ASTR615 Fall 2017 Homework #4

Due November 21, 2017

Generalize the integration package you wrote for Homework #3 so that it can solve the N-body problem in 3-D for particles of arbitrary mass using the particle-particle method with fixed timesteps.[1] The code should read the masses and initial positions and velocities of all the particles from a text file in the following input format, one particle per line: $m_i\ x_i\ y_i\ z_i\ \dot{x}_i\ \dot{y}_i\ \dot{z}_i$. The user specifies the integrator (either 2$^{\rm nd}$-order leapfrog [LF2] or 4$^{\rm th}$-order Runge-Kutta [RK4] at a minimum), the softening parameter ε, the timestep h, the number of steps n_s, the output frequency (number of steps between outputs), and the name of the initial conditions file. It is recommended to make the output format the same as the input format, with one file per output—this has the advantage that you can restart from any output step.

1. Check your code by solving the two-body problem for equal masses ($m_1 = m_2 = 1$; $G = 1$) without softening ($\varepsilon = 0$) for 100 loops around orbits of eccentricity $e = 0.5$ and $e = 0.9$, using both LF2 and RK4. Start at apocenter with separation $r = 1$ and relative speed $v = \sqrt{2(1-e)}$ (derived from the vis-viva equation). By symmetry your initial conditions for each body are then $\mathbf{r}_1 = (-\frac{1}{2}, 0, 0)$, $\mathbf{r}_2 = (\frac{1}{2}, 0, 0)$, $\mathbf{v}_1 = (0, -\frac{1}{2}v, 0)$, $\mathbf{v}_2 = (0, \frac{1}{2}v, 0)$, where we have chosen to start on the x-axis with the orbital angular momentum aligned with the positive z-axis. Use $h = 0.05$ for $e = 0.5$ and $h = 0.003$ for $e = 0.9$. Table 1 summarizes the parameters for you, and includes n_s (computed from the orbital period P; your code should calculate v and n_s internally—use the values in the table as a check). For each combination of e and integrator, generate 3 plots: 1) an orbit diagram showing x vs. y for each body; 2) a phase diagram giving r vs. $v_r = (\mathbf{v}\cdot\mathbf{r})/r$; and 3) the fractional change in total energy $E = \frac{1}{2}v_1^2 + \frac{1}{2}v_2^2 - 1/r$ as a function of time t, i.e., plot $[E(t) - E(0)]/|E(0)|$, where for this problem $E(0) = -(1+e)/2$ (see Table 1). For the first 2 plots, use points, not lines. For all the plots to look good, aim for $\sim 1{,}000$ outputs for each run. Comment on your results. An an (optional!) extra check, compare your results with the expected solution from Kepler's equation (see Class 16 exercise).

e	r	v	P	h	n_s	E(0)
0.5	1	1	$4\pi/\sqrt{27} \doteq 2.418$	0.05	4,836	-0.75
0.9	1	$1/\sqrt{5} \doteq 0.4472$	$\pi\sqrt{2000/6859} \doteq 1.696$	0.003	56,547	-0.95

Table 1: Run parameters for Problem 1. Note r and v are *relative* quantities, so divide by 2 when assigning to each body, as explained in the main text. The period $P = \pi\sqrt{2/(1+e)^3}$ for this problem. The number of steps $n_s = 100P/h$ (truncated to an integer).

[1] You are welcome to attempt something more sophisticated, such as using a tree code and/or multistepping, but that would be considerably more work!

2. Generate initial conditions of your choosing for between 100 and 1,000 particles, e.g., particles in a spherical region, particles in a disk, whatever you want. Use a range of masses. Integrate this system long enough for something "interesting" to happen. What should the timestep be? Do you need softening? How well is energy conserved? How long did it take to run? Summarize the evolution of your system in a visualization of some kind, such as a plot and/or an animation.