Logical agents

Chapter 7

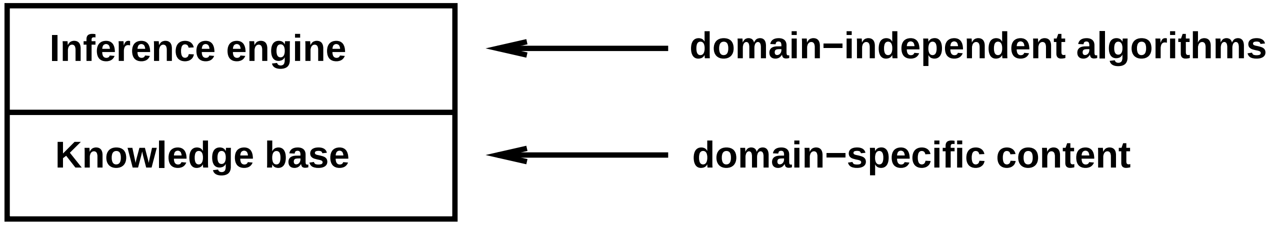
Outline

◇ Knowledge-based agents

◇ Wumpus world

◇ Inference rules and theorem proving
  – forward chaining
  – backward chaining
  – resolution

Knowledge bases



Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):

  TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

  i.e., **what they know**, regardless of how implemented

Or at the implementation level

  i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

```
function KB-AGENT( percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

The agent must be able to:

Represent states, actions, etc.

Incorporate new percepts

Update internal representations of the world

Deduce hidden properties of the world

Deduce appropriate actions

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

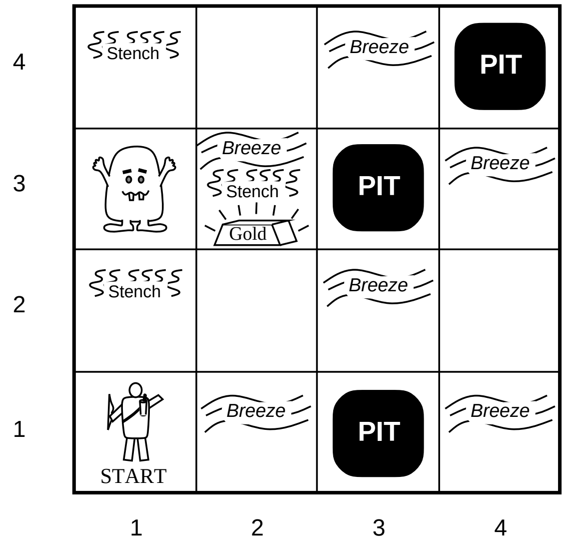
Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square



Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell

Wumpus world characterization

Observable??

Wumpus world characterization

Observable?? No—only local perception

