

AE2202 Fluid Dynamics - Homework 5

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Problem 1

The x and y components of velocity for a two-dimensional flow are $u = 6y$ ft/s and $v = 3$ ft/s where y is in feet. Determine the equation for the streamlines and sketch representative streamlines in the upper half plane.

Answer

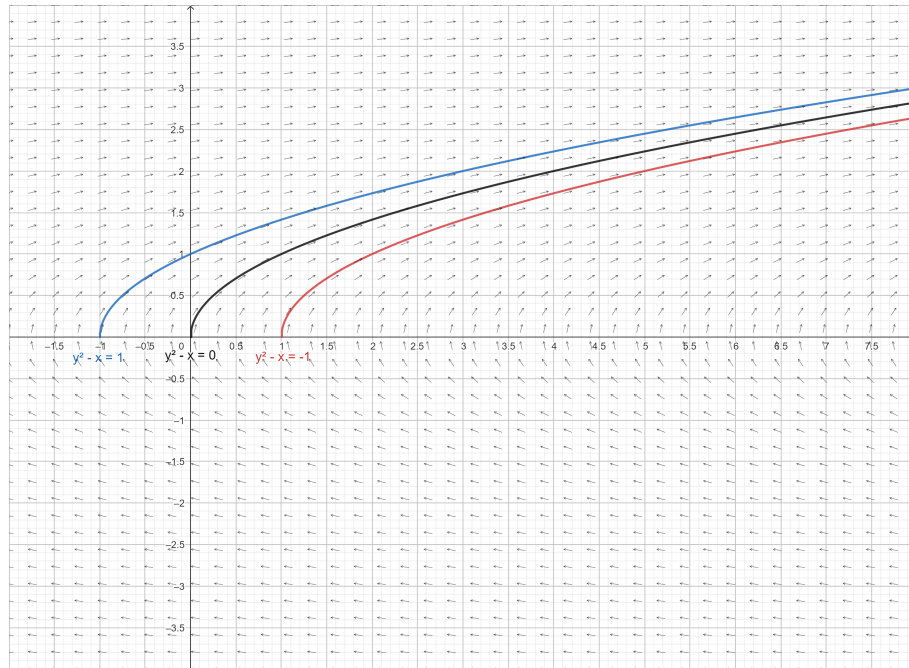
The streamline equation of streamline may be obtained by integrating:

$$\frac{dy}{dx} = \frac{v_y}{v_x} \quad (1)$$

For this case,

$$\begin{aligned} \frac{dy}{dx} &= \frac{v}{u} \\ \frac{dy}{dx} &= \frac{3}{6y} \\ \frac{dy}{dx} &= \frac{1}{2y} \\ 2y \, dy &= dx \\ y^2 &= x + C \\ y^2 - x &= C \end{aligned}$$

Sketch:



Problem 2

Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and where $v = 2cxy$, where c is a constant, are given by the equation $x^2y - y^3/3 = \text{constant}$. At which point (points) is the flow parallel to the y axis? At which point (points) is the fluid stationary?

Answer

When the point is parallel to the y axis, the velocity in x -direction should be zero, or:

$$\begin{aligned}u &= 0 \\x^2 - y^2 &= 0 \\x^2 &= y^2 \\x &= \pm y\end{aligned}$$

Thus, the flow is parallel to the y axis at $(t, \pm t)$, where $t \in \mathbb{R}$. When the point is stationary, the velocity components are $u = 0$ and $v = 0$.

$$\begin{aligned}v &= 0 \\xy &= 0 \\x = 0 \vee y &= 0\end{aligned}$$

Intersecting both solutions of $u = 0$ and $v = 0$, we get that the flow is stationary at $(x, y) = (0, 0)$.

Problem 3

The x - and y -components of a velocity field are given by $u = (V_0/l)x$ and $v = -(V_0/l)y$, where V_0 and l are constants. Plot the streamlines for this flow and determine the acceleration field.

