Effects of the correlations among partons to the collective flow in pA collisions

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I. INTRODUCTION

Collectivity in hadron-hadron scattering are measured at RHIC [1–5] and the LHC [6–14]. One of the way to investigate features of the collectivity is to measure distortions in the azimuthal distribution of produced particles which are quantified in terms of coefficients v_n of Fourier expansion of the azimuthal distribution. The v_n of the nucleus-nucleus scattering are well described by the nearly perfect relativistic fluid dynamics [15–17].

Surprisingly, these v_n are also measured in small systems such as the p-A and p-p scattering at the LHC [18]. For theoretical studies, there are roughly two types of the explanations of the collectivity in the small system: one is the initial state correlations and the other is final state correlations, see, e.g., a review Ref. [19]. The experimental results are well described by the final state correlations using the fluid dynamics [27]. But the validity of the fluid dynamics in the small system is still under discussion. For the initial state ones, there are some works based on the Color Glass Condensate framework which is the effective theory describing the high energy QCD scattering. Recently some explanations are proposed: the parton-parton correlations in the projectile proton or the correlation of parton-nucleus scattering amplitudes at finite N_c [22–25].

In this paper, we focus on the initial state correlations with interaction among the partons in the projectile proton and show the effects of them are small. The ingredients are the multi-parton correlation and the correlations between the transverse momentum and the impact parameter of the parton through the Wigner distribution of the nucleus.

The following chapter, we demonstrate that the muti-parton distribution and the elliptic Wigner distribution produce the m-th particle correlations. At the chapter 3, we show the results of the numerical calculations of $v_2\{n\}$. Finally, we conclude the paper.

II. GENERAL FORMULAS OF THE AZIMUTHAL CORRELATIONS

In this paper, we basically follow a formalism used in the paper [23] with using the multi-particle distribution. To apply the formalism to collective flow calculation we should consider not only the formalism of the cross section of two particle production as in [21] but also multi-parton production. We simply generalize the double parton distribution into the multi-parton distribution. Then the multiplicity of multi-particle productions can be written in the following form:

$$\frac{d^m N}{d^2\boldsymbol{p}_{\perp 1}\cdots d^2\boldsymbol{p}_{\perp m}} \propto \int \prod_{i=1}^{m} d^2\boldsymbol{b}_{\perp i}\; d^2\boldsymbol{x}_{\perp i} e^{i\boldsymbol{p}_{\perp i}\cdot\boldsymbol{x}_{\perp i}} F_p(\boldsymbol{b}_{\perp 1},\cdots,\boldsymbol{b}_{\perp m}) F_A(\boldsymbol{b}_{\perp 1},\cdots,\boldsymbol{b}_{\perp m},\boldsymbol{x}_{\perp 1},\cdots,\boldsymbol{x}_{\perp m}). \quad (1)$$

Where F_p is the proton multi-parton distribution and F_A is the nucleus. In this paper we concentrate on the contribution of the correlations of the multi-parton distribution besides the elliptic gluon Wigner distribution, so let us simply assume that the F_A has no correlations between the Wigner distributions:

$$F_A(\boldsymbol{b}_{\perp 1},\cdots,\boldsymbol{b}_{\perp m},\boldsymbol{x}_{\perp 1},\cdots,\boldsymbol{x}_{\perp m}) \approx \prod_{i=1}^{m} C(\boldsymbol{b}_{\perp i}^2) S_Y(\boldsymbol{x}_{\perp i},\boldsymbol{b}_{\perp i}) \quad . \quad (2)$$

Where $C(\boldsymbol{b}_{\perp i}^2)$ is effectively cutting off the upper limit of the impact parameter integrations: we use $\exp\left(-\Lambda_{nucl}^2 \boldsymbol{b}_{\perp i}^2\right)$ or $\Theta(R_{cut} - |\boldsymbol{b}_{\perp i}|)$ and confirm that the results do not changed the choices. The S-matrix S_Y is the dipole-nucleus scattering amplitude and Y is the rapidity of the nucleus. The rapidity dependence of the S-matrix obeys the Balitsky-Kovchegov equation.

