National University of Computer & Emerging Sciences, Karachi
Computer Science Department

ASSIGNEMENT-II
Probability and Statistics (MT2005)

In order to receive full credit, you must show your all necessary work.
Write all answers correct up to 3 decimal places.

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Problem 1

From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Find the probability distribution for the number of green balls.

Problem 2

An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify that this is a valid density function.
(b) Evaluate $F(x)$.
(c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

Problem 3

The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution find the

x	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	0.3	0.04

mean and variance of the discrete random variable $Z = 3X - 2$, when X represents the number of errors per 100 lines of code.

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Problem 4

Three cards are drawn without replacement from the 12 face cards (jacks, queens, and kings) of an ordinary deck of 52 playing cards. Let X be the number of kings selected and Y the number of jacks. Find

(a) the joint probability distribution of X and Y;

(b) $P[(X, Y) \in A]$, where A is the region given by $\{(x, y) \mid x + y \geq 2\}$.

Problem 5

A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let X = the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and Y = the proportion of time that the walk-up window is in use. Then the set of possible values for (X, Y) is the rectangle $D = \{(x, y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Suppose the joint pdf of (X, Y) is given by

$$f(x, y) = \begin{cases} \dfrac{6}{5}(x + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find covariance and correlation between variable x and y .

Problem 6

Find the proportion X of individuals who can be expected to respond to a certain mail-order solicitation if X has the density function

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

find $\sigma^2_{g(X)}$ for the function $g(X) = 3X^2 + 4.$

Problem 7

Given the joint density function

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, \ 2 < y < 4, \\ 0, & \text{elsewhere,} \end{cases}$$

find $P(1 < Y < 3 \mid X = 1)$.

Problem 8

A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

p(x, y)		0	1	2
	0	.10	.04	.02
x	1	.08	.20	.06
	2	.06	.14	.30

Find the Mean and variance of x and y. Are x and y independent variable?

Problem 9

Toss two fair dice. Let X denote their sum and Y the absolute value of their difference.

a) Construct Joint Probability distribution table
b) Write marginal distribution of x and y
c) Calculate mean and variance of x and y
d) Find covariance and correlation between x and y
e) Calculate

i. $P(x \le 4, y \le 2)$
ii. $P\{(x \le 5), (y \le 1)\}$
iii. $P(x < y)$
iv. $P(x + y \le 4)$
v. $P(x/y = 3)$

Problem 10

i) If a baseball player's batting average is 0.320 (32%), find the probability that the player will get at most 26 hits in 100 times at bat.

ii) A magazine reported that 6% of American drivers read the newspaper while driving. If 300 drivers are selected at random, find the probability that exactly 25 say they read the newspaper while driving.

iii) Americans consume an average of 1.64 cups of coffee per day. Assume the variable is approximately normally distributed with a standard deviation of 0.24 cup. If 500 individuals are selected, approximately how many will drink less than 1 cup of coffee per day?

iv) A sales firm receives, on average, 3 calls per hour on its toll-free number. For any given hour, find the probability that it will receive the following.

    a. At most 3 calls   *b.* At least 3 calls   *c.* 5 or more calls

v) A box contains 4 white balls, 3 red balls, and 3 blue balls. A ball is selected at random, and its color is written down. It is replaced each time. Find the probability that if 5 balls are selected, 2 are white, 2 are red, and 1 is blue.

vi) A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 15 single-vehicle traffic fatalities that occur at night on a weekend is selected, find the probability that exactly 12 involve a driver who is intoxicated.

Due Date: 9th November 2023