National University of Computer & Emerging Sciences, Karachi
Spring-2022 CS-Department
Mid-Term II
20th April, 2022, 11:30 am – 12:30 pm

Course Code: MT - 2005	**Course Name: Probability & Statistics**
Instructor Name : Mr. Osama Bin Ajaz, Mr. Muhammad Amjad, Dr. Fahad Riaz, and Mr. Muhammad Jamil Usmani	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper.
- Read each question completely before answering it. There are **4 questions and 2 pages.**
- In case of any ambiguity, you may make assumptions. But your assumption should not contradict any statement in the question paper.
- All the answers must be solved according to the sequence given in the question paper.
- Write down all answers in the Answer sheet.

Time: 60 minutes **Max. Marks = 30**

Q1) Consider the distribution function F(t) given below: **[3]**

$$F(t) = \begin{cases} 0, & t < 1 \\ \frac{1}{4}, & 1 \le t < 3 \\ \frac{1}{2}, & 3 \le t < 5 \\ \frac{3}{4}, & 5 \le t < 7 \\ 1, & t \ge 7 \end{cases}$$

Find **(a)** PMF f(t) **(b)** f (1.4 < t < 6) **(c)** f (t < 6 | t ≥ 4)

Q2) **(i)** Let's suppose a balanced coin is tossed until it turns up a head. Find the **(a)** PMF **(b)** CDF **[2]**

(ii) The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of these proportions is given by:

$$f(y) = \begin{cases} 5(1-y)^4, & 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$

(a) Verify that f(y) is a valid PDF. **[1]**

(b) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution controls? **[1]**

(c) What is the probability that a company selected at random spends more than 50% of its budget on environmental and pollution controls? **[1]**

Q3) **(i)** Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given below: **[8]**

f(x, y)		x		
		1	**2**	**3**
y	**1**	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

Calculate the correlation coefficient between X and Y.

(ii) The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function:

$$f(x,y) = \begin{cases} 8xy, & 0 \le y \le x \le 1 \\ 0, & elsewhere \end{cases}$$

(a) Find the marginal distribution of X and Y. **[2]**
(b) Find the covariance between X and Y **[4]**
(c) Are X and Y independent? **[1]**

Q4) **(i)** The probability that a computer recovers from a rare virus attack is 0.4. If 15 computers are known to have contracted with this virus, what is the probability that: **[3]**

(a) At least 13 computers survive; **(b)** From 3 to 5 computers survive.

(ii) An insurance company calculates the probabilities for the number of fatal accidents during a given year. It finds that on the average 7 accidents occur in a year. Find the probability that, in a certain year: **[1+1]**

(a)two or less accidents will occur, **(b)**more than 2 such accidents will take place.

(iii) A box of 8 screws contains 5 defective screws. If a random sample of 3 screws is selected at random. What is the probability that the number of defective screws in the sample is 2? **[2]**