Deterministic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic??

Wumpus world characterization

Observable?? No—only local perception

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Episodic?? No—sequential at the level of actions

Static??

Wumpus world characterization

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Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

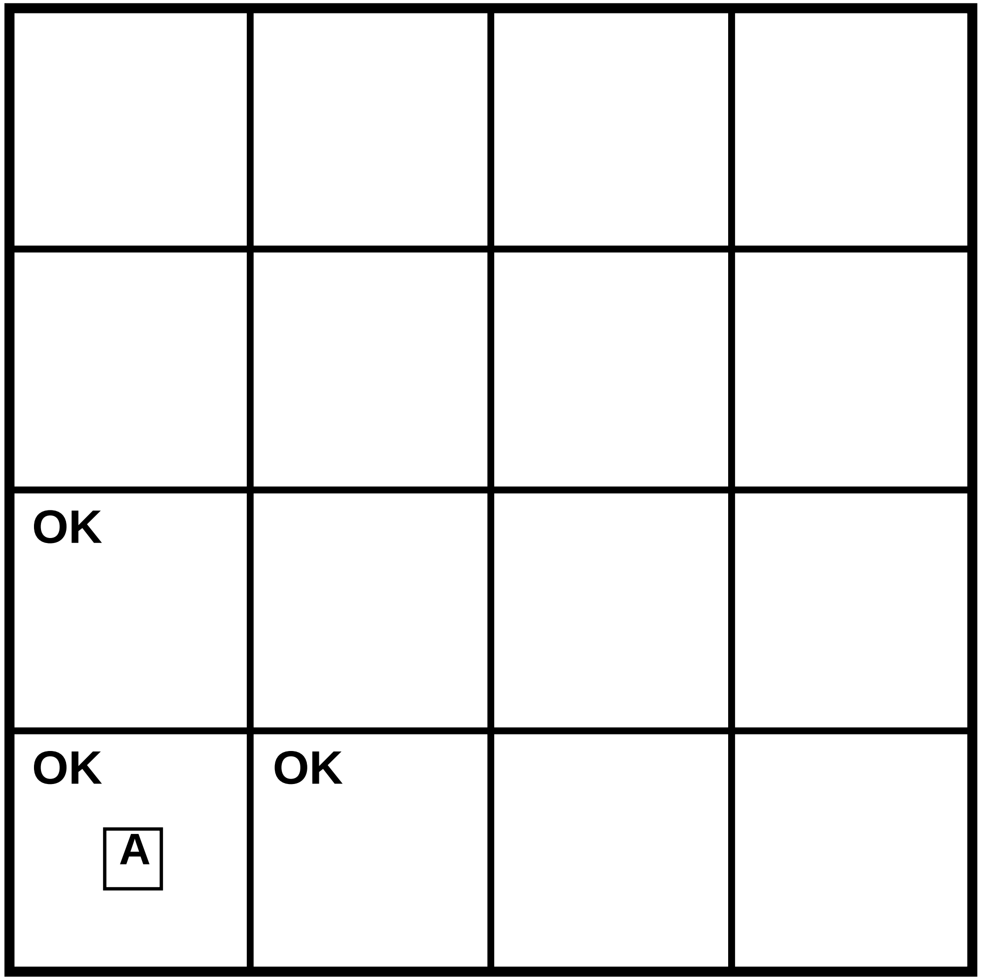
Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



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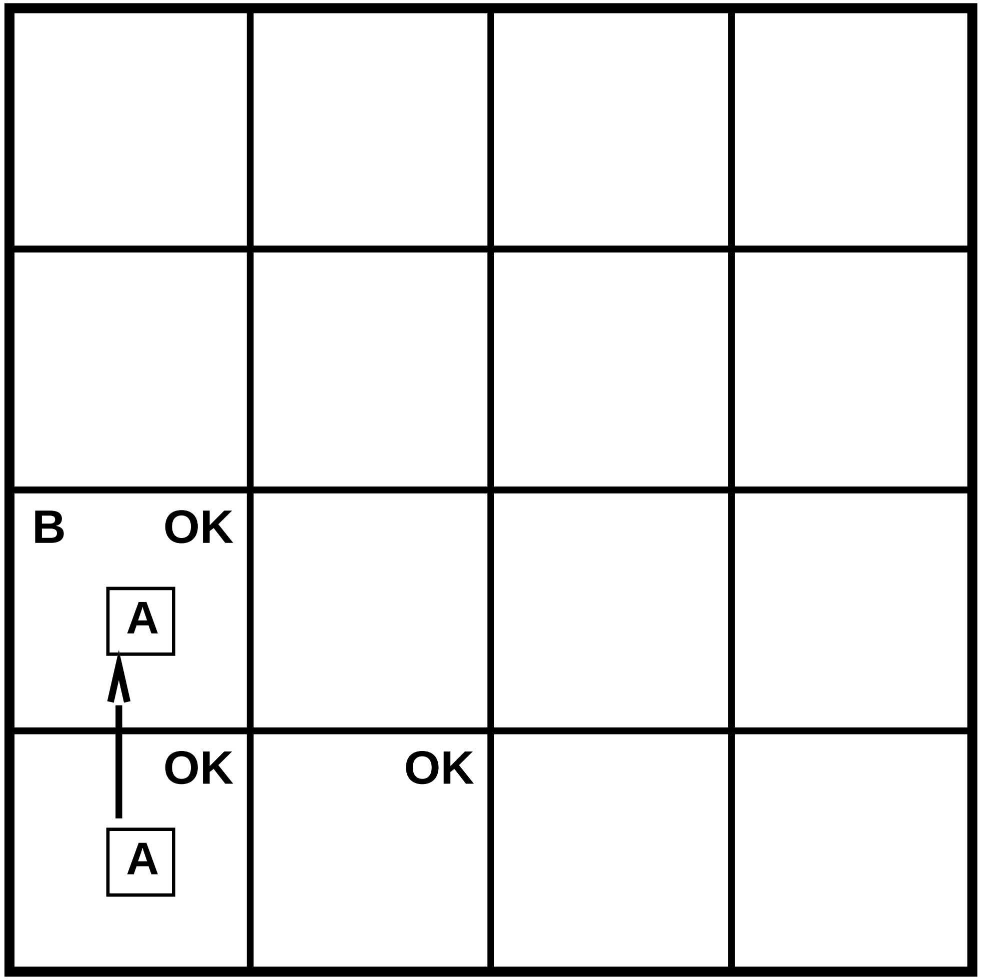

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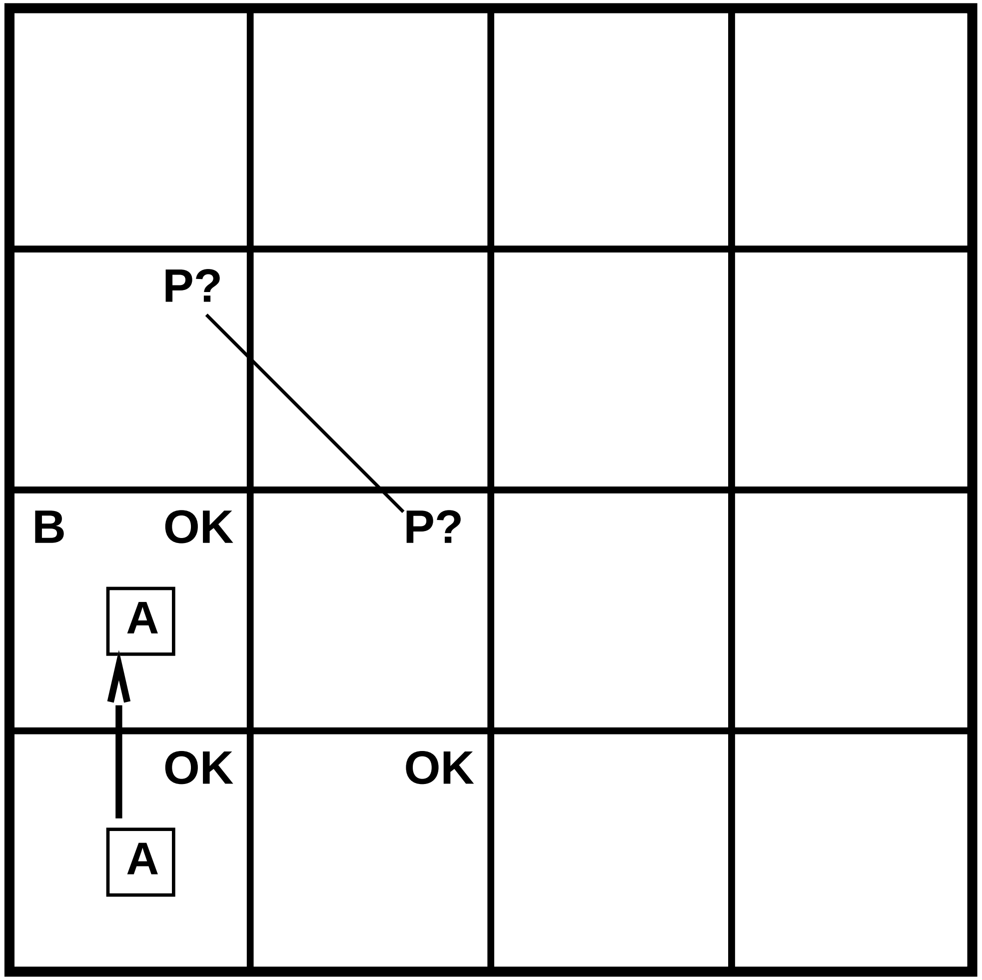

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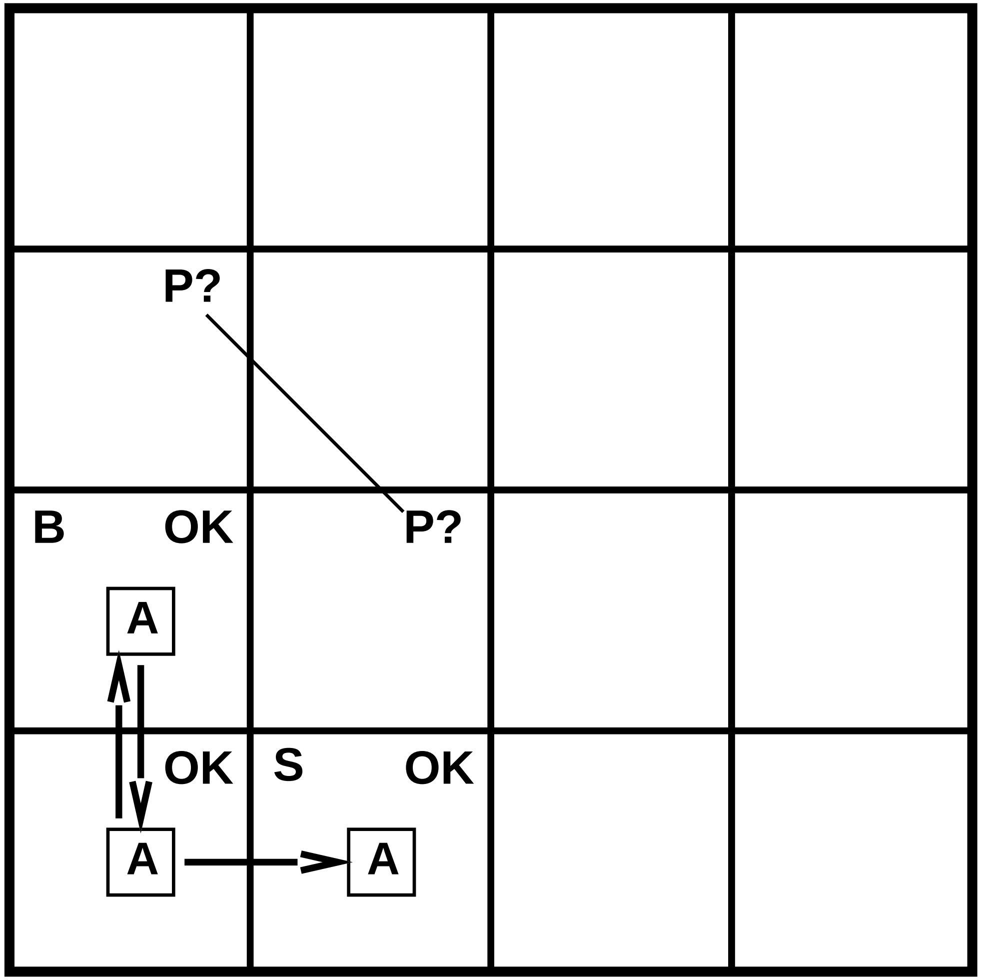

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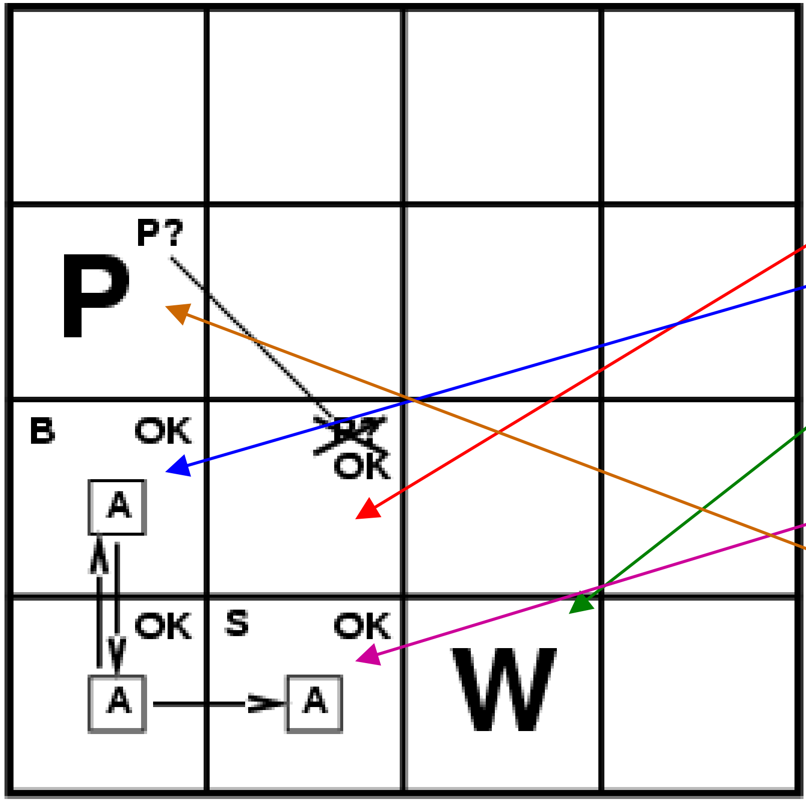
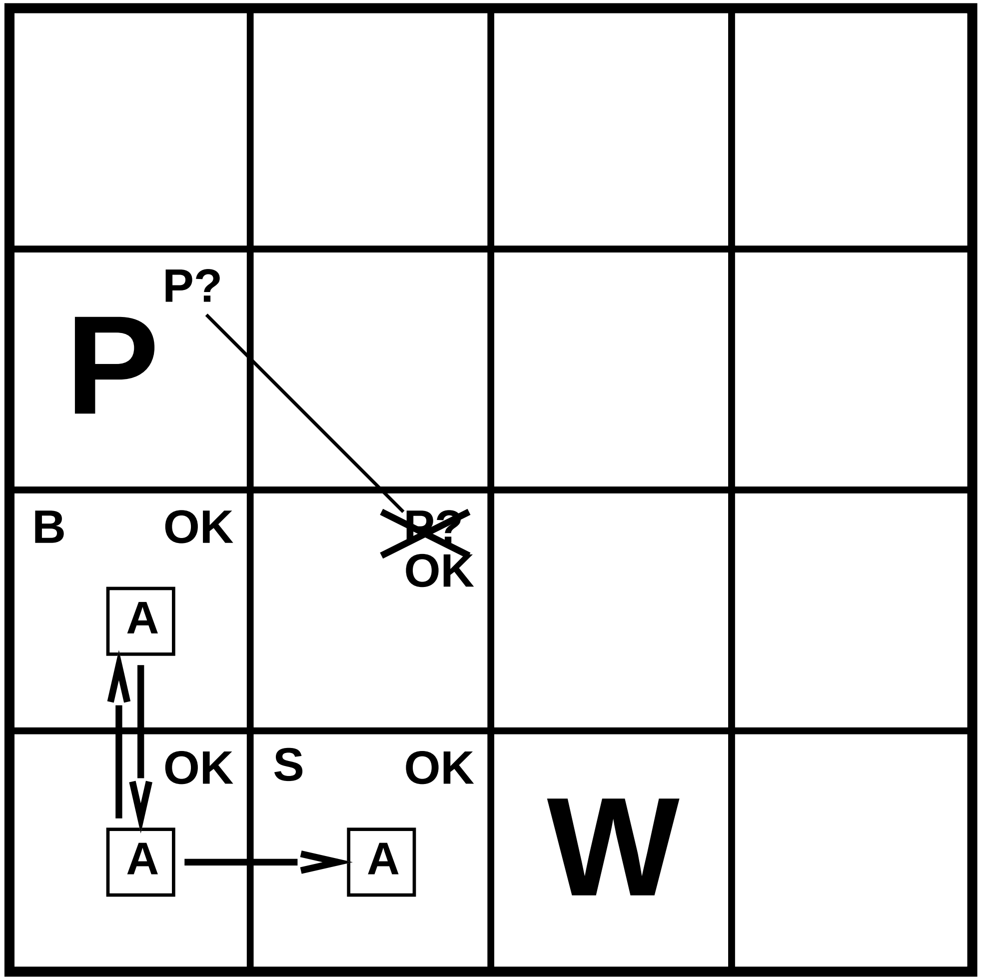


If the Wumpus were here, stench should be here. Therefore it is here.
Since, there is no breeze here, the pit must be there

We need rather sophisticated reasoning here!

