

Finding Most Left Turn

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How to find the most left turn point among the points to a reference point



To find the "most left turn" point among a set of points relative to a reference point, you can use the concept of **polar angles** or the **cross product**. Here's a step-by-step approach:

1. **Select a Reference Point:** Choose your reference point, let's call it P_{ref} .
2. **Compute Vectors:** For each point P_i in the list, compute the vector $\overrightarrow{P_{\text{ref}}P_i}$.
3. **Calculate Angles or Cross Product:**
 - **Angle Method:** Calculate the polar angle (angle with respect to the horizontal axis) for each vector $\overrightarrow{P_{\text{ref}}P_i}$. The point with the smallest angle in a counterclockwise direction from the reference point will be the "most left turn."
 - **Cross Product Method:** If you want to avoid calculating angles, you can use the cross product to determine the relative position:
$$\text{cross product} = (P_i.x - P_{\text{ref}}.x) \cdot (P_j.y - P_{\text{ref}}.y) - (P_j.x - P_{\text{ref}}.x) \cdot (P_i.y - P_{\text{ref}}.y)$$

If the cross product is positive, P_i is to the left of P_j with respect to P_{ref} . Iterate over the points to find the one with the maximum positive cross product.
4. **Choose the Point with the Maximum Left Turn:** After calculating the angles or cross products, the point that represents the largest counterclockwise turn from the horizontal (or has the largest positive cross product) is the "most left turn" point.



Provide a numerical example of that



Let's work through a numerical example using the **cross product method** to find the "most left turn" point.

Given Data

Suppose we have the following points and a reference point:

- Reference point $P_{\text{ref}} = (0, 0)$
- Other points:
 - $P_1 = (1, 2)$
 - $P_2 = (2, 1)$
 - $P_3 = (-1, 1)$
 - $P_4 = (-2, -1)$

We want to find the point that makes the most counterclockwise (left) turn from P_{ref} when compared to other points.

Step-by-Step Solution

1. **Choose a baseline direction:** Let's compare each point to the others relative to the reference point P_{ref} .
2. **Compute the Cross Product:** We'll calculate the cross product for each pair of points relative to the reference point. For simplicity, we can compare each point to the others using the cross product.

For points $P_i = (x_i, y_i)$ and $P_j = (x_j, y_j)$, the cross product $\overrightarrow{P_{\text{ref}}P_i} \times \overrightarrow{P_{\text{ref}}P_j}$ is given by:

$$(x_i - 0) \cdot (y_j - 0) - (x_j - 0) \cdot (y_i - 0) = x_i \cdot y_j - x_j \cdot y_i$$

3. **Calculate Pairwise Cross Products:**

- P_1 vs. P_2 :

$$(1 \cdot 1) - (2 \cdot 2) = 1 - 4 = -3$$

- P_1 vs. P_3 :

$$(1 \cdot 1) - (-1 \cdot 2) = 1 + 2 = 3$$

- P_1 vs. P_4 :

$$(1 \cdot -1) - (-2 \cdot 2) = -1 + 4 = 3$$

- P_2 vs. P_3 :

$$(2 \cdot 1) - (-1 \cdot 1) = 2 + 1 = 3$$

- P_2 vs. P_4 :

$$(2 \cdot -1) - (-2 \cdot 1) = -2 + 2 = 0$$

- P_3 vs. P_4 :

$$(-1 \cdot -1) - (-2 \cdot 1) = 1 + 2 = 3$$

4. **Determine the Most Left Turn:** The pairs that have the highest cross product (3) indicate the points that are "most left" relative to each other. We see that P_3 and P_4 consistently show up with a positive cross product relative to other points. Therefore, either P_3 or P_4 could represent the most left turn from the reference point depending on the direction.



Which one we consider



To determine which single point represents the "most left turn" from the reference point P_{ref} , you should find the point that, relative to a fixed baseline (like the horizontal axis), has the maximum counterclockwise angle.

Since we are using the cross product, one way to consistently find the point with the maximum left turn is by comparing each point to the positive x-axis direction vector, i.e., the direction from $P_{\text{ref}} = (0, 0)$ to $(1, 0)$.

Solution Steps with Fixed Baseline

1. **Choose Baseline Vector:** Set the vector along the positive x-axis, $\overrightarrow{P_{\text{ref}}B} = (1, 0)$, as the baseline.

