1) $V_1 = \{2, -3, 5\}$ and $V_2 = \{6, 2, 1\}$

Euclidean vector, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(6-2)^2 + (2+3)^2 + (1-5)^2}$

$= \sqrt{4^2 + 5^2 + (-4)^2}$

$= \sqrt{16 + 25 + 16}$

$= \sqrt{57}$

$= 7.54$

2) $x = \{6, -8, 0\}$

Magnitude $\|\vec{x}\| = \sqrt{(6^2) + (-8)^2 + 0^2}$

$= \sqrt{36 + 64 + 0}$

$= \sqrt{100}$

$= \underline{\underline{10}}$

Unit vector in the direction of x

$= \frac{1}{10}(6, -8, 0)$

$= \left(\frac{6}{10}, \frac{-8}{10}, \frac{0}{10}\right)$

$= \underline{\underline{\left(\frac{3}{5}, \frac{-4}{5}, 0\right)}}$

3) $A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 5 \\ 6 & 0 & -1 \end{bmatrix}$

$\det A = 3(-1-0) - 4(-2-30) + 2(0-6)$

$= -3 + 128 - 12$

$= 128 - 15$

$= \underline{\underline{113}}$

4) $B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\det B = 6 - -1$

$= 7$

$\text{Adj } B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$B^{-1} = \frac{1}{\det B} \times \text{Adj } B$

$= \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3/7 & 1/7 \\ -1/7 & 2/7 \end{bmatrix}$

Adjoint

$A^{-1} = \frac{1}{\det B} \times \text{Adj } B$

$A \cdot A^{-1} = I$

5) $V_1 = \{1, 2, -1\}$ $V_2 = \{3, -6, 2\}$

dot product V_1 \& V_2 $= (1 \times 3) + (2 \times -6) + (-1 \times 2)$

$= 3 - 12 - 2$

$= -11$

6) $V_1 = \{1, 2\}$, $V_2 = \{3, 4\}$

$V_1 \cdot V_2 = (1 \times 3) + (2 \times 4)$

$= 3 + 8$

$= 11$

$|V_1| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$|V_2| = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5$

Angle $\theta = \cos^{-1}\left(\frac{V_1 \cdot V_2}{|V_1||V_2|}\right)$

$= \cos^{-1}\left(\frac{11}{\sqrt{5} \times 5}\right) =$

$= \cancel{3} \; \underline{33.2}$

9)

7) $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$|C - \lambda \cdot I| = \left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$= \left| \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-1 \end{bmatrix} \right| = 0$$

$$= (2-\lambda)(2-\lambda) - 1 = 0 \quad -4$$

3

$$= 2^2 - 2\lambda - 2\lambda + \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$(\lambda - 3) \cdot (\lambda - 1) = 0$$