Exploring a wumpus world



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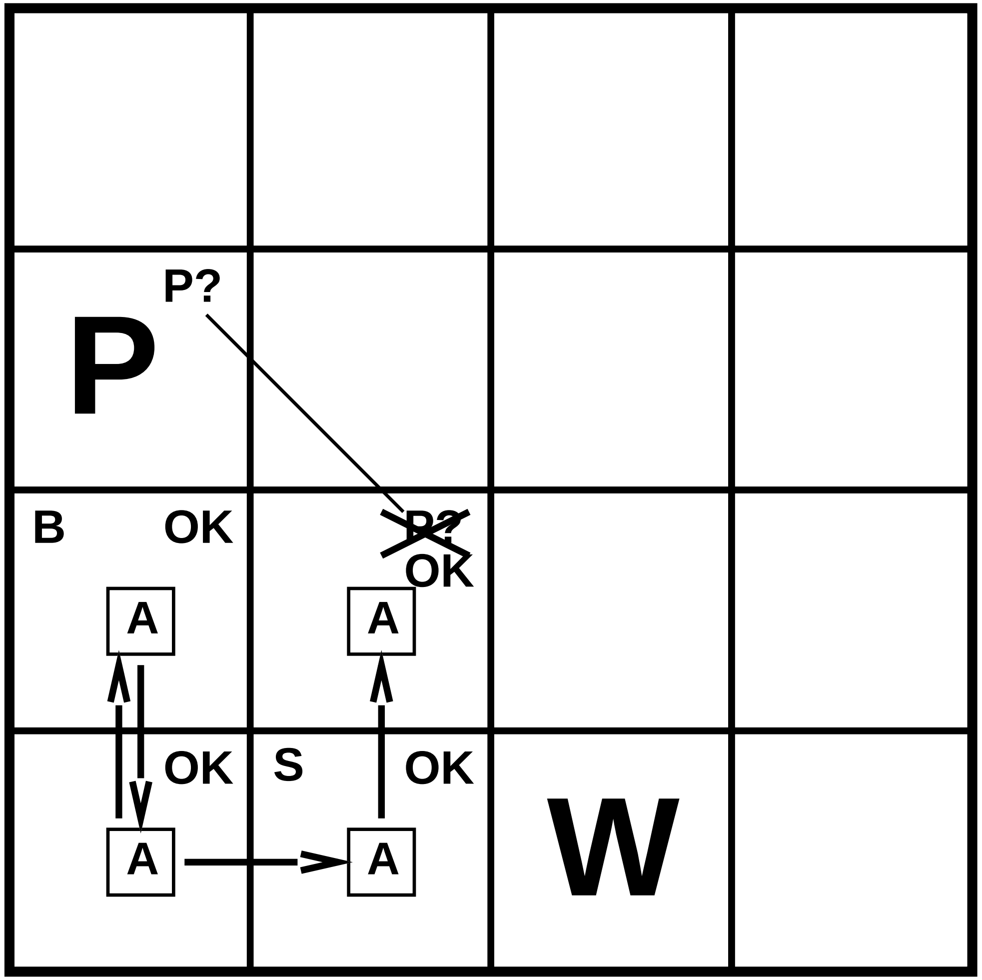

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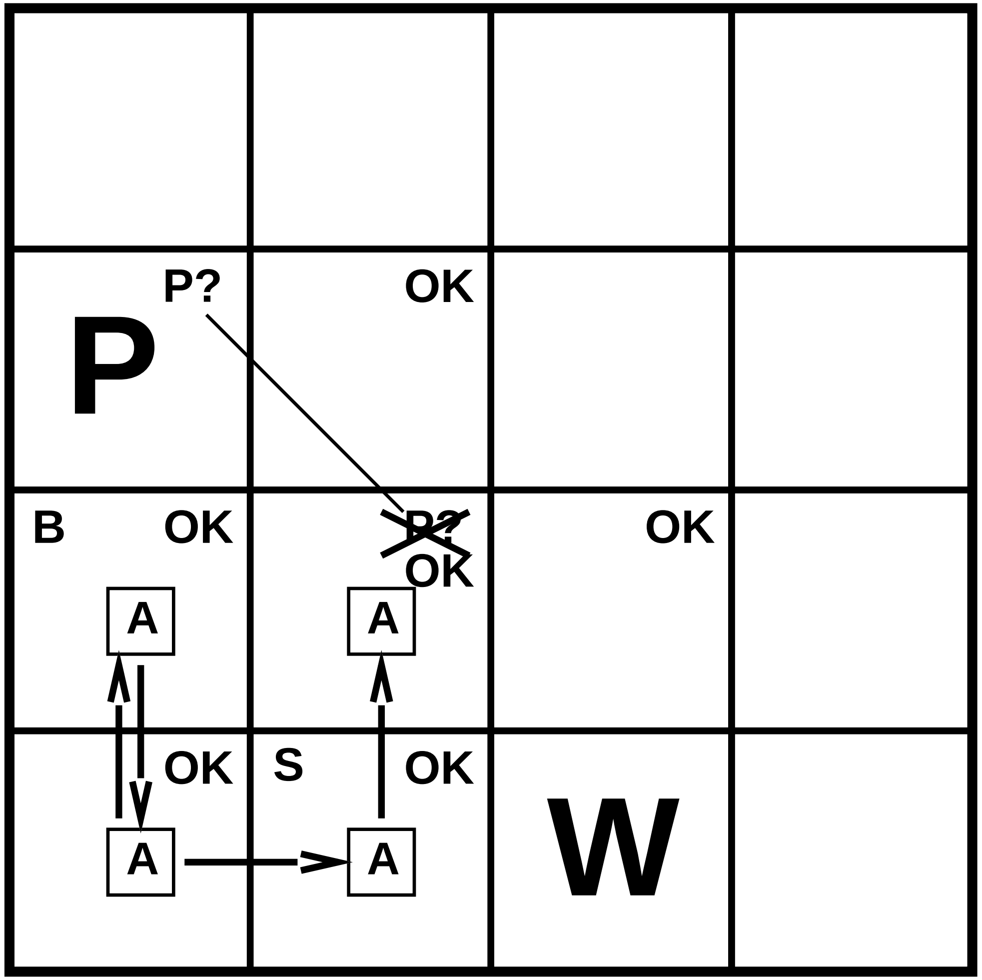

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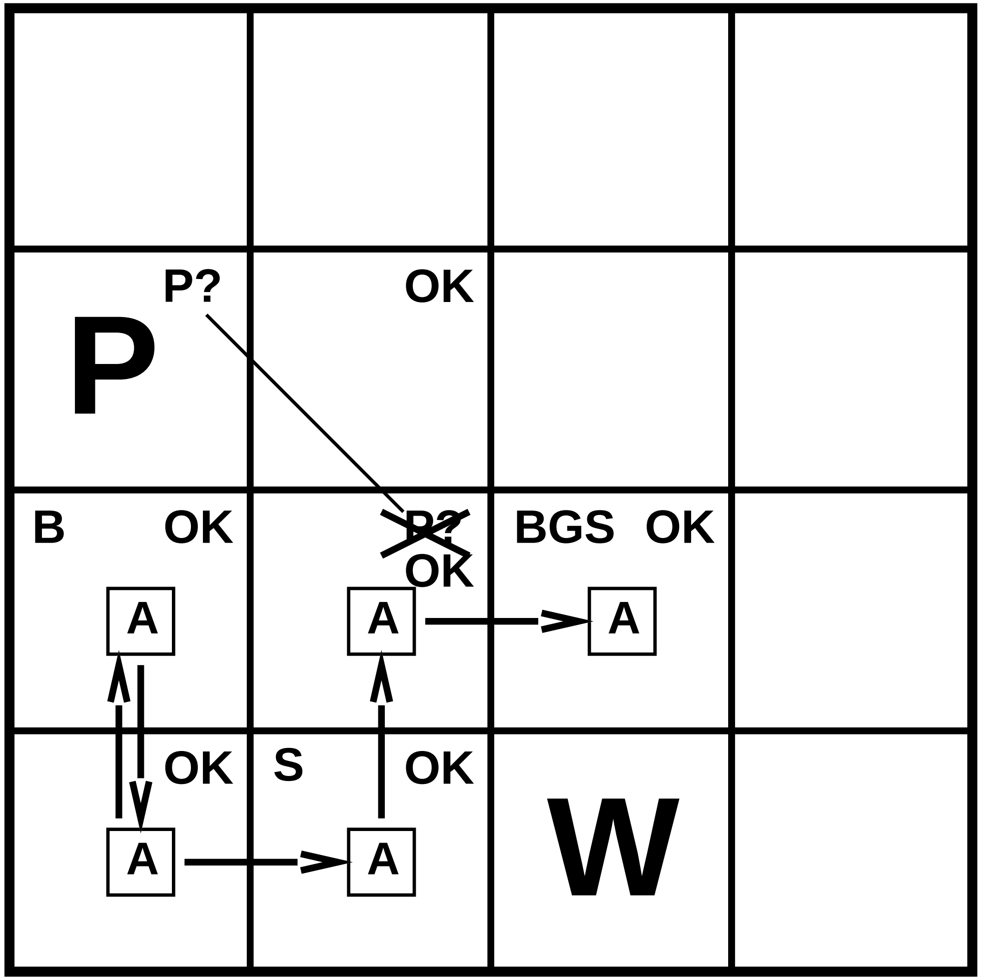
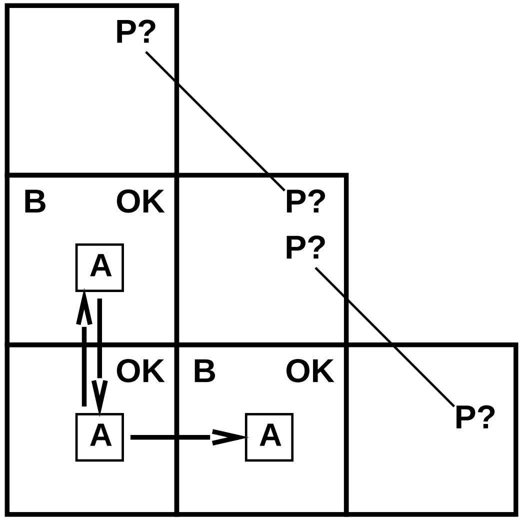
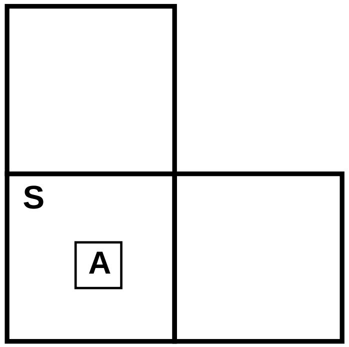


Other tight spots



Breeze in (1,2) and (2,1)
$\Rightarrow$ no safe actions

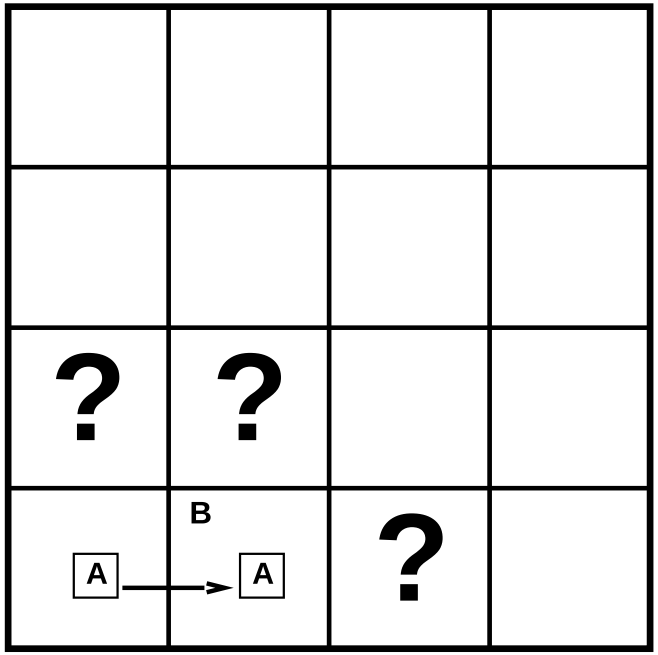
Assuming pits uniformly distributed,
(2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1)
$\Rightarrow$ cannot move

Can use a strategy of coercion:
shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Entailment in the wumpus world

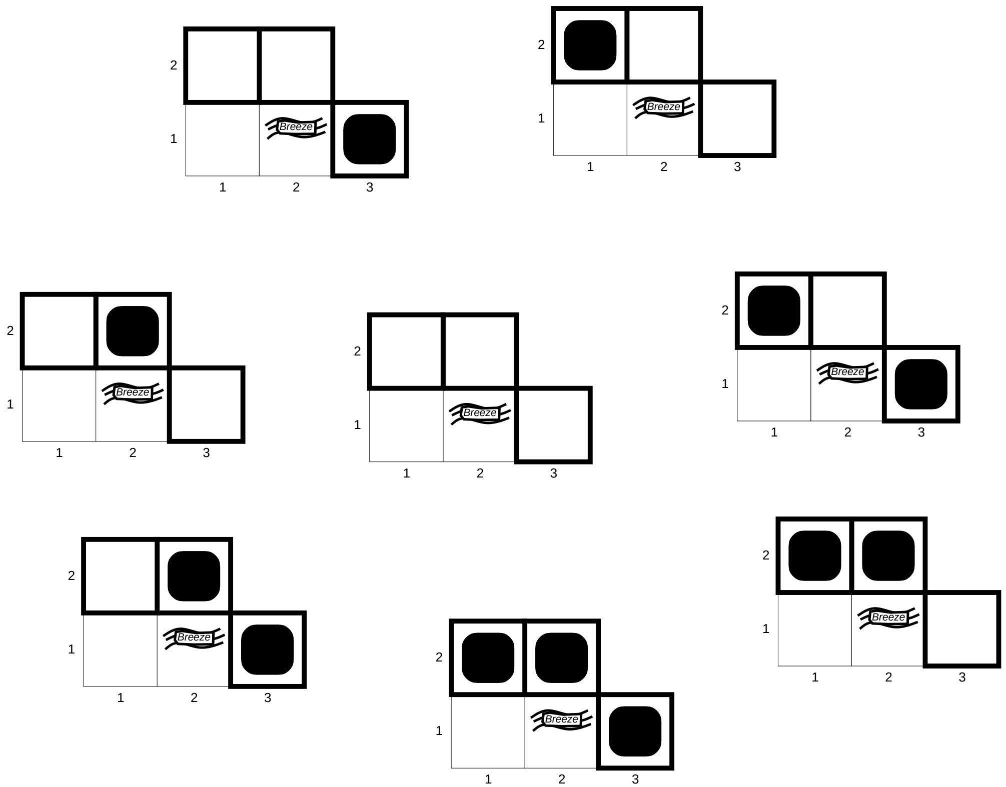


Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

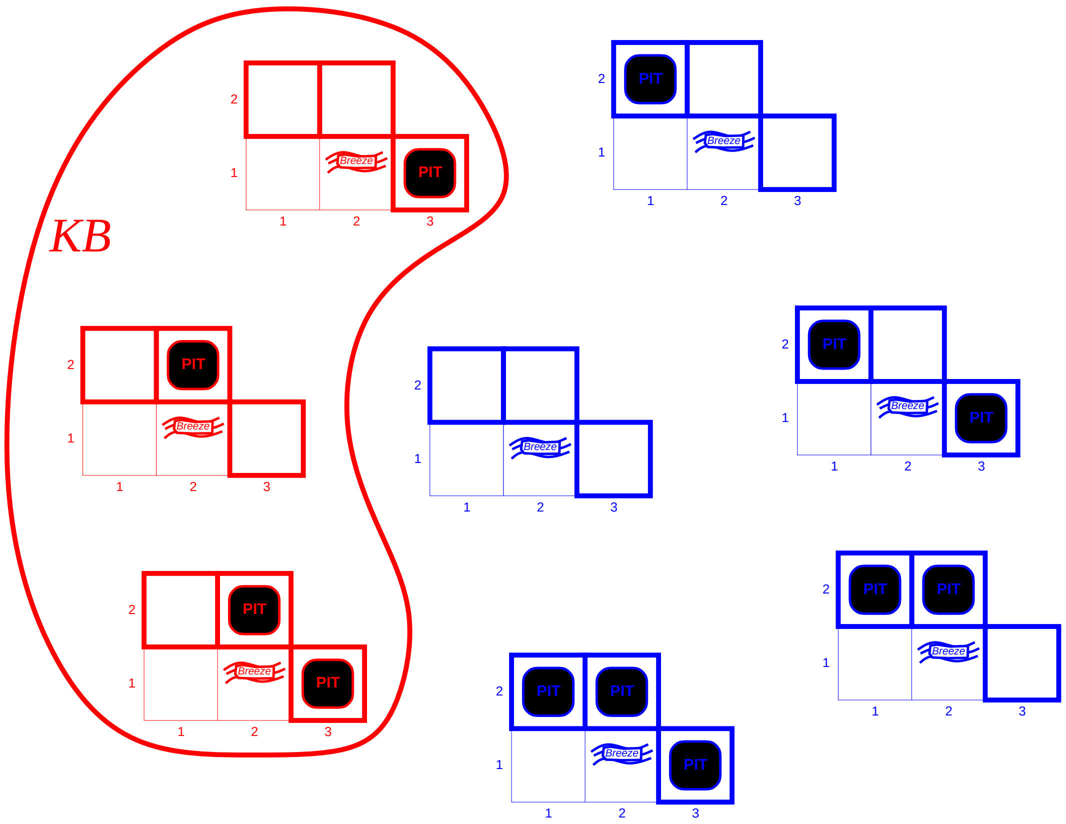
Consider possible models for ?s assuming only pits

3 Boolean choices $\Rightarrow$ 8 possible models

Wumpus models

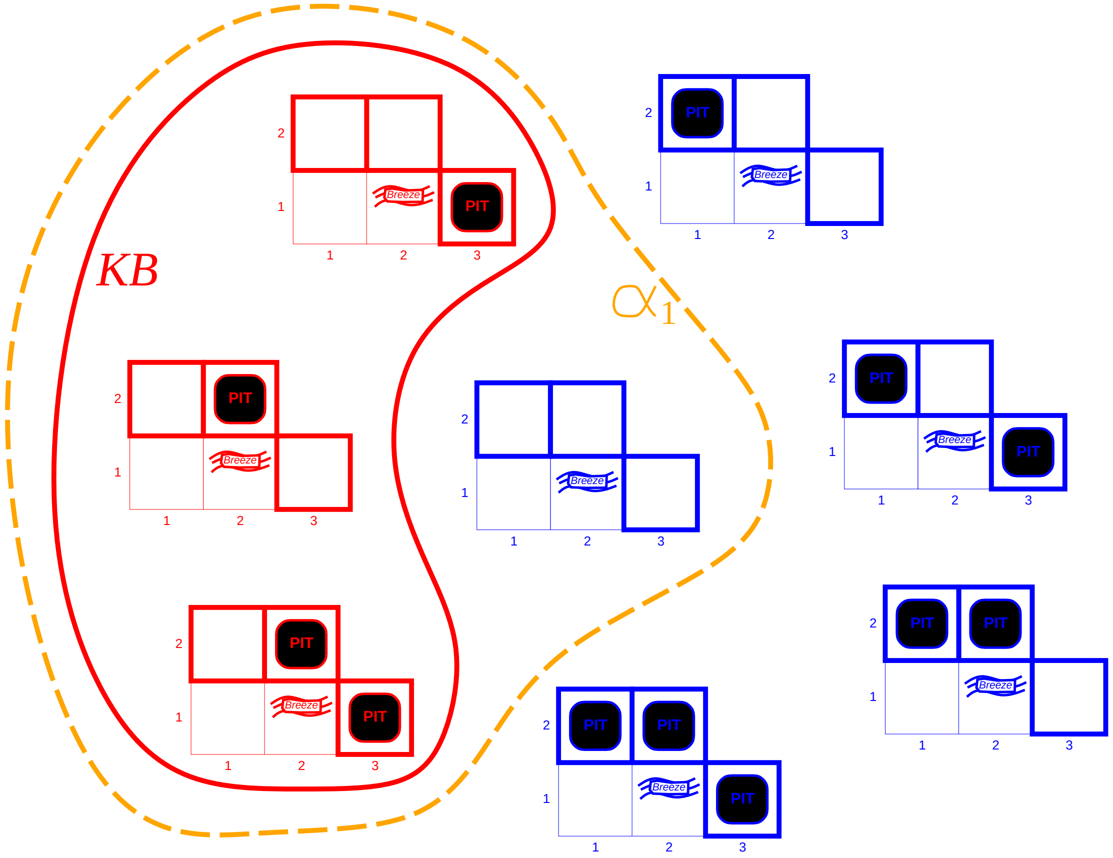


Wumpus models



KB = wumpus-world rules + observations

