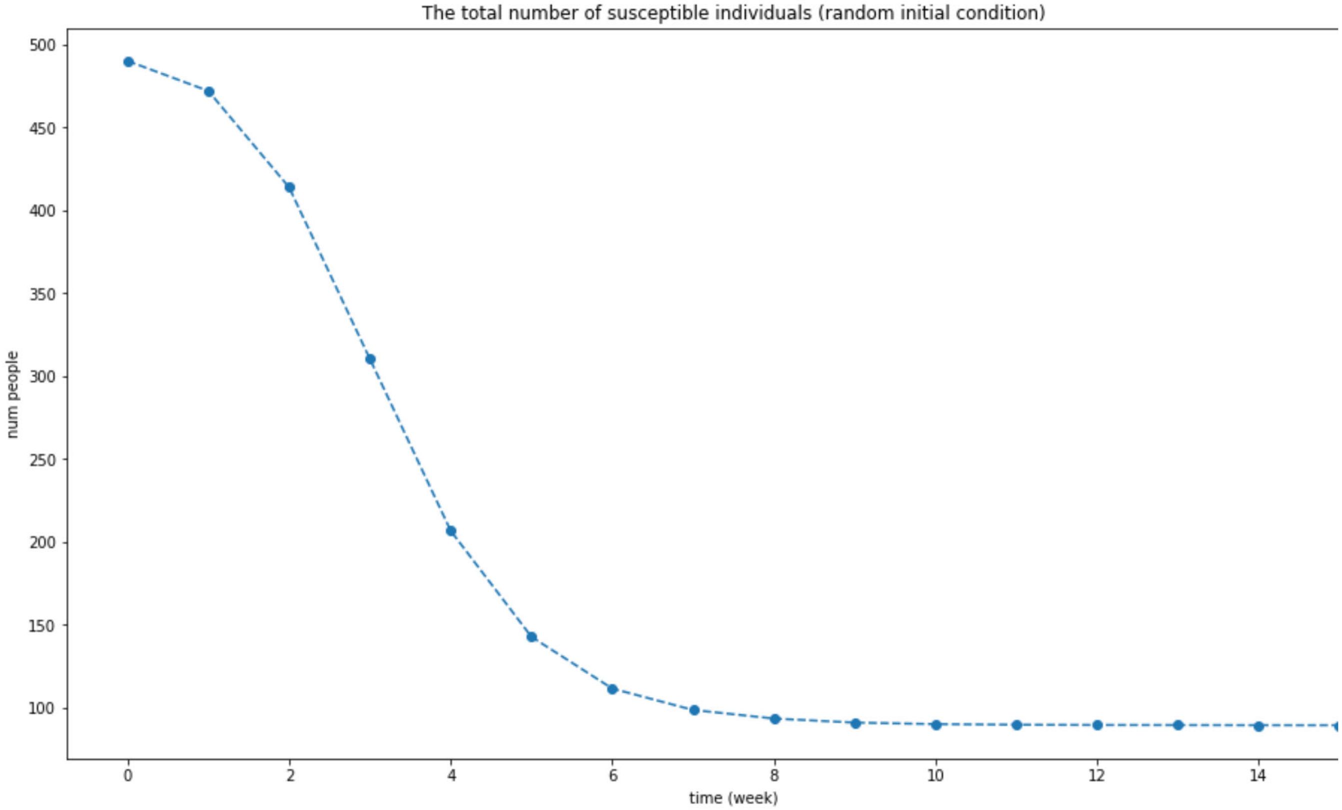
```
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
```

```
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
#ax.set_yticks(yint)
ax.set_title("The number of newly infected individuals (random initial condition)")
plt.savefig("new_inf_p2.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot avg sus',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1)
#ax.set_yticks(yint)
ax.set_title("The total number of susceptible individuals (random initial condition)")
plt.savefig("tot_sus_p2.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_inf, label='tot infected',linestyle='--', marker='o')
ax.plot(avg_rec, label='tot recovered',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (random initial condi
ax.legend(loc='best')
plt.savefig("tot_rec_inf_p2.eps",format='eps')

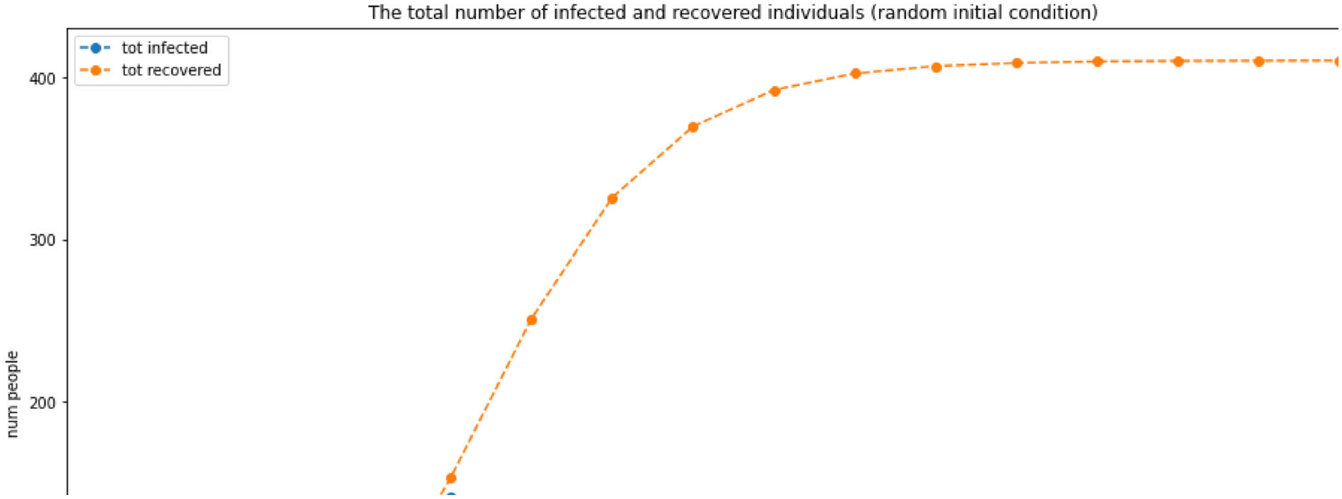
```

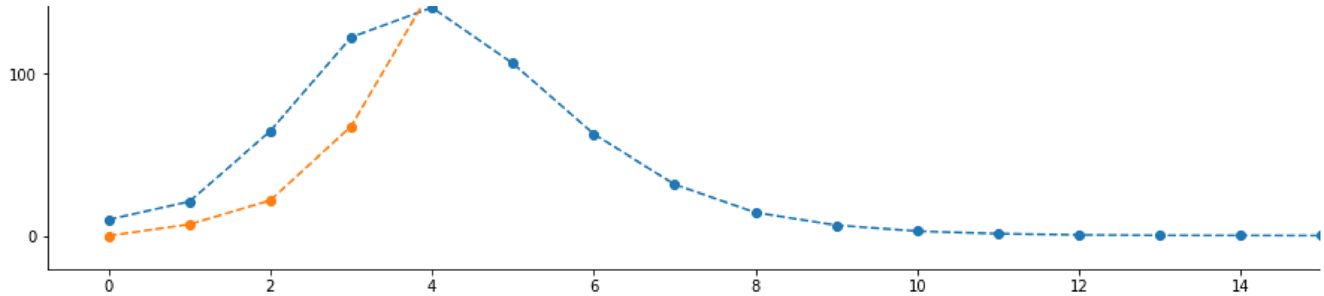


The total number of susceptible individuals (random initial condition)



