Developing a Bayesian procedure to detect breakpoints in time series

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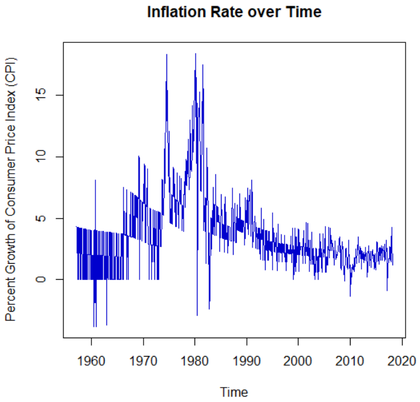
What is the problem?



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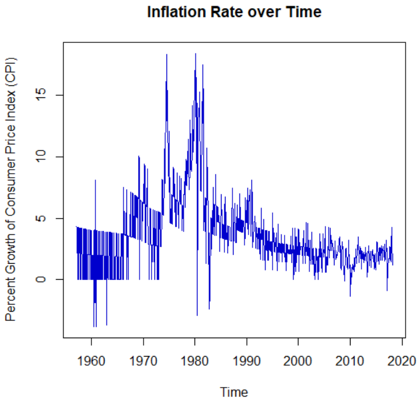

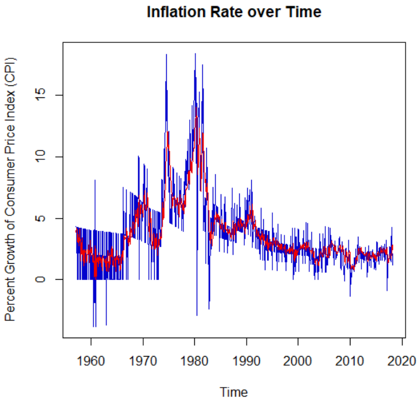

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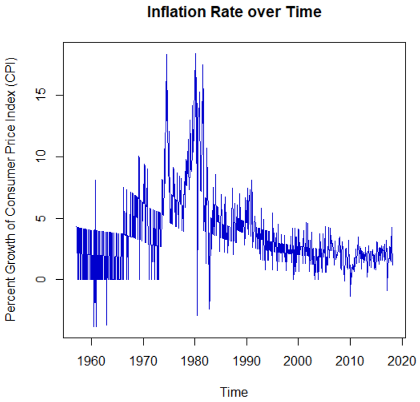

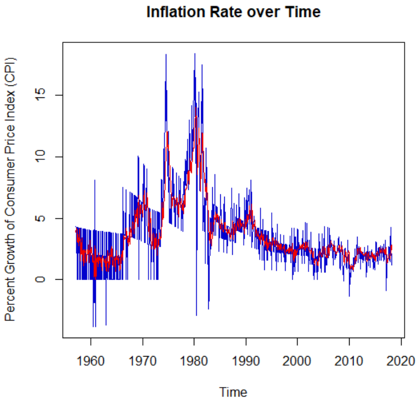

$$Y = \beta_0 + \sum_{i=1}^{n} \beta_i Y_{t-i} + \epsilon_{t-i}$$

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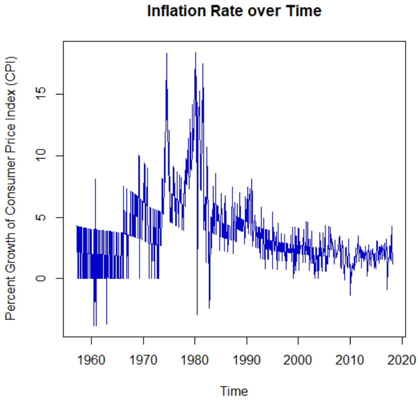
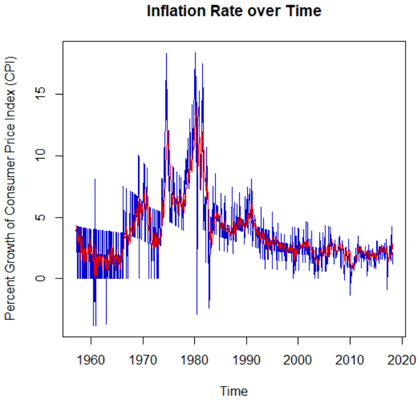