$$\underline{\underline{\lambda = 3 / \lambda = 1}}$$

Sum = -4

Prod = 3

8) Red balls = 4

Blue balls = 3

Green balls - 2

Probability of drawing a red or blue ball

$$= \frac{4+3}{4+3+2}$$

$$= 7/9$$

$$= 0.7777$$

9) $P(+|D) = 0.95$

$P(D) = 0.01$

$P(-|!D) = 0.95$

$P(+|!D) = 0.05$

$P(D|+) = ?$

Bayeans Theorem

$$P(D|+) = \frac{[P(+|D) * P(D)]}{[P(+|D) * P(D) + P(+|!D) * P(!D)]}$$

$$P(!D) = 1 - P(D)$$
$$= 0.99$$

$$P(D|+) = \frac{[0.95 \times 0.01]}{[0.95 \times 0.01 + 0.05 \times 0.99]}$$

$$= \frac{0.0095}{0.0095 + 0.0495}$$

$$= \frac{0.0095}{0.059}$$

$$= \underline{\underline{0.161}}$$

10) $P(M) = 0.70$ $P(M \cap P) = 0.30$

$P(P) = 0.50$

$\cancel{P(P \cap M) = 0.30}$

$\cancel{P(P \cup M) = 0.30}$

$\cancel{P(P \cap M) = ?}$

$\cancel{P(P \cup M) = P(M) + P(P) - P(P \cap M)}$

$\cancel{0.3 = 0.7 + 0.5 - P(P \cap M)}$

$\cancel{P(P \cap M) = 0.7 + 0.5 - 0.3}$

$$P(P|M) = \frac{P(P \cap M)}{P(M)}$$

$$= \frac{0.3}{0.7}$$

$$\doteq 0.4286$$

11) $P(H) = 0.8$

$P(T) = 0.T$

Entropy formula $\Rightarrow H(x) = -[P(H)\log_2 P(H) + P(T)\log_2 P(T)]$

$$= -[0.8 \log_2 0.8 + 0.2 \log_2 0.2]$$

$$H(x) = -[0.8 \times (-0.3219) + 0.2 \times (-2.3219)]$$

$$H(x) = -[-0.25752 - 0.46438]$$

$$H(x) = 0.7219$$

12) $X = \{1, 2, 3, 4\}$

$p(x=1) = 0.1$

$p(X=2) = 0.2$

$p(X=3) = 0.3$

$p(X=4) = 0.4$

~~$p(x=5) = 0.5$~~

Expected value of x, $E(X) = (1\times0.1) + (2\times0.2) + (3\times0.3) + (4\times0.4) + \cancel{(5\times0.5)}$

$= 0.1 + 0.4 + 0.9 + 1.6$

$= 3.0$

13) Sum greater than 8 are

(3,6), (4,5), (4,6), (5,4), (5,5), (5,6)

(6,3), (6,4), (6,5), (6,6)

Total out when 2 dices are rolled $= 36$

probability of sum than 8

$= \frac{10}{36} = 5/18$

$= 0.277$

14) $P(H) = 0.6$

No. of flip $= 10$

$P(T) = 1 - P(H) = 0.4$

$\log_2 P(T)]$

By Applying Binomial formula

$$p(x=7) = \frac{10!}{7!\,(10-3!)} (0.6)^7 (0.4)^{10-7}$$

$$= \frac{10!}{7!\,3!} (0.6)^7 (0.4)^3$$

$$= 120 \times 0.0279936 \times 0.064$$

$$= 0.214$$

15) $v_1 = (2,4)$

$v_2 = (1,2)$

$$v = \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix}$$

$\det v = 4 - 4$

$= \underline{\underline{0}}$

$\therefore$ linearly dependant

16) $a = (1,-2,3)$ $b = (4,0,-1)$

$a \cdot b = 4 + 0 - 3$

$= 1$

17) $P(A) = 4/52$

$P \cap B$

or $-P \cup B$

$= 1/13$

$P(H) = 13/52$

$= 1/4$

$P(A \cap B) = 1/52$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{13} + \frac{1}{4} - \frac{1}{52}$

$= \frac{16}{52}$

$= 0.307692$

18) $V = \{2, -3, z\}$

$u = \{1, 4, 5\}$

$z = ?$

$\vec{v}$ & $\vec{u}$ are orthogonal $(\because v \cdot u = 0)$

$\vec{v} \cdot \vec{u} = (2 \times 1) + (-3 \times 4) + (z \times 5)$

$\vec{v} \cdot \vec{u} = 2 + (-12) + 5z$

$\vec{v} \cdot \vec{u} = -10 + 5z = 0$

$\Rightarrow 5z = 10$

$z = \frac{10}{5}$

$= 2$

19) $P(R) = 0.3$

$P(U) = 0.6$

$P(R \cap U) = 0.2$

$$P(R/U) = \frac{P(R \cap U)}{P(U)}$$

$$= \frac{0.2}{0.6}$$

$$= 0.333$$

20) $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$$|C - \lambda I| = \left| \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix}$$

$$= (2-\lambda)(3-\lambda) = 0$$

$$(\lambda-2)(\lambda-3) = 0$$

$\lambda = 2 \,/\, \lambda = 3$