Summary of Time and Space Complexity for NP-Hard Problems

Problem	Worst-case Time Complexity	Best-case Time Complexity	Worst-case Space Complexity	Best-case Space Complexity
Traveling Salesman	O(n!) or O(2^n * n)	Can be polynomial for small inputs or specific heuristics	O(n * 2^n) (dynamic programming)	O(n) (for greedy heuristics)
Knapsack Problem	O(n * W) (dynamic programming)	O(n) for greedy heuristic	O(n * W) (dynamic programming)	O(n) (for greedy approach)
Vertex Cover	O(2^n) or O(n!)	Polynomial for small graphs	O(n) for some special cases (like bounded treewidth)	O(n) for bounded treewidth graphs
Subset Sum Problem	O(2^n)	O(n) for small problems	O(n * W) (if using dynamic programming)	O(n) for specific instances
Graph Coloring	O(2^n) or O(n!)	Polynomial for certain graph classes	O(n^2) (if using DP for small graphs)	O(n) for specific graph structures
Independent Set	O(2^n)	Polynomial for certain graph types	O(2^n) or O(n)	O(n) for specific instances
Set Cover	O(2^m) or O(m * 2^m)	Polynomial for small k or special cases	O(2^m) or O(m * n)	O(m + n)
Clique Decision	O(2^n)	Polynomial for sparse graphs	O(2^n) or O(n * 2^n)	O(n) or better for sparse graphs
3-Satisfiability (3-SAT)	O(2^n)	Polynomial for specific instances	O(2^n)	O(n)

National University of Computer and Emerging Sciences
Karachi Campus

Design and Analysis of Algorithms (CS2009)

Date: Dec 23rd 2024

Course Instructor(s)

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Final Exam Paper Part-B

Total Time (Hrs):	**3**
Total Marks:	**50**
Total Questions:	**8**

Do not write below this line

Attempt all the questions.

CLO #1: To apply acquired knowledge to solve computing problems complexities and proofs

Question 3 [5 Marks]: A programmer is tasked with optimizing a computationally heavy data processing workflow. They are using System Z, which operates at a speed of 4 billion instructions per second (4 GIPS). The programmer has three software solutions to process a dataset of 1,000,000 records. Each solution has a different running time complexity as follows: The time complexity of Solution A is (n^2), the time complexity of Solution B is $(n log_2 n)$ and the time complexity of Solution C is $(20\, n(log_2 n)^2)$.

a) Calculate the number of instructions required to process the dataset for each solution. Show all your calculations. **[2.25 Marks]**
b) Compute the execution time (in seconds) for each solution. **[1.5 Marks]**
c) Compare the performance of the three solutions and explain the impact of running time complexity on execution time. **[1.25 Marks]**

CLO #1: To construct and analyze real world problems solutions using different algorithms design techniques. i.e. Brute Force, Divide and Conquer, Dynamic Programming, Greedy Algorithms.

Question 4 [5 Marks]: A shopping mall has introduced a reward system where customers can redeem reward coins of denominations 1, 5, 6, and 8 to receive discounts. A customer has collected 20 coins in total and wants to redeem them in such a way that the total value equals 11, using the minimum number of coins.

a) Design an $O(n^2)$ dynamic programming based algorithm and write down its recurrence relation **[1 Mark]**
b) Use the dynamic programming approach to determine the minimum number of reward coins required to achieve a total value of 11. Provide the complete DP table for all amounts from 0 to 11. Identify the denominations of coins used in the optimal solution. **[3 Marks]**
c) What if you have to select maximum number of coins to make a total. **[1 Mark]**

CLO #3: To evaluate generic algorithmic solutions such as sorting, searching and graphs applied to real-world problems

Question 5 [5 Marks]: A logistics company uses drones to deliver parcels between four warehouses: (A, B, C, D). The energy cost for flying between these warehouses is shown in the graph in Figure 1. Some routes have negative energy costs due to favorable conditions like wind assistance or advanced energy saving technologies.

