Determination of the Center of Rotation in CT

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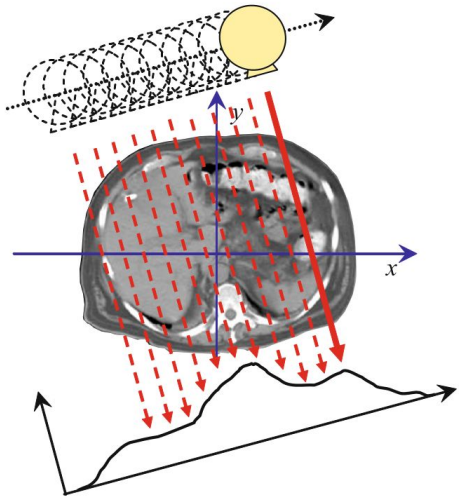
Image Acquisition in CT

Parallel Beam Geometry

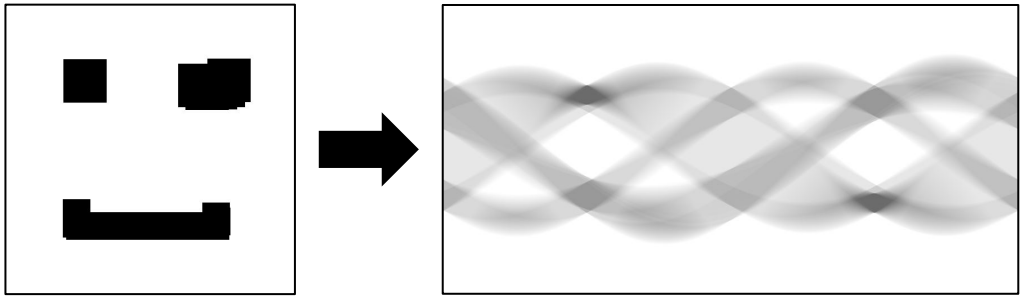


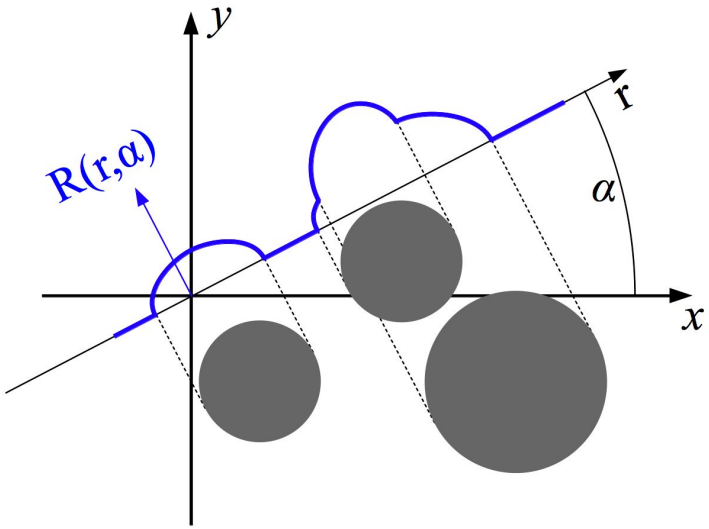
[0]

Radon Transform

$$Rf(r,\alpha) = \int_{-\infty}^{\infty} f(r\cos\alpha + t\sin\alpha, r\sin\alpha - t\cos\alpha)\,\mathrm{d}t$$

Sinogram



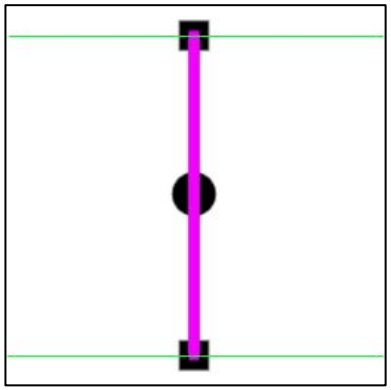


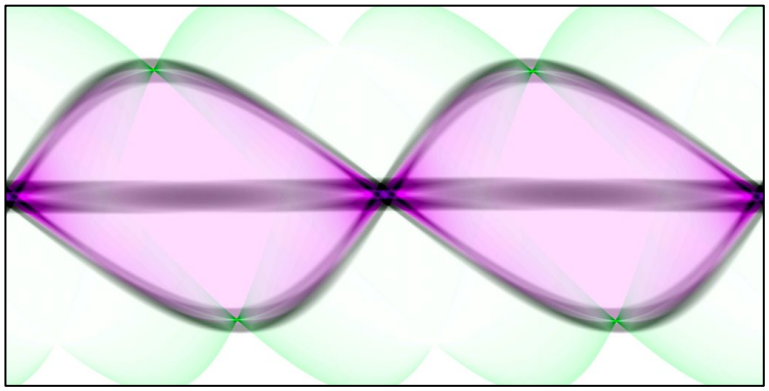
[1]

