Determination of Center of Rotation in Computed Tomography

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Abstract—The abstract goes here.

Index Terms—Computer Science, Image Processing, Computed Tomography, Tomographic Reconstruction

1 Introduction

In computed tomography (CT), a set of set of projections is acquired by meassuring the attenuation of X-rays through the scanned object at different rotation angles. The X-rays are emmited by a source on one side of the scanned object, intersect with the object, and hit a detector on the other side. The resulting meassurement of the detector is a projection of the object's internals. Rotating the object and repeating the process results in a set of projections, which can be used to reconstruct an image of the internal structure of the scanned object. For this reconstruction it is mandatory to know the center of rotation (COR), or rather the location of its projection, as slight deviations from the actual COR produce severe artifacts in the reconstructed image. In general, the COR is assumed to be projected onto the center of the detector, but this assumption can be violated on badly calibrated CT systems. A correct calibration can also become almost impossible when the X-ray source is very close to the scanned object, as is the case for industrial scanners with resolutions on the micrometer scale [1]. In this paper, different techniques and algorithms for the determination of the center of rotation are explained, which can be found in the literature.

2 Prerequisites

In order to explain the problem of inaccurate COR assumption in tomographic reconstruction, some prerquisite knowledge about CT will be briefly explained in this section.

2.1 Image Acquisition in CT

This subsection will briefly explain how images are typically aquired in CT.

In X-ray imaging, an X-ray source (e.g. X-ray tube) is used to generate photons which then run through an object. The capability of the photons to penetrate matter, depends on the material, which means that the decrease in number of photons along the incident beam is different for different materials. The decrease in the number of photons, or attenuation, results from the photons being absorbed or scattered as they travel through the matters [2]. Using an X-ray detector, the photon count after matter penetration can be measured and thus the attenuation along the beam from the source to the detector. The measured quantity can be interpreted as the projection of all attenuations along the ray.

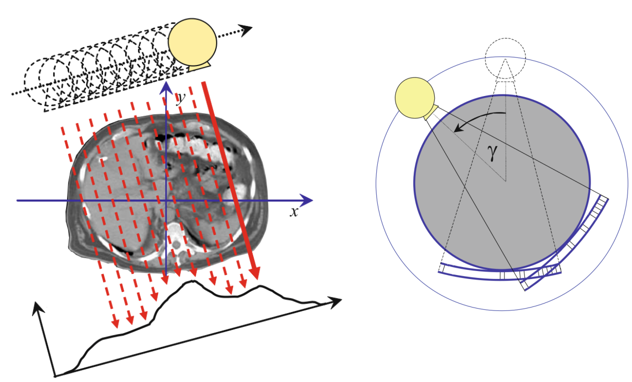
However, a single projection is not sufficient to deduce the attenuation strengths within the object. To locate regions of high and low attenuation, multiple projections from different sides of the object are required. Therefore the object is placed on a rotary table or the imaging aparatus (X-ray source and detector) is rotated around the object.

2.2 CT Geometries

This subsection will briefly explain the most common geometries in CT, that is the setup of X-ray source and detector and the ray projections which result from it.

2.2.1 Parallel Beam

The simplest geometry in CT (from a mathematical point of view) is the parallel beam geometry. Here all rays are parallel that are emitted from the same side of the object. Figure 1a shows a schematic of this geometry with the resulting signal.



(a) Parallel Beam (b) Fan Beam

Fig. 1. Parallel beam and fan beam geometry. In parallel beam the source moving perpendicular to the ray direction for sampling a 1-dimensional image (graph). In fan beam, rays are not perpendicular but instead emmited in fan blade form. γ is the rotation angle at which the signal is obtained. [3], [4]

This projection geometry can be mathematically expressed with the Radon transform. When we think of the object as a function of attenuation coefficients $f(x, y)$ and the measured signal as a function of projected coefficients $R(r, \gamma)$ where r is the position on the detector and γ is the rotation angle at which the signal is obtained, then

$$R(r, \gamma) = \int_{-\infty}^{\infty} f(r\ cos(\gamma) + t\ sin(\gamma), r\ sin(\gamma) - t\ cos(\gamma))\ dt. \tag{1}$$

The parameter t is the position on the ray that hits the detector at r and angle γ, which makes it clear that the projection $R(r, \gamma)$ is the integral along a ray through the object $f(x, y)$. The sampling of an object in this fashion, results in a so called sinogram. The contribution of a specific area of the object to the individual projections, follows a sinusoidal path along the γ-axis as can be seen in fig. 2

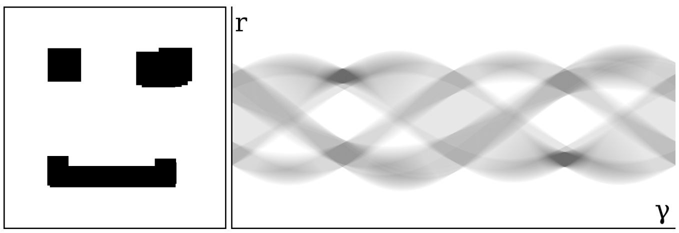


Fig. 2. A "smilie" phantom and its corresponding sinogram with $\gamma \in [0°, 360°]$. An imaginary CT would have started with source on top and detector on bottom and then rotated clockwise around the phantom.

Unfortuantely, parallel beam geometry is impractical as a pencil beam X-ray source (source emmiting only one ray in a specific direction) would have to be moved parallel to the detector to collect all samples for a single rotation angle, which is very time consuming. However, parallel beam geometry occurs at synchrotron facilities [**?**].

2.2.2 Fan Beam

A more practical geometry is the fan beam, in which a source emits a flat beam that extends to the sides (like a fan blade).

2.2.3 Cone Beam

2.3 Reconstruction

2.3.1 Filtered Back Projection

3 COR Determination

3.1 Sinogram Based Methods

3.1.1 High Density Feature

3.1.2 Center of Mass

3.1.3 Cross-Correlation

3.2 Reconstruction Based Methods

3.2.1 Integral of Negativity

3.2.2 Histogram Entropy

4 Conclusion

The conclusion goes here.

Appendix A

References

[1] S. G. Azevedo, D. J. Schneberk, J. P. Fitch, and H. E. Martz, "Calculation of the rotational centers in computed tomography sinograms," *IEEE Transactions on Nuclear Science*, vol. 37, no. 4, pp. 1525–1540, Aug 1990.

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