AE2202 Fluid Dynamics - Homework 5

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Problem 1

The x and y components of velocity for a two-dimensional flow are $u = 6y$ ft/s and $v = 3$ ft/s where y is in feet. Determine the equation for the streamlines and sketch representative streamlines in the upper half plane.

Answer

The streamline equation of streamline may be obtained by integrating:

$$\frac{dy}{dx} = \frac{v_y}{v_x} \tag{1}$$

For this case,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{v}{u} \\
\frac{dy}{dx} &= \frac{3}{6y} \\
\frac{dy}{dx} &= \frac{1}{2y} \\
2y\ dy &= dx \\
y^2 &= x + C \\
y^2 - x &= C
\end{aligned}$$

Sketch:

Problem 2

Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and where $v = 2cxy$, where c is a constant, are given by the equation $x^2y - y^3/3 = constant$. At which point (points) is the flow parallel to the y axis? At which point (points) is the fluid stationary?

Answer

When the point is parallel to the y axis, the velocity in $x-$direction should be zero, or:

$$\begin{aligned} u &= 0 \\ x^2 - y^2 &= 0 \\ x^2 &= y^2 \\ x &= \pm y \end{aligned}$$

Thus, the flow is parallel to the y axis at $(t, \pm t)$, where $t \in \mathbb{R}$. When the point is stationary, the velocity components are $u = 0$ and $v = 0$.

$$\begin{aligned} v &= 0 \\ xy &= 0 \\ x = 0 \ &\vee \ y = 0 \end{aligned}$$

Intersecting both solutions of $u = 0$ and $v = 0$, we get that the flow is stationary at $(x, y) = (0, 0)$.

Problem 3

The $x-$ and $y-$components of a velocity field are given by $u = (V_0/l)x$ and $v = -(V_0/l)y$, where V_0 and l are constants. Plot the streamlines for this flow and determine the acceleration field.

Answer

We may find the streamline equation by integrating:

$$\frac{dy}{dx} = \frac{v_y}{v_x} \tag{2}$$

For this case,

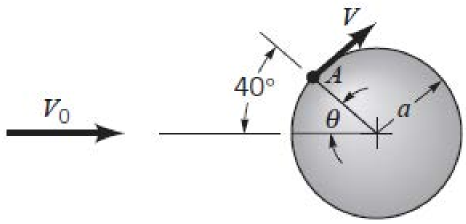
$$\begin{aligned}
\frac{dy}{dx} &= \frac{v}{u} \\
\frac{dy}{dx} &= \frac{-(V_0/l)y}{(V_0/l)x} \\
\frac{dy}{dx} &= \frac{-y}{x} \\
\frac{dy}{y} &= \frac{-dx}{x} \\
\ln|y| &= -\ln|x| + C \\
\ln|y| + \ln|x| &= C \\
\ln|xy| &= C \\
|xy| &= e^C \equiv K \\
|xy| &= K
\end{aligned}$$

Sketch: On the other hand, the acceleration field may be found by differentiating the velocity field:

$$\begin{aligned}
\mathbf{a} &= \frac{d\mathbf{v}}{dt} \\
&= \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \\
&= \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{v}}{\partial x} \cdot v_x + \frac{\partial \mathbf{v}}{\partial y} \cdot v_y \\
&= \begin{bmatrix} \frac{\partial v_x}{\partial t} \\ \frac{\partial v_y}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{\partial v_x}{\partial x} \\ \frac{\partial v_y}{\partial x} \end{bmatrix} \cdot v_x + \begin{bmatrix} \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial y} \end{bmatrix} \cdot v_y \\
&= \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial t} \\ \frac{\partial[-(V_0/l)y]}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial x} \\ \frac{\partial[-(V_0/l)y]}{\partial x} \end{bmatrix} \cdot (V_0/l)x + \begin{bmatrix} \frac{\partial[(V_0/l)x]}{\partial y} \\ \frac{\partial[-(V_0/l)y]}{\partial y} \end{bmatrix} \cdot -(V_0/l)y \\
&= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} (V_0/l) \\ 0 \end{bmatrix} \cdot (V_0/l)x + \begin{bmatrix} 0 \\ -(V_0/l) \end{bmatrix} \cdot -(V_0/l)y \\
&= \begin{bmatrix} (V_0/l)^2 x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (V_0/l)^2 y \end{bmatrix} \\
\mathbf{a} &= (V_0/l)^2 \begin{bmatrix} x \\ y \end{bmatrix}
\end{aligned}$$

Problem 4

A fluid flows past a sphere with an upstream velocity of $V_0 = 40$ m/s as shown in the following figure. From a more advanced theory, it is found that the speed of the fluid along the front part of the sphere is $V = \frac{3}{2}V_0 \sin\theta$. Determine the streamwise and normal components of acceleration at point A if the radius of the sphere is $a = 0.20$ m.



