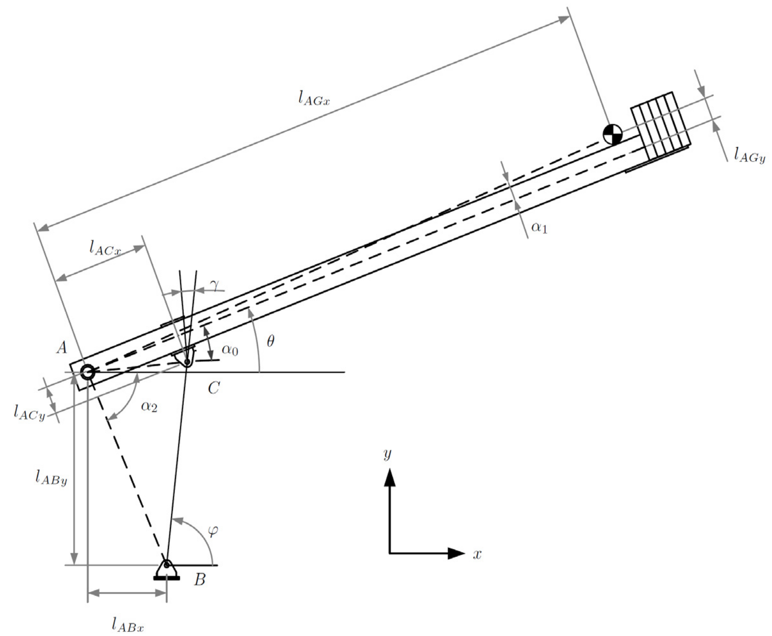
>



Mechanical Model og Green Crane @ UiA Laboratory

> $restart;$

> $with(LinearAlgebra):$

> $with(Physics):$

Translational Position

> $R_p := \langle\langle \cos(\theta_p(t)+\alpha_1), \sin(\theta_p(t)+\alpha_1)\rangle | \langle -\sin(\theta_p(t)+\alpha_1), \cos(\theta_p(t)+\alpha_1)\rangle\rangle :$

> $L_{\mathrm{AG}} := \langle\langle \mathrm{L}_{AGx}, \mathrm{L}_{AGy}\rangle\rangle :$

> $x_{cm} := Multiply(R_p, L_{\mathrm{AG}})[1, 1]:$

> $y_{cm} := Multiply(R_p, L_{\mathrm{AG}})[2, 1]:$

Translational Velocity

> $x_{cmd} := diff(x_{\mathrm{cm}}, t):$

> $y_{cmd} := diff(y_{\mathrm{cm}}, t):$

> $v_{\mathrm{cm}} := \langle\langle x_{cmd}, y_{cmd}\rangle\rangle :$

Angular Velocity

> $\omega_p := \frac{d}{dt}\theta_p(t):$

Kinetic Energy

> $K_l := \frac{1}{2}\cdot m_{cm}\cdot Multiply\left(Transpose\left(v_{\mathrm{cm}}\right), v_{\mathrm{cm}}\right)[1,1] + \frac{1}{2}\cdot j_{\mathrm{cm}}\cdot\omega_p^2:$

Potential Energy

> $P_l := m_{\mathrm{cm}}\cdot g\cdot y_{\mathrm{cm}}:$

Lagrangian

> $L_l := K_l - P_l:$

> $\tau_{\mathrm{A1}} := simplify\left(diff\left(diff\left(L_l, \frac{d}{dt}\theta_p(t)\right), t\right) - diff\left(L_l, \theta_p(t)\right), 'trig'\right):$

Joint A Torque

> $\tau_{\mathrm{A}} := collect\left(collect\left(collect\left(\tau_{\mathrm{A1}}, \frac{d^2}{dt^2}\theta_p(t)\right), \frac{d}{dt}\theta_p(t)\right), g\right):$

Joint-to-Actuator Kinematics

> $restart;$

> $L_{cyl} := x_p(t) + L_{\min}:$

> $\alpha_4 := \arccos\left(\frac{L_{AC}^2 + L_{AB}^2 - L_{cyl}^2}{2\cdot L_{AC}\cdot L_{AB}}\right) - \alpha_2:$

> $\theta_p := \alpha_4 + \alpha_0:$

> $diff\left(\theta_p, t\right):$

> $diff\left(diff\left(\theta_p, t\right), t\right):$

Simplified Expressions

> $\theta_d := \frac{2\,L_{cyl}\,x_d}{L_{AC}\,L_{AB}\sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\,L_{AB}^2}}}:$

