Propositional Logic

Russell and Norvig:

Chapter 7, Sections 7.1 – 7.5.1

Slides by Jean-Claude Latombe, from an introductory AI course given at Stanford University. Used (and adapted) with permission.

Important Concepts in AI

- The Representation of Knowledge about the world

- The Reasoning Process to make use of it

Types of Agents

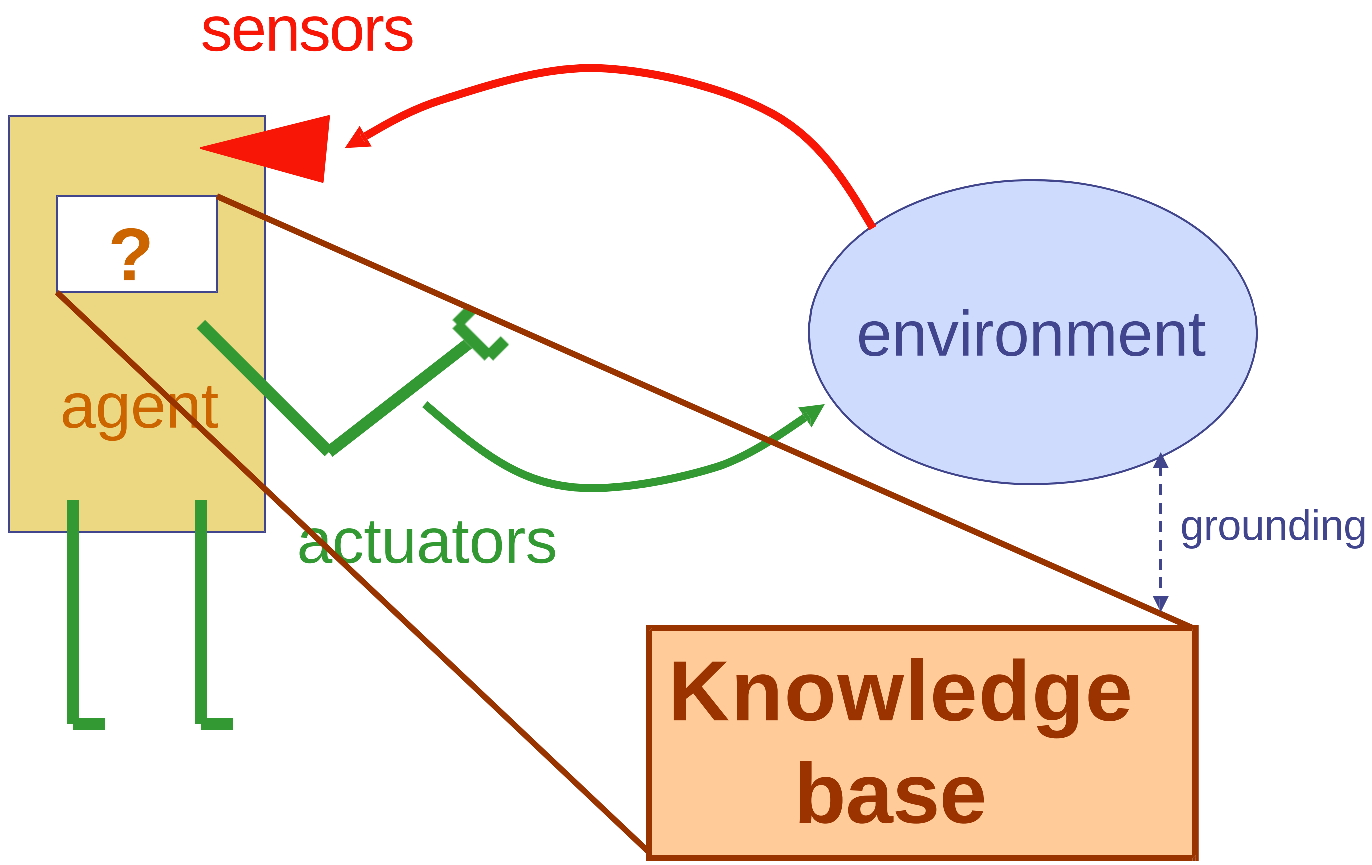
- Reflex Agent
  - Dumb luck
- Problem-solving Agent
  - Specific and inflexible
- Knowledge-based agent
  - General and flexible

Partially Seen Environments

- Knowledge-based Agents can combine
  - General Knowledge
  - Current Percepts

To infer hidden aspects!

