National University of Computer & Emerging Sciences, Karachi
FAST School of Computing (BCY & BAI)
Final Examination, Spring-2023
Tuesday, June 06, 2023 08:30 am to 11:30 am

Course Code: MT- 1004	Course Name: Linear Algebra
Instructor Names :	Dr. Nazish Kanwal.
Student Roll No:	Section No: BAI-2A & BCY-2A

Instructions:

1. Answer all questions on answer script . Credit will be awarded for correct content and clarity of presentation.
2. There are 10 questions and 3 pages. Solve all parts of a question in sequence.

Time: 3 hours. Max Points : 100

Question 1: ....CLO 1................................ *13 points*

(a) $\boxed{\text{5 points}}$ Solve the linear system $AX = b$ by *Cramer's rule* whose coefficient matrix and matrix b are given below:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}.$$

(b) $\boxed{\text{2 points}}$ Find $\det(4A)$ for matrix given in part(a).

(c) For square matrix

$$C = \begin{bmatrix} 1 & 0 & -5 \\ -2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

i. $\boxed{\text{3 points}}$ Check whether C is orthogonal matrix? Justify your answer.

ii. $\boxed{\text{3 points}}$ If C is orthogonal matrix, find its inverse.

Question 2: ....CLO 1................................ *8 points*

(a) Let $\mathcal{V}$ be the set of all positive real numbers, the addition and scalar multiplication in $\mathcal{V}$ are defined as:

$$u \oplus v = uv \quad \text{and} \quad k \otimes v = u^k.$$

Evaluate the following:

i. $\boxed{\text{3 points}}$ $k \otimes (u \oplus v)$ for $k = 5, u = -3$ & $v = \frac{1}{2}$.

ii. $\boxed{\text{2 points}}$ $u \oplus (-u)$, for $u = 2$. (Hint: $-u = u^{-1} = \frac{1}{u}$)

(b) $\boxed{\text{3 points}}$ Use subspace test to determine whether the set W is a subspace of $M_{3\times 3}$.
W = {The set of all 3×3 matrices A such that $tr(A) = 0$}.
(Hint: $tr(A)$ is the sum of main diagonal entries of matrix A.)

Question 3: .... CLO 2 .... *6 points*

Let $T_A : \mathsf{R}^2 \longrightarrow \mathsf{R}^3$ be multiplication by A that is, $T_A(x) = Ax$, where

$$A = \begin{bmatrix} \text{a} & \text{b} \\ \text{c} & \text{d} \\ \text{g} & \text{h} \end{bmatrix}, \text{ where } a, b, c, d, g, h \text{ are real numbers.}$$

Let $e_1 = (1, 0)$ & $e_2 = (0, 1)$ be the standard basis vectors for R^2. Find the following:
(i) $T_A(2e_1)$ (ii) $T_A\,(3e_2 - 2e_1)$.

Question 4: .... CLO 2 .... *10 points*

Let

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 2 & 8 & 0 & 1 & 2 \\ 0 & 4 & -6 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $\boxed{\text{7 points}}$ Find the bases for the,
(i) row space of B (ii) column space of B (iii) null space of B

(b) $\boxed{\text{1 point}}$ Find the rank of B.

(c) $\boxed{\text{2 points}}$ Find the following:
(i) nullity of B (ii) nullity of B^T.

Question 5: .... CLO 2 .... *6 points*

For the given linear system :

$$\begin{aligned} 3x + 2y - z &= 2, \\ 6x + 4y - 2z &= 4 \\ -3x - 2y + z &= -2. \end{aligned}$$

(a) $\boxed{\text{4 points}}$ Find the vector form of the general solution of the given system ($AX = b$).

(b) $\boxed{\text{2 points}}$ Use the result of part(a) to find the vector form of the general solution of corresponding homogeneous system ($AX = 0$).

Question 6: .... CLO 2 .... *19 points*

Let

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

(a) $\boxed{\text{3 points}}$ Find the eigen values of A.

(b) $\boxed{\text{4 points}}$ Find the algebraic and geometric multiplicity of each eigen value of A.

(c) $\boxed{\text{2 points}}$ For each eigen value λ , find the rank of $\lambda I - A$.

(d) $\boxed{\text{2 points}}$ Is A is diagonalizable? justify your answer.

(e) $\boxed{\text{2 points}}$ If A is diagonalizable, find a matrix P that diagonalize A.

(f) $\boxed{\text{3 points}}$ Find $P^{-1}AP$.

(g) $\boxed{\text{3 points}}$ Compute A^{11}.

Question 7: ....CLO 2................................ *10 points*

Let $\langle u,\ v\rangle$ be the standard inner product on $\mathsf{M}_{2\times2}$.

$$U = \begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -5 & 1 \\ 6 & 2 \end{bmatrix}.$$

(a) $\boxed{\text{6 points}}$ Compute the following.
(i) $\langle 4u,\ v\rangle$ (ii) $\| \ 3u - 2v \ \|$

(b) $\boxed{\text{4 points}}$ Find the cosine of angle between the vector u and v.

Question 8: .... CLO 2 .... *8 points*

Let $\mathsf{S} = \{v_1, v_2, v_3\}$, where $v_1 = (0,\ -\frac{3}{5},\ \frac{4}{5}),\ v_2 = (1, 0, 0),\ v_3 = (0,\ \frac{4}{5},\ \frac{3}{5})$.

(a) $\boxed{\text{5 points}}$ Determine whether the set S is orthonormal.

(b) $\boxed{\text{3 points}}$ Find the coordinate vector $(u)_{\mathsf{S}}$ for $u = (1,\ \frac{14}{5},\ -\frac{2}{5})$.

Question 9: ....CLO 2................................ *14 points*

(a) $\boxed{\text{8 points}}$ Let R^4 have a Euclidean inner product, and $u_1 = (1, 1, 1, 1),\ u_2 = (0, 1, 1, 1,),$ $u_3 = (0, 0, 1, 1)$ be the basis for a subspace of R^4. Use *Gram-Schmidth* process to transform the basis $\{u_1,\ u_2,\ u_3\}$ into orthonormal basis.

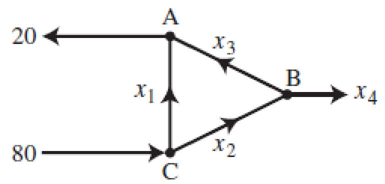
(b) $\boxed{\text{6 points}}$ Let u_1, u_2, u_3 (are basis vectors for subspace of R^4) given in part(a), and q_1, q_2, q_3 are the orthonorml basis vectors obtained in part(a). Let

$$A = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}, \quad \text{and} \quad Q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$$

Find QR-composition of A.

Question 10: ....CLO 3................................ *6 points*

For the network shown in the figure. Assuming that the flows are all non-negative.



i. $\boxed{\text{4 points}}$ Find the general solution of the network flow.

ii. $\boxed{\text{2 points}}$ what is the largest possible value for x_3.