For instance, the drones are equipped with solar panels, and while they usually consume battery power during flights, certain routes benefit from optimal sunlight angles. On these routes, the solar panels charge the batteries instead of depleting them, resulting in a net energy gain. These scenarios are represented by negative energy costs in the graph. The company seeks answers to the following questions:

a) What is the minimum energy cost for transporting goods between all pairs of warehouses? **[2.5 Marks]**
b) What are the most energy-efficient delivery routes between each pair of warehouses? **[2.5 Marks]**

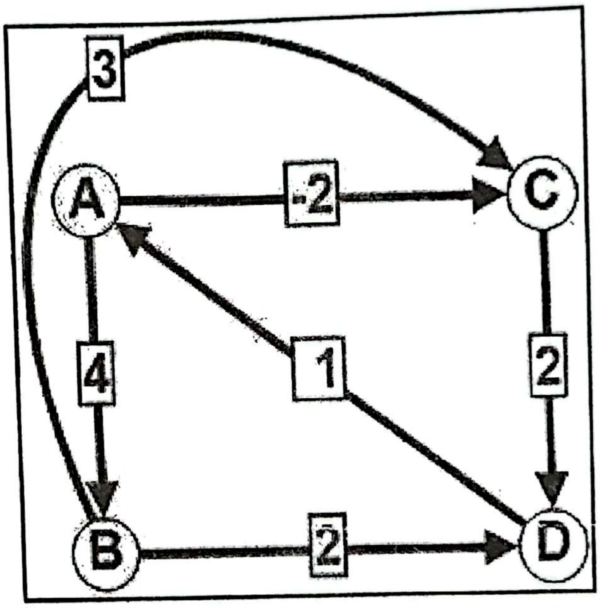

Figure 1 The directed graph showing the connections between the warehouses

CLO #4: To construct and analyze real world problems solutions using different algorithms design techniques i.e. Brute Force, Divide and Conquer, Dynamic Programming, Greedy Algorithms.

Question 6 [5 Marks]: Demand for multimedia, combining audio, video, and data streams over a network, is rapidly increasing. Some of the most popular uses of multimedia are real-time interactive applications such as desktop video and audio conferencing, collaborative engineering, shared white boards, transmission of university lectures to a remote audience, and animated simulations. With the advent of real-time interactive applications, **delay constraint (Δ)** is also an important objective along with minimizing bandwidth cost. Delay constraint means that a packet must be reached within that interval. Figure below shows an example graph. Each edge consists of 2 values (**first one is bandwidth cost** and **second one is delay**. Figure 1 shows the multicast graph where "a" is the source and **{b, c, d, f}** are the destinations. **{e, h, g}** are the intermediate nodes i.e. they can be used in multicast tree only if help in optimizing bandwidth cost under some delay constraint. Multicast communication means instead of sending packets to all destinations, packets will be send to only selected destinations. In the example below, main objective is to minimize the bandwidth cost under delay constraint. This problem is NP-Complete.