Knowledge-Based Agent



Types of Knowledge

- Procedural, e.g.: functions
  Such knowledge can only be used in one way -- by executing it

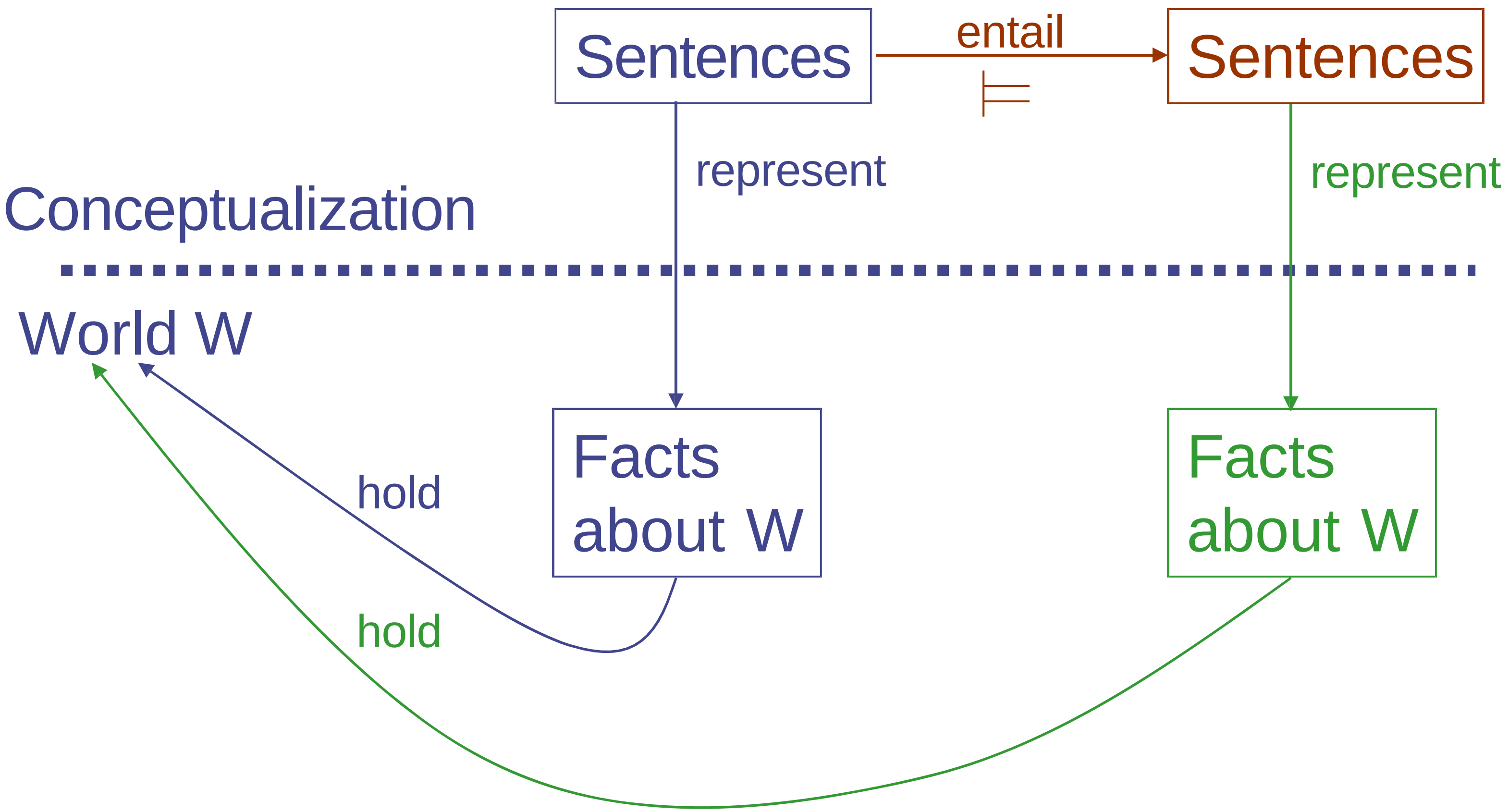
- Declarative, e.g.: constraints
  It can be used to perform many different sorts of inferences

