Assignment 3 - Q2.

Date: D D M M Y Y

Q2a) Computing Gradients

$$e_{ij}^2 = \left(r_{ij} - \sum_k p_{ik} q_{kj}\right)^2$$

Partial derivative wrt P

$$\frac{\partial e_{ij}^2}{\partial p_{ik}} = -2\left(r_{ij} - \sum_k p_{ik} q_{kj}\right) q_{kj}$$

→ replacing with e_{ij}

$$\frac{\partial e_{ij}^2}{\partial p_{ik}} = -2 e_{ij} q_{kj}$$

Partial Derivative wrt q

$$\frac{\partial e_{ij}^2}{\partial q_{kj}} = -2\left(r_{ij} - \sum_k p_{ik} q_{ki}\right)(p_{ik})$$

$$\frac{\partial e_{ij}^2}{\partial q_{kj}} = -2 e_{ij} p_{ik}$$

Updated values of P & q

$$p_{ik}' = p_{ik} + \alpha \frac{\partial}{\partial p_{ik}} e_{ij}^2 = p_{ik} + 2\alpha e_{ij} q_{kj}$$

$$q_{kj}' = q_{kj} + \alpha \frac{\partial}{\partial q_{kj}} e_{ij}^2 = q_{ki} + 2\alpha e_{ij} p_{ik}$$

b) Adding Biases

$$\hat{r}_{ij} = u_i + I_j + \sum_k p_{ik} q_{kj}$$

$$e_{ij}^2 = \left[r_{ij} - \left(\sum_k p_{ik} q_{ki} + u_i + I_j\right)\right]^2$$

Partial Derivative wrt u

$$\frac{\partial e_{ij}^2}{\partial u_i} = -2\left(r_{ij} - \left(\sum_k p_{ik} q_{kj} + u_i + I_j\right)\right) = -2 e_{ij}$$

Partial Derivative wrt I

$$\frac{\partial e_{ij}^2}{\partial I_j} = -2\left(r_{ij} - \left(\sum_k p_{ik} q_{ki} + u_i + I_j\right)\right) = -2 e_{ij}$$

Updated values of biases, u & I

$$u_i' = u_i - \alpha \frac{\partial}{\partial u_i} e_{ij}^2 = u_i + 2\alpha e_{ij}$$

$$I_j' = I_j' - \alpha \frac{\partial}{\partial I_j} e_{ij}^2 = I_j + 2\alpha e_{ij}$$