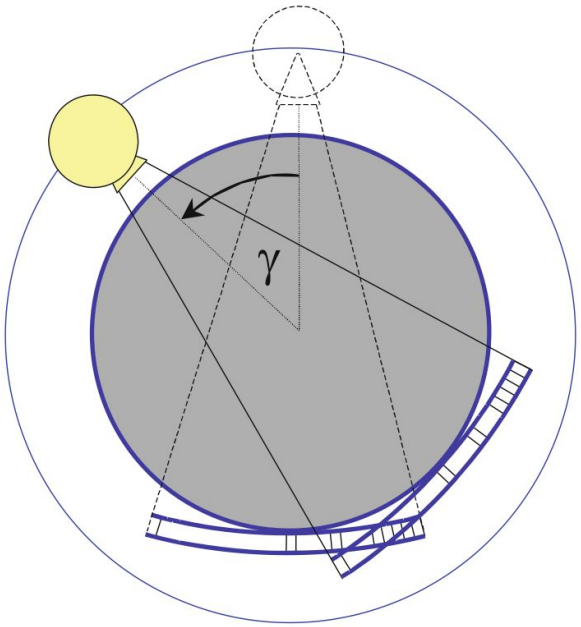
Image Acquisition in CT 2

Fan Beam Geometry

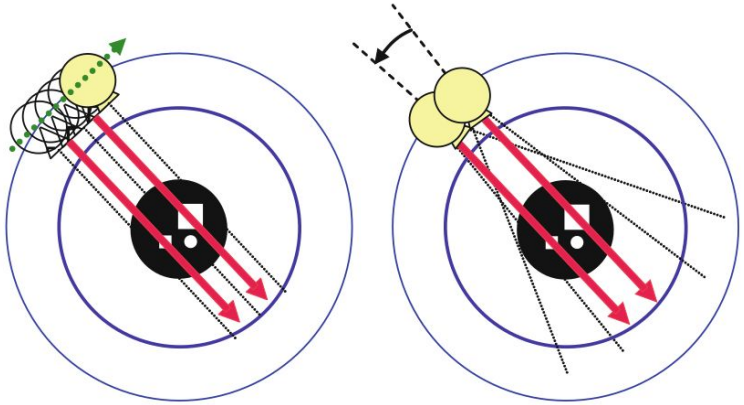
Rebinning to Parallel Beam







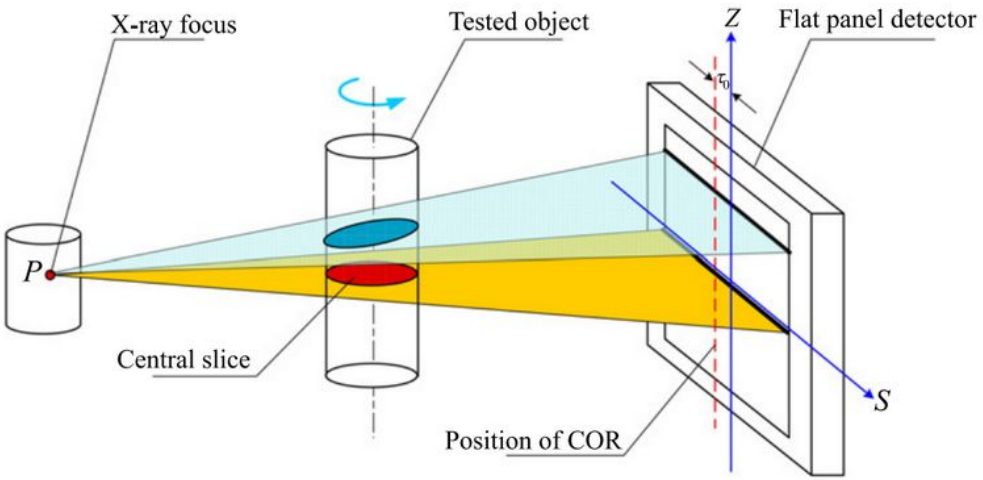
[2]