Logic

Logic is a declarative language to:

- Assert sentences representing facts that hold in a world **W** (these sentences are given the value true)
- Deduce the true/false values to sentences representing other aspects of **W**

World-Representation



Examples of Logics

- Propositional calculus

  $A \wedge B \Rightarrow C$

- First-order predicate calculus

  $(\forall x)(\exists y)\ Mother(y,x)$

- Logic of Belief

  `B(John,Father(Zeus,Cronus))`

Symbols of PL

- Connectives: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- Propositional symbols:
  - Can be either true or false: P, Q, R, …
  - Fixed meaning: *True*, *False*

Syntax of PL

- sentence $\rightarrow$ atomic sentence | complex sentence
- atomic sentence $\rightarrow$ *True* | *False* | P | Q | R | …
- complex sentence $\rightarrow$ $\neg$ sentence

  | (sentence $\wedge$ sentence)

  | (sentence $\vee$ sentence)

  | (sentence $\Rightarrow$ sentence)

  | (sentence $\Leftrightarrow$ sentence)

Syntax of PL

- sentence → atomic sentence | complex sentence
- atomic sentence → *True* | *False* | P | Q | R | …
- complex sentence → ¬ sentence

  | (sentence ∧ sentence)

  | (sentence ∨ sentence)

  | (sentence ⇒ sentence)

  | (sentence ⇔ sentence)

- Examples:
  - $((P \wedge Q) \Rightarrow R)$
  - $(A \Rightarrow B) \vee (\neg C)$

Counter examples:

$(A \wedge \Rightarrow R)$

$(A\ B) \vee (\neg C)$

Order of Precedence

- $\neg \quad \wedge \quad \vee \quad \Rightarrow \quad \Leftrightarrow$
- Examples:
  - $\neg A \vee B \Rightarrow C$ is equivalent to $((\neg A) \vee B) \Rightarrow C$
  - $A \Rightarrow B \Rightarrow C$ is incorrect

$(A \Rightarrow B) \Rightarrow C$

$A \Rightarrow (B \Rightarrow C)$

Model

- Assignment of a truth value – *true* or *false* – to every atomic sentence
- Examples:
  - Let A, B, C, and D be the propositional symbols
  - m = {A=true, B=false, C=false, D=true} is a model
  - m’ = {A=true, B=false, C=false} is not a model
- With n propositional symbols, one can define 2^n models

What Worlds Does a Model Represent?

A model represents any world in which some fact represented by a proposition A having the value *True* holds and some fact represented by a proposition B having the value *False* does not hold (where only A and B are symbols)

m={A=*True*, B=*False*} →

<u>Any world</u> where A represents a held fact and B represents a fact that doesn't hold

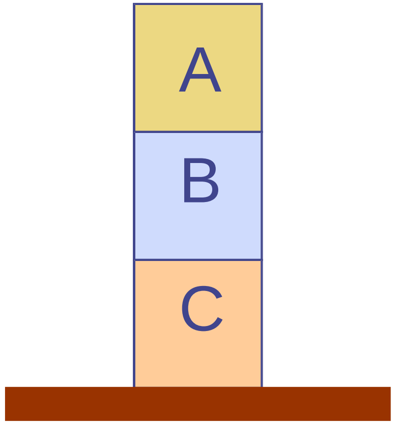
A model represents infinitely many worlds

Compare!

prop.symb.