Answer

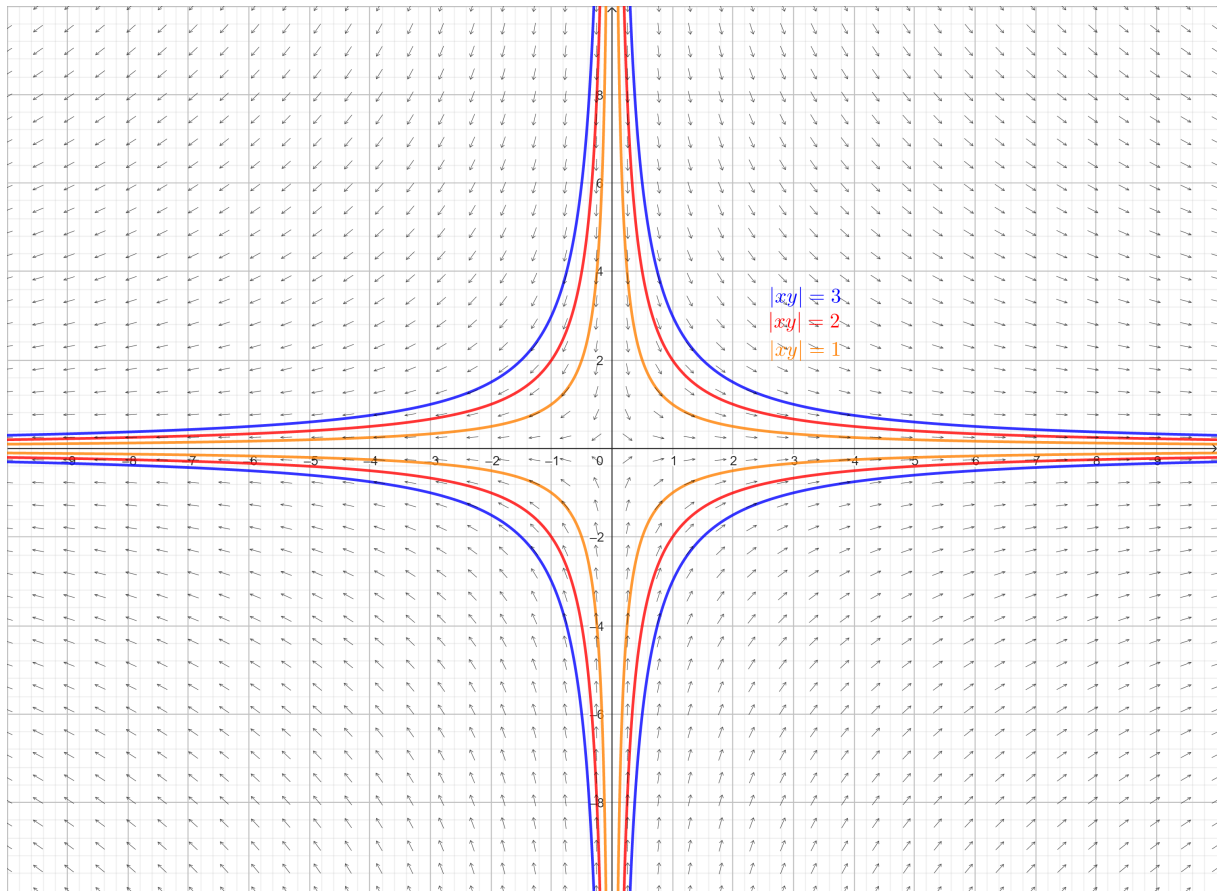
We may find the streamline equation by integrating:

$$\frac{dy}{dx} = \frac{v_y}{v_x} \quad (2)$$

For this case,

$$\begin{aligned} \frac{dy}{dx} &= \frac{v}{u} \\ \frac{dy}{dx} &= \frac{-(V_0/l)y}{(V_0/l)x} \\ \frac{dy}{dx} &= \frac{-y}{x} \\ \frac{dy}{y} &= \frac{-dx}{x} \\ \ln |y| &= -\ln |x| + C \\ \ln |y| + \ln |x| &= C \\ \ln |xy| &= C \\ |xy| &= e^C \equiv K \\ |xy| &= K \end{aligned}$$

Sketch:

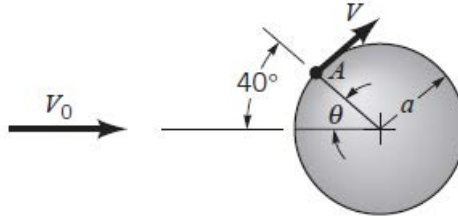


On the other hand, the acceleration field may be found by differentiating the velocity field:

$$\begin{aligned}
\mathbf{a} &= \frac{d\mathbf{v}}{dt} \\
&= \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \\
&= \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{v}}{\partial x} \cdot v_x + \frac{\partial \mathbf{v}}{\partial y} \cdot v_y \\
&= \begin{bmatrix} \frac{\partial v_x}{\partial t} \\ \frac{\partial v_y}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{\partial v_x}{\partial x} \\ \frac{\partial v_y}{\partial x} \end{bmatrix} \cdot v_x + \begin{bmatrix} \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial y} \end{bmatrix} \cdot v_y \\
&= \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial t} \\ \frac{\partial[-(V_0/l)y]}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial x} \\ \frac{\partial[-(V_0/l)y]}{\partial x} \end{bmatrix} \cdot (V_0/l)x + \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial y} \\ \frac{\partial[-(V_0/l)y]}{\partial y} \end{bmatrix} \cdot -(V_0/l)y \\
&= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} (V_0/l) \\ 0 \end{bmatrix} \cdot (V_0/l)x + \begin{bmatrix} 0 \\ -(V_0/l) \end{bmatrix} \cdot -(V_0/l)y \\
&= \begin{bmatrix} (V_0/l)^2 x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (V_0/l)^2 y \end{bmatrix} \\
\mathbf{a} &= (V_0/l)^2 \begin{bmatrix} x \\ y \end{bmatrix}
\end{aligned}$$

Problem 4

A fluid flows past a sphere with an upstream velocity of $V_0 = 40$ m/s as shown in the following figure. From a more advanced theory, it is found that the speed of the fluid along the front part of the sphere is $V = \frac{3}{2}V_0 \sin \theta$. Determine the streamwise and normal components of acceleration at point A if the radius of the sphere is $a = 0.20$ m.