> $\theta_{dd} := \frac{2\,x_d^2}{L_{AC}\,L_{AB}\sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\,L_{AB}^2}}} + \frac{2\,L_{cyl}\,x_{dd}}{L_{AC}\,L_{AB}\sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\,L_{AB}^2}}} - \frac{4\,L_{cyl}^2\,x_d^2\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)}{L_{AC}^3\,L_{AB}^3\left(4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\,L_{AB}^2}\right)^{3/2}}:$

> $restart;$

> $\tau_A := \left(\cos\left(\theta_p + \alpha_1\right) L_{AGx}\,m_{cm} - \sin\left(\theta_p + \alpha_1\right) L_{AGy}\,m_{cm}\right) g + \left(L_{AGx}^2\,m_{cm} + L_{AGy}^2\,m_{cm} + j_{cm}\right)\theta_{dd}:$

Torque Arm

> $L_{cyl} := x_p + L_{\min}:$

Relevant Angles

> $\alpha_0 := \arctan\left(\frac{L_{ACy}}{L_{ACx}}\right):$

> $\alpha_1 := \arctan\left(\frac{L_{AGy}}{L_{AGx}}\right):$

> $\alpha_2 := \arctan\left(\frac{L_{ABy}}{L_{ABx}}\right):$

> $\alpha_3 := \arccos\left(\frac{L_{cyl}^2 + L_{AB}^2 - L_{AC}^2}{2 \cdot L_{cyl} \cdot L_{AB}}\right):$

> $\alpha_4 := \arccos\left(\frac{L_{AC}^2 + L_{AB}^2 - L_{cyl}^2}{2 \cdot L_{AC} \cdot L_{AB}}\right) - \alpha_2:$

> $\theta_p := \alpha_4 + \alpha_0:$

> $\theta_d := \frac{2\, L_{cyl}\, x_d}{L_{AC}\, L_{AB} \sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\, L_{AB}^2}}}:$

> $$\theta_{dd} := \frac{2\, x_d^2}{L_{AC}\, L_{AB} \sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\, L_{AB}^2}}} + \frac{2\, L_{cyl}\, x_{dd}}{L_{AC}\, L_{AB} \sqrt{4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\, L_{AB}^2}}} - \frac{4\, L_{cyl}^2\, x_d^2 \left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)}{L_{AC}^3\, L_{AB}^3 \left(4 - \frac{\left(L_{AB}^2 + L_{AC}^2 - L_{cyl}^2\right)^2}{L_{AC}^2\, L_{AB}^2}\right)^{3/2}}:$$

> $L_{arm} := L_{AB} \cdot \sin\left(\alpha_3\right):$

Cylinder Output Force

> $F_{cyl1} := simplify\left(solve\left(\tau_{\mathrm{A}} = L_{arm} \cdot F_{cyl}, F_{cyl}\right), 'symbolic'\right):$

Generalized Force Balance Formulation

> $$M_{eq} := -\frac{4\left(L_{AGx}^2\, L_{cyl}\, m_{cm} + L_{AGy}^2\, L_{\mathrm{cyl}}\, m_{cm} + L_{\mathrm{cyl}}\, j_{cm}\right)\left(L_{cyl}\right)}{\left(L_{AB} - L_{cyl} - L_{AC}\right)\left(L_{AB} - L_{cyl} + L_{AC}\right)\left(L_{cyl} + L_{AB} + L_{AC}\right)\left(L_{cyl} + L_{AB} - L_{AC}\right)}:$$

> $$C_{eq} := \frac{4\left(\left(L_{AGx}^2 + L_{AGy}^2\right) m_{cm} + j_{cm}\right) L_{cyl}\left(-L_{AB}^2 + L_{AC}^2 + L_{cyl}^2\right)\left(L_{AB}^2 - L_{AC}^2 + L_{cyl}^2\right)}{\left(L_{cyl} - L_{AC} - L_{AB}\right)^2\left(L_{cyl} + L_{AC} - L_{AB}\right)^2\left(L_{cyl} + L_{AC} + L_{AB}\right)^2\left(L_{cyl} - L_{AC} + L_{AB}\right)^2}:$$