- BLOCK(A), BLOCK(B), BLOCK(C)
- ON(A,B), ON(B,C), ONTABLE(C)
- ON(A,B) $\wedge$ ON(B,C) $\Rightarrow$ ABOVE(A,C)

ABOVE(A,C)



- HUMAN(A), HUMAN(B), HUMAN(C)
- CHILD(A,B), CHILD(B,C), BLOND(C)
- CHILD(A,B) $\wedge$ CHILD(B,C) $\Rightarrow$ GRAND-CHILD(A,C)

GRAND-CHILD(A,C)

Semantics of PL

- It specifies how to determine the truth value of any sentence in a model m
- The truth value of *True* is *True*
- The truth value of *False* is *False*
- The truth value of each atomic sentence is given by m
- The truth value of every other sentence is obtained recursively by using truth tables

Truth Tables

A	B	¬A	A ∧ B	A ∨ B	A⇒B
True	*True*	*False*	*True*	*True*	*True*
True	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*False*	*True*
False	*True*	*True*	*False*	*True*	*True*

Truth Tables

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \Rightarrow B$
True	*True*	*False*	*True*	*True*	*True*
True	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*False*	*True*
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False	*True*	*True*	*False*	*True*	*True*

About ⇒

- ODD(5)⇒CAPITAL(Japan,Tokyo)
- EVEN(5)⇒SMART(Sam)
- Read A⇒B as:
  "If A IS *True*, then I claim that B is *True*, otherwise I make no claim."

Example

Model: A=*True*, B=*False*, C=*False*, D=*True*

$$\underbrace{(\underbrace{\underbrace{\underbrace{\neg A}_{F} \lor B}_{F} \Rightarrow C}_{T}) \Rightarrow \underbrace{D \land A}_{T}}_{T}$$

Definition: If a sentence **s** is true in a model **m**, then **m** is said to be a **model** of **s**

A Small Knowledge Base

1. Battery-OK ∧ Bulbs-OK⇒Headlights-Work
2. Battery-OK ∧ Starter-OK ∧¬Empty-Gas-Tank⇒ Engine-Starts
3. Engine-Starts ∧¬Flat-Tire⇒Car-OK
4. Headlights-Work
5. ¬Car-OK

Sentences 1, 2, and 3 → Background knowledge

Sentences 4 and 5 → Observed knowledge

Model of a KB

- Let KB be a set of sentences
- A model m is a model of KB iff it is a model of all sentences in KB, that is, all sentences in KB are true in m

Satisfiability of a KB

A KB is satisfiable iff it admits at least one model; otherwise it is unsatisfiable

KB1 = $\{P, \neg Q \land R\}$ is satisfiable

KB2 = $\{\neg P \lor P\}$ is satisfiable

valid sentence or tautology

KB3 = $\{P, \neg P\}$ is unsatisfiable

Logical Entailment

A sentence follows logically from another sentence

$\boldsymbol{\alpha} \models \boldsymbol{\beta}$ if and only if,

in every model in which $\boldsymbol{\alpha}$ is true,

$\boldsymbol{\beta}$ is also true

Logical Entailment

- KB : set of sentences
- $\boldsymbol{\alpha}$: arbitrary sentence
- KB **entails** $\boldsymbol{\alpha}$ – written KB $\models \boldsymbol{\alpha}$ – iff every model of KB is also a model of $\boldsymbol{\alpha}$

Logical Entailment

- KB : set of sentences
- $\boldsymbol{\alpha}$: arbitrary sentence
- KB **entails** $\boldsymbol{\alpha}$ – written KB $\models \boldsymbol{\alpha}$ – iff every model of KB is also a model of $\boldsymbol{\alpha}$
- Alternatively, KB $\models \boldsymbol{\alpha}$ iff
  - {KB, $\neg\boldsymbol{\alpha}$} is unsatisfiable
  - KB $\Rightarrow \boldsymbol{\alpha}$ is valid

Logical Equivalence

- Two sentences **α** and **β** are logically equivalent – written **α ≡ β** -- iff they have the same models, i.e.:

$$\alpha \equiv \beta \text{ iff } \alpha \models \beta \text{ and } \beta \models \alpha$$