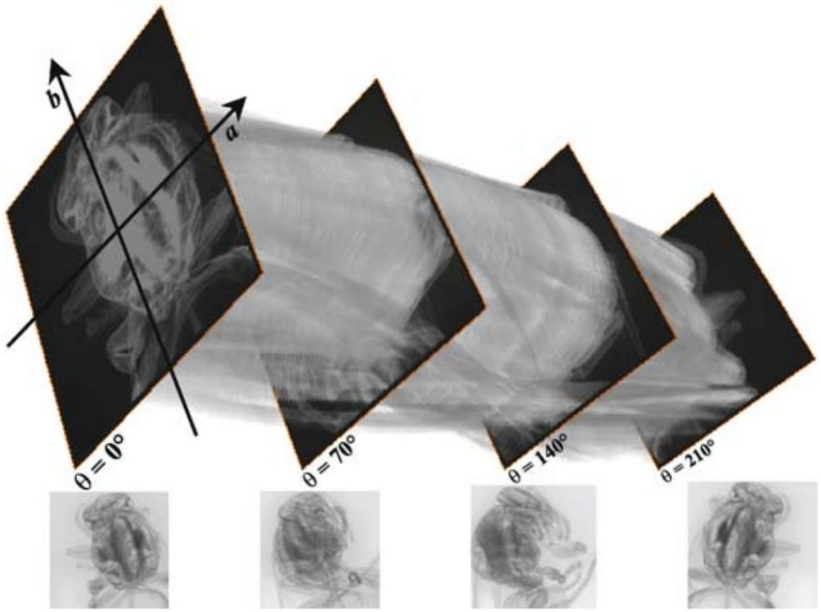
[3]

Image Acquisition CT 3

Cone Beam Geometry



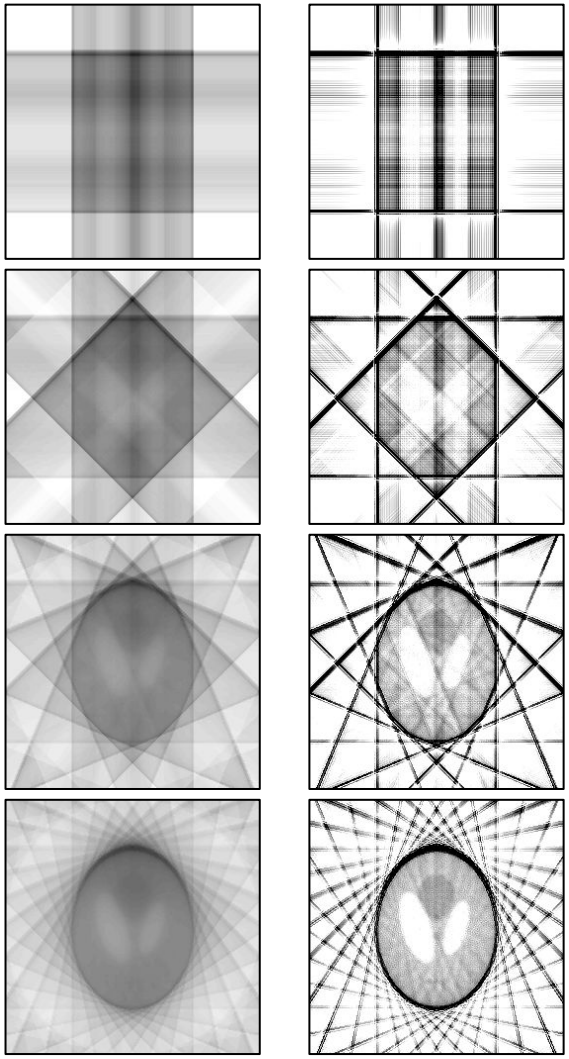
[5]



[6]