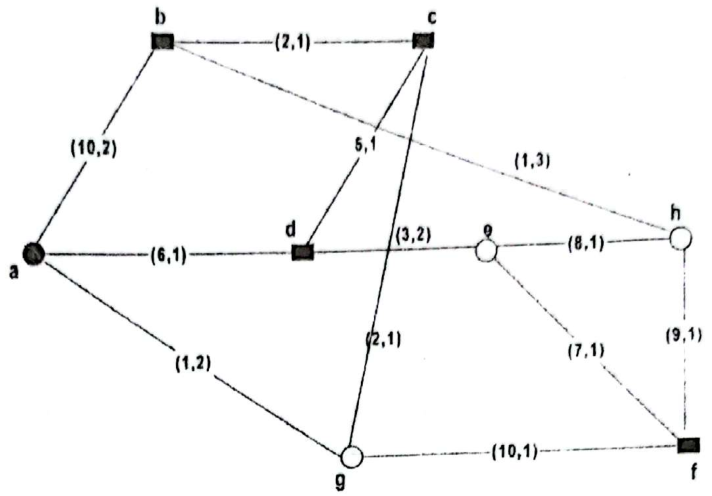

Figure 1: Multicast Graph

Algorithm proposed by V. P. Kompella, J. C. Pasquale and G. C. Polyzos, "Multicast Routing for Multimedia Communication," IEEE/ ACM Transaction on Networking is one of the most popular greedy algorithm to find the optimal bandwidth from source to selected destinations under some constraint. Algorithm 1 is the pseudocode of the algorithm and let's assume input graph of **Figure 1** with delay constraint $\Delta = 5$. You are required to complete the following tasks

(a) What is the size of V′ for input graph in Figure 1 and list the elements in V′? Remember, V is initialized in Algorithm 1. **[0.5 Mark]**

(b) Compute all pairs constrained least cost and delay paths i.e. $P_c\{v,w\}$ and $P_D\{v,w\}$. You can use compute shortest path from source "a" to other destinations using Dijkstra Algorithm using bandwidth cost. Rest of the values are pre-computed and given below. **[1.5 Marks]**

$P_c\{a,b\} = 5$	$P_D\{a,b\} = 4$	$\Pi(a,b) = \{a,g,c,b\}$
$P_c\{a,c\} = ?$ 3	$P_D\{a,c\} = ?$ 4	$\Pi(a,c) = ?$ a,g,c
$P_c\{a,d\} = ?$ 6	$P_D\{a,d\} = ?$ 1	$\Pi(a,d) = ?$ a,d
$P_c\{a,f\} = ?$ 11	$P_D\{a,f\} = ?$ 3	$\Pi(a,f) = ?$ a,g,f
$P_c(b,c) = 2$	$P_D(b,c) = 1$	$\Pi(b,c) = \{b,c\}$
$P_c(b,d) = 7$	$P_D(b,d) = 2$	$\Pi(b,d) = \{b,c,d\}$
$P_c(b,f) = 10$	$P_D(b,f) = 4$	$\Pi(b,f) = \{b,h,f\}$
$P_c(c,d) = 5$	$P_D(c,d) = 1$	$\Pi(c,d) = \{c,d\}$
$P_c(c,f) = 12$	$P_D(c,f) = 5$	$\Pi(c,f) = \{c,b,h,f\}$
$P_c(d,f) = 10$	$P_D(d,f) = 3$	$\Pi(d,f) = \{d,e,f\}$

CLO #1: To apply acquired knowledge to solve computing problems complexities and proofs

Question 9 [5 Marks]: Briefly answer the following questions

a. Why it is important to find approximate solutions for NP Complete Problems. **[1 Mark]**
b. Write down any one real life application of Maximum sub-array. **[1 Mark]**
c. Suppose you want to lay pipelines in a building such that minimum pipe and other materials are consumed covering every desired location. Which technique from algorithms course will you employ and why? **[1 Mark]**
d. Differentiate between decision and optimization based problems? **[1 Mark]**
e. Does there any chance exist of solving an NP-Hard problem by deterministic polynomial time algorithm? **[1 Mark]**

CLO #4: To construct and analyze real world problems solutions using different algorithms design techniques i.e. Brute Force, Divide and Conquer, Dynamic Programming, Greedy Algorithms.

Question 10 [5 Marks]: Write pseudocode for a divide-and-conquer algorithm **MATRIX-ADD-RECURSIVE** that sums two n x n matrices A and B by partitioning each of them into **four** $n/2 \ x \ n/2$ submatrices and then recursively summing corresponding pairs of sub-matrices. Assume that matrix partitioning uses $\theta(1)$-time index calculations.

a. Write a recurrence for the worst-case running time of MATRIX-ADD-RECURSIVE, and solve your recurrence. **[2.5 Marks]**
b. What happens if you use $\theta(n^2)$-time copying to implement the partitioning instead of index calculations? **[2.5 Marks]**