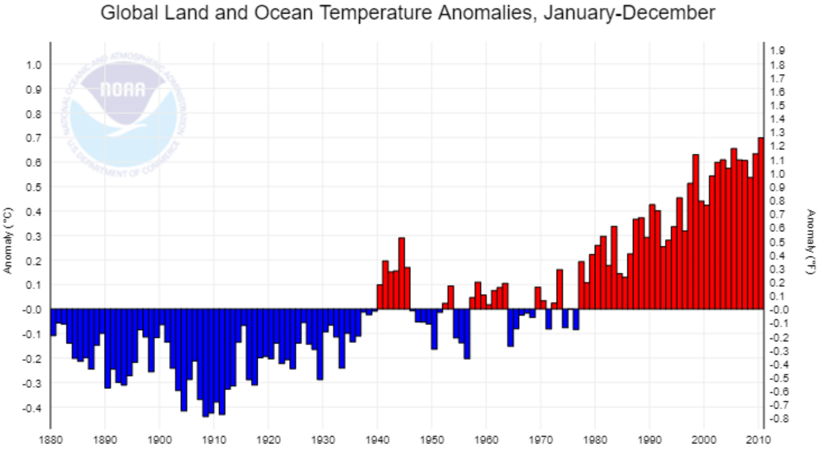


9 coefficients for autoregressive model:
1: 0.210967388; 2: 0.246292153; 3: 0.095668132;
4: -0.006722439; 5: 0.086106156; 6: 0.083077353;
7: 0.021451801; 8: 0.089686225; 9: 0.097260118

Goal

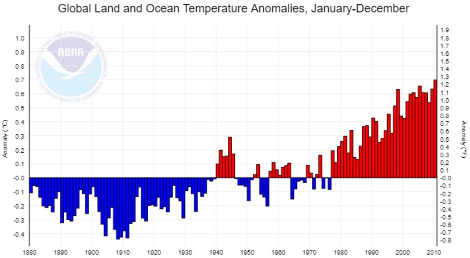
To develop a better quantitative method for locating break points in time series data.

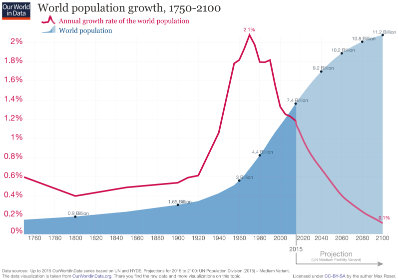
Other Data



Images courtesy of NOAA and Our World in Data.

Other Rate





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Current Strategies

Bai-Perron Test (1998):

- Frequentist approach to identify significant changes in a time series.
- Search through every single possible break point and determine the best model off of the residual sum of squares.
- Conditions on the number of breaks specified by the user the output is a single model.
- It has multiple downsides however.

Current Strategies

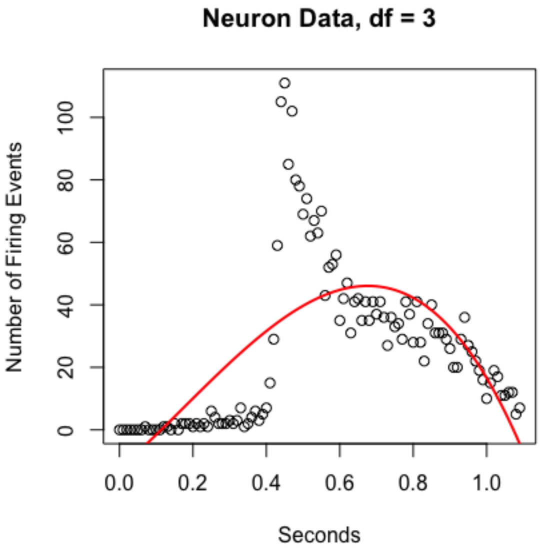
Curve Fitting with Splines

- Uses cubic polynomial piece wise functions that are connected at certain points, called knots.
- How many knots and where they are located determine the fit.