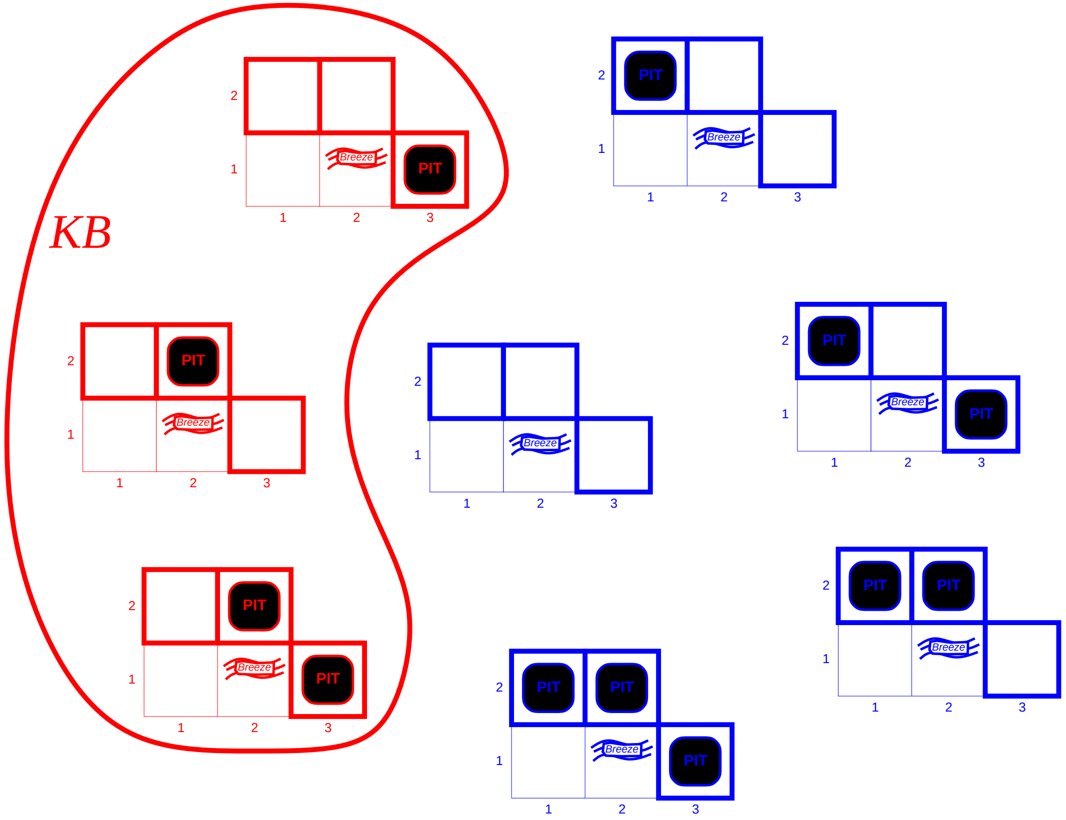
Wumpus models



KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking

Wumpus models



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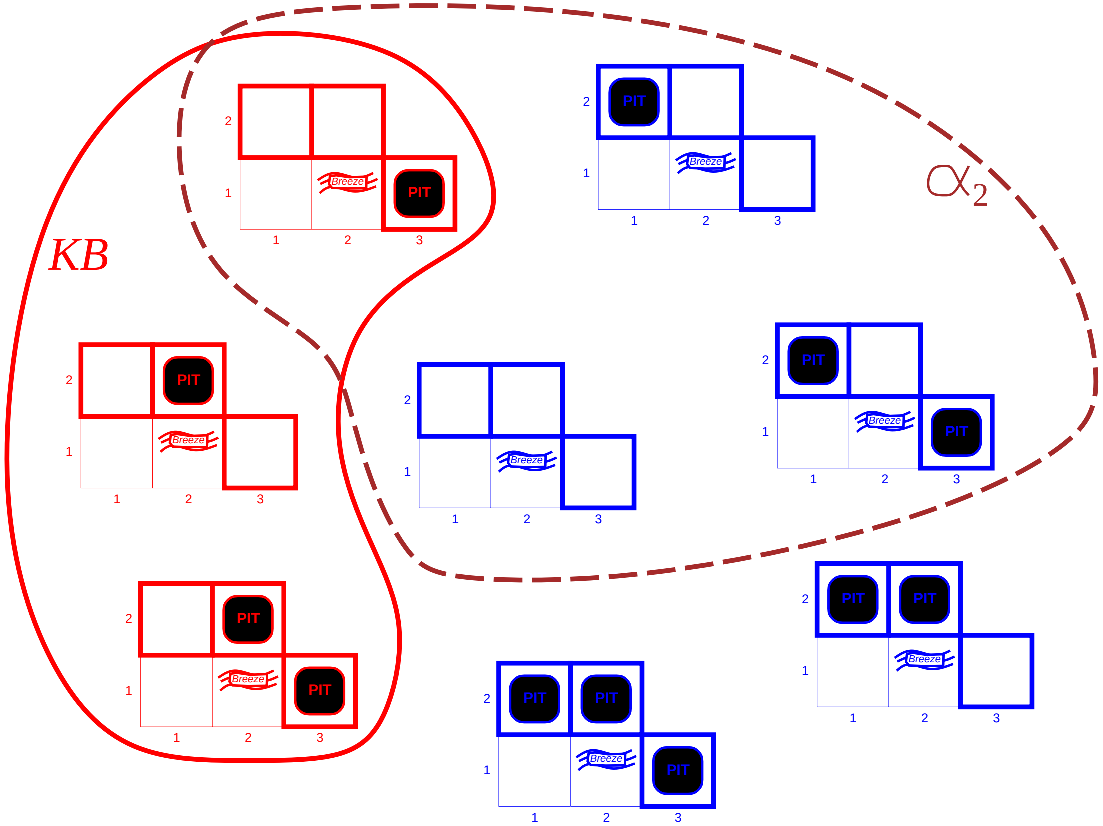

KB = wumpus-world rules + observations

α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

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Implicit information: "There is no other reason for breeze than a pit in an adjacent square."

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"Pits cause breezes in adjacent squares"
Implicit information: "There is no other reason for breeze than a pit in an adjacent square."

$$B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \lor P_{2,1})$$
$$B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

"A square is breezy **if and only if** there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

R_1: not $P_{1,1}$

R_2: $B_{1,1}$ iff ($P_{1,2}$ or $P_{2,1}$)

R_3: $B_{2,1}$ iff ($P_{1,1}$ or $P_{2,2}$ or $P_{3,1}$)

R_4: not $B_{1,1}$

R_5: $B_{2,1}$

Inference by enumeration

Depth-first enumeration of all models is sound and complete.