The total number of infected and recovered individuals (random initial condition)





Realistic Initial Condition

```
sus_list = []
inf_list = []
rec_list = []
new_inf_list = []

k = 6
beta = 0.3
rho = 0.7
n=500
G2 = generate_graph(k,n, verbose=False)

init_state = [0]*n

inf_init = [0]+list(dict(G2.adjacency())[0].keys())[:7]
for i in inf_init:
    init_state[i] = 1

N = 100
for i in range(N):
    #print("Simulation #{}".format(i+1))
    weeks, newly_infected_list = sim_SIR_epidemic(G2, n_weeks=15, initial_states=init_stat
    susceptibles, infected, recovered = get_sir_count(weeks)
    #print('newly infected: {}\n'.format(newly_infected_list))
    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    new_inf_list.append(newly_infected_list)
```

```
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
```

```
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
#ax.set_yticks(yint)
ax.set_title("The number of newly infected individuals (realistic initial condition)")
plt.savefig("new_inf_p2_real.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot avg sus',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1)
#ax.set_yticks(yint)
ax.set_title("The total number of susceptible individuals (realistic initial condition)")
plt.savefig("tot_sus_p2_real.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_inf, label='tot infected',linestyle='--', marker='o')
ax.plot(avg_rec, label='tot recovered',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (realistic initial co
ax.legend(loc='best')
plt.savefig("tot_rec_inf_p2_real.eps",format='eps')
```

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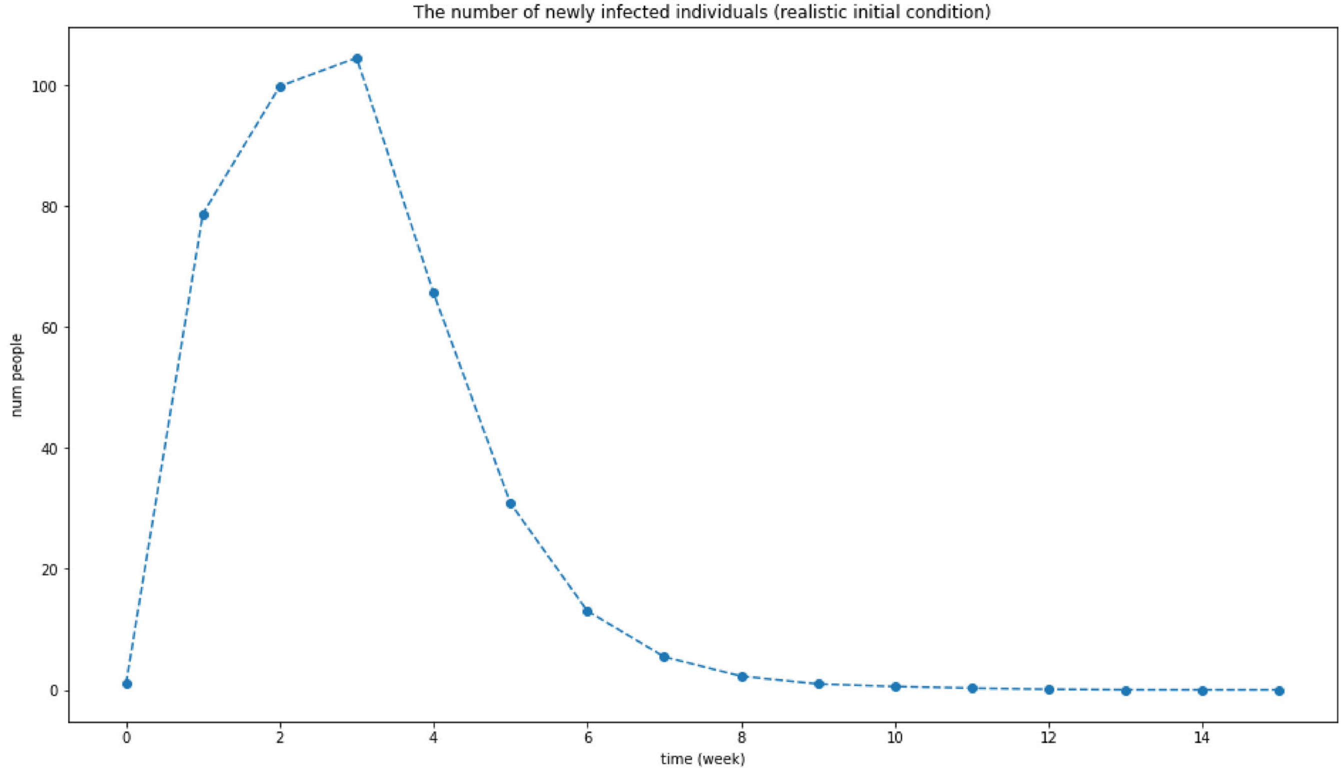