2. **Calculate Cross Product with Each Point:**

The cross product between each point P_i and the baseline vector $\overrightarrow{P_{\text{ref}}B}$ is calculated as:

$$\text{Cross Product} = (P_i.x - 0) \cdot (0) - (1) \cdot (P_i.y - 0) = -P_i.y$$

Calculating this for each point:

- For $P_1 = (1, 2)$: Cross product = -2
- For $P_2 = (2, 1)$: Cross product = -1
- For $P_3 = (-1, 1)$: Cross product = -1
- For $P_4 = (-2, -1)$: Cross product = 1

3. **Determine the Most Left Turn Point:**

The point with the **highest positive cross product** value relative to the baseline vector $\overrightarrow{P_{\text{ref}}B}$ is $P_4 = (-2, -1)$. This point represents the "most left turn" from the reference point.

So, $P_4 = (-2, -1)$ is the point that makes the most left turn relative to P_{ref} .



What if we have same values for 2 points



If two points have the same cross product value relative to the reference point and baseline vector, this means they are aligned in the same direction relative to the reference point. In such cases, you can choose the "most left turn" point by applying a secondary criterion, typically **distance from the reference point**.

Here's how to handle it:

1. **Primary Criterion (Cross Product):** Identify points with the maximum positive cross product relative to the baseline.
2. **Secondary Criterion (Distance):** Among points with the same cross product value, choose the point that is **farthest** from the reference point. This is because the farthest point will create a larger turn angle visually and geometrically.

Formula for Distance

For any point $P_i = (x_i, y_i)$, the distance from the reference point $P_{\text{ref}} = (0, 0)$ is:

$$\text{Distance} = \sqrt{x_i^2 + y_i^2}$$

Example with Two Points Having Same Cross Product Value

Suppose we have points P_1 and P_2 both with a cross product value of 3.

- **Point 1:** $P_1 = (1, 3)$

$$\text{Distance} = \sqrt{1^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10} \approx 3.16$$

- **Point 2:** $P_2 = (2, 3)$

$$\text{Distance} = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.61$$

Since P_2 has the same cross product but is farther from the reference point, P_2 **would be chosen as the "most left turn" point**.

This approach ensures that if multiple points lie in the same direction, the point farthest from the reference is selected as the "most left turn" point.



What if the secondary criteria is polar angle



If the secondary criterion is the **polar angle**, then among points with the same cross product value, you would choose the point with the **smallest polar angle relative to the horizontal axis** (positive x-axis). This ensures that the chosen point is the most counterclockwise (leftward) relative to the reference point.

Steps Using Polar Angle as the Secondary Criterion

1. **Primary Criterion (Cross Product):** First, identify points with the maximum positive cross product relative to the baseline (or reference direction).
2. **Secondary Criterion (Polar Angle):** For points with the same cross product value, calculate the polar angle of each point relative to the reference point P_{ref} .

The polar angle θ of a point $P_i = (x_i, y_i)$ with respect to $P_{\text{ref}} = (0, 0)$ is given by:

$$\theta = \arctan \left(\frac{y_i}{x_i} \right)$$

Note: Adjust the angle based on the quadrant if x_i or y_i is negative to get a correct polar angle.

3. **Choose the Point with the Smallest Polar Angle:** Among points with the same cross product, select the point with the smallest θ (polar angle), which will be the most counterclockwise point relative to the reference.

Example with Two Points Having the Same Cross Product Value

Let's say two points, P_1 and P_2 , both have the same maximum cross product value.

- **Point 1:** $P_1 = (1, 3)$

$$\theta_1 = \arctan \left(\frac{3}{1} \right) \approx 71.6^\circ$$

- **Point 2:** $P_2 = (2, 3)$

$$\theta_2 = \arctan \left(\frac{3}{2} \right) \approx 56.3^\circ$$

Since $\theta_2 < \theta_1$, P_2 **would be chosen as the "most left turn" point** because it has the smaller polar angle, making it more counterclockwise relative to the reference point.

This approach, with the polar angle as the secondary criterion, ensures that if multiple points lie in similar directions, the point with the smallest polar angle (most counterclockwise) is selected as the "most left turn" point.