function TT-ENTAILS?(KB, α) **returns** *true* or *false*
    inputs: KB, the knowledge base, a sentence in propositional logic
            α, the query, a sentence in propositional logic

    $symbols \leftarrow$ a list of the proposition symbols in KB and α
    return TT-CHECK-ALL($KB, \alpha, symbols, [\,]$)

---

function TT-CHECK-ALL($KB, \alpha, symbols, model$) **returns** *true* or *false*
    if EMPTY?($symbols$) **then**
        if PL-TRUE?($KB, model$) **then return** PL-TRUE?($\alpha, model$)
        else return *true*
    else do
        $P \leftarrow$ FIRST($symbols$); $rest \leftarrow$ REST($symbols$)
        return TT-CHECK-ALL($KB, \alpha, rest$, EXTEND($P, true, model$)) **and**
                    TT-CHECK-ALL($KB, \alpha, rest$, EXTEND($P, false, model$))

$O(2^n)$ for n symbols; problem is **co-NP-complete**

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules
- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications

  Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in n)

improved backtracking, e.g., Davis–Putnam–Logemann–Loveland

heuristic search in model space (sound but incomplete)

e.g., min-conflicts-like hill-climbing algorithms

Forward and backward chaining

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- ♢ proposition symbol; or
- ♢ (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C, (B \Rightarrow A), (C \land D \Rightarrow B)$

$\Rightarrow$ Every set of Horn clauses has at least one model.

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \land \cdots \land \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in **linear** time.

Forward chaining algorithm