> $G_{eq} :=$

$$-\left(2\sqrt{-\left(L_{cyl} + L_{AC} - L_{AB}\right)\left(L_{cyl} - L_{AC} - L_{AB}\right)\left(L_{cyl} - L_{AC} + L_{AB}\right)\left(L_{cyl} + L_{AC} + L_{AB}\right)}\, g\, m_{cm}\right.$$

$$\left(\cos\left(\arccos\left(\frac{1}{2}\,\frac{L_{AB}^2+L_{AC}^2-(L_{cyl})^2}{L_{AC}L_{AB}}\right)-\alpha_2+\alpha_0+\alpha_1\right)L_{AGx}\right.$$
$$\left.\left.-\sin\left(\arccos\left(\frac{1}{2}\,\frac{L_{AB}^2+L_{AC}^2-(L_{cyl})^2}{L_{AC}L_{AB}}\right)-\alpha_2+\alpha_0+\alpha_1\right)L_{AGy}\right)(L_{cyl})\right)\Big/\left(\left(L_{cyl}+L_{AC}\right.\right.$$
$$\left.\left.-L_{AB}\right)\left(L_{cyl}\text{-}L_{AC}-L_{AB}\right)\left(L_{cyl}-L_{AC}+L_{AB}\right)\left(L_{cyl}+L_{AC}+L_{AB}\right)\right):$$

> $F_{cyl} := M_{eq}\cdot x_{dd} + C_{eq}\cdot x_d^2 + G_{eq}:$

Verification of Generalized Force Balance Formulation

> $factor(F_{cyl1} - F_{cyl})$

$$0 \tag{1}$$

Lengths & Mass Properties

> $L_{AGx} :=$ L_AGx :

> $L_{AGy} :=$ L_AGy :

> $L_{ACx} :=$ L_ACx :

> $L_{ACy} :=$ L_ACy :

> $L_{ABx} :=$ L_ABx :

> $L_{ABy} :=$ L_ABy :

> $L_{AB} := L\_AB :$

> $L_{AC} := L\_AC :$

> $L_{AG} :=$ L_AG :

> $m_{cm} :=$ m_cm :

> $x_p := x :$

> $j_{cm} :=$ J_cm :

> $L_{\min} :=$ L_min :

Conversion to Matlab Code

> $CodeGeneration['Matlab'](G_{eq})$

```
cg = -0.2e1 * sqrt(-(x - L_AB + L_min + L_AC) * (x + L_min - L_AC
- L_AB) * (x + L_AB + L_min - L_AC) * (x + L_AB + L_min + L_AC))
* g * m_cm * (cos(acos((L_AB ^ 2 + L_AC ^ 2 - (x + L_min) ^ 2) /
L_AC / L_AB / 0.2e1) - atan(L_ABy / L_ABx) + atan(L_ACy / L_ACx)
+ atan(L_AGy / L_AGx)) * L_AGx - sin(acos((L_AB ^ 2 + L_AC ^ 2 -
(x + L_min) ^ 2) / L_AC / L_AB / 0.2e1) - atan(L_ABy / L_ABx) +
atan(L_ACy / L_ACx) + atan(L_AGy / L_AGx)) * L_AGy) * (x + L_min)
/ (x - L_AB + L_min + L_AC) / (x + L_min - L_AC - L_AB) / (x +
L_AB + L_min - L_AC) / (x + L_AB + L_min + L_AC);
```

> $CodeGeneration['Matlab'](M_{eq})$

```
cg0 = -4 * (L_AGx ^ 2 * (x + L_min) * m_cm + L_AGy ^ 2 * (x +
L_min) * m_cm + (x + L_min) * J_cm) * (x + L_min) / (-x + L_AB -
L_min - L_AC) / (-x + L_AB - L_min + L_AC) / (x + L_AB + L_min +
L_AC) / (x + L_AB + L_min - L_AC);
```

> $CodeGeneration['Matlab'](C_{eq})$

```
cg1 = 4 * ((L_AGx ^ 2 + L_AGy ^ 2) * m_cm + J_cm) * (x + L_min) *
(-L_AB ^ 2 + L_AC ^ 2 + (x + L_min) ^ 2) * (L_AB ^ 2 - L_AC ^ 2 +
```

```
(x + L_min) ^ 2) / (x + L_min - L_AC - L_AB) ^ 2 / (x - L_AB +
L_min + L_AC) ^ 2 / (x + L_AB + L_min + L_AC) ^ 2 / (x + L_AB +
L_min - L_AC) ^ 2;
```

```
>
```