

$$I_0(\boldsymbol{x}), I_1(\boldsymbol{x}) : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\boldsymbol{u} = \begin{pmatrix} u(\boldsymbol{x}) \\ v(\boldsymbol{x}) \end{pmatrix} : \Omega \rightarrow \mathbb{R}^2$$

$$I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) = I_0(\boldsymbol{x})$$

$$E(\boldsymbol{u}) = D(\boldsymbol{u}) + \alpha \cdot R(\boldsymbol{u})$$

$$\int_{\Omega} p(\boldsymbol{u}) \; dx$$

$$p(\boldsymbol{u}) = (\; I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - I_0(\boldsymbol{x}) \;)^2$$

$$p(\boldsymbol{u}) = ||\; \nabla I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - \nabla I_0(\boldsymbol{x}) \; ||_2^2$$

$$p(\boldsymbol{u}) = ||\; \nabla u \; ||_2^2 + ||\; \nabla v \; ||_2^2$$

$$D(\boldsymbol{u}) = \int_{\Omega} \Psi_D(p_{brightness}(\boldsymbol{u})) + \gamma \cdot \Psi_D(p_{gradient}(\boldsymbol{u})) \; dx$$

$$p_{brightness}(\boldsymbol{u}) = (\; I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - I_0(\boldsymbol{x}) \;)^2$$

$$p_{gradient}(\boldsymbol{u}) = ||\; \nabla I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - \nabla I_0(\boldsymbol{x}) \; ||_2^2$$

$$\Psi_D(s^2) = 2\epsilon^2\sqrt{1+s^2/\epsilon^2}$$

$$S_w(\mathbf{x}) = \int w(\tau) \begin{pmatrix} I_x(\mathbf{x} - \tau)^2 & I_x(\mathbf{x} - \tau)I_y(\mathbf{x} - \tau) \\ I_x(\mathbf{x} - \tau)I_y(\mathbf{x} - \tau) & I_y(\mathbf{x} - \tau)^2 \end{pmatrix} d\tau$$

$$S_w = Q\Lambda Q^\top = \begin{pmatrix} \mathbf{r}_1 & \mathbf{r}_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \mathbf{r}_1^\top \\ \mathbf{r}_2^\top \end{pmatrix}$$

$$R_1(\mathbf{u}) = \int_{\Omega} S_1(\mathbf{u}) \, dx = \int_{\Omega} \sum_{l=1}^2 \Psi_l \left((\mathbf{r}_l^\top \nabla u)^2 + (\mathbf{r}_l^\top \nabla v)^2 \right) \, dx$$

$$\Psi_1(s^2) = \epsilon^2 \ln(1 + s^2/\epsilon^2)$$

$$\Psi_2(s^2) = 2\epsilon^2 \sqrt{1 + s^2/\epsilon^2}$$

$$R_2(\mathbf{u}) = \int_{\Omega} \inf_{\mathbf{a}, \mathbf{b}} \left[S_2(\mathbf{u}, \mathbf{a}, \mathbf{b}) + \beta \cdot S_{\text{aux}}(\mathbf{a}, \mathbf{b}) \right] \, dx$$

$$S_2(\mathbf{u}, \mathbf{a}, \mathbf{b}) = \sum_{l=1}^2 \Psi_l \left((\mathbf{r}_l^\top (\nabla u - \mathbf{a}))^2 + (\mathbf{r}_l^\top (\nabla v - \mathbf{b}))^2 \right)$$

$$S_{\text{aux}}(\mathbf{a}, \mathbf{b}) = \sum_{l=1}^2 \Psi_l \left(\sum_{k=1}^2 (\mathbf{r}_k^\top \mathcal{J} \mathbf{a} \, \mathbf{r}_l)^2 + (\mathbf{r}_k^\top \mathcal{J} \mathbf{b} \, \mathbf{r}_l)^2 \right)$$

$$R(\mathbf{u}) = \int_{\Omega} \inf_{\mathbf{a}, \mathbf{b}} \left[S_1(\mathbf{u}) + S_2(\mathbf{u}, \mathbf{a}, \mathbf{b}) + \beta \cdot S_{\text{aux}}(\mathbf{a}, \mathbf{b}) \right] \, dx$$

$$R(\mathbf{u}, c) = \int_{\Omega} \inf_{\mathbf{a}, \mathbf{b}} \left[c \cdot S_1(\mathbf{u}) + (1 - c) \cdot S_2(\mathbf{u}, \mathbf{a}, \mathbf{b}) + \beta \cdot S_{\text{aux}}(\mathbf{a}, \mathbf{b}) + \phi_\lambda(c) \right] \, dx$$

$$c = \frac{1}{1 + e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\Omega|} \int_{\Omega} S_2 - S_1 \, dx$$

$$c = \frac{1}{1 + e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\Omega|} \int_{\Omega} S_2 - S_1 \, dx$$

$$c_{\text{local}}(\mathbf{x}) = \frac{1}{1 + e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + S_2(\mathbf{x}) - S_1(\mathbf{x})$$

$$c_{\text{nonlocal}}(\mathbf{x}) = \frac{1}{|\mathcal{N}(\mathbf{x})|} \int_{\mathcal{N}(\mathbf{x})} \frac{1}{1 + e^{-\Delta/\lambda}} \, d\mathbf{y} \quad \text{with} \quad \Delta = T + \frac{1}{|\mathcal{N}(\mathbf{x})|} \int_{\mathcal{N}(\mathbf{x})} S_2 - S_1 \, d\mathbf{y}$$

$$c_{\text{nonlocal}}(\mathbf{x}) = \frac{1}{1 + e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\mathcal{N}(\mathbf{x})|} \int_{\mathcal{N}(\mathbf{x})} S_2 - S_1 \, d\mathbf{y}$$

$$z : \Omega \rightarrow \mathbb{R} \quad (\text{a level-set function})$$

$$c_{\text{region}}(z) = \frac{1}{1 + e^{-z/\lambda}} \quad \text{and selection term } \Phi_{\lambda}(c) \text{ replaced by } |\nabla c_{\text{region}}(z)|$$