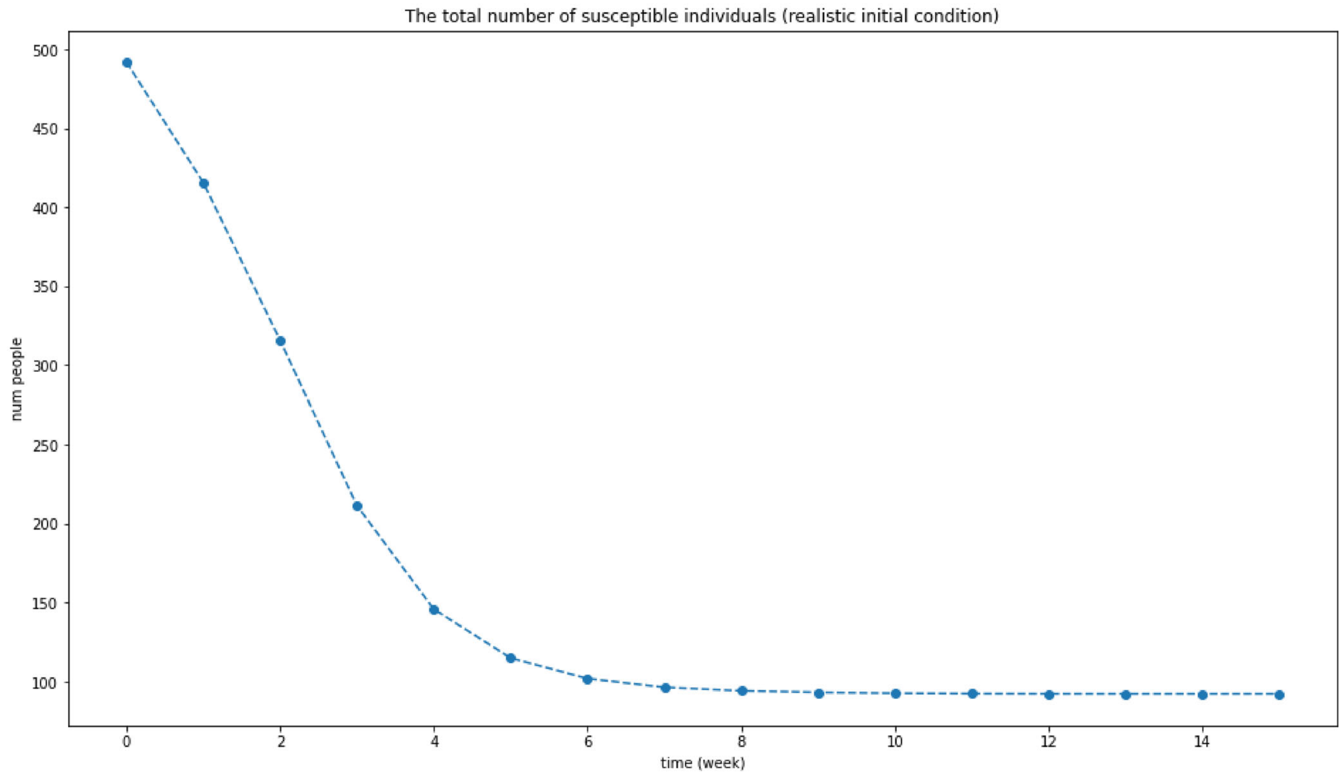

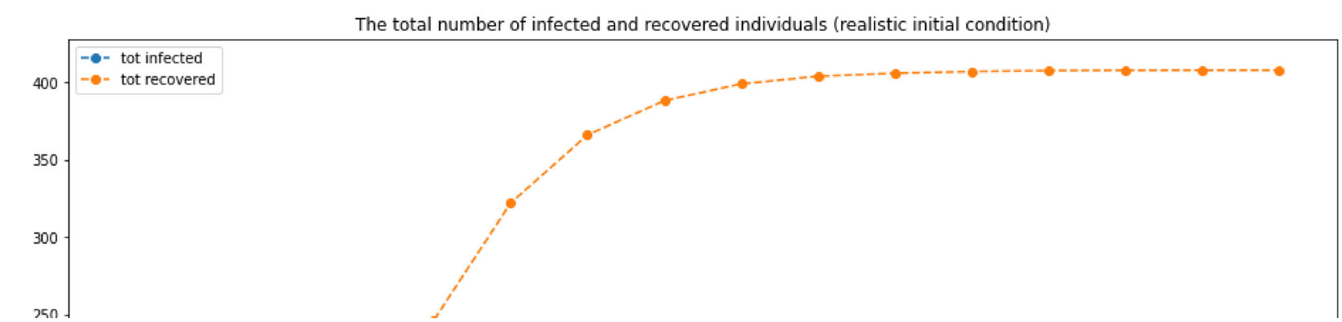

Problem 3


```
def sim_SIRV_epidemic(G, initial_infections, vacc, n_weeks = 5, initial_states = 'random',
    '''
    G: networkx.classes.graph.Graph, graph
    initial_infections: int, number of initally infected nodes
    vacc: list, the fraction of the total population vaccinated during each week
    n_weeks: int, number of simulation weeks
    initial_states: list, initial state of agents or default 'random' initialize sick peop
    random_sick: int, how many sick agents if initial state is randomly generated, default
    beta: float, the probability that the infection is spread from an infected individual
    rho: float, the probability that an infected individual will recover, default 0.7
    vacc: list, vaccinated people percentage
        until_end: boolean, indicates whether the simulation continues until the end or fi
    '''

    # Number of people
    n_agents = len(G)

    A_ = [0,1,2,3] # 0-S, 1-I, 2-R, 3-V

    states = [0 for _ in range(n_agents)] # 0-S, 1-I, 2-R, 3-V

    if initial_states == 'random':
        choices = np.random.choice(G.nodes(),size = random_sick, replace= False)
        for i in choices:
            states[i] = 1
        initial_states = states.copy()

    old_states = initial_states.copy()

#     print('Initial states of agents: {}'.format(initial_states))

    W = nx.convert_matrix.to_numpy_matrix(G)

    weeks = []
    weeks.append(initial_states.copy())


    newly_infected_list = [initial_infections]
```

```

    unv_people = list(G.nodes) # unvaccinated people
    newly_vaccinated_list = [0]
    vacc2 = [vacc[i+1] - vacc[i] for i in range(len(vacc)-1)]

    for week in range(n_weeks):
        if not until_end:
            if not 1 in states:
                #print('No one is sick! \nGREAT SUCCESS!')
                break

        n_tobe_vac = int(n_agents*vacc2[week]/100) # number of people to be vaccinated in
        tobe_vac_list = np.random.choice(unv_people, size = n_tobe_vac, replace = False) #
        newly_vaccinated_list.append(n_tobe_vac)
        for agent in tobe_vac_list:
            old_states[agent] = 3
            states[agent] = 3
            unv_people.remove(agent) # remove agent from unvaccinated people's list
#         print("States: {}, \nWeek: {}".format(states, week+1))

        newly_infected = 0
        for agent in range(n_agents):
            # if S:
            if old_states[agent] == 0:
                ngh_states = np.squeeze(np.asarray(W[agent]))*old_states # get neighbors'
                m = collections.Counter(ngh_states)[1] # count sick neighbors

                if np.random.rand() < (1-(1-beta)**m):
                    states[agent] = 1 # get sick
                    newly_infected += 1
            # if I:
            elif old_states[agent] == 1:
                if np.random.rand() < rho:
                    states[agent] = 2 # get recovered

        weeks.append(states.copy())
        newly_infected_list.append(newly_infected)

        old_states = states.copy()
#         print(old_states)
    return weeks, newly_infected_list, newly_vaccinated_list
        # weeks = weeks[:5]
```

```
def get_sirv_count(weeks):
    '''
    weeks: list, list of states through weeks
    '''
    susceptibles = [collections.Counter(np.asarray(w))[0] for w in weeks]
    infected = [collections.Counter(np.asarray(w))[1] for w in weeks]
    recovered = [collections.Counter(np.asarray(w))[2] for w in weeks]
    vaccinated = [collections.Counter(np.asarray(w))[3] for w in weeks]
```

```
    #print('susceptibles: {}, \ninfected: {}, \nrecovered: {}, \nvaccinated: {}'.format(su
    return susceptibles, infected, recovered, vaccinated
```

```
sus_list = []
inf_list = []
rec_list = []
vac_list = []
new_inf_list = []
new_vac_list = []
k = 6
n = 500
beta = 0.3
rho = 0.7
n_weeks = 15

vacc_t = [0, 5, 15, 25, 35, 45, 55, 60, 60, 60, 60, 60, 60, 60, 60]
vacc = [0]
vacc.extend(vacc_t)

G3 = generate_graph(k,n, verbose=False)
N = 100
for i in range(N):
    #print("Simulation #{}".format(i+1))
    weeks, newly_infected_list, newly_vaccinated_list = sim_SIRV_epidemic(G3, 10, vacc, n_
    susceptibles, infected, recovered, vaccinated = get_sirv_count(weeks)
    #print('newly infected: {}'.format(newly_infected_list))
    #print('newly vaxed: {}\n'.format(newly_vaccinated_list))

    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    vac_list.append(vaccinated)

    new_inf_list.append(newly_infected_list)
    new_vac_list.append(newly_vaccinated_list)
```

```
avg_new_vac = np.mean(np.array(new_vac_list), axis=0)
avg_vac = np.mean(np.array(vac_list), axis=0)
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
ax.plot(avg_new_vac, label='newly vaxxed',linestyle='--', marker='o')

ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
#ax.set_yticks(yint)
```

```
ax.legend(loc='best')

ax.set_title("The number of newly infected and vaccinated individuals (random initial cond
plt.savefig("new_inf_p3.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot sus',linestyle='--', marker='o')
ax.plot(avg_vac, label='tot vaxxed',linestyle='--', marker='o')
ax.legend(loc='best')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1)
#ax.set_yticks(yint)
ax.set_title("The total number of susceptible and vaccinated individuals (random initial c
plt.savefig("tot_sus_p3.eps",format='eps')
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(range(n_weeks+1),avg_inf, label='tot infected', linestyle='--', marker='o')
ax.plot(range(n_weeks+1),avg_rec, label='tot recovered', linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (random initial condi
ax.legend(loc='best')
plt.savefig("tot_rec_inf_p3.eps",format='eps')

