Trajectory Generation

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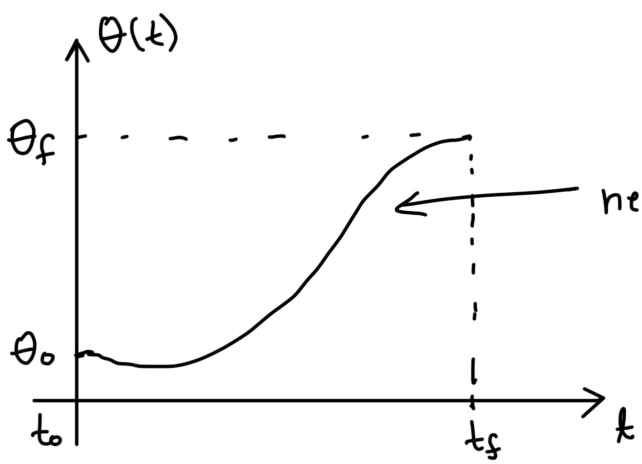
Trajectory Generation → Joint Space → Single Time Interval → Polynomials → 3rd order Cubic

Roadmap diagram

Joint Space Schemes – Cubic Polynomials – Zero Velocity

Problem – define a function for each joint such that its value at t_0 is the initial position of the joint and at t_f is the desired goal position of the joint



need to find this!

Constraints:

$\theta(0) = \theta_0$

$\theta(t_f) = \theta_f$

$\dot{\theta}(0) = 0$

$\dot{\theta}(t_f) = 0$

Define $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ (1)

joint velocity: $\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$ (2)

acceleration: $\ddot{\theta}(t) = 2a_2 + 6a_3 t$ (3)

Combining (1), (2), (3) with constraints we get:

$$\theta_0 = a_0$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$

$$0 = a_1$$

$$0 = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

$$\begin{bmatrix} \theta_0 \\ \theta_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 0 & 0 \\ 0 & t_f & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Solving the equation for a_i, $i = 0,1,2,3$

we get:

$$\boxed{\begin{aligned} a_0 &= \theta_0 \\ a_1 &= 0 \\ a_2 &= \frac{3}{t_f^2}(\theta_f - \theta_0) \\ a_3 &= -\frac{2}{t_f^3}(\theta_f - \theta_0) \end{aligned}}$$

Max angular velocity at $t_f/2$ $\rightarrow$ plug into eqn. (2)

$$\Rightarrow \dot{\theta}_{max}(t = t_f/2) = \cancel{2}\cdot\frac{3}{t_f^{\cancel{2}}}(\theta_f - \theta_0)\cdot\frac{\cancel{t_f}}{\cancel{2}} - 3\cdot\frac{\cancel{2}}{t_f^{\cancel{3}}}(\theta_f - \theta_0)\frac{\cancel{t_f^2}}{\cancel{4}\,2}$$

$$= \frac{3(\theta_f - \theta_0)}{t_f} - \frac{3(\theta_f - \theta_0)}{2t_f}$$

$$= \underline{\frac{3(\theta_f - \theta_0)}{2t_f}}$$

Max & min angular acceleration at $t=0$ and $t=t_f$

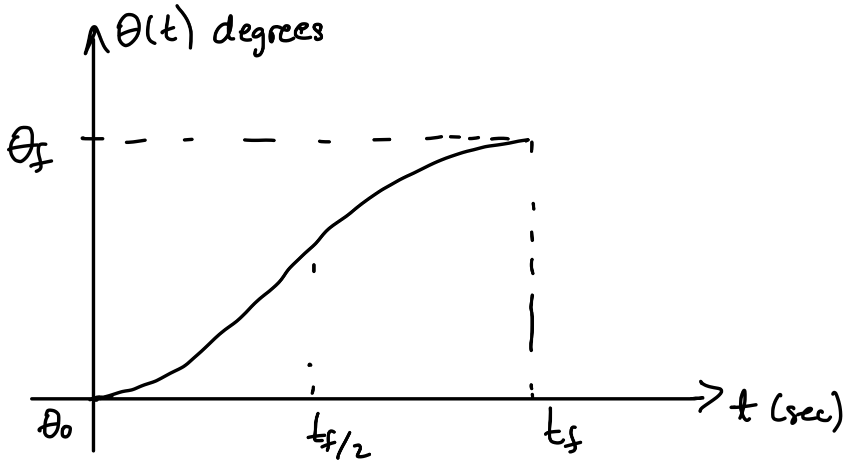
$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