function PL-FC-ENTAILS?(KB, q) **returns** $true$ or $false$
  inputs: KB, the knowledge base, a set of propositional Horn clauses
    q, the query, a proposition symbol
  local variables: $count$, a table, indexed by clause, initially the number of premises
    $inferred$, a table, indexed by symbol, each entry initially $false$
    $agenda$, a list of symbols, initially the symbols known in KB

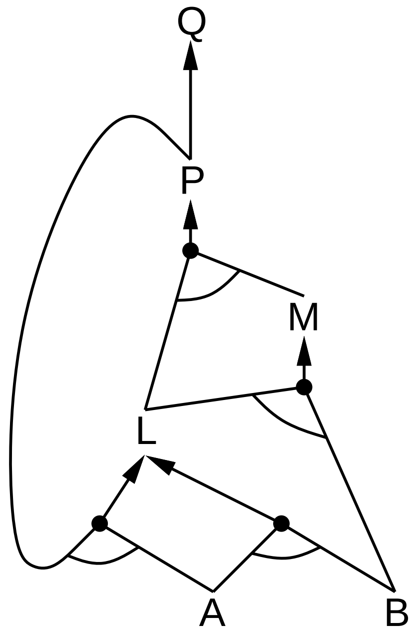
  while $agenda$ is not empty **do**
    $p \leftarrow$ POP($agenda$)
    unless $inferred[p]$ **do**
      $inferred[p] \leftarrow true$
      for each Horn clause c in whose premise p appears **do**
        decrement $count[c]$
        if $count[c] = 0$ **then do**
          add c.Conclusion to agenda

  return $false$

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining example

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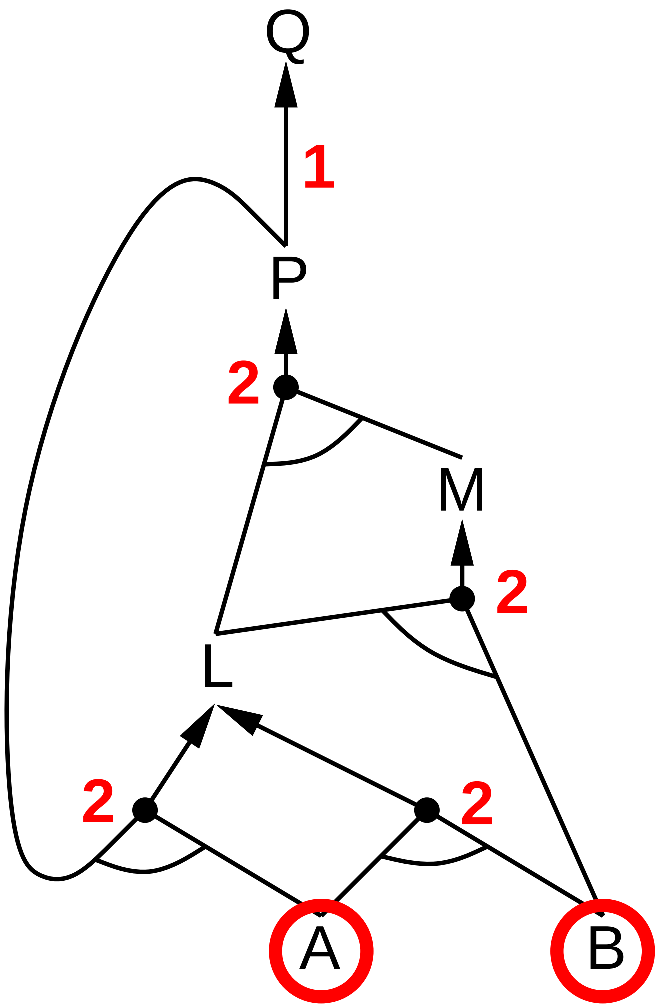

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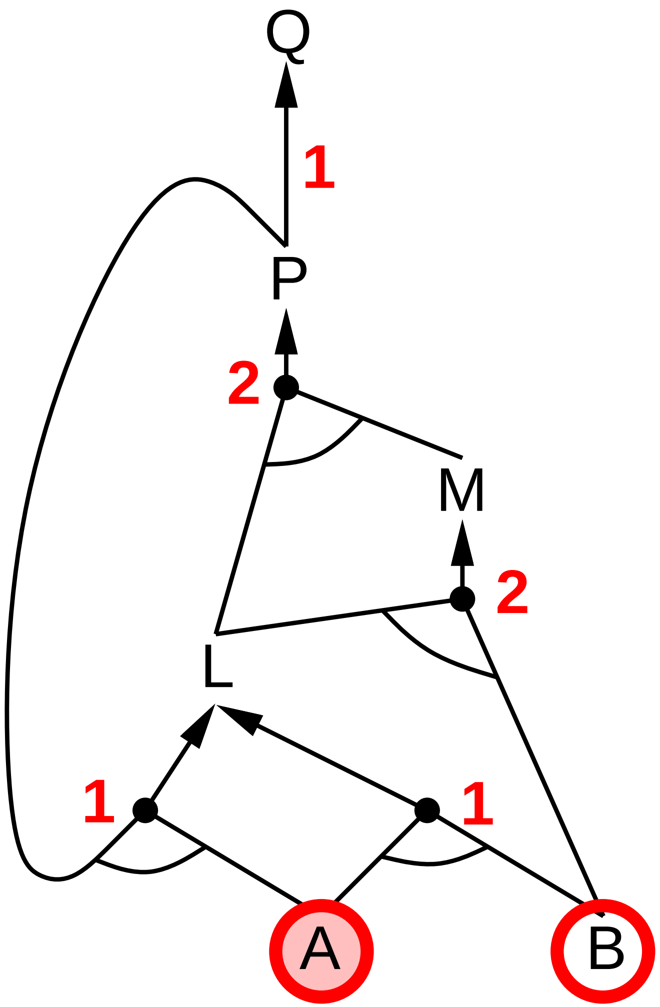

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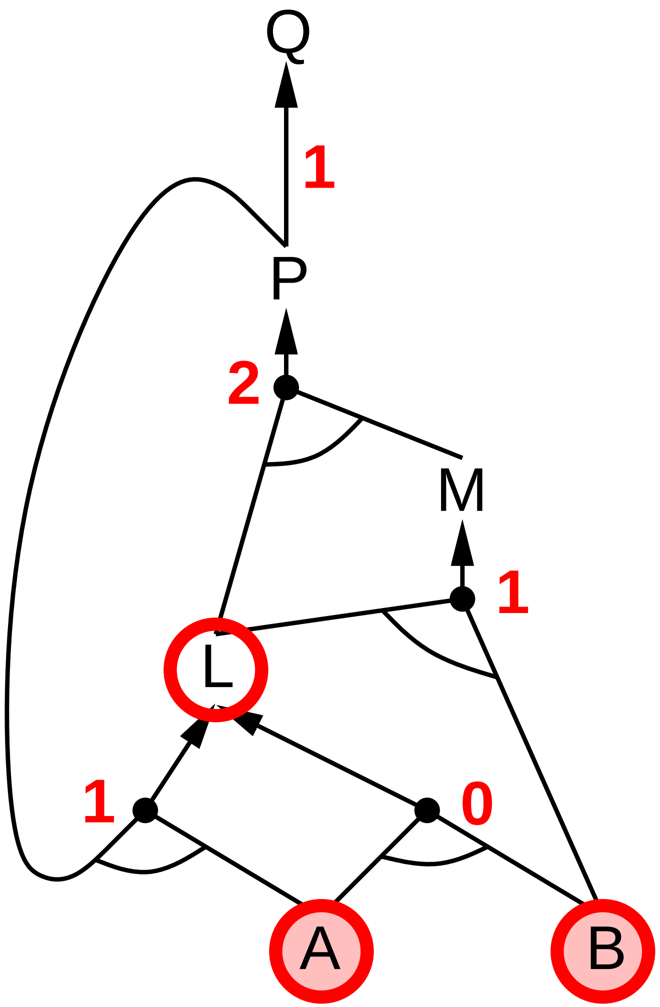

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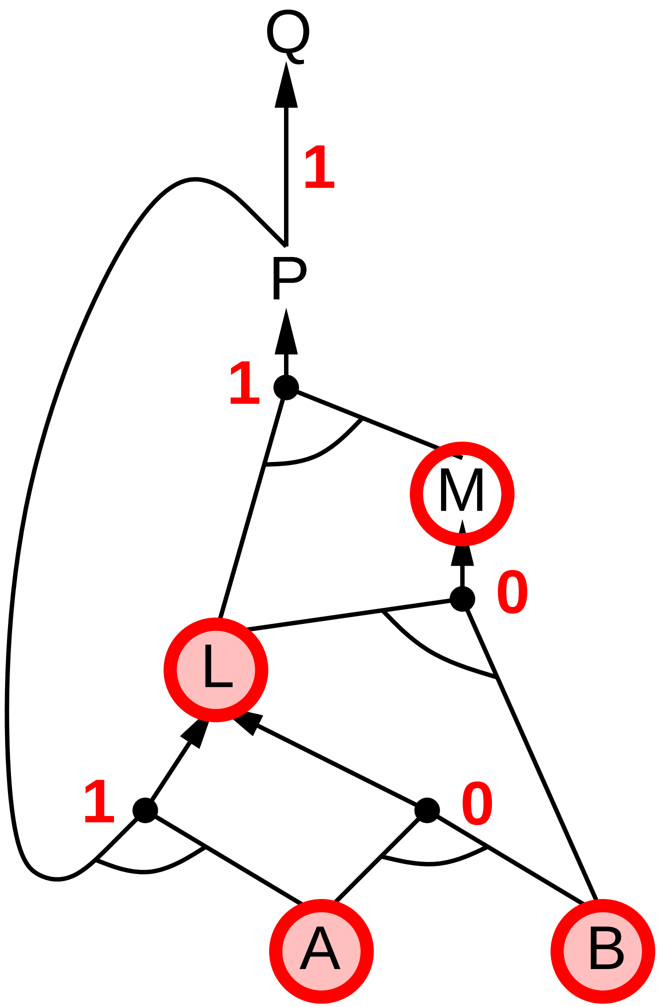

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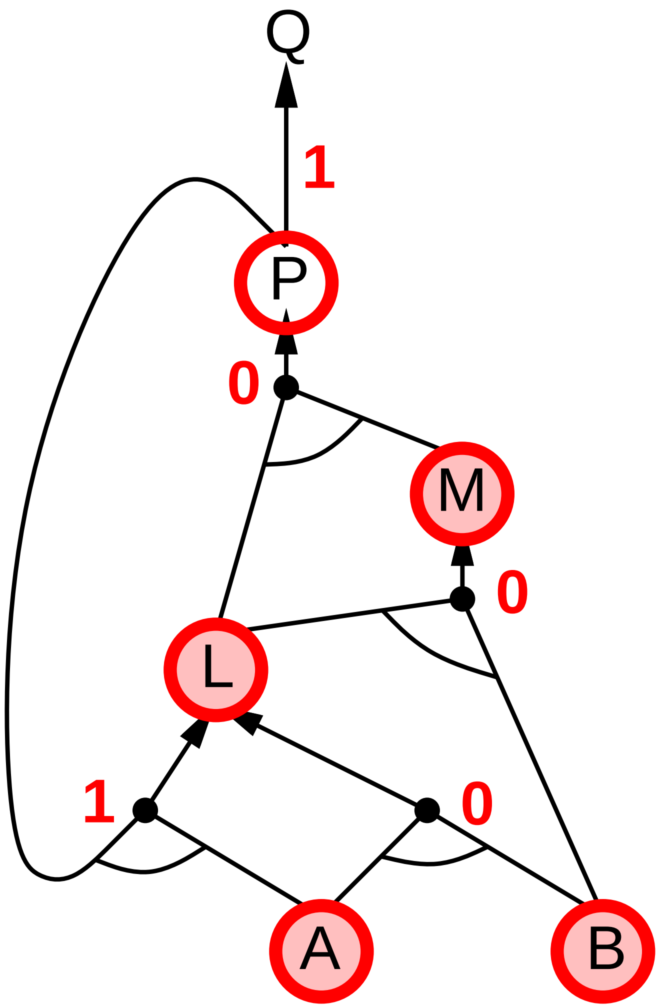

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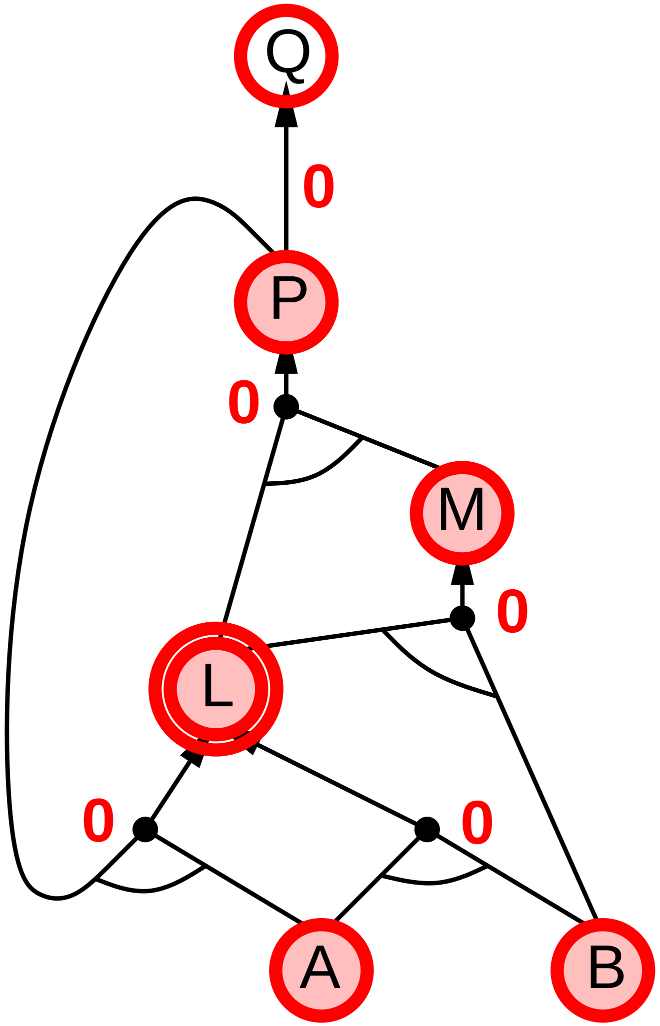

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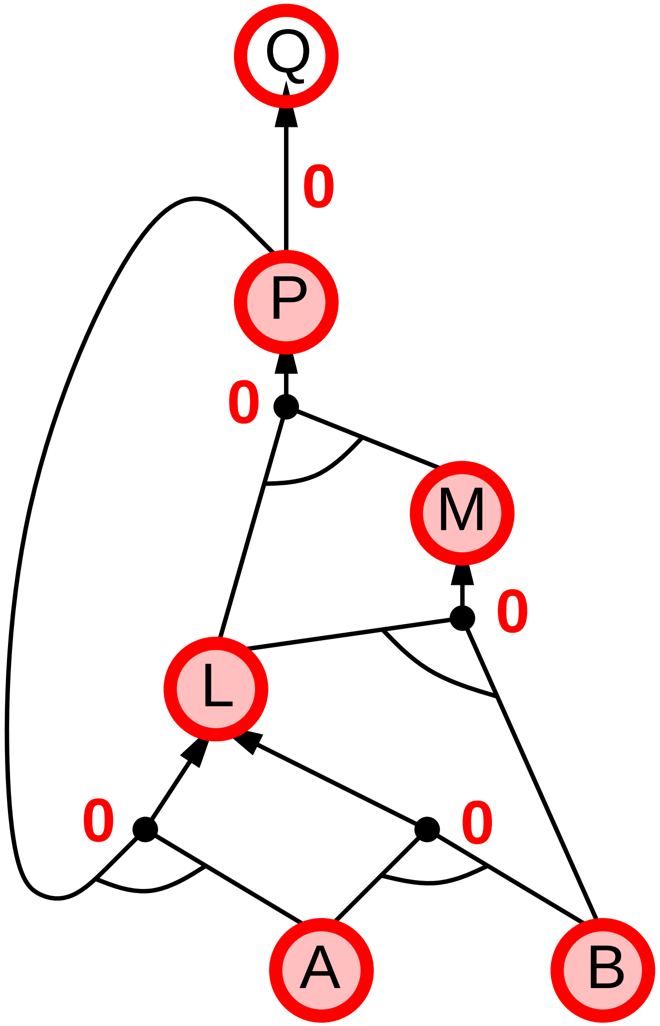

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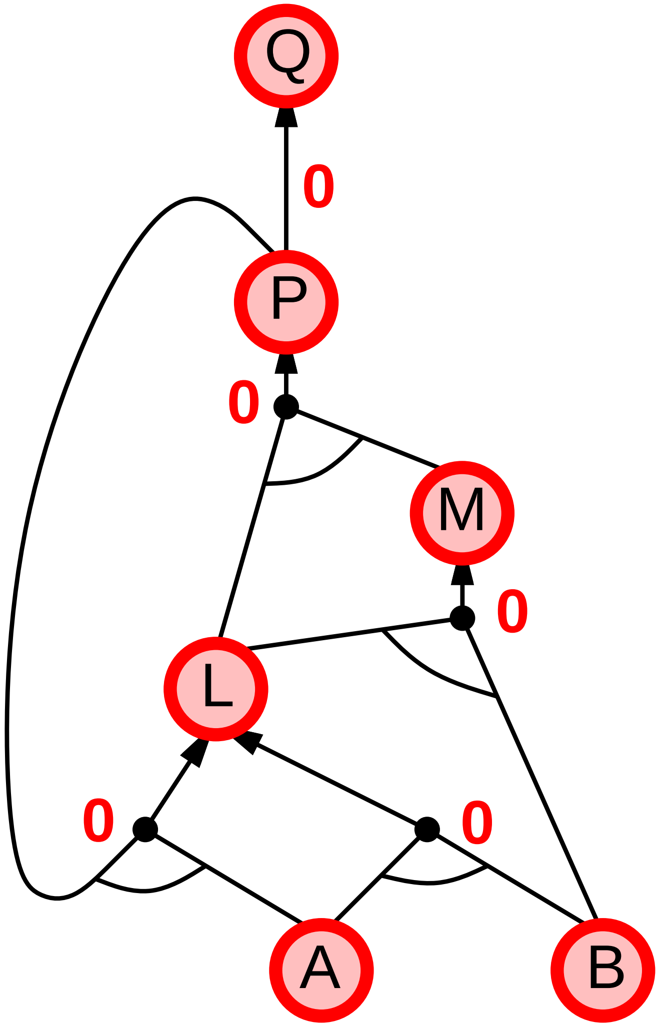

Backward chaining

Idea: work backwards from the query q:
  to prove q by BC,
    check if q is known already, or
    prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
  1) has already been proved true, or
  2) has already failed

Backward chaining example

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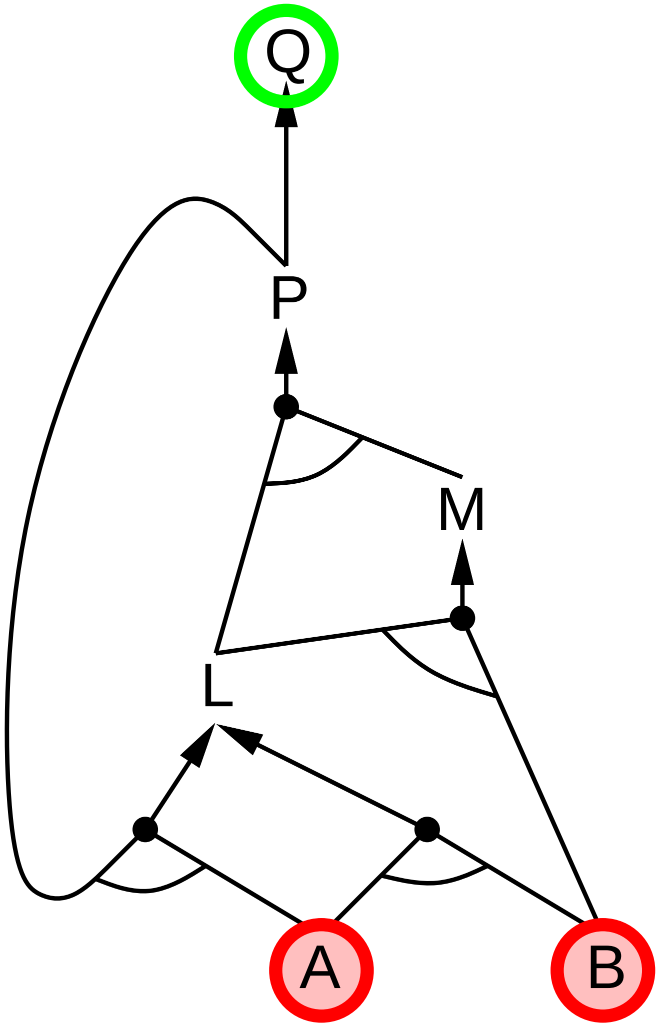

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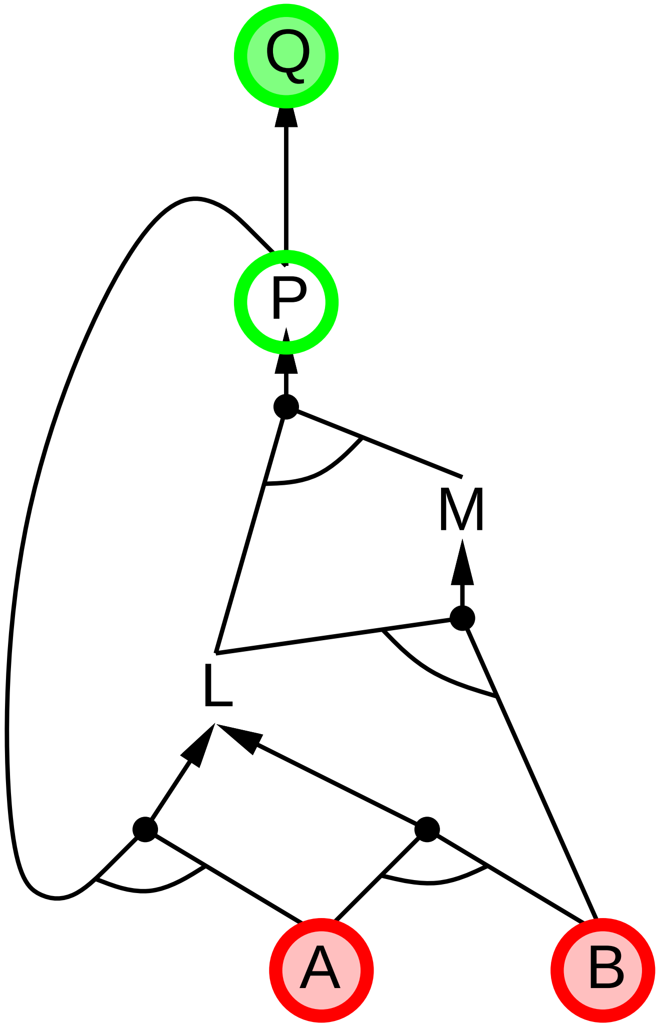

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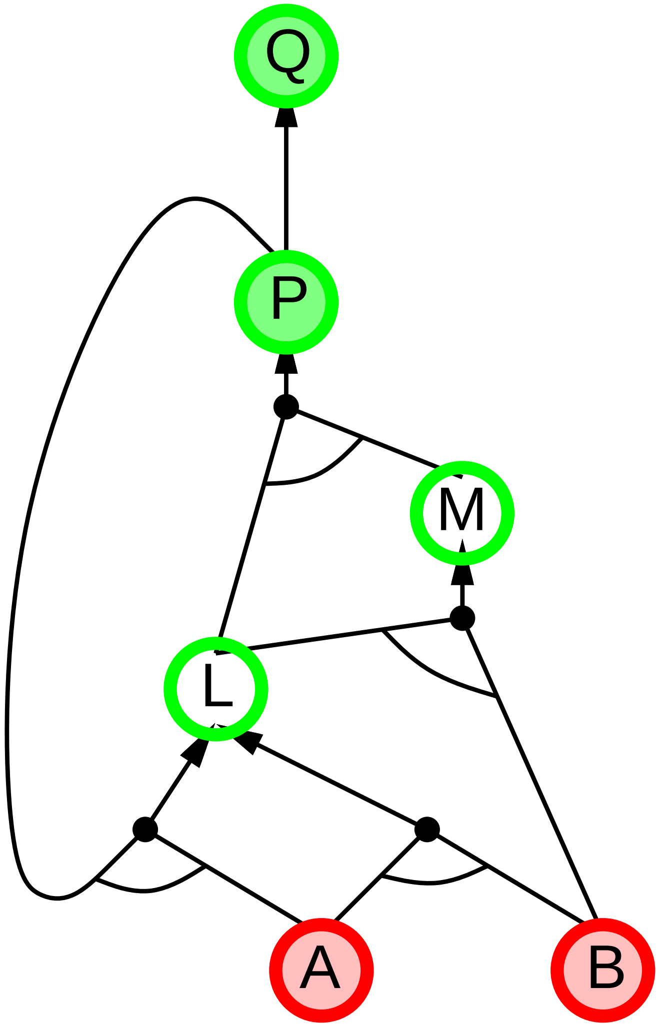

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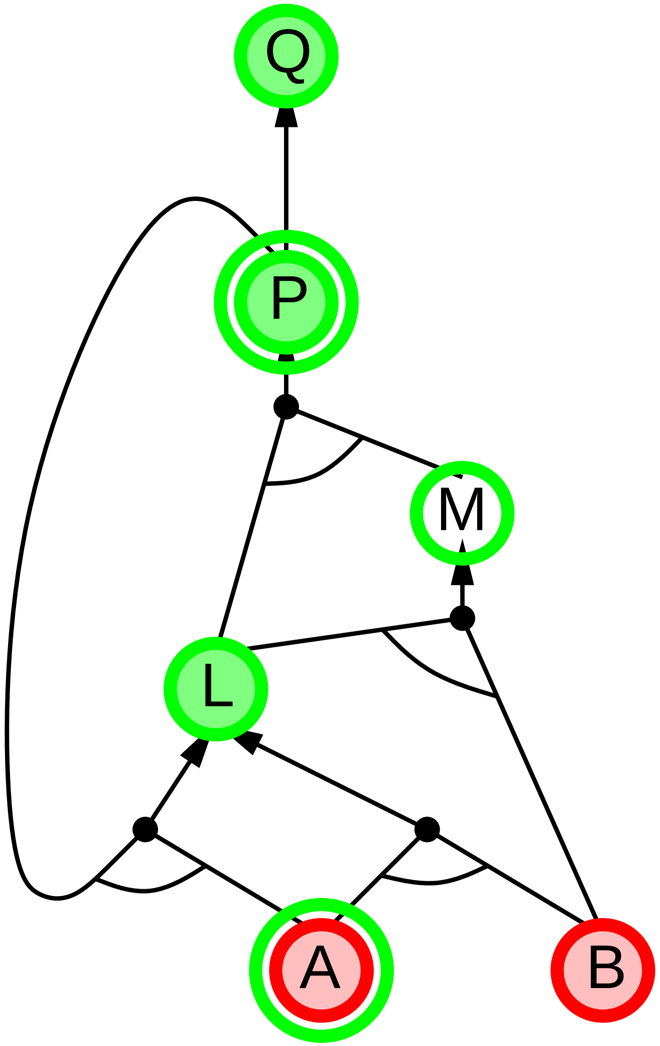

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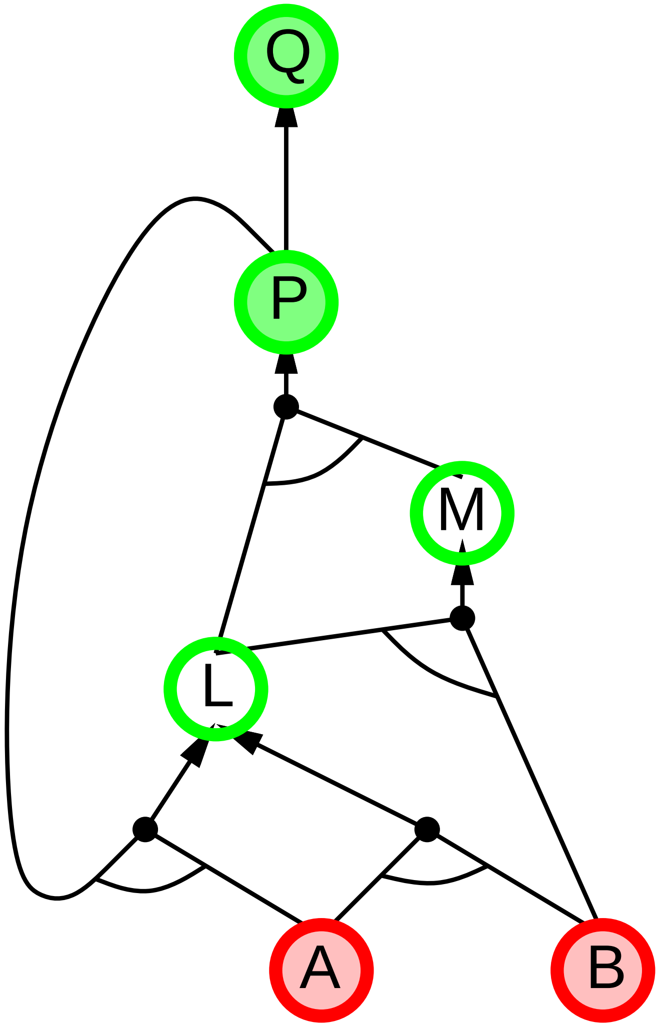

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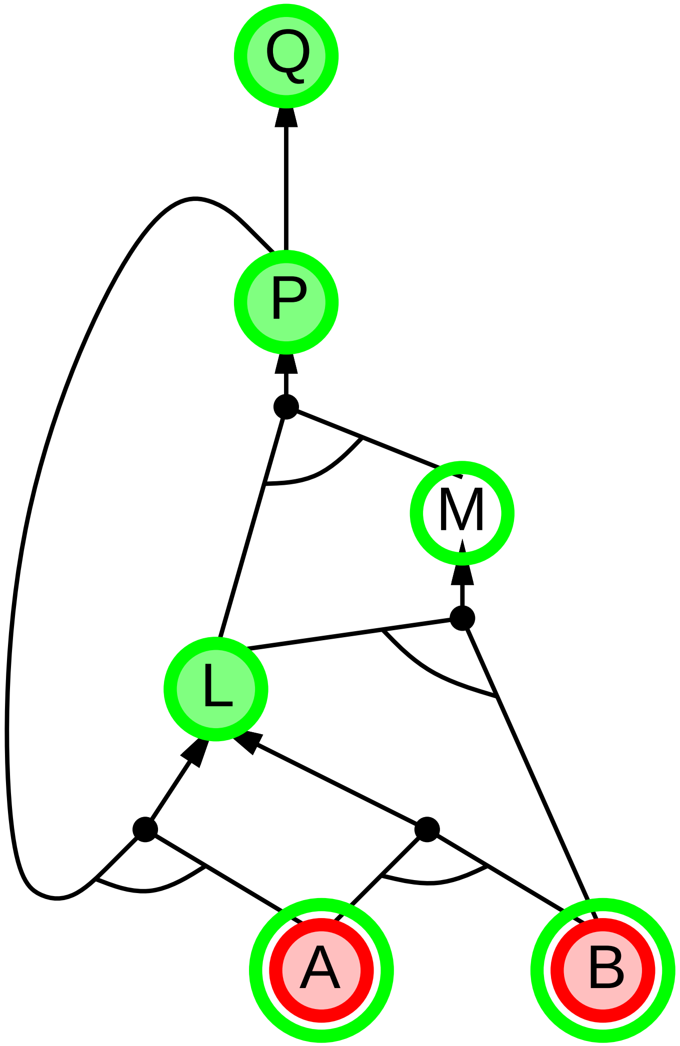

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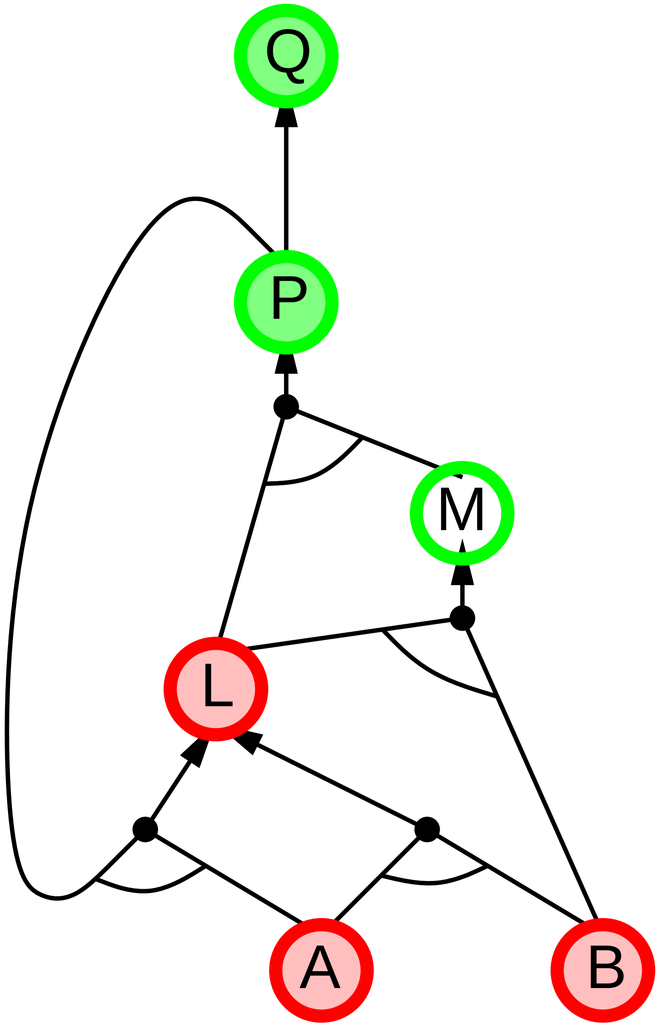

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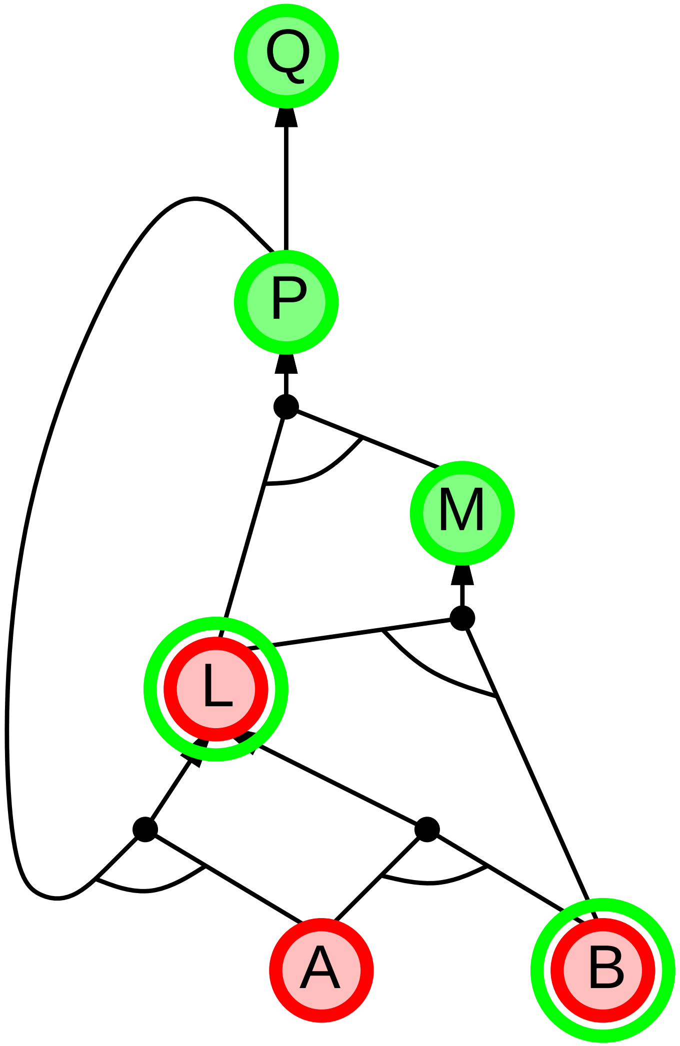
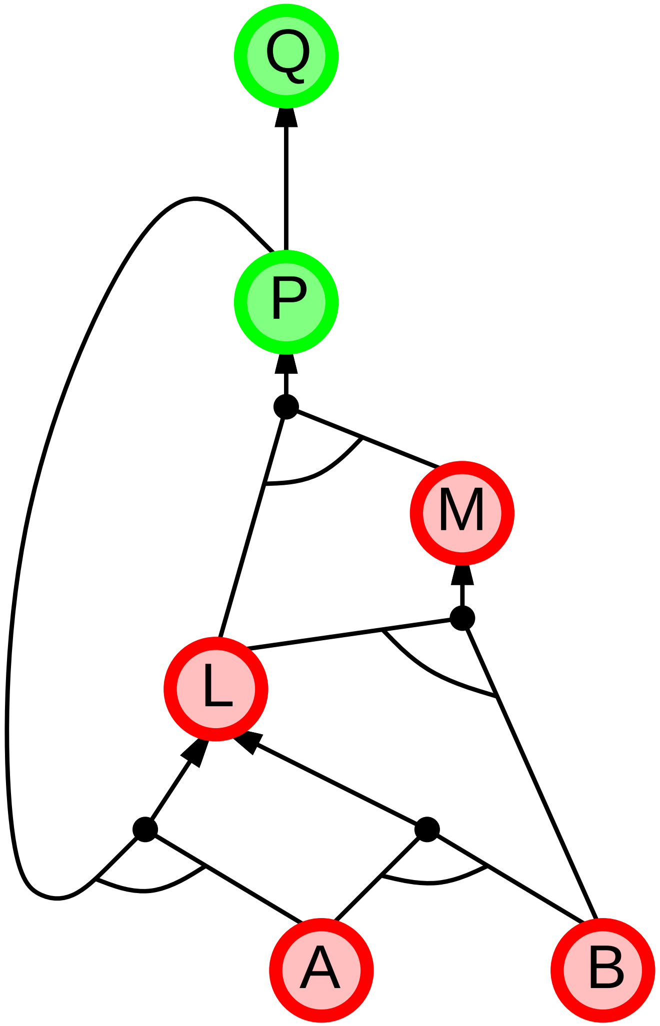
A
B



Backward chaining example

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B



Backward chaining example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

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$A \wedge P \Rightarrow L$

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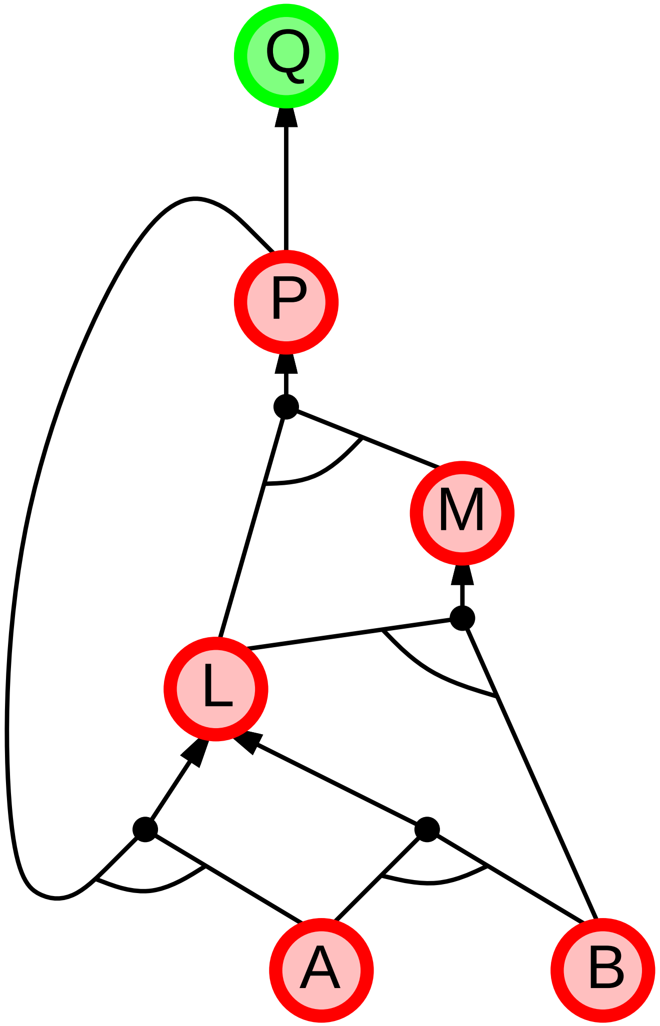
A

B



Backward chaining example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

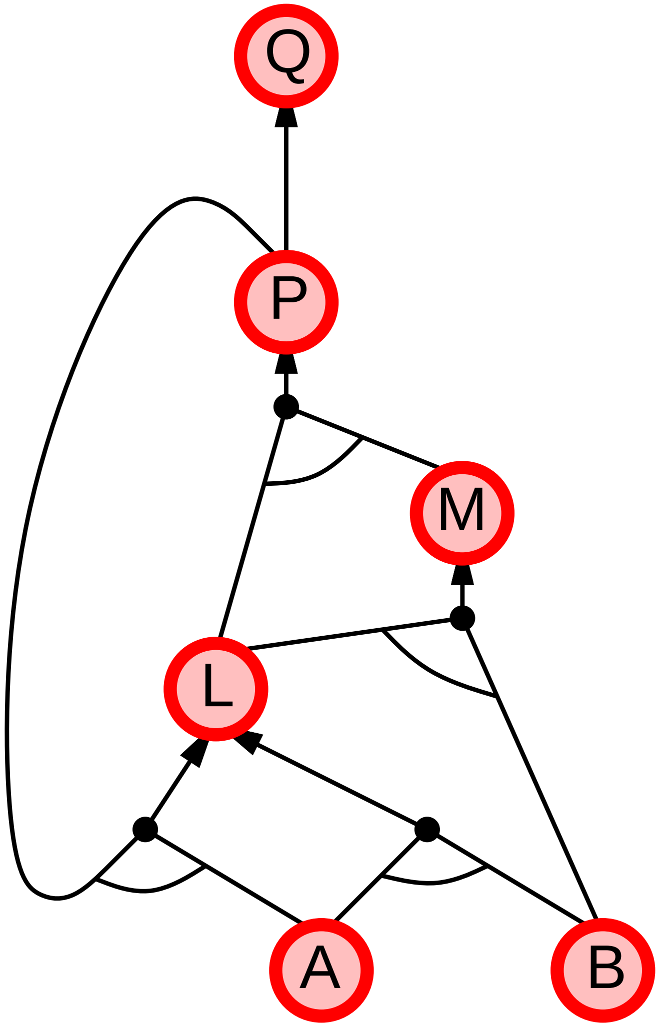
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB

Resolution

Proof algorithm for general propositional logic (as opposed to horn logic).

Conjunctive Normal Form (CNF—universal)

$$\textbf{conjunction} \text{ of } \underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$$