```

```
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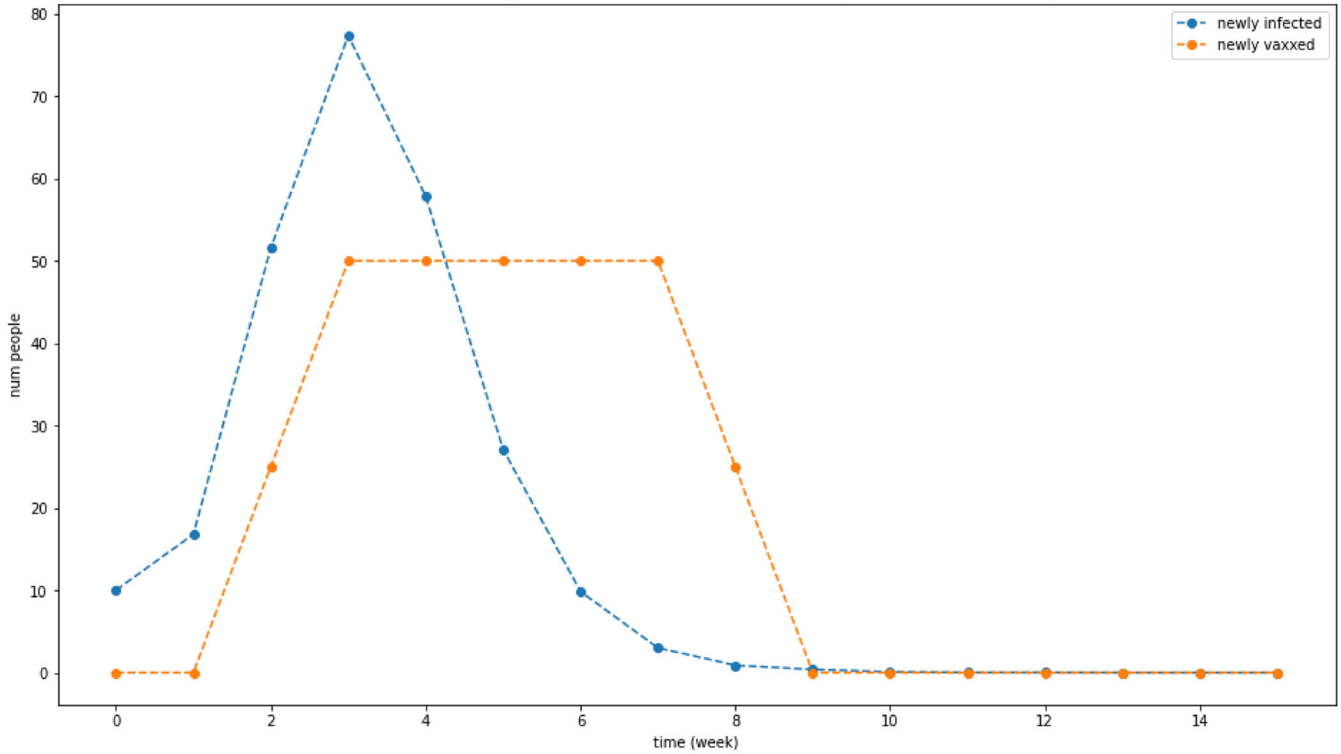
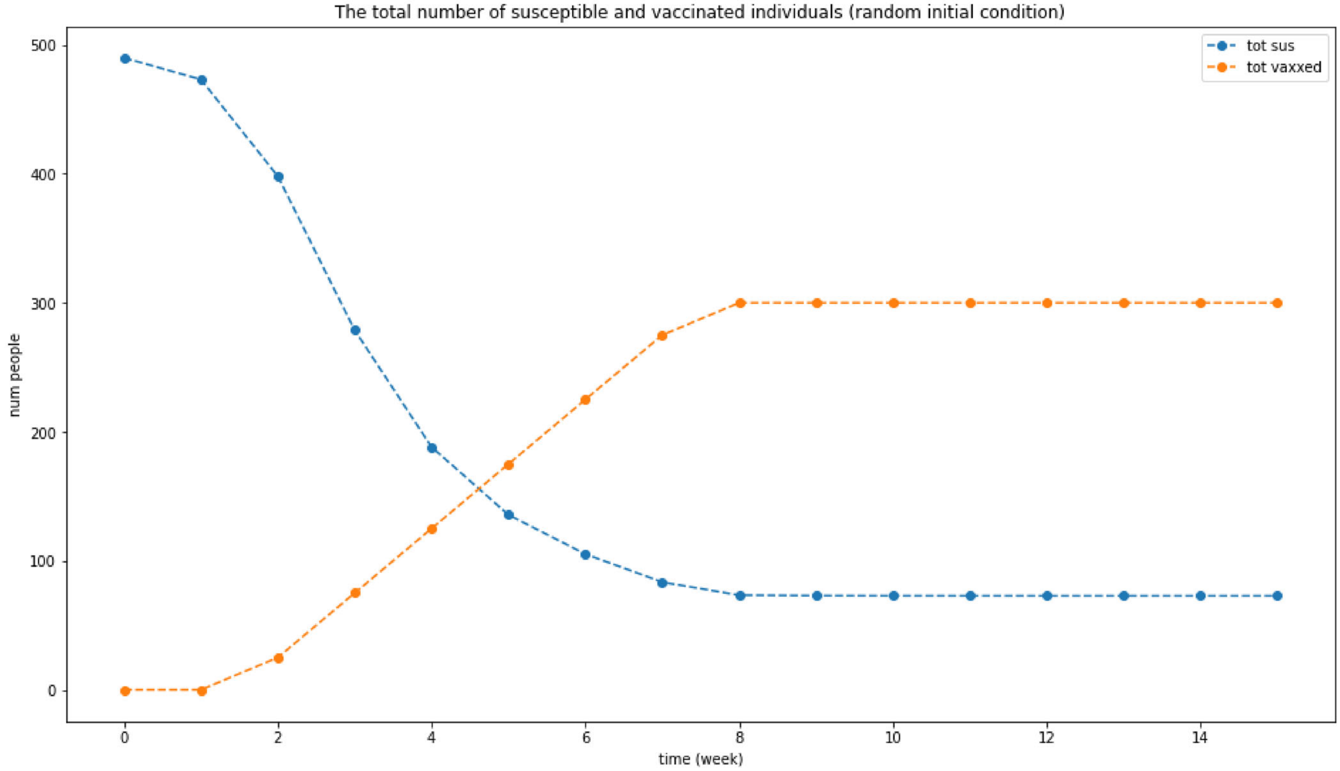
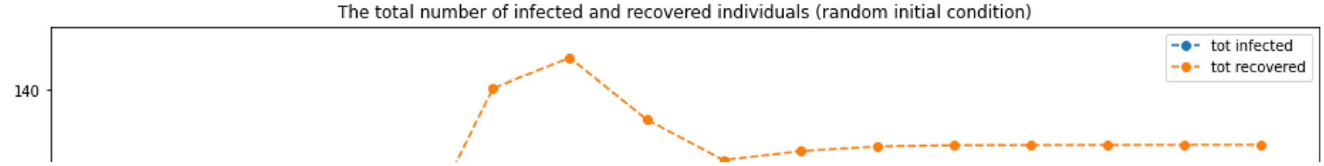
The number of newly infected and vaccinated individuals (random initial condition)

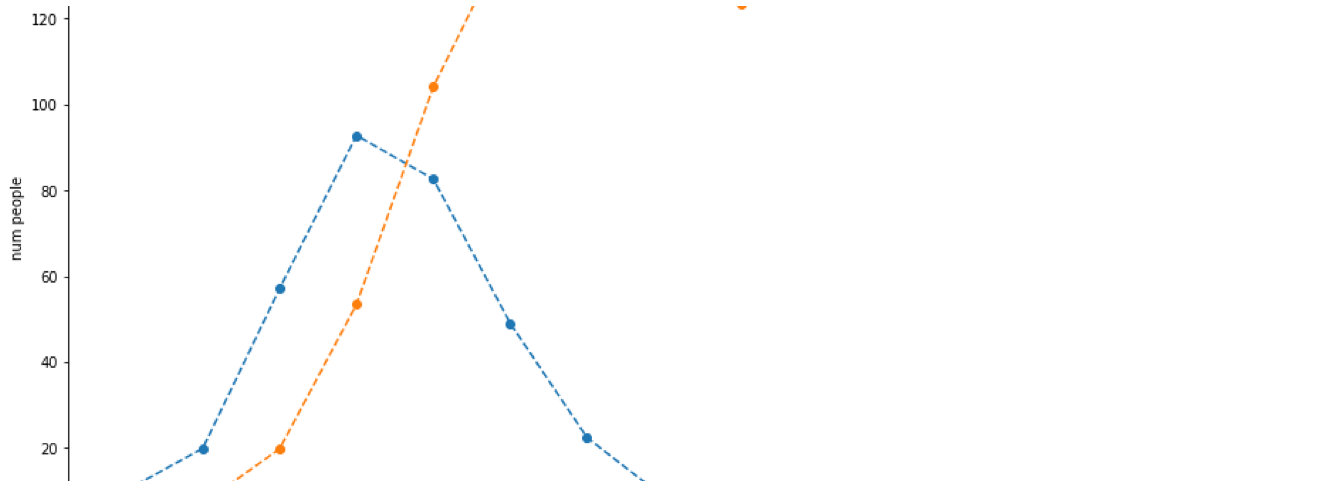


The total number of susceptible and vaccinated individuals (random initial condition)



The total number of infected and recovered individuals (random initial condition)





Problem 4

