MTH 837: Numerical Methods for Stochastic PDEs

DEPARTMENT OF MATHEMATICS, UNIVERSITY AT BUFFALO – SUNY

Year: 2015 Spring
Course Schedule: MWF 9:00AM – 9:50AM (TBA)
Location: Room 150 @ Math Building

Instructor: Jae-Hun Jung
Office location: Mathematics Building 328
Phone number: 716-645-8814
Office hours: Tue 10:00 – 12:00, 1:00 -- 3:00 or by appointment
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Course Description: The uncertainty quantification is one of the most important research areas in applied mathematics today. Uncertainty is everywhere. Most physical, biological, social, and financial processes are commonly characterized by uncertainties and it is important to understand how such uncertainties propagate and determine the global solution structures. Accordingly the mathematical models dealing with these processes should be able to include uncertainty terms in them. In this course, numerical methods that deal with such uncertainties embedded in partial differential equations (PDEs) are introduced. The PDEs contain various uncertainties in various forms such as the random coefficients, random boundary/initial conditions, etc. We will learn how to deal with random randomness in PDEs and how to compute the stochastic solutions. Particularly we are interested in the generalized polynomial chaos methods for solving stochastic PDEs. The generalized polynomial chaos method is to utilize the orthogonal polynomials to project the random space to a regular space. By such a projection, the polynomial chaos method can transform stochastic PDEs into deterministic PDEs. By doing this, the conventional PDE solvers can be still used. Related computational issues such as the sparse grid algorithm will be introduced. This course will be composed of the instructor's lecture and students' presentation. Students are required to present almost every week and also required to conduct a small research on stochastic PDEs.

Textbook:

- O.P. LeMaitre and O.M. Kino, Spectral Methods for Uncertainty Quantification with Applications to Computational Fluid Dynamics, Springer.
- D. Xiu, Numerical Methods for Stochastic Computations, A Spectral Method Approach, Princeton UP.

Course assignments, due dates and grading:

- Homework Assignments/Reading/Presentation 60%
- Final Project 40%

The weekly plan of the class is composed of the instructor's lecture and students' presentation. A list of papers related to the weekly course materials will be provided weekly. Students are supposed to pick one paper from the list to read and present in the following week in the class. If the homework is assigned instead of the list of papers, students will do their homework and there will be no presentation in the following week. Students will also do their research project and are required to submit their final project paper by May 16.

A(92-100), A-(88-91), B+(84-87), B(80-83), B-(75-79), C+(70-74), C(65-69), C-(60-64), D+(56-59), D(53-55).

Important dates

- *Monday, Jan. 26* Class begins
- *Monday March 16 – Saturday March 21* Spring Recess
- *Friday, May 8* Last day of Class
- *Saturday, May 16* Final Project Paper Due

Students with Disabilities: The University accommodates students with disabilities. The University accommodates students whose religious obligations conflict with attendance, submitting assignments, or completing scheduled tests and examinations. Please let me know in advance, preferably in the first week of class, if you will require any accommodation on these grounds. Students who plan to be absent for varsity athletics, family obligations, or other similar commitments, cannot assume they will be accommodated, and should discuss their commitments with the instructor before the drop date.

Academic Dishonesty: The policy of UB academic integrity will be strictly applied. Students of academic dishonesty will get a mark of 0 in the course and there will be a possible suspension or expulsion from the university. Please review the UB policy on cheating, plagiarism, and other forms of academic dishonesty.

TENTATIVE SCHEDULE

The outlined schedule below is an estimate only, and *may* change as the semester progresses.

Introduction to Stochastic PDEs
- Monte Carlo
- KL Transformation
- Generalized Polynomial Chaos Methods

Basic Concepts of Probability Theory
- Measurable Space
- Probability Space
- Random Variables

Orthogonal Projection
- Orthogonal Polynomials
- Orthogonal Projection

Stochastic Process
- Random Process
- Gaussian/Non-Gaussian Process
- Karhunen-Loeve Expansion
- Monte Carlo Methods
- Enhanced Sampling

- Polynomial Chaos Methods
    - Homogenous Chaos
    - Generalized Polynomial Chaos
    - Galerkin/Collocation Approach
- gPC for Hyperbolic PDEs
    - gPC formulation for hyperbolic PDEs
    - Re-parameterization/initialization for Time-Dependent PDEs
    - Discontinuity in Random Space
- Computational Complexity
    - gPC on Sparse Grid
    - Smolyak Formulation
    - Orthogonalization on Sparse Grid
    - Non-polynomial basis for Sparse Grid
    - Non-structured Nodes: Cubature
- Wavelet and Multi-Resolution Analysis
    - The Wiener-Haar Expansion
    - Multi-resolution and Multi-wavelet Basis
- Uncertainty Analysis for Fractional PDEs
    - Fractional Calculus
    - Fractional Diffusion-Reaction Equations