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- Examples:
  - $(\alpha \land \beta) \equiv (\beta \land \alpha)$
  - $\alpha \Rightarrow \beta \equiv \neg\alpha \lor \beta$
  - $\neg(\alpha \land \beta) \equiv \neg\alpha \lor \neg\beta$
  - $\neg(\alpha \lor \beta) \equiv \neg\alpha \land \neg\beta$

Logical Equivalence

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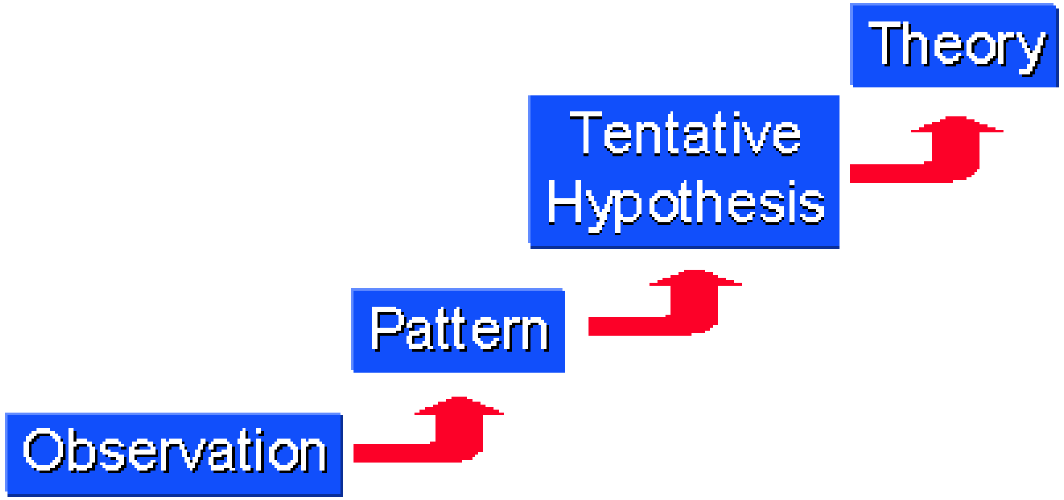
  $$\alpha \equiv \beta \text{ iff } \alpha \models \beta \text{ and } \beta \models \alpha$$

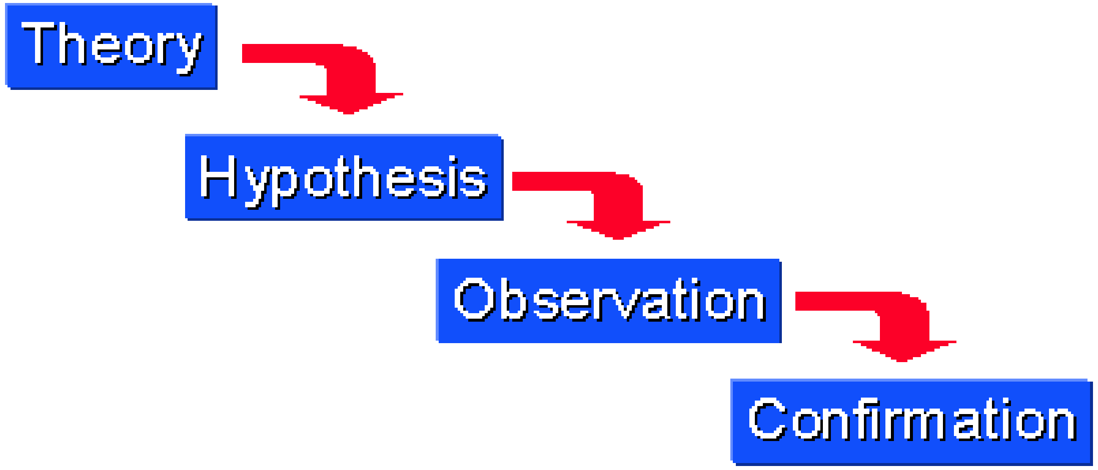
- Examples:
  - $(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$
  - $\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$
  - $\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$
  - $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$
- One can always replace a sentence by an equivalent one in a KB