```
vacc = [5, 9, 16, 24, 32, 40, 47, 54, 59, 60, 60, 60, 60, 60, 60, 60]

 # we assume that during the week 0 nobody is vaccinated and
 # add 0'th week in order to calculate percentage of newly vaccinated people during week 1
I0 = [1, 1, 3, 5, 9, 17, 32, 32, 17, 5, 2, 1, 0, 0, 0, 0]
n_agents = 934

k0 = 10
beta0 = 0.3
rho0 = 0.6

dk = 1
dbeta = 0.1
drho = 0.1
```

```
def RMSE(I, I0):
    return np.sqrt(np.mean((np.array(I)-np.array(I0))**2))
```

```
def find_pand_params(n_agents = 934, k0 = 10, beta0 = 0.3, rho0 = 0.6,
                     dk_ = 1, dbeta_ = 0.1, drho_ = 0.1, p=0.5, graph_type = 'pref_attach'
    RMSE_list = []
    N = 10 # number of simulations

    k_opt = k0
    beta_opt = beta0
    rho_opt = rho0

    dk = dk_
    dbeta = dbeta_
    drho = drho_
    min_RMSE = np.inf

```

```
    opt_comb  = [k_opt, beta_opt, rho_opt]
    optimization_option = kwargs.get('opt_option', None)


    print("Initial hyperparameter combination: {}".format(opt_comb))

    iteration = 1
    while True:
        print("Iteration {}".format(iteration))

        #print("Current best combination: {}".format(opt_comb))
        k = [k_opt, k_opt-dk, k_opt+dk]
        beta = [beta_opt, np.round(beta_opt-dbeta,2), np.round(beta_opt+dbeta,2)]
        rho = [rho_opt, np.round(rho_opt-drho, 2), np.round(rho_opt+drho, 2)]

        combs = []
        for i in range(len(k)):
            for j in range(len(beta)):
                for l in range(len(rho)):
                    combs.append([k[i], beta[j], rho[l]])

        for comb in combs:
            k = comb[0]
            beta = comb[1]
            rho = comb[2]




            if graph_type == 'pref_attach':
                G4 = generate_graph(k, n_agents)
            else:
                raise NotImplementedError("The graph model is not valid!")


            new_inf_list = []
            for i in range(N):
                _, newly_infected_list, _ = sim_SIRV_epidemic(G4, 1, vacc,
                                                              n_weeks=15,
                                                              initial_states='random',
                                                              random_sick=1,
                                                              beta=beta,
                                                              rho=rho)
                #print('newly infected: {}'.format(newly_infected_list))

                new_inf_list.append(newly_infected_list.copy())

            I_pred = np.mean(np.array(new_inf_list), axis=0)
            #print(I_pred)
            RMSE_c = RMSE(I_pred, I0)
            if RMSE_c < min_RMSE:
```

```
                best_comb = comb
                min_RMSE = RMSE_c
            print('The Current Comb.: {}, RMSE: {}'.format(comb, RMSE_c))
        RMSE_list.append(min_RMSE)
        print('\nThe Current Best Comb.: {}, RMSE: {}'.format(best_comb, min_RMSE))


    #     print('RMSEs:', RMSE_list)

        if (best_comb == opt_comb):
            print('Convergence reached!')
            break
        opt_comb = best_comb.copy()
        if optimization_option == None:
            k_opt = best_comb[0]
            beta_opt = best_comb[1]
            rho_opt = best_comb[2]
        elif optimization_option == 'halving':
            if beta_opt == best_comb[1]:
                dbeta /= 2
            if rho_opt == best_comb[2]:
                drho /= 2
            k_opt = best_comb[0]
            beta_opt = best_comb[1]
            rho_opt = best_comb[2]
        iteration += 1
    return RMSE_list, opt_comb


```