Reconstruction via Back Projection

BP FBP

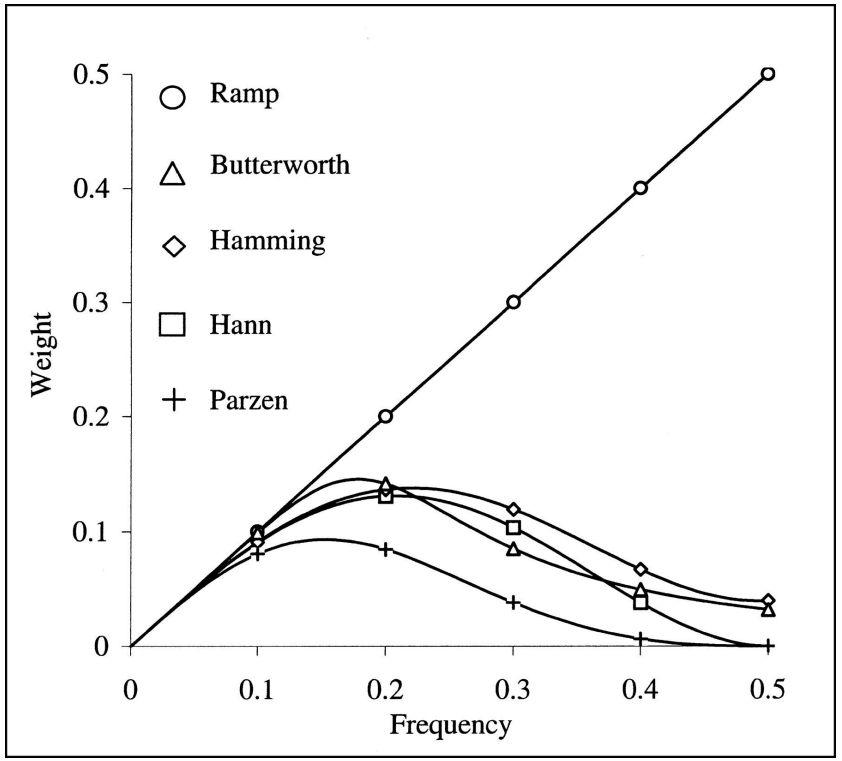


Projections are ‘smeared’ over the image in respective direction

Low Frequencies are amplified in unfiltered BP
➡ need for a high pass filter

Original

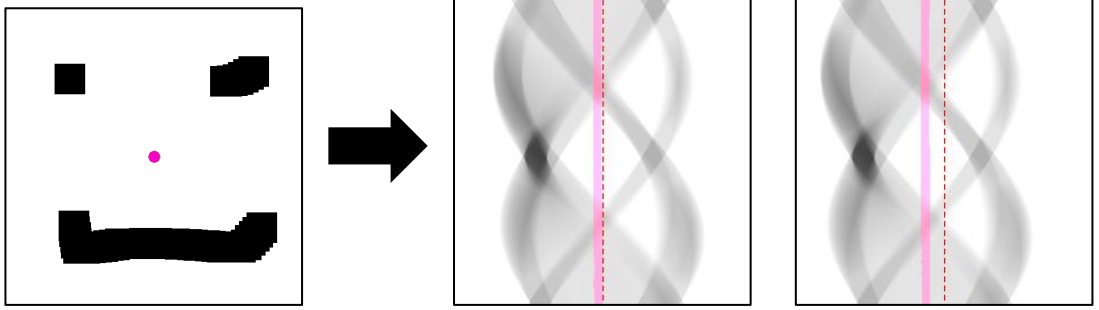
[7]



[8]