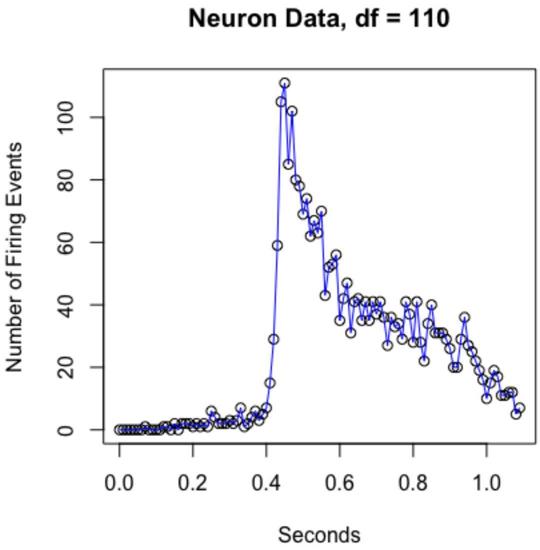
Generalized B Spline Function

$f(x) = \sum_{j=1}^{k} \beta_j B_{jk}(x)$

Current Strategies



Current Strategies



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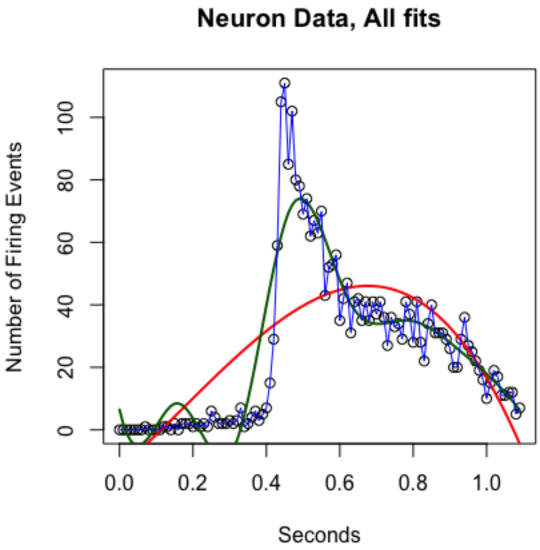

Current Strategies

Bayesian Adaptive Regression Splines (BARS):

- A stochastic process that proposes new set of knots for the splines.
- This process involves adding, subtracting, and moving knots to obtain a draw from the possible fits to the model.
- The optimal fit is obtained by averaging the different fits.

Project Outline

Project Format:

1. Initial Knot/Breakpoint Problem
2. Modification of BARS to propose a new set of knots
3. Get $\theta = (K, \tau, \beta, \sigma)$ and find parameter distributions
4. Evaluate new proposed fit using Metropolis-Hastings
5. Repeat steps 2 through 4

Initial Breakpoint Problem

Determining the initial placement of the first or first couple of knots

Previous Approaches:

- random
- middle placement

New Approach: Bai-Perron (frequentist method)

- Places the initial breakpoints strategically to improve the model.

Modification of BARS

Modification: Autoregressive (AR) rather then cubic splines

AR(1) Model

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon_t$$

Modification: Adding, Subtracting, and Moving

- Adding, subtracting, and moving breakpoints all have their own probability of occurring.
- Proposed addition will be based on probability and current location of other breakpoints.
    - Changes are necessary because BARS proposes breakpoints that are too close together which is not appropriate for most time series.

Going Forward

- Develop adding and subtracting algorithms for BARS modification
- Figure out the algorithms for the best initial breakpoint(s) position(s)
- Lots of coding and simulations to R!!!