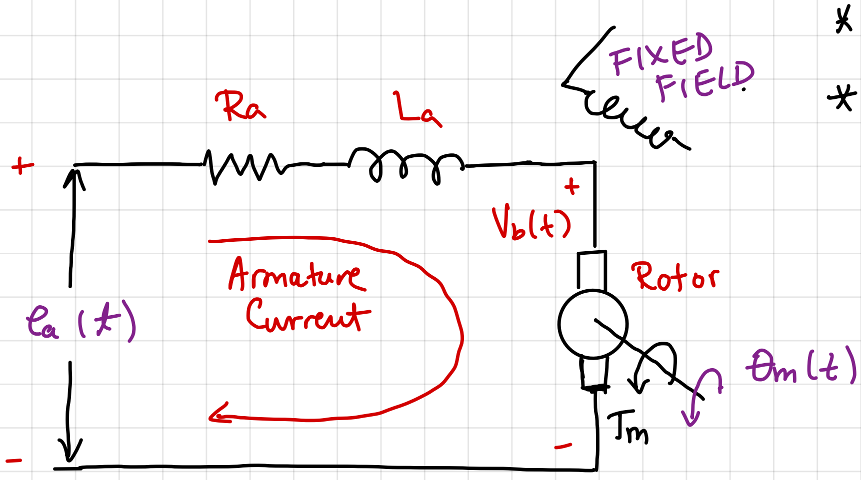
DC Servo Motor Transfer Function

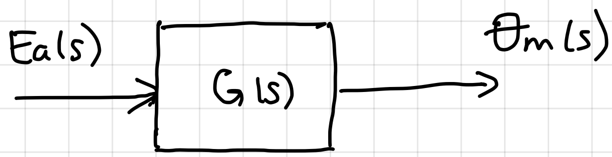
(armature controlled)

* rotational position $\theta_m(t)$
* angular velocity of motor $\omega(t)$



$$\boxed{V_b(t) \propto \omega(t)}$$

back emf



$$\boxed{G(s) = \frac{\theta_m(s)}{E_a(s)}}$$

$f(t) \rightarrow F(s)$

$f'(t) = sF(s) \ldots$

$f''(t) = s^2F(s) \ldots$

$$V_b(t) = K_b \frac{d\theta_m(t)}{dt}$$

s domain : $V_b(s) = K_b \, s \, \theta_m(s)$ (1)

KVL : $e_a(t) = i_a(t) R_a + L_a \dfrac{di_a(t)}{dt} + V_b(t)$

s domain : $E_a(s) = I_a(s) R_a + L_a s I_a(s) + V_b(s)$

$$E_a(s) = (R_a + L_a s) I_a(s) + V_b(s)$$

plug in (1) $\longrightarrow$

$$\boxed{E_a(s) = (R_a + L_a s) I_a(s) + K_b \, s \, \theta_m(s)} \quad (2)$$

Torque of the motor depends on the armature current

$\tau_m \sim i_a$

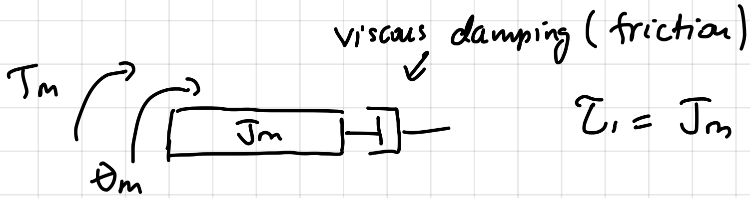
- torque depends on two components : inertia and friction

$$\tau_m(t) = K_t \, i_a(t)$$

s domain : $\boxed{T_m(s) = K_t \, I_a(s)} \Rightarrow I_a(s) = \dfrac{T_m(s)}{K_t}$

$$E_a(s) = (R_a + L_a s)\, I_a(s) + K_b s\, \theta_m(s)$$

$$E_a(s) = (R_a + L_a s) \cdot \frac{T_m(s)}{K_t} + K_b s\, \theta_m(s) \qquad (3)$$



$$\tau_1 = J_m \cdot \alpha = J_m \cdot \frac{d\omega}{dt}$$

$$\tau_m = J_m \frac{d\omega}{dt} + D_m \omega = J_m \ddot{\theta}_m(t) + D_m \dot{\theta}_m(t)$$

Inertia + Friction

s domain: $\boxed{T_m(s) = J_m(s) \cdot s^2 \theta_m(s) + D_m s\, \theta_m(s)}$ plug into $\rightarrow$ eqn. (3)

$$E_a(s) = (R_a + L_a s) \cdot \frac{J_m(s)\, s^2 \theta_m(s) + D_m s\, \theta_m(s)}{K_t} + K_b s\, \theta_m(s)$$

$$E_a = \frac{R_a + L_a s}{K_t} \cdot (J_m s^2 + D_m s) \cdot \theta_m + K_b s\, \theta_m$$

$$E_a = \frac{(R_a + L_a s)(J_m s^2 + D_m s) + K_t K_b s}{K_t} \theta_m$$

$$G(s) = \frac{\theta_m}{E_a} = \frac{K_t}{(R_a + L_a s)(J_m s^2 + D_m s) + K_t K_b s}$$

L_a - very small $\rightarrow$ can ignore

$$G(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{K_t}{R_a(J_m s^2 + D_m s) + K_t K_b s}$$

$$G(s) = \frac{\frac{K_t}{J_m R_a}}{s\left[s + \frac{K_b K_t}{J_m R_a} + \frac{D_m}{J_m}\right]}$$

TF of a DC servo motor

$$\boxed{G(s) = \frac{\frac{K_t}{J_m R_a}}{s\left[s + \frac{1}{J_m}\left(\frac{K_b K_t}{R_a} + D_m\right)\right]}}$$