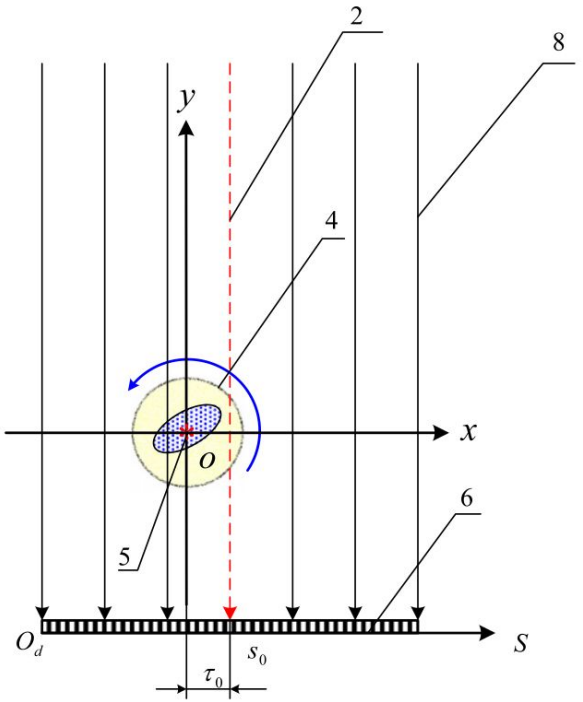
Displacement of Center of Rotation

Shift in Sinogram



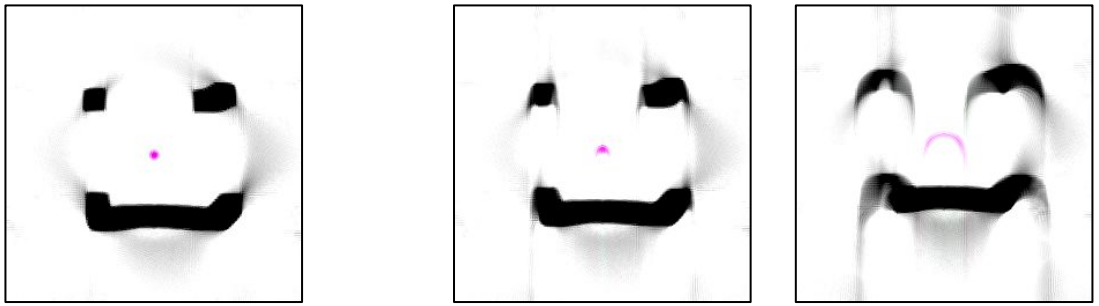
shift: 4px

shift: 16px



[9]

Resulting Artifacts in Reconstruction



FBP of unshifted

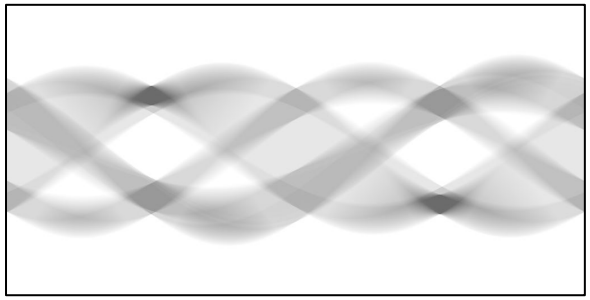
FBP of 4px shift

FBP of 16px shift

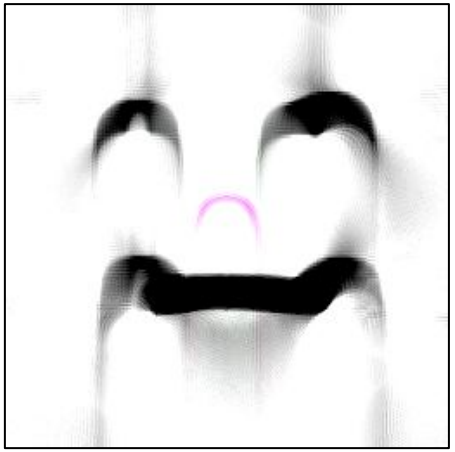
Determination of Center of Rotation (COR)