Answer

At A, the speed of fluid along the front part of the sphere is

$$\begin{aligned} V_A &= \frac{3}{2}V_0 \sin\theta_A \\ &= \frac{3}{2}(40 \text{ m/s}) \sin 40^\circ \\ &= 60 \sin 40^\circ \text{ m/s} \end{aligned}$$

Let **s** be the unit vector in the direction of the streamwise velocity, **n** be the unit vector in the direction of the normal velocity. The velocity of the fluid at a certain point on the sphere is

$$\mathbf{v} = V\hat{\mathbf{s}} \tag{3}$$

Therefore, the acceleration of the fluid at that certain point is

$$\begin{aligned} \mathbf{a} &= \frac{d\mathbf{v}}{dt} \\ &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \\ &= V\frac{\partial \hat{\mathbf{s}}}{\partial t} + \frac{\partial V}{\partial t}\hat{\mathbf{s}} + V\frac{\partial \mathbf{v}}{\partial s} \\ &= V \cdot \frac{V}{a}\hat{\mathbf{n}} + \mathbf{0} + V\frac{\partial \mathbf{v}}{\partial \theta}\frac{\partial \theta}{\partial s} \\ &= \frac{V^2}{a}\hat{\mathbf{n}} + V\frac{\partial V}{\partial \theta} \cdot \frac{1}{a}\hat{\mathbf{s}} \end{aligned}$$

Therefore, our streamwise acceleration is

$$\begin{aligned} a_s &= \frac{V}{a} \cdot \frac{\partial V}{\partial \theta} \\ &= \frac{V}{a} \cdot \frac{\partial \left(\frac{3}{2}V_0 \sin\theta\right)}{\partial \theta} \\ &= \frac{\frac{3}{2}V_0 \sin\theta}{a} \cdot \frac{3}{2}V_0 \cos\theta \\ &= \frac{9}{4}\frac{{V_0}^2}{a} \sin\theta\cos\theta \\ (a_s)_A &= \frac{9}{4}\frac{(40 \text{ m/s})^2}{0.20 \text{ m}} \sin 40^\circ \cos 40^\circ \\ &= 8863.270 \text{ m/s} \end{aligned}$$

and our normal acceleration is

$$\begin{aligned} a_n &= \frac{V^2}{a} \\ (a_n)_A &= \frac{(60 \sin 40^\circ)^2}{0.2 \text{ m}} \\ &= 7437.166 \text{ m/s} \end{aligned}$$

Problem 5

Assume the temperature of the exhaust in an exhaust pipe can be approximated by $T = T_0\left(1 + ae^{-bx}\right)[1 + c\cos\omega t]$, where $T_0 = 100$°C, $a = 3$, $b = 0.03\text{m}^{-1}$, $c = 0.05$, and $\omega = 100$ rad/s. If the exhaust speed is a constant 3 m/s, determine the time rate of change of temperature of the fluid particles at $x = 0$ and $x = 4$ m when $t = 0$.

Answer

As the exhaust come from a pipe, it only should move in one direction. We may determine the time rate of change of temperature of the fluid particles by using the following equation:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \tag{4}$$

For the first term,

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{\partial}{\partial t}\left[T_0\left(1 + ae^{-bx}\right)[1 + c\cos\omega t]\right]\\ &= T_0\left(1 + ae^{-bx}\right)(-\omega c\sin\omega t)\end{aligned}$$

For the second term,

$$\begin{aligned}\mathbf{v} \cdot \nabla T &= v_x \cdot \frac{\partial T}{\partial x}\\ &= (u) \cdot T_0\left(-abe^{-bx}\right)[1 + c\cos\omega t]\\ &= -T_0 u\left(abe^{-bx}\right)[1 + c\cos\omega t]\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{DT}{Dt} &= T_0\left(1 + ae^{-bx}\right)(-\omega c\sin\omega t) - T_0 u\left(abe^{-bx}\right)[1 + c\cos\omega t]\\ &= -T_0\left[\left(1 + ae^{-bx}\right)(\omega c\sin\omega t) + u\left(abe^{-bx}\right)[1 + c\cos\omega t]\right]\end{aligned}$$

Thus, for $t = 0$ and $x = 0$,

$$\begin{aligned}\frac{DT}{Dt} &= -(100 + 273.15)[(1 + 3e^{-0})(100 \cdot 0.05\sin 0) + 3\left(0.03e^{-0}\right)(1 + 0.05\cos 0)]\\ &= -105.788 \text{ K/s}\end{aligned}$$

and for $x = 4$ m,

$$\begin{aligned}\frac{DT}{Dt} &= -(100 + 273.15)[(1 + 3e^{-0.03\cdot 4})(100 \cdot 0.05\sin 0) + 3\left(0.03e^{-0.03\cdot 4}\right)(1 + 0.05\cos 0)]\\ &= -93.826 \text{ K/s}\end{aligned}$$