

Line Selection CCW Method



Which two lines should be selected among the three below?

$L1 = \{(2,2), (2,5)\}$, $L2 = \{(1,1), (4,5)\}$, $L3 = \{(3,2), (3,4)\}$ using ccw orientation cross product method



To determine which two lines should be selected using the **Counter-Clockwise (CCW) orientation cross product method**, the basic approach is to calculate the cross products for the vectors formed by the endpoints of the lines. Based on the orientation, you can determine if the vectors (and hence the lines) form a counterclockwise or clockwise turn.

Let's compute this for the three given lines $L1$, $L2$, and $L3$.

1. Convert the lines into vectors:

- For $L1$: Vector from $(2, 2)$ to $(2, 5)$ is $(0, 3)$.
- For $L2$: Vector from $(1, 1)$ to $(4, 5)$ is $(3, 4)$.
- For $L3$: Vector from $(3, 2)$ to $(3, 4)$ is $(0, 2)$.

2. Calculate cross products:

The cross product of two 2D vectors (x_1, y_1) and (x_2, y_2) is calculated as:

$$\text{Cross product} = x_1 \cdot y_2 - y_1 \cdot x_2$$

Now, we compute the cross products between the vectors formed by the endpoints of the lines:

• Cross product of $L1$ and $L2$:

$$(0, 3) \times (3, 4) = (0 \cdot 4) - (3 \cdot 3) = -9$$

The negative result indicates a clockwise turn.

• Cross product of $L1$ and $L3$:

$$(0, 3) \times (0, 2) = (0 \cdot 2) - (3 \cdot 0) = 0$$

The result is zero, indicating the vectors are collinear.

• Cross product of $L2$ and $L3$:

$$(3, 4) \times (0, 2) = (3 \cdot 2) - (4 \cdot 0) = 6$$

The positive result indicates a counterclockwise turn.

3. Conclusion:

- $L1$ and $L2$ form a clockwise turn.
- $L1$ and $L3$ are collinear.
- $L2$ and $L3$ form a counterclockwise turn.

Since we are looking for lines that form a counterclockwise orientation, the two lines to select are $L2$ and $L3$.



Compute intersection b/w i) $L1, L2$ ii) $L1, L3$ iii) $L2, L3$ and any which return 0 will be selected
 e.g. `intersect(L1, L2)`
`test1 = ccw((2,2), (2,5), (1,1)) * ccw((2,2), (2,5), (4,5)) = -3`, so true and intersection
 Line1 and Line3 will return False How?



To compute the intersection between two lines using the **Counter-Clockwise (CCW) orientation method**, we use the following steps for each pair of lines:

1. **Check for intersection between two line segments.**
2. Apply the CCW test to determine if the two segments intersect.

The CCW test for three points $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is determined by the sign of the expression:

$$CCW(A, B, C) = (y_3 - y_1) \cdot (x_2 - x_1) - (x_3 - x_1) \cdot (y_2 - y_1)$$

If the result is:

- Positive: Points make a counterclockwise turn.
- Negative: Points make a clockwise turn.
- Zero: Points are collinear.

We will compute this for the three pairs of lines $(L1, L2)$, $(L1, L3)$, $(L2, L3)$, and check if any pair does **not** intersect (returns ``False``).

i) Intersection between $L1$ and $L2$

- $L1 = \{(2, 2), (2, 5)\}$
- $L2 = \{(1, 1), (4, 5)\}$

Test 1:

- Compute $CCW((2, 2), (2, 5), (1, 1))$:

$$(1 - 2) \cdot (2 - 2) - (1 - 2) \cdot (5 - 2) = 0 - (-3) = 3$$

This is positive, so it's counterclockwise.

- Compute $CCW((2, 2), (2, 5), (4, 5))$:

$$(5 - 2) \cdot (2 - 2) - (4 - 2) \cdot (5 - 2) = 0 - (2 \cdot 3) = -6$$

This is negative, so it's clockwise.

Test 2:

- Compute $CCW((1, 1), (4, 5), (2, 2))$:

$$(2 - 1) \cdot (4 - 1) - (2 - 1) \cdot (5 - 1) = (1 \cdot 3) - (1 \cdot 4) = -1$$

This is negative, so it's clockwise.

- Compute $CCW((1, 1), (4, 5), (2, 5))$:

$$(5 - 1) \cdot (4 - 1) - (2 - 1) \cdot (5 - 1) = (4 \cdot 3) - (1 \cdot 4) = 8$$

This is positive, so it's counterclockwise.

Since the results of both tests show different signs (i.e., one is positive and one is negative), the lines **intersect**.

ii) Intersection between $L1$ and $L3$

- $L1 = \{(2, 2), (2, 5)\}$
- $L3 = \{(3, 2), (3, 4)\}$

Test 1:

- Compute $CCW((2, 2), (2, 5), (3, 2))$:

$$(2 - 2) \cdot (2 - 2) - (3 - 2) \cdot (5 - 2) = 0 - (1 \cdot 3) = -3$$

This is negative, so it's clockwise.

- Compute $CCW((2, 2), (2, 5), (3, 4))$:

$$(4 - 2) \cdot (2 - 2) - (3 - 2) \cdot (5 - 2) = 0 - (1 \cdot 3) = -3$$

This is negative, so it's also clockwise.

Since both tests return the **same sign**, the lines **do not intersect** (they are parallel).

iii) Intersection between $L2$ and $L3$

- $L2 = \{(1, 1), (4, 5)\}$
- $L3 = \{(3, 2), (3, 4)\}$

Test 1:

- Compute $CCW((1, 1), (4, 5), (3, 2))$:

$$(2 - 1) \cdot (4 - 1) - (3 - 1) \cdot (5 - 1) = (1 \cdot 3) - (2 \cdot 4) = -5$$

This is negative, so it's clockwise.

- Compute $CCW((1, 1), (4, 5), (3, 4))$:

$$(4 - 1) \cdot (4 - 1) - (3 - 1) \cdot (5 - 1) = (3 \cdot 3) - (2 \cdot 4) = 9 - 8 = 1$$

This is positive, so it's counterclockwise.

Test 2:

- Compute $CCW((3, 2), (3, 4), (1, 1))$:

$$(1 - 2) \cdot (3 - 3) - (1 - 3) \cdot (4 - 2) = 0 - (-2 \cdot 2) = 4$$

This is positive, so it's counterclockwise.

- Compute $CCW((3, 2), (3, 4), (4, 5))$:

$$(5 - 2) \cdot (3 - 3) - (4 - 3) \cdot (4 - 2) = 0 - (1 \cdot 2) = -2$$

This is negative, so it's clockwise.

Since the results of both tests show different signs (i.e., one is positive and one is negative), the lines **intersect**.

Final Selection:

- $L1$ and $L2$ intersect.
- $L1$ and $L3$ do not intersect (test returned false).
- $L2$ and $L3$ intersect.

Thus, based on the condition, **Line 1 and Line 3** will be selected as they do not intersect.

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