Algorithms can be split into two groups

Sinogram based methods



Reconstruction based methods

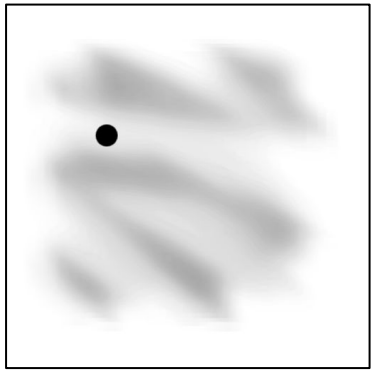


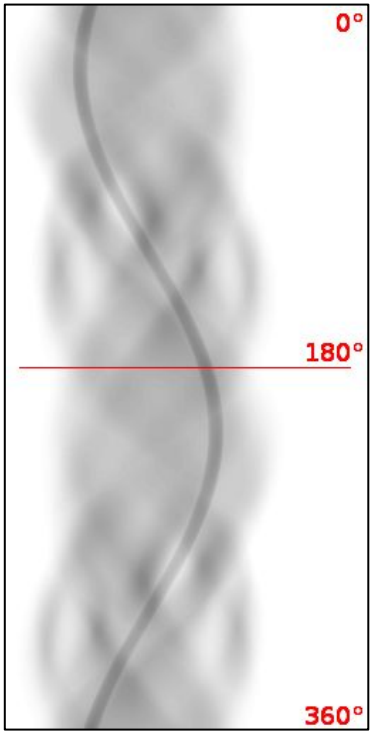
Determination of COR using Sinogram

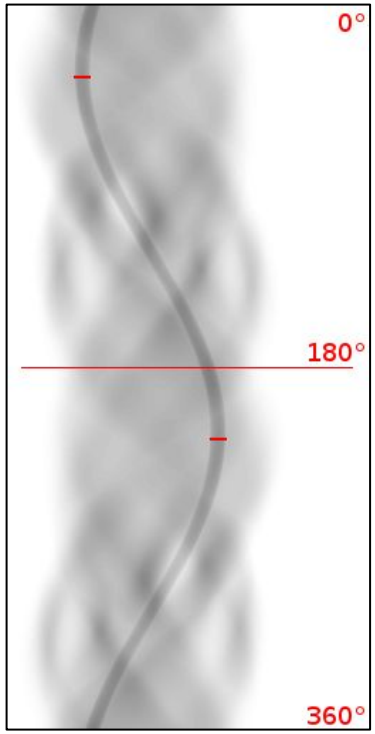
locating a high density feature in opposing parallel projections

