CS 132 – Geometric Algorithms

Homework 2

Due: Friday June 2, 2017 by 11:59pm

Once again, before starting, we encourage you to look at and solve the "Practice Problems" that appear before the exercises. The solutions appear after the exercises.

The problems from LAA 5th edition, Section 1.2 (pages 21-23), and 1.3 (pages 32-35).

1. Find a polynomial of degree 2 (a polynomial of the form $f(t) = a + bt + ct^2$) whose graph goes through the points $(1,-1),(2,3)$, and $(3,13)$ by setting up and solving a linear system. Show the linear system as well as the solution. Hint: The points are simply pairs of the form $(t, f(t))$, for example, $f(3) = 13$.
2. Find the circle that runs through the points $(5,5),(4,6)$, and $(6,2)$. Write your equation in the form $a + bx + cy + x^2 + y^2 = 0$. Find the center and radius of this circle. Hint: The points are of the form (x,y) - plug each point into the equation to generate your system of equations.
3. LAA 1.2 Exercise 2, page 22.
4. LAA 1.2 Exercise 4, page 22.
5. LAA 1.2 Exercise 8, page 22.
6. LAA 1.3 Exercise 2, page 32.
7. LAA 1.3 Exercise 10, page 32.
8. LAA 1.3 Exercise 12, page 32.
9. LAA 1.3 Exercise 14, page 32.
10. LAA 1.3 Exercise 28, page 33. For part c just write the equation.

Computational.

1. Implement a set of functions in `python` to analyze and solve linear systems. Before starting, read the document on Piazza "Linear Algebra in Python."

   Solving a linear system using Gaussian Elimination requires implementing three functions:

   (a) forward elmination (Stage 1)

   (b) testing for consistency

   (c) backsubstitution (Stage 2)

   I have written the code for forward elimination. Your job is to provide the rest of the code and use the resulting set of functions to analyze some linear systems. (In case it helps, the code you have to write is (quite a bit) shorter than the code I have already supplied!)

   In `hw2GECode.py` I have provided the function:

   ```
   forwardElimination(B)
   ```

which takes a matrix (numpy array) `B` and returns a new matrix that is the row echelon form of `B`. There are also helper functions in that file, for doing things like swapping rows of a matrix, doing row reduction, etc.

You need two write two functions:

```
inconsistentSystem(B)
```

which takes a matrix `B` in echelon form, and returns True if it represents an inconsistent system, and False otherwise; and

```
backsubstitution(B)
```

which returns the reduced row echelon form matrix of `B`.

Using these, you should be able to analyze a matrix representing a linear system as follows:

```
AEchelon = forwardElimination(A)
if (! inconsistentSystem(AEchelon)):
    AReducedEchelon = backsubstitution(AEchelon)
```

and then by inspecting `AReducedEchelon` you should be able to write down the solution set for the linear system.

Hints. First, make sure you understand the writeups "Notes on Python." and "Linear Algebra in Python." Second, I encourage you to read carefully the code that is already provided before starting to write your own code. Most of what you need to do can be patterned after code that is already provided, as long as you understand what the provided code is doing!

One of the main things to notice is that there is a function provided for you that does row reduction. If you look at this function you will see that it specifically handles floating point inaccuracies. So if you need to do row reduction in your code, you should use that function rather than coding something else yourself.

Here is my #1 piece of advice for success on this and future programming assignments: work incrementally, writing and testing small bits of code at a time. Don't implement the whole thing and then try to debug it; that will take much more time than if you break the process into smaller pieces and make sure they work at each stage.

What to submit. Submit a listing of your code, which should be added to the file `hw2GECode.py` (the function templates are already supplied). It is a strict requirement that you comment your code sufficiently for a reader to understand what you are doing easily.

Additionally, you are to submit a listing of your functions operating on seven sample problems. These are:

- First, solve the following linear system:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -16 \\ 5x_1 + 4x_2 + 6x_3 &= -41 \\ 10x_1 + 9x_2 + 8x_3 &= -50 \end{aligned}$$

- Second, analyze the six linear systems found in the files `h2m1.txt, h2m2.txt, h2m3.txt,h2m4.txt,h2m5.` and `h2m6.txt`. To read these in as numpy arrays in `python,` you can use code like this:

  ```
  A = np.loadtxt('h2m1.txt')
  ```

  For each system, state whether the system is consistent, and if so, write down the solution set.