$$I_0(\boldsymbol{x}), I_1(\boldsymbol{x}) : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$$

$$\boldsymbol{u} = \begin{pmatrix} u(\boldsymbol{x}) \\ v(\boldsymbol{x}) \end{pmatrix} : \Omega \to \mathbb{R}^2$$

$$I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) = I_0(\boldsymbol{x})$$

$$E(\boldsymbol{u}) = D(\boldsymbol{u}) + \alpha \cdot R(\boldsymbol{u})$$

$$\int_\Omega p(\boldsymbol{u})\ dx$$

$$p(\boldsymbol{u}) = (\ I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - I_0(\boldsymbol{x})\ )^2$$

$$p(\boldsymbol{u}) = ||\ \nabla I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - \nabla I_0(\boldsymbol{x})\ ||_2^2$$

$$p(\boldsymbol{u}) = ||\ \nabla u\ ||_2^2 + ||\ \nabla v\ ||_2^2$$

$$D(\boldsymbol{u}) = \int_\Omega \Psi_D(p_{brightness}(\boldsymbol{u})) + \gamma \cdot \Psi_D(p_{gradient}(\boldsymbol{u}))\ dx$$

$$p_{brightness}(\boldsymbol{u}) = (\ I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - I_0(\boldsymbol{x})\ )^2$$

$$p_{gradient}(\boldsymbol{u}) = ||\ \nabla I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - \nabla I_0(\boldsymbol{x})\ ||_2^2$$

$$\Psi_D(s^2) = 2\epsilon^2 \sqrt{1 + s^2/\epsilon^2}$$

$$S_w(\boldsymbol{x}) = \int w(\tau) \begin{pmatrix} I_x(\boldsymbol{x}-\tau)^2 & I_x(\boldsymbol{x}-\tau)I_y(\boldsymbol{x}-\tau) \\ I_x(\boldsymbol{x}-\tau)I_y(\boldsymbol{x}-\tau) & I_y(\boldsymbol{x}-\tau)^2 \end{pmatrix} \, d\tau$$

$$S_w = Q\Lambda Q^\top = \begin{pmatrix} \boldsymbol{r}_1 & \boldsymbol{r}_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{r}_1^\top \\ \boldsymbol{r}_2^\top \end{pmatrix}$$

$$R_1(\boldsymbol{u}) = \int_\Omega S_1(\boldsymbol{u}) \, dx = \int_\Omega \sum_{l=1}^{2} \Psi_l\Big((\boldsymbol{r}_l^\top \nabla u)^2 + (\boldsymbol{r}_l^\top \nabla v)^2 \Big) \, dx$$

$$\Psi_1(s^2) = \epsilon^2 \ln(1 + s^2/\epsilon^2)$$

$$\Psi_2(s^2) = 2\epsilon^2 \sqrt{1 + s^2/\epsilon^2}$$

$$R_2(\boldsymbol{u}) = \int_\Omega \inf_{\boldsymbol{a},\boldsymbol{b}} \Big[S_2(\boldsymbol{u},\boldsymbol{a},\boldsymbol{b}) + \beta \cdot S_{\text{aux}}(\boldsymbol{a},\boldsymbol{b}) \Big] \, dx$$

$$S_2(\boldsymbol{u},\boldsymbol{a},\boldsymbol{b}) = \sum_{l=1}^{2} \Psi_l\Big((\boldsymbol{r}_l^\top(\nabla u - \boldsymbol{a}))^2 + (\boldsymbol{r}_l^\top(\nabla v - \boldsymbol{b})) \Big)$$

$$S_{\text{aux}}(\boldsymbol{a},\boldsymbol{b}) = \sum_{l=1}^{2} \Psi_l\Big(\sum_{k=1}^{2} (\boldsymbol{r}_k^\top \mathcal{J}\boldsymbol{a} \, \boldsymbol{r}_l)^2 + (\boldsymbol{r}_k^\top \mathcal{J}\boldsymbol{b} \, \boldsymbol{r}_l)^2 \Big)$$

$$R(\boldsymbol{u}) = \int_\Omega \inf_{\boldsymbol{a},\boldsymbol{b}} \Big[S_1(\boldsymbol{u}) + S_2(\boldsymbol{u},\boldsymbol{a},\boldsymbol{b}) + \beta \cdot S_{\text{aux}}(\boldsymbol{a},\boldsymbol{b}) \Big] \, dx$$

$$R(\boldsymbol{u},c) = \int_\Omega \inf_{\boldsymbol{a},\boldsymbol{b}} \Big[c \cdot S_1(\boldsymbol{u}) + (1-c) \cdot S_2(\boldsymbol{u},\boldsymbol{a},\boldsymbol{b}) + \beta \cdot S_{\text{aux}}(\boldsymbol{a},\boldsymbol{b}) + \phi_\lambda(c) \Big] \, dx$$

$$c = \frac{1}{1 + e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\Omega|} \int_\Omega S_2 - S_1 \, dx$$

$$c = \frac{1}{1+e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\Omega|}\int_\Omega S_2 - S_1 \; dx$$

$$c_{\text{local}}(\boldsymbol{x}) = \frac{1}{1+e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + S_2(\boldsymbol{x}) - S_1(\boldsymbol{x})$$

$$c_{\text{nonlocal}}(\boldsymbol{x}) = \frac{1}{|\mathcal{N}(\boldsymbol{x})|}\int_{\mathcal{N}(\boldsymbol{x})} \frac{1}{1+e^{-\Delta/\lambda}} \; d\boldsymbol{y} \quad \text{with} \quad \Delta = T + \frac{1}{|\mathcal{N}(\boldsymbol{x})|}\int_{\mathcal{N}(x)} S_2 - S_1 \; d\boldsymbol{y}$$

$$c_{\text{nonlocal}}(\boldsymbol{x}) = \frac{1}{1+e^{-\Delta/\lambda}} \quad \text{with} \quad \Delta = T + \frac{1}{|\mathcal{N}(\boldsymbol{x})|}\int_{\mathcal{N}(x)} S_2 - S_1 \; d\boldsymbol{y}$$

$z : \Omega \to \mathbb{R}$ (a level-set function)

$c_{\text{region}}(z) = \dfrac{1}{1+e^{-z/\lambda}}$ and selection term $\Phi_\lambda(c)$ replaced by $|\nabla c_{\text{region}}(z)|$

$$f(\boldsymbol{x}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \frac{\partial f}{\partial \boldsymbol{x}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$f(\boldsymbol{x}) = \begin{pmatrix} -x_1 \\ -x_2 \end{pmatrix} \qquad \frac{\partial f}{\partial \boldsymbol{x}} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$