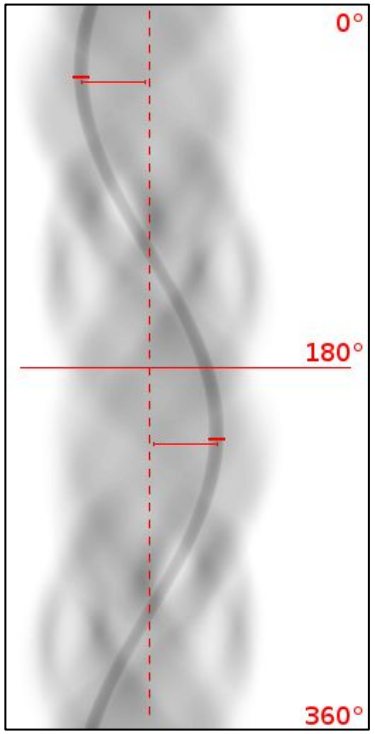
→ peak in sinogram

→ average positions of peaks is center of rotation





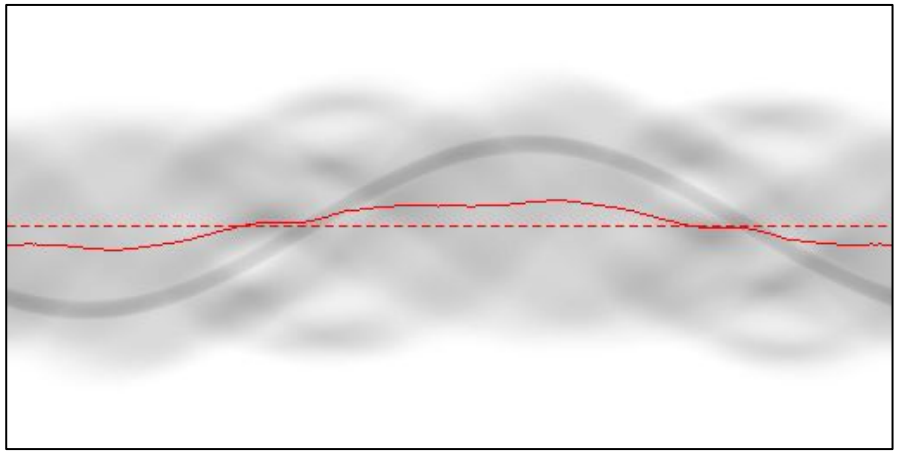




Determination of COR using Sinogram

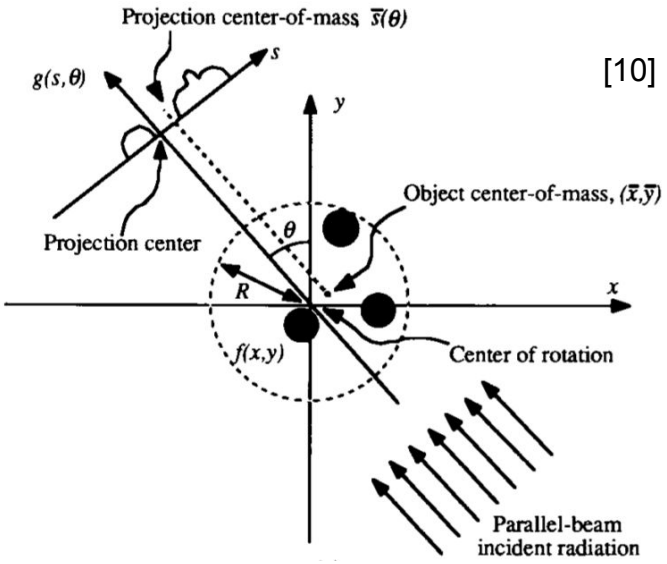
Determining Center of Mass

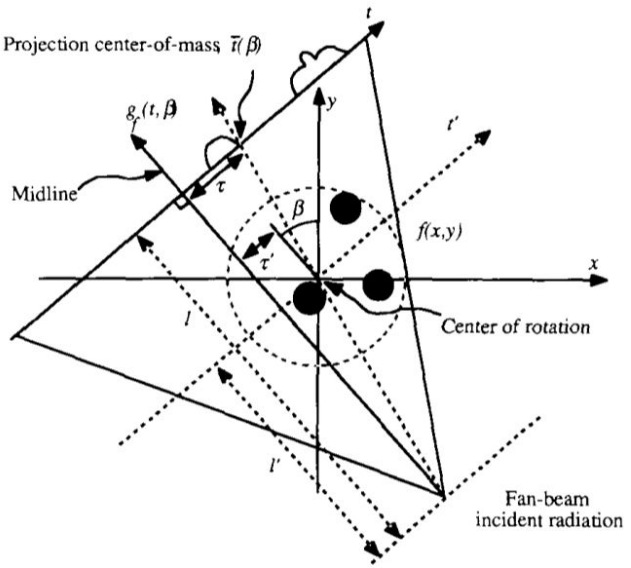
- Parallel beam through object's center of mass hits projection's center of mass



- for fan beam: algorithm can not be used. Object's COM is not projected to COM in sinogram.

[10]





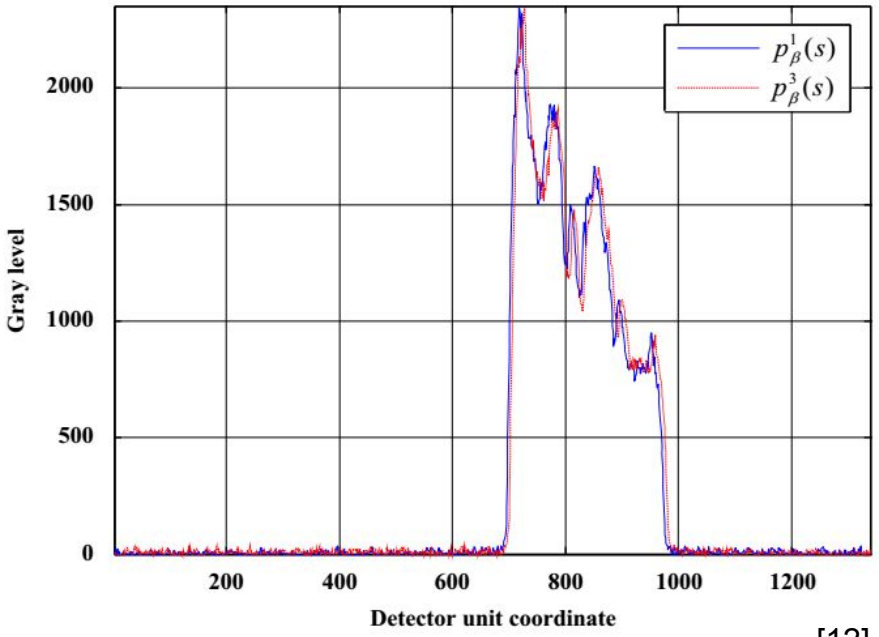
Determination of COR using Sinogram

Cross correlation of lower and upper half of sinogram yields shift of COR

[11]

$p_\beta^1(s)$ **sub-sinogram**

$p_\beta^3(s)$ **sub-sinogram**



[12]

Determination of COR using Reconstruction

Iterative/adaptive algorithm to approximate optimal center of rotation

Need for scoring metrics to compare quality of reconstruction

```
corToTry = {...}
scores = {}
for (cor in corToTry) do:
    image = reconstructImg(cor)
    score = generateScore(image, metric)
    scores.append(score)
end for

index = indexOfMinimum(scores)
cor = corToTry[index]
```