We approximate the multi-parton distribution as a convolution of single parton distributions.

$$F_p(\boldsymbol{b}_{\perp 1},\cdots,\boldsymbol{b}_{\perp m}) \approx \int d\boldsymbol{b}_\perp\ f_p(\boldsymbol{b}_\perp)\prod_{i=2}^{m} f_p(\boldsymbol{b}_\perp - \boldsymbol{b}_{\perp 1} + \boldsymbol{b}_{\perp i}) \quad . \tag{3}$$

We assume a Gaussian distribution for the impact parameter dependent single parton distributions $f_p(\boldsymbol{b}_\perp) = \exp\left(-\frac{\boldsymbol{b}_\perp^2}{B}\right)$. This integration variable $\boldsymbol{b}_\perp$ is the distance between parton and the projectile proton, and $\boldsymbol{b}_{\perp i}$ is parton and the target nucleus. For example:

$$F_p(\boldsymbol{b}_{\perp 1},\boldsymbol{b}_{\perp 2}) \propto \exp\left(-\frac{(\boldsymbol{b}_{\perp 1}-\boldsymbol{b}_{\perp 2})^2}{2B}\right) \tag{4}$$

$$F_p(\boldsymbol{b}_{\perp 1},\boldsymbol{b}_{\perp 2},\boldsymbol{b}_{\perp 3},\boldsymbol{b}_{\perp 4}) \propto \exp\left(-\frac{1}{4B}\sum_{i<j}(\boldsymbol{b}_{\perp i}-\boldsymbol{b}_{\perp j})^2\right) \tag{5}$$

To quantitatively evaluate the correlations we introduce the nth-moment of the m-particle correlation:

$$\kappa_n\{m\} := \prod_{i=1}^{m}\int \frac{d^2\boldsymbol{p}_{\perp i}}{(2\pi)^2} e^{in(-1)^{i+1}\phi_i}\frac{d^m N}{d^2\boldsymbol{p}_{\perp 1}\cdots d^2\boldsymbol{p}_{\perp m}} \quad , \tag{6}$$

and then define two and four particle cumulants measured in experiments:

$$c_n\{2\} = \langle e^{in(\phi_1-\phi_2)}\rangle = \frac{\kappa_2\{2\}}{\kappa_0\{2\}} \tag{7}$$

$$c_n\{4\} = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle - 2c_n\{2\}^2 = \frac{\kappa_2\{4\}}{\kappa_0\{4\}} - 2\frac{\kappa_2\{2\}^2}{\kappa_0\{2\}^2} \quad . \tag{8}$$

The nth-Fourier harmonic coefficients are defined:

$$v_n\{2\} = \sqrt{c_n\{2\}} \tag{9}$$

$$v_n\{4\} = (-c_n\{4\})^{\frac{1}{4}} \quad . \tag{10}$$

In this paper, we assume that the S-matrix has SO(3) symmetry so the S-matrix can be represented as $S_Y(\boldsymbol{x}_\perp,\boldsymbol{b}_\perp) = S_Y(|\boldsymbol{x}_\perp|,|\boldsymbol{b}_\perp|,\cos(2\phi))$. Where the ϕ is the angle between $\boldsymbol{x}_\perp$ and $\boldsymbol{b}_\perp$. Then the Fourier expansion of the S-matrix is $S = S_0 + 2\sum_{n=1}^{\infty}\cos(2n\phi)S_{2n}$. The elliptic part S_2 is related to the elliptic Wigner distribution [26]. The κ can be written as:

$$\begin{aligned}\kappa_n\{2m\} = D_n(2\pi)^{2m}&\int\prod_{i=1}^{2m} d^2\boldsymbol{b}_{\perp i} F_p(\boldsymbol{b}_{\perp 1},\cdots,\boldsymbol{b}_{\perp 2m})\\ &\times\cos(n(\sum_{i=0}^{2m}(-1)^i\phi_i))\prod_{i=1}^{2m}C(\boldsymbol{b}_{\perp i}^2)\int_0^\infty dr_i\ e^{-\epsilon r_i^2}p_{up}A_n(p_{pu}r_i)S_n(r_i,b_i) \quad .\end{aligned} \tag{11}$$

Where p_{pu} is an upper limit of the $\boldsymbol{p}_\perp$ integrations. For example, the A_n and S_n at $n=0,2$ are written as:

$$A_0(x) = J_1(x) \quad , \quad S_0(r,b) = \int_0^{2\pi} d\phi\ S_Y(r,b,\cos(2\phi)) \quad ,$$

$$A_2(x) = \frac{2-2J_0(x)-xJ_1(x)}{x} \quad , \quad S_2(r,b) = \int_0^{2\pi} d\phi\ S_Y(r,b,\cos(2\phi))\cos(2\phi) \quad . \tag{12}$$

Where the J_n is Bessel function. The general expressions of A_n are derived at [24].