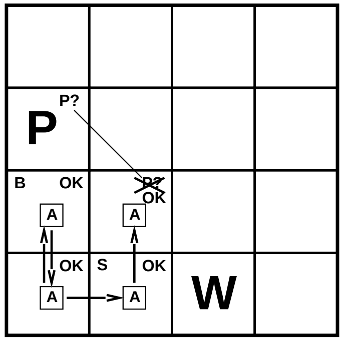
E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \lor \cdots \lor \ell_k, \qquad m_1 \lor \cdots \lor m_n}{\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n}$$

where $m_j = \neg \ell_i$. E.g.,

$$\frac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Conversion of any formula to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \;\Rightarrow\; \beta) \land (\beta \;\Rightarrow\; \alpha)$.

   $(B_{1,1} \;\Rightarrow\; (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \;\Rightarrow\; B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \lor \beta$.

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law ($\lor$ over $\land$) and flatten:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Resolution algorithm

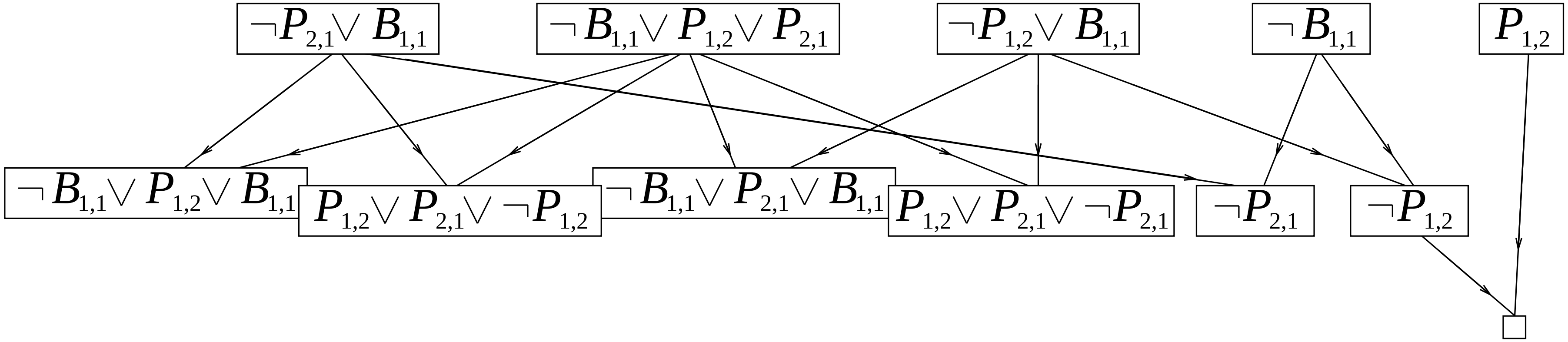
Proof $KB \models \alpha$ by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable.
General method: Apply resolution inference rule to all pairs of known clauses and add results to the known clauses until the empty clause ($\equiv False$) can be derived.

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
      α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\}$
  loop do
    for each C_i, C_j **in** *clauses* **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if *resolvents* contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false*
    $clauses \leftarrow clauses \cup new$

Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
$\alpha = \neg P_{1,2}$



Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses
Resolution is complete for propositional logic

Propositional logic lacks expressive power