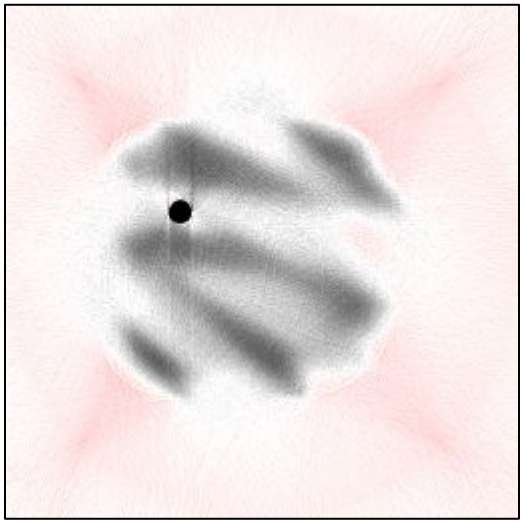
Determination of COR using Reconstruction

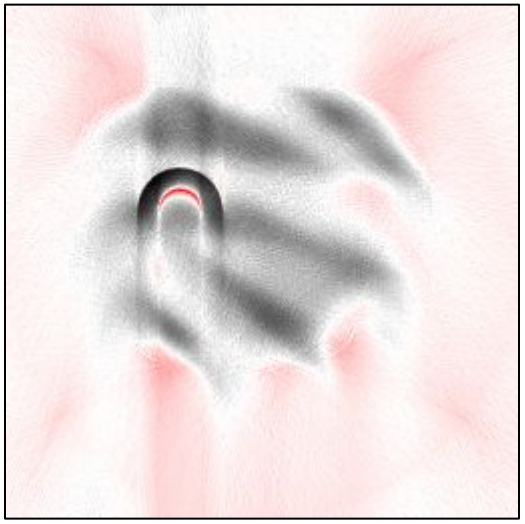
Integral of Negativity

$$Q_{IN}(\tilde{f}) = -\frac{1}{m_0}\int\int u[-\tilde{f}(x,y)]\tilde{f}(x,y)\mathrm{d}x\mathrm{d}y$$

$$u(\alpha) = \begin{cases} 1: & \alpha \geqslant 0 \\ 0: & \text{else} \end{cases}$$

$$m_0 = \int\int f(x,y)\mathrm{d}x\mathrm{d}y = \int p_\theta(t)\mathrm{d}t$$





Summary

- Projections from multiple angles result in a sinogram
- Reconstruction of original image via filtered back projection of sinogram

- faulty assumption of COR in reconstruction introduces artifacts
- methods for COR determination are either sinogram or reconstruction based

- COR from sinogram using
  - a high density feature
  - the center of mass
  - cross-correlation
- COR from reconstruction using an image metric like integral of negativity

References

[0] Buzug, Thorsten. Computed Tomography (2008), fig. 4.10

[1] https://commons.wikimedia.org/w/index.php?title=File:Radon_transform_projection.png&oldid=125078898

[2] Buzug, Thorsten. Computed Tomography (2008), fig. 7.10 (without labels)

[3] Buzug, Thorsten. Computed Tomography (2008), fig. 7.11

[4] ~~Buzug, Thorsten. Computed Tomography (2008), fig. 7.12 (sinograms only, inverted color)~~

[5] Yang et al. 2011. 'A new method to determine the center of rotation shift in 2D-CT scanning system using image cross correlation' , fig. 4

[6] Buzug, Thorsten. Computed Tomography (2008), fig. 8.27

[7] https://commons.wikimedia.org/w/index.php?title=File:SheppLogan_Phantom.svg&oldid=210578173

[8] Bruyant P.P. 'Analytic and iterative reconstruction algorithms in SPECT.', fig 12

[9] Yang et al. 2011. 'A new method to determine the center of rotation shift in 2D-CT scanning system using image cross correlation' , fig. 1

[10] Azevedo et al. 1990. 'Calculation of the rotational centers in computed tomography sinograms', fig. 2

[11] Yang et al. 2011. 'A new method to determine the center of rotation shift in 2D-CT scanning system using image cross correlation' , fig. 6

[12] Yang et al. 2011. 'A new method to determine the center of rotation shift in 2D-CT scanning system using image cross correlation' , fig. 7