III. B-JIMWLK CALCULATION .... UNDER CONSTRUCTION ....

When we would like to use the solution of the B-JIMWLK equation, we have to use $S(\boldsymbol{x}-\boldsymbol{y},(\boldsymbol{x}+\boldsymbol{y})/2) = S_o(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{N_c}\langle V^\dagger(\boldsymbol{x})V(\boldsymbol{y})\rangle$ on lattice. The integrations in the relative distance should be changed to integrations in $\boldsymbol{x} = \boldsymbol{r}/2$. Then the integrations become:

$$\begin{aligned}\int_0^\infty dr\; e^{-\epsilon r^2} p_{up} A_n(p_{pu} r) S_n(r,b) &= \int d^2\boldsymbol{r}_\perp\; e^{-\epsilon \boldsymbol{r}_\perp^2} \frac{p_{up}}{|\boldsymbol{r}_\perp|} A_n(p_{up}|\boldsymbol{r}_\perp|) S(\boldsymbol{r},\boldsymbol{b}) \cos 2n\phi \\ &= 4\int d^2\boldsymbol{x}_\perp\; e^{-4\epsilon(\boldsymbol{x}_\perp)^2} \frac{p_{up}}{2|\boldsymbol{x}_\perp|} A_n(2p_{up}|\boldsymbol{x}_\perp|) \\ &\qquad \times S_o(\boldsymbol{b}_\perp+\boldsymbol{x}_\perp, \boldsymbol{b}_\perp-\boldsymbol{x}_\perp) \cos 2n\phi \end{aligned} \tag{13}$$

Where $\cos\phi = \frac{\boldsymbol{r}_\perp\cdot\boldsymbol{b}_\perp}{|\boldsymbol{r}_\perp||\boldsymbol{b}_\perp|} = \frac{(\boldsymbol{x}_\perp)\cdot\boldsymbol{b}_\perp}{|\boldsymbol{x}_\perp||\boldsymbol{b}_\perp|}$, and we can express the $\cos n\phi$ as polynomials of $\cos\phi$ with using the Chebyshev polynomials. If $\boldsymbol{b}_\perp+\boldsymbol{x}_\perp$ or $\boldsymbol{b}_\perp-\boldsymbol{x}_\perp$ is over the domain of the lattice, we take $V(\boldsymbol{b}_\perp+\boldsymbol{x}_\perp), V(\boldsymbol{b}_\perp-\boldsymbol{x}_\perp) = 1$ and becomes $S=0$ because of the gauge invariance.

IV. NUMERICAL CALCULATION

We use solutions of a impact-parameter dependent Balitsky-Kovchegov equation derived in the [26] and set the initial conditions $S_{Y=0} = e^{-6d^2}$. To get reasonable results for κ_n we first perform the r_i and ϕ_i integrations then use the Monte-Carlo method for the $\boldsymbol{b}_{\perp i}$ integrations in which we average over 2.0×10^{10} samples. We set $B = 1/2$ to match the same parameters in the previous work. We set $C(\boldsymbol{b}_{\perp i}^2) = \exp\left(-\Lambda_{nucl}^2 \boldsymbol{b}_{\perp i}^2\right)$. We show some

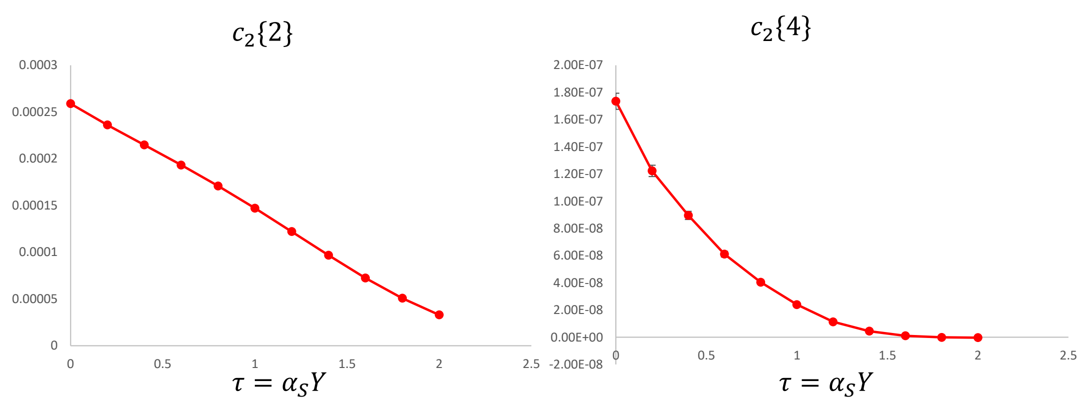

FIG. 1. $c_2\{2\}$ (left) and $c_2\{4\}$ (right) in $\Lambda_{nucl}^2 = \frac{1}{72B}$.

We show the Fourier harmonics at Fig.3. The magnitude of the $v_2\{n\}$ are ten times smaller than the measured ones.

