Statistical Distribution

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Introduction

Statistical distributions are divided into two main categories: discrete and continuous distributions. Each distribution can be characterized by four aspects: the probability mass function (or probability density function), the cumulative distribution function, the mean, and the variance. Below, we will discuss seven common statistical distributions.

1 Bernoulli Distribution

The Bernoulli distribution is a discrete distribution that models a random experiment with two possible outcomes: success (1) and failure (0).

- **Probability Mass Function (PMF):**

$$f(x) = p^x(1-p)^{1-x}, \quad x \in \{0, 1\}$$

- **Cumulative Distribution Function (CDF):**

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

- **Mean:**

$$\mu = p$$

- **Variance:**

$$\sigma^2 = p(1-p)$$

2 Binomial Distribution

The Binomial distribution models the number of successes in a fixed number of independent Bernoulli trials.

- **PMF:**

$$f(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

- **CDF:**

$$F(x) = \sum_{k=0}^{x} \binom{n}{k} p^k(1-p)^{n-k}$$

- **Mean:**

$$\mu = np$$

- **Variance:**

$$\sigma^2 = np(1-p)$$

3 Poisson Distribution

The Poisson distribution is a discrete distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space.

- **PMF:**
$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$$
- **CDF:**
$$F(x) = e^{-\lambda} \sum_{k=0}^{x} \frac{\lambda^k}{k!}$$
- **Mean:**
$$\mu = \lambda$$
- **Variance:**
$$\sigma^2 = \lambda$$

4 Normal Distribution

The Normal distribution is a continuous distribution characterized by its bell-shaped curve.

- **PDF:**
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
- **CDF:**
$$F(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$
- **Mean:**
$$\mu = \mu$$
- **Variance:**
$$\sigma^2 = \sigma^2$$

5 Exponential Distribution

The Exponential distribution is a continuous distribution that models the time between events in a Poisson process.

- **PDF:**
$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$
- **CDF:**
$$F(x) = 1 - e^{-\lambda x}$$
- **Mean:**
$$\mu = \frac{1}{\lambda}$$
- **Variance:**
$$\sigma^2 = \frac{1}{\lambda^2}$$

6 Uniform Distribution

The Uniform distribution is a continuous distribution where all outcomes are equally likely.

- **PDF:**
$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$
- **CDF:**
$$F(x) = \frac{x-a}{b-a}, \quad a \leq x \leq b$$
- **Mean:**
$$\mu = \frac{a+b}{2}$$
- **Variance:**
$$\sigma^2 = \frac{(b-a)^2}{12}$$

7 Geometric Distribution

The Geometric distribution models the number of trials until the first success in a sequence of independent Bernoulli trials.

- **PMF:**
$$f(x) = p(1-p)^{x-1}, \quad x = 1, 2, \ldots$$
- **CDF:**
$$F(x) = 1 - (1-p)^x$$
- **Mean:**
$$\mu = \frac{1}{p}$$
- **Variance:**
$$\sigma^2 = \frac{1-p}{p^2}$$

Conclusion

Understanding these distributions is fundamental in statistics, as they provide the basis for various statistical methods and analyses.