

2. **Sort the other points by their polar angle relative to P_5 .** The points sorted by angle will allow us to traverse the boundary of the convex hull in a counter-clockwise direction.
3. **Build the convex hull** by iterating through the sorted points and checking for left or right turns.
For each point:
 - Add the point to the hull if it makes a left turn.
 - If a right turn is made, remove the last point in the hull until a left turn is obtained.

Given points:

- $P_1 = (0, 0)$
- $P_2 = (1, 2)$
- $P_3 = (2, 1)$
- $P_4 = (2, -1)$
- $P_5 = (1, -2)$ (starting point)

Let's go through these steps with the updated starting point, P_5 .

1. Sorting by polar angle relative to P_5 :

The sorted points by angle will be determined by calculating the angle each point makes with P_5 . After sorting, let's assume the points are ordered as P_4, P_3, P_2, P_1 .

2. Building the convex hull:

- Start with P_5 and add each point sequentially, checking turns.
- The resulting points that remain after checking turns will form the convex hull.

The convex hull points, in order, should yield the boundary vertices. Let me calculate it manually if you want detailed polar angle and cross-product values for each step.



calculate it manually if you want detailed polar angle and cross-product values for each step.



Let's go through the Graham scan algorithm step by step, manually calculating the polar angles and cross products.

Given points:

- $P_1 = (0, 0)$
- $P_2 = (1, 2)$
- $P_3 = (2, 1)$
- $P_4 = (2, -1)$
- $P_5 = (1, -2)$ (starting point)

Step 1: Sorting Points by Polar Angle with $P5$

To calculate the polar angle of each point relative to $P5 = (1, -2)$, we'll use the formula:

$$\text{angle} = \arctan\left(\frac{y - y_{\text{start}}}{x - x_{\text{start}}}\right)$$

Let's calculate these:

- **Angle of $P1 = (0, 0)$ with $P5$:**

$$\text{angle} = \arctan\left(\frac{0 - (-2)}{0 - 1}\right) = \arctan\left(\frac{2}{-1}\right) \approx -63.4^\circ$$

- **Angle of $P2 = (1, 2)$ with $P5$:**

$$\text{angle} = \arctan\left(\frac{2 - (-2)}{1 - 1}\right) = \text{undefined (vertical line)} = 90^\circ$$

- **Angle of $P3 = (2, 1)$ with $P5$:**

$$\text{angle} = \arctan\left(\frac{1 - (-2)}{2 - 1}\right) = \arctan\left(\frac{3}{1}\right) \approx 71.6^\circ$$

- **Angle of $P4 = (2, -1)$ with $P5$:**

$$\text{angle} = \arctan\left(\frac{-1 - (-2)}{2 - 1}\right) = \arctan(1) = 45^\circ$$

After sorting by angle, we get the following order for the points:

$$P1, P4, P3, P2$$

Step 2: Constructing the Convex Hull

1. **Initialize** the hull with the starting point $P5$: $\text{Hull} = [P5]$
2. **Add points in sorted order**, checking turns using the cross product. The cross product of vectors \overrightarrow{OA} and \overrightarrow{OB} (where O , A , and B are points) is calculated as:

$$(a_x - o_x) \times (b_y - o_y) - (a_y - o_y) \times (b_x - o_x)$$

- **Add $P1$:** No previous points to compare, so add directly. $\text{Hull}: [P5, P1]$

- **Add $P4$:**

$$\text{Cross Product} = (0 - 1) \times (-1 - (-2)) - (0 - (-2)) \times (2 - 1) = 1 \quad (\text{Left turn})$$

$$\text{Add } P4: \text{Hull}: [P5, P1, P4]$$

- **Add $P3$:**

$$\text{Cross Product} = (0 - 1) \times (1 - (-1)) - (-1 - 0) \times (2 - 0) = -4 \quad (\text{Right turn})$$

Since it's a right turn, remove $P4$ from the hull and check again with the remaining points.
Repeat the process until we get left turns only.

By following this, we can determine the points forming the convex hull. Let me know if you'd like the full set of cross products calculated for verification of each turn.



The full set of cross products calculated for verification of each turn.



Let's go through each step in detail, calculating all cross products to verify left or right turns for constructing the convex hull.

Points:

- $P1 = (0, 0)$
- $P2 = (1, 2)$
- $P3 = (2, 1)$
- $P4 = (2, -1)$
- $P5 = (1, -2)$ (starting point)

After sorting the points by angle relative to $P5$, the order is:

1. $P1 = (0, 0)$
2. $P4 = (2, -1)$
3. $P3 = (2, 1)$
4. $P2 = (1, 2)$

Building the Convex Hull

We start with $P5$ as the first point in the hull.

1. Initialize the Hull:

- Start with $P5$: Hull = $[P5]$

2. Add $P1$:

- There are no previous points to check, so we add $P1$ directly.
- **Updated Hull:** Hull = $[P5, P1]$

3. Add $P4 = (2, -1)$:

- Calculate the cross product for $P5$, $P1$, and $P4$ to check the turn:

$$\text{Cross Product} = (0 - 1) \times (-1 - (-2)) - (0 - (-2)) \times (2 - 1) = (-1) \times 1 - 2 \times 1 =$$

- **Right turn** (cross product is negative), so remove $P1$ from the hull.
- **Updated Hull:** $Hull = [P5]$
- Add $P4$ to the hull.
- **Updated Hull:** $Hull = [P5, P4]$

4. Add $P3 = (2, 1)$:

- Calculate the cross product for $P5, P4$, and $P3$:

$$\text{Cross Product} = (2 - 1) \times (1 - (-2)) - (-1 - (-2)) \times (2 - 1) = 1 \times 3 - 1 \times 1 = 3 -$$

- **Left turn** (cross product is positive), so we add $P3$ to the hull.
- **Updated Hull:** $Hull = [P5, P4, P3]$

5. Add $P2 = (1, 2)$:

- Calculate the cross product for $P4, P3$, and $P2$:

$$\text{Cross Product} = (2 - 2) \times (2 - (-1)) - (1 - (-1)) \times (1 - 2) = 0 \times 3 - 2 \times -1 = 0 -$$

- **Left turn** (cross product is positive), so we add $P2$ to the hull.
- **Updated Hull:** $Hull = [P5, P4, P3, P2]$

Final Convex Hull

The final convex hull is:

$$P5 = (1, -2), P4 = (2, -1), P3 = (2, 1), P2 = (1, 2)$$

These points represent the vertices of the convex polygon that encloses all given points.

ChatGPT can make mistakes. Check important info.