Drawing conclusions

Induction



Deduction



https://conjointly.com/kb/deduction-and-induction/

Deductive reasoning

Basic form of reasoning:

- Start out with a general statement or hypothesis
- Examine the possibilities of reaching a specific, logical conclusion

Test hypotheses and theories that predict certain outcomes if they are correct

Going from the general — the theory — to the specific — the observations

Many forms, three examples:

- Syllogism
- Modus ponens
- Modus tollens

Deductive reasoning - Syllogism

Typically a three-part logical argument
Two premises are combined to arrive at a conclusion
Major premise and minor premise used to reach a logical conclusion
Common term that appears in both premises but not the conclusion
Structure:

Major premise	$\boldsymbol{p = q}$	All humans are mortal.
Minor premise	$\boldsymbol{q = r}$	Socrates is human.
Conclusion	$\therefore \boldsymbol{p = r}$	Therefore, Socrates is mortal.

Deductive reasoning - Syllogism

Conditional or hypothetical syllogism
Structure:

Major premise	$\boldsymbol{p} \rightarrow \boldsymbol{q}$	If I eat too much candy, I will get cavities.
Minor premise	$\boldsymbol{q} \rightarrow \boldsymbol{r}$	If I get cavities, I must go to the dentist.
Conclusion	$\therefore \boldsymbol{p} \rightarrow \boldsymbol{r}$	Therefore, if I eat too much candy, I must go to the dentist.

Deductive reasoning - Modus Ponens

First premise is a conditional statement
Second statement affirms that the first part of the conditional statement applies
Deduce that the second part of the first statement applies
Structure:

$\boldsymbol{p \rightarrow q}$	If a body of water freezes at -1° C, then it is freshwater
$\boldsymbol{p}$	A body of water froze at -1° C
$\therefore \boldsymbol{q}$	The body of water is freshwater

Deductive reasoning - Modus Tollens

The opposite of modus ponens
The second premise negates the first part of the conditional
Deduce that the first part of the conditional does not apply
Law of the contrapositive
Structure:

$p \rightarrow q$	If a body of water is freshwater, then it freezes at -1° C
$\neg q$	A body of water did not freeze at -1° C
$\therefore \neg p$	The body of water is not freshwater

Deductive reasoning

Advantages:

Airtight form of inference in the appropriate context, such as a mathematical proof

Flaws:

Relies on strong assumptions about how the world works

The premises must be true

Limited range of applicability

All men are mortal. My dog is mortal. Therefore, my dog is a man.

1. Some nice people are teachers.
2. Some people with red hair are nice.
3. Therefore, some teachers have red hair.

1. Some trees are tall things.
2. Some tall things are buildings.
3. Therefore, some trees are buildings.

Inductive reasoning

Extracts a likely premise from specific and limited observations

Inductive logic:

There is data and conclusions are drawn from it

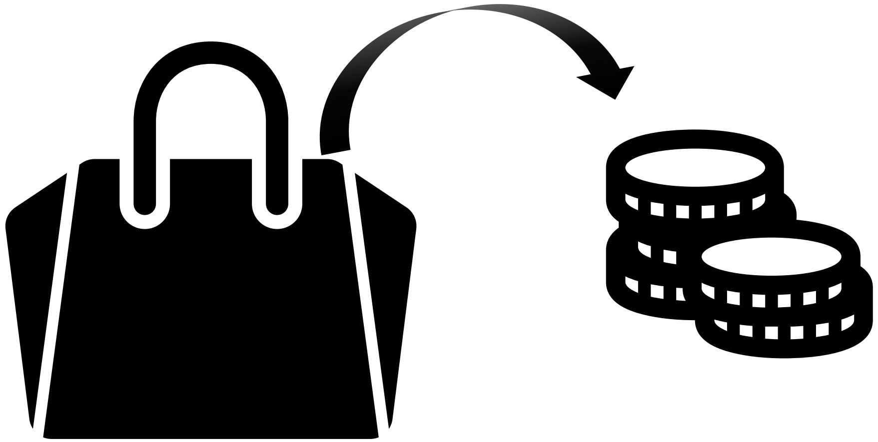
Go from the specific to the general

1. Make many observations
2. Find a pattern
3. Make a generalization
4. Infer an explanation or theory

Inductive reasoning

Conclusions made with inductive logic depend on the completeness of the observations:

