Extended Master Method

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$$T(n) = a\,T\left(\frac{n}{b}\right) + \theta\left(n^k \log^p n\right)$$

Master's Theorem

Here, a >= 1, b > 1, k >= 0 and p is a real number.

- To solve recurrence relations using Master's theorem, we compare a with b^k.
- Then, we follow the following cases-

Extended Master Method (cont)

Case-01:

- If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

Case-02:

- If $a = b^k$ and
  - If $p < -1$, then $T(n) = \theta(n^{\log_b a})$
  - If $p = -1$, then $T(n) = \theta(n^{\log_b a}.\log^2 n)$
  - If $p > -1$, then $T(n) = \theta(n^{\log_b a}.\log^{p+1} n)$

Case-03:

If $a < b^k$ and
  - If $p < 0$, then $T(n) = O(n^k)$
  - If $p >= 0$, then $T(n) = \theta(n^k \log^p n)$

Example-1

Ex: Solve the following recurrence relation using Master's theorem- **$T(n) = 2T(n/2) + n\log n$**

- **Solution:** We compare the given recurrence relation with $T(n) = aT(n/b) + \theta\,(n^k \log^p n)$.
- Then, we have- $a = 2$, $b = 2$, $k = 1$, $p = 1$
- Now, $a = 2$ and $b^k = 2^1 = 2$.
- Clearly, $a = b^k$.
- So From Case 2, Since $p = 1$, so we have-
- $T(n) = \theta\,(n^{\log_b a} . \log^{p+1} n)$
- $T(n) = \theta\,(n^{\log_2 2} . \log^{1+1} n)$

Example-1

- Thus $\mathbf{T(n) = \theta\ (n\log^2 n)}$

Example-2

Ex: Solve the following recurrence relation using Master's theorem- **$T(n) = 2T(n/4) + n^{0.51}$**

- **Solution:** We compare the given recurrence relation with $T(n) = aT(n/b) + \theta(n^k \log^p n)$.
- Then, we have- $a = 2$, $b = 4$, $k = 0.51$, $p = 0$
- Now, $a = 2$ and $b^k = 4^{0.51} = 2.0279$.
- Clearly, $a < b^k$.
- So From Case 3, Since $p = 0$, so we have-
- $T(n) = \theta(n^k \log^p n)$
- $T(n) = \theta(n^{0.51} \log^0 n)$

Example-2

- Thus **$T(n) = \theta\ (n^{0.51})$**

Example-3

Ex: Solve the following recurrence relation using Master's theorem- **$T(n) = \sqrt{2}T(n/2) + \log n$**

- **Solution:** We compare the given recurrence relation with $T(n) = aT(n/b) + \theta(n^k \log^p n)$.
- Then, we have- $a = \sqrt{2}$, $b = 2$, $k = 0$, $p = 1$
- Now, $a = \sqrt{2} = 1.414$ and $b^k = 2^0 = 1$.
- Clearly, $a > b^k$.
- So From Case 1,
- $T(n) = \theta(n^{\log_b a}) = \theta(n^{\log_2 \sqrt{2}})$
- $T(n) = \theta(n^{1/2})$

Example-3

- Thus $\mathbf{T(n) = \theta(\sqrt{n})}$

Example-4

Ex: $T(n) = 0.5T(n/2) + 1/n$

Example-4

- **Solution:**

$T(n) = 0.5T(n/2) + 1/n \Rightarrow$ Does not apply $(a < 1)$

Example-5

Ex: $T(n) = 2^n T(n/2) + n^n$

Example-5

- **Solution:**

$T(n) = 2^n T(n/2) + n^n \Rightarrow$ Does not apply (a is not constant)

Example-6

$T(n) = 64T(n/8) - n^2 \log n$

Example-6

- **Solution:**

$T(n) = 64T(n/8) - n^2 \log n \Rightarrow$ Does not apply ($f(n)$ is not positive)

Example-7

$$T(n) = 3T(n/3) + \sqrt{n}$$

Example-7

- Solution:

$T(n) = 3T(n/3) + \sqrt{n} \Rightarrow T(n) = \Theta(n)$ (Case 1)

Example-8

$T(n) = \sqrt{2}\, T(n/2) + \log n$

Example-8

- Solution:

$T(n) = \sqrt{2}T(n/2) + \log n \Rightarrow T(n) = \Theta(\sqrt{n})$ (Case 1)

Example-9

$T(n) = 4T(n/2) + n/\log n$

Example-9

- Solution:

$T(n) = 4T(n/2) + n/\log n \Rightarrow T(n) = \Theta(n^2)$ (Case 1)

Example-10

$T(n) = 3T(n/3) + n/2$

Example-10

- Solution:

$T(n) = 3T(n/3) + n/2 \Rightarrow T(n) = \Theta(n \log n)$ (Case 2)

Thank You