Conclusions and outlooks. First, we formulate the n-th moment of the m-particle correlation from the multi-parton distribution and the elliptic Wigner distribution. Calculating $c_2\{2\}$ and $c_2\{4\}$, we find that the collective flow is generated from the correlations of multi-parton distribution and the elliptic Wigner distribution. We can conclude that The effects of the nucleon structures impact on the collective phenomena. This corrections are small but non-negligible impact on the precise description of the collective flow.

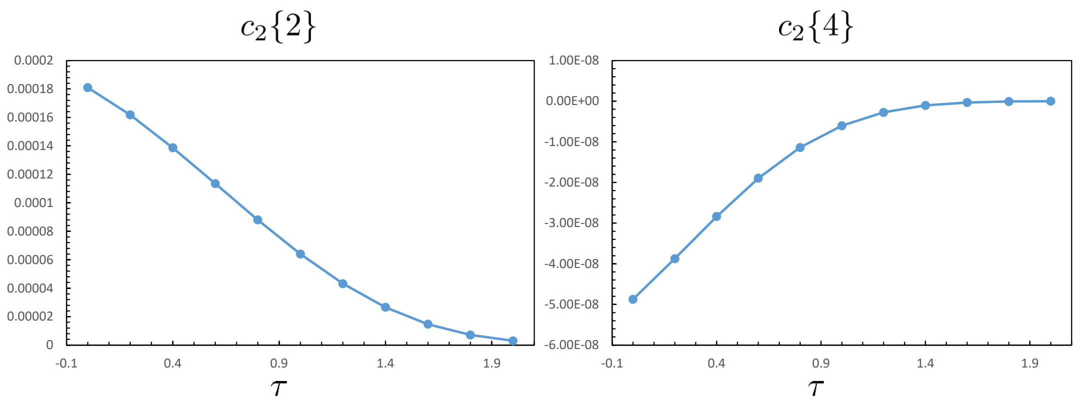

FIG. 2. $c_2\{2\}$ (left) and $c_2\{4\}$ (right) in $\Lambda^2_{nucl} = \frac{1}{2B}$.

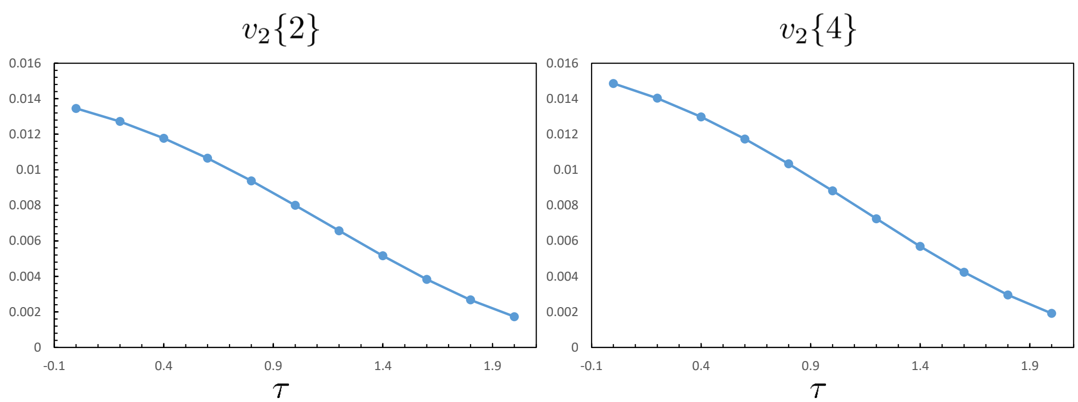

FIG. 3. $v_2\{2\}$ (left) and $v_2\{4\}$ (right) in $\Lambda^2_{nucl} = \frac{1}{2B}$.

Appendix A: Some tricks for numerical calculation

We show the formula to get good numerical behavior.

$$\int_0^\infty dr_i\ e^{-\epsilon r_i^2} p_{up} J_1(p_{up} r_i) \int_0^{2\pi} d\phi_i = 2\pi \int_0^\infty dx\ e^{-\frac{\epsilon}{p_{up}^2} x^2} J_1(x)$$
$$= \pi^{3/2} \frac{|p_{up}|}{\sqrt{\epsilon}} e^{-\frac{p_{up}^2}{8\epsilon}} I_0\left(\frac{p_{up}^2}{8\epsilon}\right) \tag{A1}$$

Appendix B: An estimation of the error in the Monte-Carlo simulation

$$\int dx\ f(x)p(x) \simeq \frac{1}{N}\sum_i f(x_i) = \langle f \rangle \tag{B1}$$

If the $p(x)$ is normalized to 1, the error of this expectation value is

$$\Delta\langle f \rangle = \frac{\sqrt{\mathrm{Var}(f)}}{\sqrt{N}} = \frac{\sqrt{\langle f^2 \rangle - \langle f \rangle^2}}{\sqrt{N-1}}, \tag{B2}$$

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