```
RMSE_list, opt_comb = find_pand_params(opt_option='halving')
```

```
Initial hyperparameter combination: [10, 0.3, 0.6]
Iteration 1
The Current Comb.: [10, 0.3, 0.6], RMSE: 37.26508083984255
The Current Comb.: [10, 0.3, 0.5], RMSE: 32.59538886100302
The Current Comb.: [10, 0.3, 0.7], RMSE: 35.213278745382404
The Current Comb.: [10, 0.2, 0.6], RMSE: 18.21098569545317
The Current Comb.: [10, 0.2, 0.5], RMSE: 9.066593902894295
The Current Comb.: [10, 0.2, 0.7], RMSE: 7.731995214690708
The Current Comb.: [10, 0.4, 0.6], RMSE: 70.04001534837067
The Current Comb.: [10, 0.4, 0.5], RMSE: 65.84742781612658
The Current Comb.: [10, 0.4, 0.7], RMSE: 57.61515100214526
The Current Comb.: [9, 0.3, 0.6], RMSE: 34.561702070355274
The Current Comb.: [9, 0.3, 0.5], RMSE: 43.049767711336145
The Current Comb.: [9, 0.3, 0.7], RMSE: 33.78029196143811
The Current Comb.: [9, 0.2, 0.6], RMSE: 11.842138742642733
The Current Comb.: [9, 0.2, 0.5], RMSE: 4.74453633140268
The Current Comb.: [9, 0.2, 0.7], RMSE: 9.10078980089091
The Current Comb.: [9, 0.4, 0.6], RMSE: 40.759615429981665
The Current Comb.: [9, 0.4, 0.5], RMSE: 60.97862842832725
The Current Comb.: [9, 0.4, 0.7], RMSE: 46.63494532000654
```

```
The Current Comb.: [11, 0.3, 0.6], RMSE: 37.85368211944513
The Current Comb.: [11, 0.3, 0.5], RMSE: 59.38258898869264
The Current Comb.: [11, 0.3, 0.7], RMSE: 49.6093615157462
The Current Comb.: [11, 0.2, 0.6], RMSE: 13.64388599336714
The Current Comb.: [11, 0.2, 0.5], RMSE: 21.714554450874648
The Current Comb.: [11, 0.2, 0.7], RMSE: 15.683689776324957
The Current Comb.: [11, 0.4, 0.6], RMSE: 65.86649850265309
The Current Comb.: [11, 0.4, 0.5], RMSE: 56.3484915503512
The Current Comb.: [11, 0.4, 0.7], RMSE: 67.35662086981502

The Current Best Comb.: [9, 0.2, 0.5], RMSE: 4.74453633140268
Iteration 2
The Current Comb.: [9, 0.2, 0.5], RMSE: 6.748657273858259
The Current Comb.: [9, 0.2, 0.4], RMSE: 18.843632876916278
The Current Comb.: [9, 0.2, 0.6], RMSE: 8.252386018600923
The Current Comb.: [9, 0.1, 0.5], RMSE: 11.68647722797593
The Current Comb.: [9, 0.1, 0.4], RMSE: 11.295519023046262
The Current Comb.: [9, 0.1, 0.6], RMSE: 11.524484804102958
The Current Comb.: [9, 0.3, 0.5], RMSE: 41.775875215248334
The Current Comb.: [9, 0.3, 0.4], RMSE: 41.72259579652254
The Current Comb.: [9, 0.3, 0.6], RMSE: 30.444118890189614
The Current Comb.: [8, 0.2, 0.5], RMSE: 11.848945100725212
The Current Comb.: [8, 0.2, 0.4], RMSE: 7.212445840351247
The Current Comb.: [8, 0.2, 0.6], RMSE: 8.097877808413756
The Current Comb.: [8, 0.1, 0.5], RMSE: 10.779668362245658
The Current Comb.: [8, 0.1, 0.4], RMSE: 10.685124004895778
The Current Comb.: [8, 0.1, 0.6], RMSE: 11.889780275513925
The Current Comb.: [8, 0.3, 0.5], RMSE: 12.061819929015687
The Current Comb.: [8, 0.3, 0.4], RMSE: 29.994676611025497
The Current Comb.: [8, 0.3, 0.6], RMSE: 27.53270782178898
The Current Comb.: [10, 0.2, 0.5], RMSE: 6.362045661577728
The Current Comb.: [10, 0.2, 0.4], RMSE: 24.98893505133822
The Current Comb.: [10, 0.2, 0.6], RMSE: 6.287139651701719
The Current Comb.: [10, 0.1, 0.5], RMSE: 11.892907550300725
The Current Comb.: [10, 0.1, 0.4], RMSE: 6.76248844730991
The Current Comb.: [10, 0.1, 0.6], RMSE: 10.840779492268995
The Current Comb.: [10, 0.3, 0.5], RMSE: 33.34647657549445
The Current Comb.: [10, 0.3, 0.4], RMSE: 44.82474205168391
```

```
k = opt_comb[0]
beta = opt_comb[1]
rho = opt_comb[2]

G4 = generate_graph(k, n_agents, verbose=False)
N = 100
sus_list = []
inf_list = []
rec_list = []
vac_list = []
new_inf_list = []
new_vac_list = []
for n in range(N):
    #print("Simulation #{}".format(n+1))
    weeks, newly_infected_list, newly_vaccinated_list = sim_SIRV_epidemic(G4, 1, vacc,
```

```
                                                                          n_weeks=15,
                                                                          initial_states='
                                                                          random_sick=1,
                                                                          beta=beta,
                                                                          rho=rho)
    susceptibles, infected, recovered, vaccinated = get_sirv_count(weeks)
    #print('newly infected: {}'.format(newly_infected_list))
#     print('newly vaxed: {}\n'.format(newly_vaccinated_list))

    sus_list.append(susceptibles)
    inf_list.append(infected)
    rec_list.append(recovered)
    vac_list.append(vaccinated)

    new_inf_list.append(newly_infected_list)
    new_vac_list.append(newly_vaccinated_list)
```

```
avg_new_vac = np.mean(np.array(new_vac_list), axis=0)
avg_vac = np.mean(np.array(vac_list), axis=0)
avg_sus = np.mean(np.array(sus_list), axis=0)
avg_inf = np.mean(np.array(inf_list), axis=0)
avg_rec = np.mean(np.array(rec_list), axis=0)
avg_new_inf = np.mean(new_inf_list, axis=0)
fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_new_inf, label='newly infected',linestyle='--', marker='o')
ax.plot(avg_new_vac, label='newly vaxxed',linestyle='--', marker='o')
ax.legend(loc='best')

ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_new_inf)), math.ceil(max(avg_new_inf))+1)
#ax.set_yticks(yint)
ax.set_title("The number of newly infected and vaccinated individuals (pref attach model)"
plt.savefig("new_inf_p4.eps",format='eps')

fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_sus, label='tot sus',linestyle='--', marker='o')
ax.plot(avg_vac, label='tot vaxxed',linestyle='--', marker='o')
ax.legend(loc='best')
ax.set(xlabel='time (week)', ylabel='num people')
yint = range(int(min(avg_sus)), math.ceil(max(avg_sus))+1)
#ax.set_yticks(yint)
ax.legend(loc='best')
ax.set_title("The total number of susceptible and vaccinated individuals (pref attach mode
plt.savefig("tot_sus_p4.eps",format='eps')

fig, ax= plt.subplots(figsize=(16,9))
ax.plot(avg_inf, label='tot infected',linestyle='--', marker='o')
ax.plot(avg_rec, label='tot recovered',linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The total number of infected and recovered individuals (pref attach model)")
ax.legend(loc='best')
```

```
plt.savefig("tot_rec_inf_p4.eps",format='eps')

fig, ax= plt.subplots(figsize=(16,9))
ax.plot(I0, label='real new inf', linestyle='--', marker='o')
ax.plot(avg_new_inf, label='estimated new inf', linestyle='--', marker='o')
ax.set(xlabel='time (week)', ylabel='num people')
ax.set_title("The comparison of the real and estimated newly infected indivuals using the
ax.legend(loc='best')
plt.savefig("real_data_comp_p4.eps",format='eps')
```

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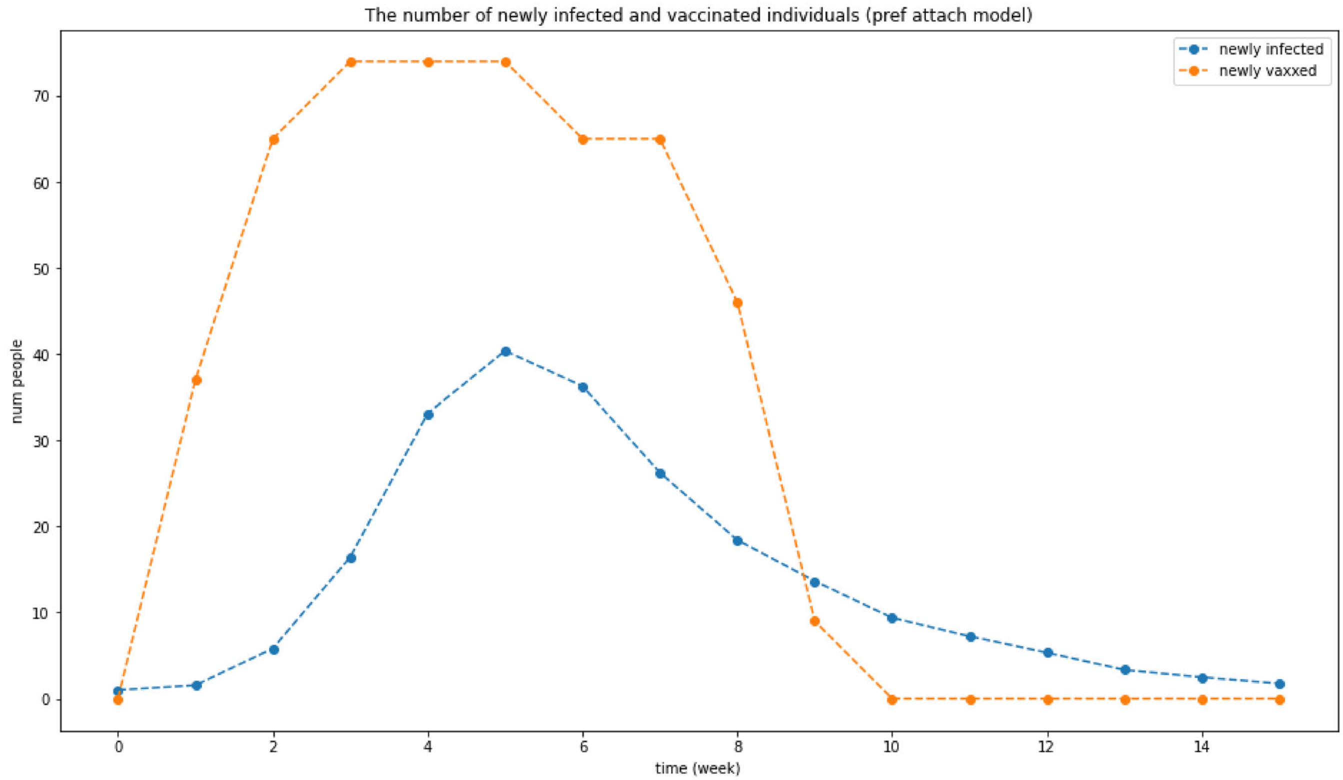

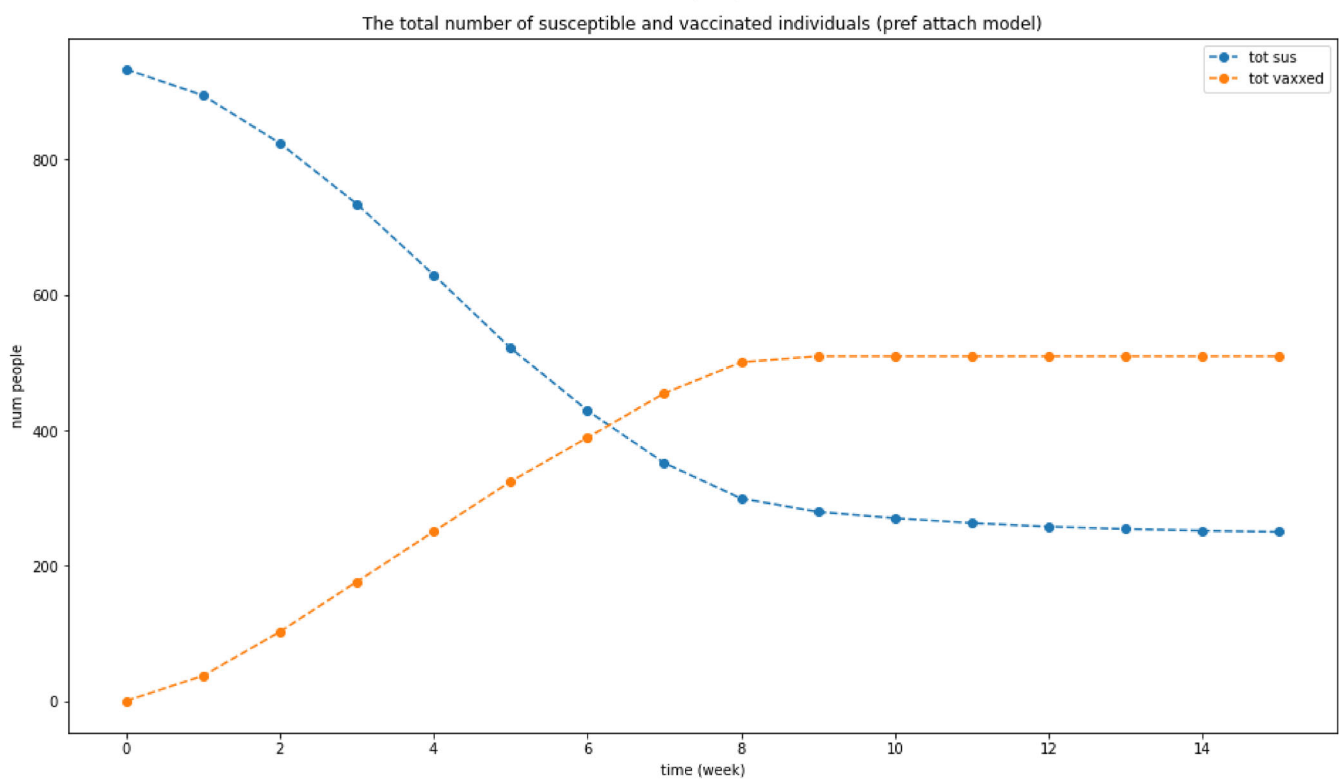

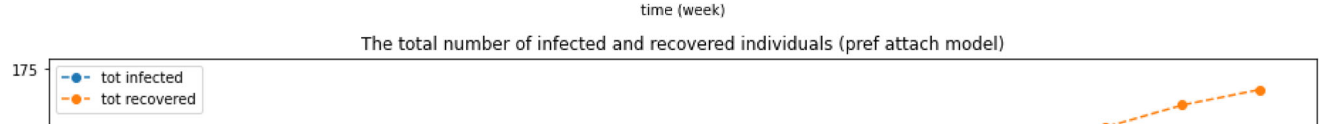

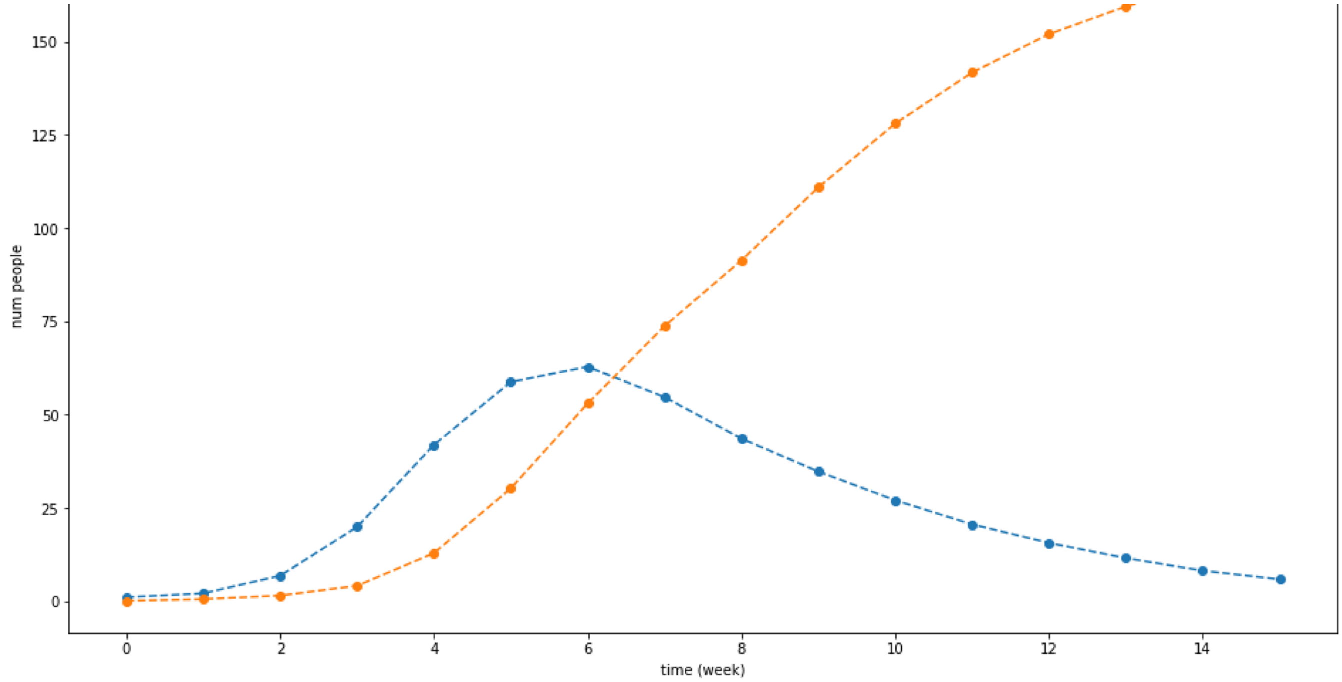

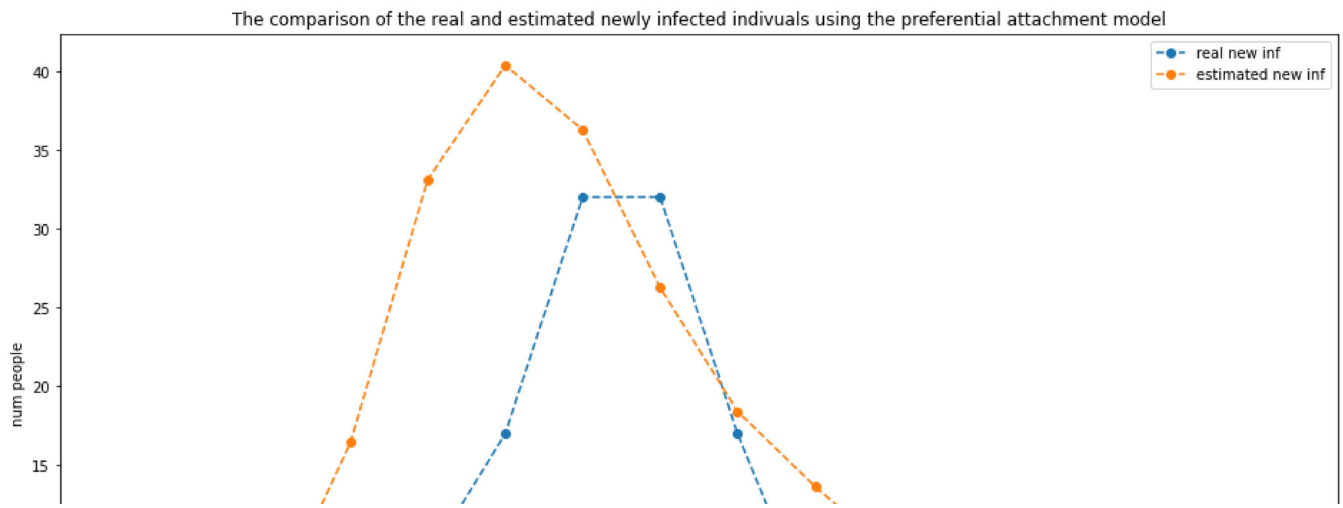

Problem 5

```
def watts_strogats_model_sim(n_agents = 934, k0 = 10, beta0 = 0.3, rho0 = 0.6,
                     dk_ = 1, dbeta_ = 0.1, drho_ = 0.1, restart_iters=2, **kwargs):
    RMSE_list = []
    N = 10 # number of simulations

    k_opt = k0
    beta_opt = beta0
    rho_opt = rho0

    dk = dk_
    dbeta = dbeta_
    drho = drho_
    min_RMSE = np.inf

    opt_comb  = [k_opt, beta_opt, rho_opt]
```