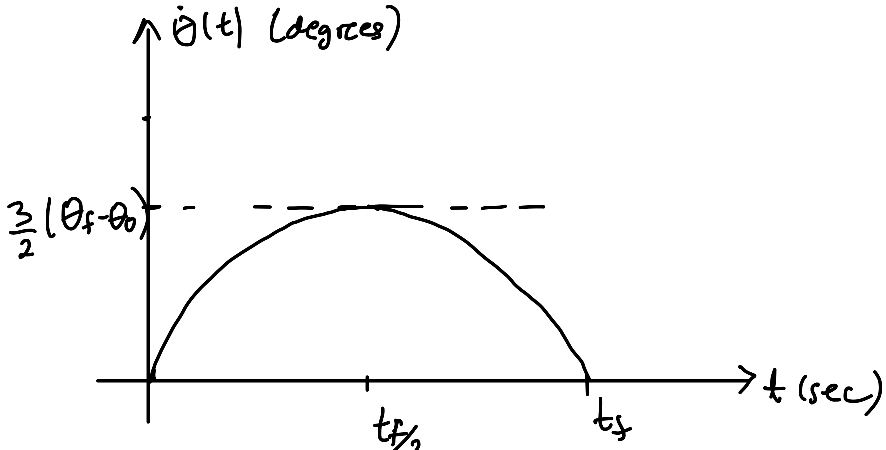
$$t=0 \Rightarrow \ddot{\theta}_{max}(t=0) = 2 \times \frac{3}{t_f^2}(\theta_f-\theta_0) = \frac{6}{t_f^2}(\theta_f-\theta_0)$$

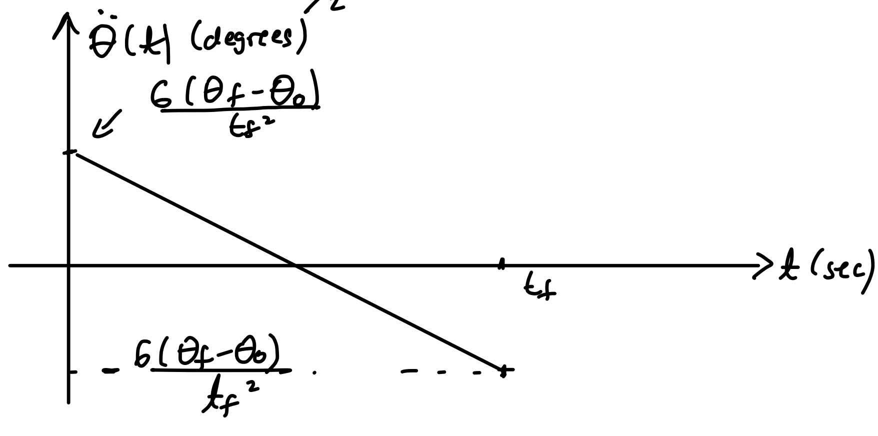
$$t=t_f \Rightarrow \ddot{\theta}_{min}(t=t_f) = 2 \times \frac{3}{t_f^2}(\theta_f-\theta_0) - 6 \cdot \frac{2}{t_f^{\cancel{3}\,2}}(\theta_f-\theta_0) \times \cancel{t_f}$$

$$= \frac{6(\theta_f-\theta_0)}{t_f^2} - \frac{12(\theta_f-\theta_0)}{t_f^2}$$

$$= -\frac{6(\theta_f-\theta_0)}{t_f^2}$$







$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

$$\boxed{\begin{aligned} a_0 &= \theta_0 \\ a_1 &= 0 \\ a_2 &= \frac{3}{t_f^2}(\theta_f - \theta_0) \\ a_3 &= -\frac{2}{t_f^3}(\theta_f - \theta_0) \end{aligned}}$$

We will use cubic polynomial function to move the joints in a smooth manner from initial position to final position in a set time.