ALGORITHM ANALYSIS:

A COMPARATIVE STUDY OF BAM, SAM, AND SAMK

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Overview

Introduction	• Define the problem • Define the solution
Methods	• Discuss testing conditions • Define the algorithms implemented
Discussion	• Discuss reasoning behind implementation choices • Interpret and synthesize findings
Results	• Regression Analysis
Conclusion	• What we learned

Problem

- Complex operations tend to require a high usage of time and space
  - Such as Matrix Multiplication!
- Practical scenarios require quick implementations!

Solution

- Find the matrix multiplication algorithm that uses time and space most efficiently.
- Implementations of Matrix Multiplication:
  - *BAM*: Basic Matrix Multiplication Algorithm
  - *SAM*: Strassen’s Algorithm
  - *SAMk*: Strassen’s Algorithm with small problem cutoff k

Methods

- How?
  - Examine the results of the BAM, SAM, SAMk algorithm on matrices of various sizes.
    - Which algorithm effectively uses space? Time? Both?
  - Use the results to determine the optimal cutoff value for k
  - Observe where n reaches a crossover value when SAMk overtakes BAM

- Language: Python
- System: UMBC GL Server
- Time and Space:
  - Time:
    - Python function “python –m timeit” returns the running time of a code section
  - Space:
    - malloc
- Input Sizes Tested: 2 through 1028
- Number of Trials:

Matrix Multiplication Algorithms: BAM

- Basic Summation: $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{ki}$
  - Runtime: $\theta(n^3)$
- $BAM(A[\,][n], B\ [n][\,])$

```
    $C[\,][\,] = C(n, n)$
    $for\ x\ =\ 0\ to\ n\ -\ 1$
            $for\ y\ =\ 0\ to\ n - 1$
            $total\ =\ 0$
            $\text{for w}\ =\ \text{0 to z} - 1$
                    $\text{total}\ =\ \text{total}\ +\ A_{wi} \times B_{wi}$
            $C_{ij}\ =\ sum$
    $return\ C$
```

Matrix Multiplication Algorithms: SAM

- Developed by Volker Strassen in 1969
- Reduced matrix multiplication run time from $O(n^3)$ to $O(n^{\log 7})$
- Conditional on the basis that both matrices A and B to be multiplied together are square matrices.
  - In our case, the input was square matrices so we did not have to pad them with zeros

Matrix Multiplication Algorithms: SAMk

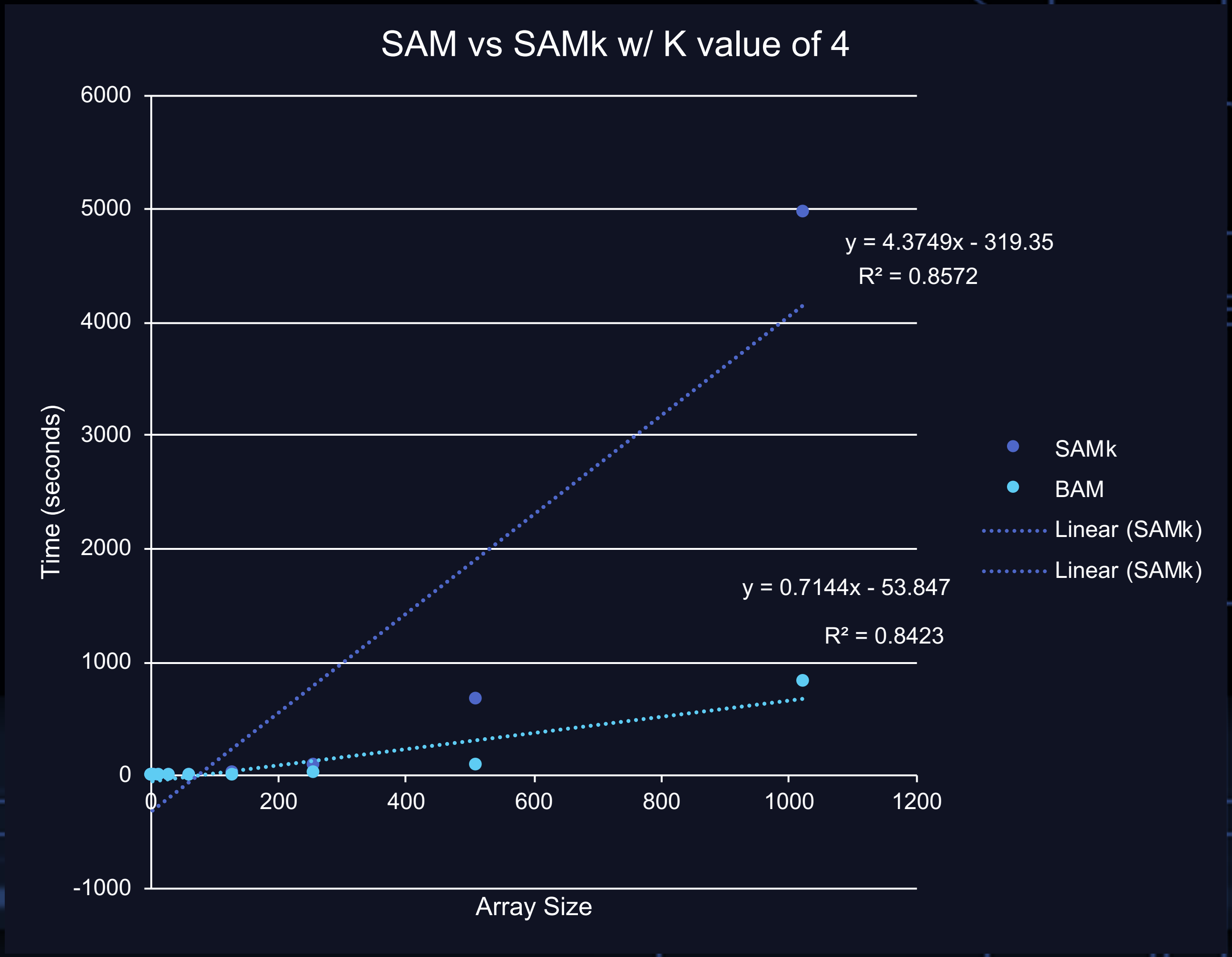
- Modified version of Strassen’s original algorithm
- An improvement over SAM
  - Algorithm solves all of its subproblems less than the small problem cutoff k using the BAM algorithm implementation.
  - k is a parameter passed into the function
- Optimal value of k was found through regression analysis