The conclusions are only as reliable as the observations



- Three coins drawn from a purse, all pennies
- Therefore, the only coins in the purse are pennies
- Inductive reasoning does not guarantee that the conclusion will be true

Inductive reasoning – Generalization

Draw conclusions based on recurring patterns or repeated observations

Data: Every dog I meet is friendly.

Conclusion: Dogs are usually friendly.

Data: I tend to catch colds when people around me are sick.

Conclusion: Colds are infectious.

Problems:

- Over-generalization
- Stereotyping

Disproved by providing contradictory evidence or examples

Inductive reasoning – Causal

Seeks to make cause-effect connections
Used unconsciously all the time

Everything outside is wet in the morning. It must have rained in the night.

Two tests of causal reasoning:

- The cause must have a *direct* relationship on the effect
- The cause must be *strong* enough to make the effect

Correlations are *not* causal

1. All of my white clothes turn pink when I put a red cloth in the washing machine with them.
2. My white clothes don’t turn pink when I wash them on their own.
3. Putting colorful clothes with light colors **causes** the colors to run and stain the light-colored clothes.

1. Every swan I’ve seen is white
2. All swans are white

Inference Rule

- An inference rule $\{\xi, \psi\} \vdash \phi$ consists of 2 sentence patterns ξ and ψ called the conditions and one sentence pattern ϕ called the conclusion

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- An inference rule $\{\xi, \psi\} \vdash \phi$ consists of 2 sentence patterns ξ and ψ called the conditions and one sentence pattern ϕ called the conclusion
- If ξ and ψ match two sentences of KB then the corresponding ϕ can be inferred according to the rule

Example: Modus Ponens

$$\{\ \alpha \Rightarrow \beta\ ,\ \alpha\ \}\ \vdash\ \beta$$

$$\{\xi,\ \psi\}\ \vdash \phi$$

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$$\{\xi,\ \psi\}\ \vdash \phi$$

Battery-OK ∧Bulbs-OK⇒Headlights-Work

Battery-OK ∧Starter-OK ∧¬Empty-Gas-Tank⇒Engine-Starts

Engine-Starts ∧¬Flat-Tire⇒Car-OK

Battery-OK ∧Bulbs-OK

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Example: Modus Ponens

$$\{\ \alpha \Rightarrow \beta\ ,\ \alpha\ \}\ \vdash\ \beta$$

$$\{\xi,\ \psi\}\ \vdash \phi$$

$\boxed{\text{Battery-OK} \land \text{Bulbs-OK}} \Rightarrow \boxed{\text{Headlights-Work}}$

Battery-OK $\land$ Starter-OK $\land \neg$ Empty-Gas-Tank $\Rightarrow$ Engine-Starts

Engine-Starts $\land \neg$ Flat-Tire $\Rightarrow$ Car-OK

Battery-OK $\land$ Bulbs-OK

Example: Modus Ponens

$$\{ \alpha \Rightarrow \beta , \alpha \} \vdash \beta$$

$$\{\xi, \psi\} \vdash \phi$$

Battery-OK $\wedge$ Bulbs-OK $\Rightarrow$ Headlights-Work

Battery-OK $\wedge$ Starter-OK $\wedge \neg$ Empty-Gas-Tank $\Rightarrow$ Engine-Starts