Answer

At A , the speed of fluid along the front part of the sphere is

$$\begin{aligned} V_A &= \frac{3}{2} V_0 \sin \theta_A \\ &= \frac{3}{2} (40 \text{ m/s}) \sin 40^\circ \\ &= 60 \sin 40^\circ \text{ m/s} \end{aligned}$$

Let \mathbf{s} be the unit vector in the direction of the streamwise velocity, \mathbf{n} be the unit vector in the direction of the normal velocity. The velocity of the fluid at a certain point on the sphere is

$$\mathbf{v} = V \hat{\mathbf{s}} \quad (3)$$

Therefore, the acceleration of the fluid at that certain point is

$$\begin{aligned} \mathbf{a} &= \frac{d\mathbf{v}}{dt} \\ &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \\ &= V \frac{\partial \hat{\mathbf{s}}}{\partial t} + \frac{\partial V}{\partial t} \hat{\mathbf{s}} + V \frac{\partial \mathbf{v}}{\partial s} \\ &= V \cdot \frac{V}{a} \hat{\mathbf{n}} + \mathbf{0} + V \frac{\partial \mathbf{v}}{\partial \theta} \frac{\partial \theta}{\partial s} \\ &= \frac{V^2}{a} \hat{\mathbf{n}} + V \frac{\partial V}{\partial \theta} \cdot \frac{1}{a} \hat{\mathbf{s}} \end{aligned}$$

Therefore, our streamwise acceleration is

$$\begin{aligned} a_s &= \frac{V}{a} \cdot \frac{\partial V}{\partial \theta} \\ &= \frac{V}{a} \cdot \frac{\partial (\frac{3}{2} V_0 \sin \theta)}{\partial \theta} \\ &= \frac{\frac{3}{2} V_0 \sin \theta}{a} \cdot \frac{3}{2} V_0 \cos \theta \\ &= \frac{9}{4} \frac{V_0^2}{a} \sin \theta \cos \theta \\ (a_s)_A &= \frac{9}{4} \frac{(40 \text{ m/s})^2}{0.20 \text{ m}} \sin 40^\circ \cos 40^\circ \\ &= 8863.270 \text{ m/s}^2 \end{aligned}$$

and our normal acceleration is

$$\begin{aligned} a_n &= \frac{V^2}{a} \\ (a_n)_A &= \frac{(60 \sin 40^\circ)^2}{0.2 \text{ m}} \\ &= 7437.166 \text{ m/s}^2 \end{aligned}$$

Problem 5

Assume the temperature of the exhaust in an exhaust pipe can be approximated by $T = T_0 (1 + ae^{-bx}) [1 + c \cos \omega t]$, where $T_0 = 100^\circ\text{C}$, $a = 3$, $b = 0.03\text{m}^{-1}$, $c = 0.05$, and $\omega = 100 \text{ rad/s}$. If the exhaust speed is a constant 3 m/s , determine the time rate of change of temperature of the fluid particles at $x = 0$ and $x = 4 \text{ m}$ when $t = 0$.

Answer

As the exhaust come from a pipe, it only should move in one direction. We may determine the time rate of change of temperature of the fluid particles by using the following equation:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \quad (4)$$

For the first term,

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial}{\partial t} [T_0 (1 + ae^{-bx}) [1 + c \cos \omega t]] \\ &= T_0 (1 + ae^{-bx}) (-\omega c \sin \omega t) \end{aligned}$$

For the second term,

$$\begin{aligned} \mathbf{v} \cdot \nabla T &= v_x \cdot \frac{\partial T}{\partial x} \\ &= (u) \cdot T_0 (-abe^{-bx}) [1 + c \cos \omega t] \\ &= -T_0 u (abe^{-bx}) [1 + c \cos \omega t] \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{DT}{Dt} &= T_0 (1 + ae^{-bx}) (-\omega c \sin \omega t) - T_0 u (abe^{-bx}) [1 + c \cos \omega t] \\ &= -T_0 [(1 + ae^{-bx}) (\omega c \sin \omega t) + u (abe^{-bx}) [1 + c \cos \omega t]] \end{aligned}$$

Thus, for $t = 0$ and $x = 0$,

$$\begin{aligned} \frac{DT}{Dt} &= -(100 + 273.15)[(1 + 3e^{-0})(100 \cdot 0.05 \sin 0) + 3 (0.03e^{-0}) (1 + 0.05 \cos 0)] \\ &= -105.788 \text{ K/s} \end{aligned}$$

and for $x = 4 \text{ m}$,

$$\begin{aligned} \frac{DT}{Dt} &= -(100 + 273.15)[(1 + 3e^{-0.03 \cdot 4})(100 \cdot 0.05 \sin 0) + 3 (0.03e^{-0.03 \cdot 4}) (1 + 0.05 \cos 0)] \\ &= -93.826 \text{ K/s} \end{aligned}$$