Memory Management

- How we handle memory:
  - Memory is handled by python and stored in blocks
  - Care was taken to minimize copying data within reason
  - The numpy library was used for several operations, numpy optimizes data types to in general use less memory than a simple array.
- Issues we encountered:
  - We ran into some issues with the data types used in a traditional python lists as the BAM algorithm was producing lists with values larger than what was allowed. Simple casting fixed this issue.

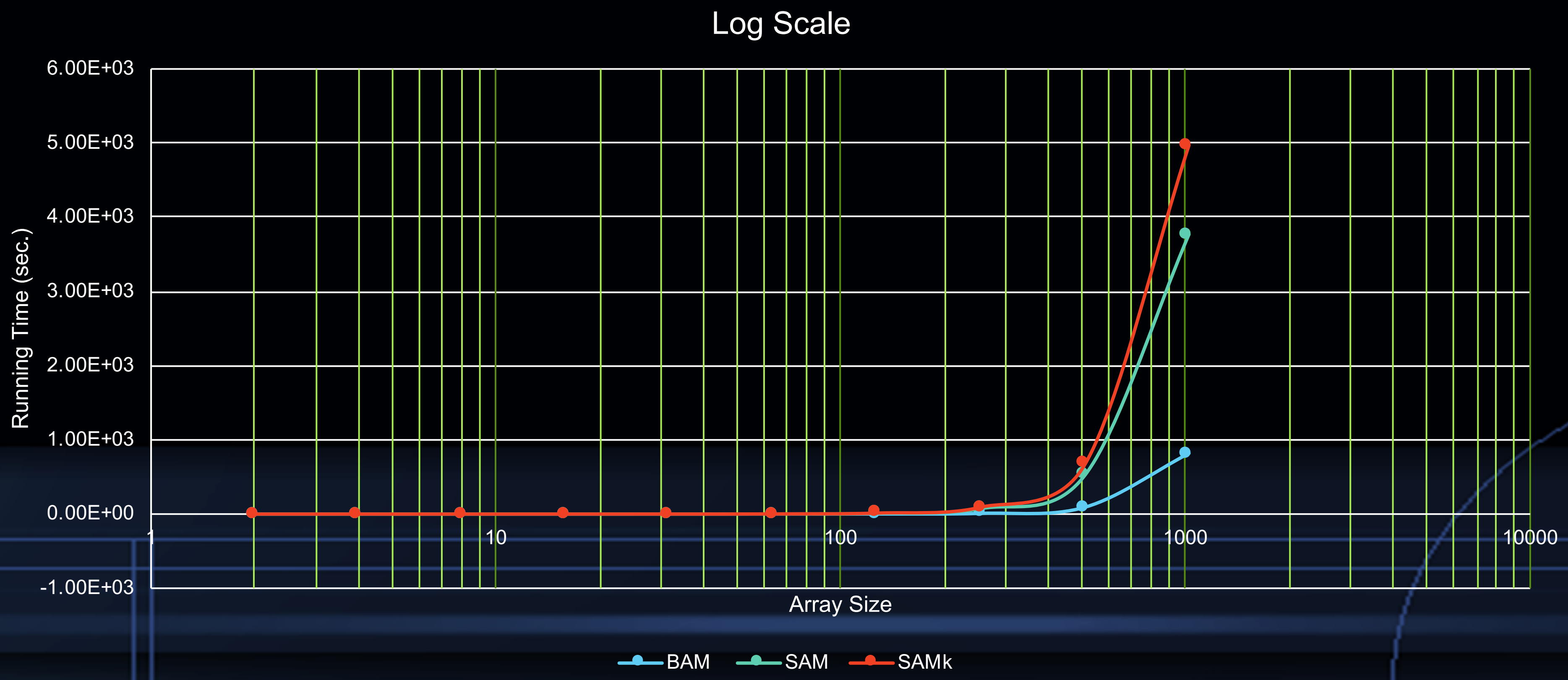
Results

- Optimal Value of k: 4
  - Running time determined:
- For what input size N_0 does the running time of SAMk cross the running time of BAM?


SAM vs SAMk w/ K value of 4

Data Interpretation

- What were our findings?



Regression Analysis

- Constant factors in the theoretical running time of SAMk that best match our data
- Resulting running time:

Conclusion