Engine-Starts $\wedge \neg$ Flat-Tire $\Rightarrow$ Car-OK

Battery-OK $\wedge$ Bulbs-OK

Headlights-Work

Example: Modus Tolens

$$\{\ \alpha \Rightarrow \beta\ ,\ \neg\beta\ \}\quad \vdash\ \neg\alpha$$

Engine-Starts $\wedge\neg$Flat-Tire $\Rightarrow$ Car-OK
$\neg$Car-OK

Example: Modus Tolens

$$\{\ \alpha \Rightarrow \beta\ ,\ \neg\beta\ \}\quad \vdash\ \neg\alpha$$

Engine-Starts ∧¬Flat-Tire ⇒ Car-OK

¬Car-OK

¬(Engine-Starts ∧¬Flat-Tire)

Example: Modus Tolens

$$\{\ \alpha \Rightarrow \beta\ ,\ \neg\beta\ \} \quad \vdash\ \neg\alpha$$

Engine-Starts ∧¬Flat-Tire⇒Car-OK
¬Car-OK
¬(Engine-Starts ∧¬Flat-Tire) ≡ ¬Engine-Starts ∨ Flat-Tire

Other Examples

- $\{\alpha,\beta\} \vdash \alpha \wedge \beta$
- $\{\alpha\wedge\beta,.\} \vdash \alpha$
- $\{\alpha\wedge\beta,.\} \vdash \beta$
- Etc …

Inference

- I: Set of inference rules
- KB: Set of sentences
- Inference is the process of applying successive inference rules from I to KB, each rule adding its conclusion to KB

Example

1. Battery-OK $\wedge$ Bulbs-OK $\Rightarrow$ Headlights-Work
2. Battery-OK $\wedge$ Starter-OK $\wedge \neg$ Empty-Gas-Tank $\Rightarrow$ Engine-Starts
3. Engine-Starts $\wedge \neg$ Flat-Tire $\Rightarrow$ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. $\neg$Empty-Gas-Tank
8. $\neg$Car-OK

Example

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3. Engine-Starts $\wedge \neg$ Flat-Tire $\Rightarrow$ Car-OK
4. Headlights-Work
5. Battery-OK
6. Starter-OK
7. $\neg$Empty-Gas-Tank
8. $\neg$Car-OK
9. Battery-OK $\wedge$ Starter-OK $\leftarrow$ (5+6)

Example

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7. $\neg$Empty-Gas-Tank
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9. Battery-OK $\wedge$ Starter-OK $\leftarrow$ (5+6)
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9. Battery-OK $\wedge$ Starter-OK $\leftarrow$ (5+6)
10. Battery-OK $\wedge$ Starter-OK $\wedge \neg$ Empty-Gas-Tank $\leftarrow$ (9+7)
11. Engine-Starts $\leftarrow$ (2+10)

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11. Engine-Starts $\leftarrow$ (2+10)
12. $\neg$Engine-Starts $\vee$ Flat-Tire $\leftarrow$ (3+8)

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9. Battery-OK $\wedge$ Starter-OK ← (5+6)
10. Battery-OK $\wedge$ Starter-OK $\wedge \neg$ Empty-Gas-Tank ← (9+7)
11. Engine-Starts ← (2+10)
12. Engine-Starts $\Rightarrow$ Flat-Tire ← (3+8)
13. Flat-Tire ← (11+12)

Soundness

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- All inference rules previously given are sound, e.g.:
  modus ponens: $\{\alpha \Rightarrow \beta\ ,\ \alpha\} \ \vdash \beta$
- The following rule:

  $$\{\alpha \vee \beta\ ,\ .\} \ \vdash \ \neg\alpha \vee \neg\beta$$

  …is unsound

Completeness

- A set of inference rules is complete if every entailed sentences can be obtained by applying some finite succession of these rules
- Modus ponens alone is not complete, e.g.:
  from $A \Rightarrow B$ and $\neg B$, we cannot get $\neg A$
  (needed Modus Tolens for that)

Proof

The proof of a sentence **α** from a set of sentences KB is the derivation of **α** by applying a series of sound inference rules

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12. Engine-Starts $\Rightarrow$ Flat-Tire $\leftarrow$ (3+8)
13. Flat-Tire $\leftarrow$ (11+12)

Summary

- Knowledge representation
- Propositional Logic
- Truth tables
- Model of a KB
- Satisfiability of a KB
- Logical entailment
- Inference rules
- Proof