

Latent variables affecting mean and covariace

$$p(g) = \text{Gam}(g; \alpha_g, \theta_g) \quad (1)$$

$$p(z) = \text{Gam}(z; \alpha_z, \theta_z) \quad (2)$$

$$p(x \mid v, z) = \mathcal{N}(x; zAv, \sigma_x I) \quad (3)$$

$$p_c(v \mid g) = \mathcal{N}(v; 0, \sum_{k=1}^K g_k C_k) \quad (4)$$

$$p_m(v \mid g) = \mathcal{N}(v; Bg, \sigma_v I) \quad (5)$$

$$p(v \mid x) = \iint_{-\infty}^{\infty} p(v \mid x, g, z) p(g) p(z) dg dz \quad (6)$$

we can do this by either numerical integration or sampling

$$p(v \mid x) \approx \sum_{z \in [z_{min}, z_{max}]} \sum_{g \in [g_{min}, g_{max}]} p(v \mid x, g, z) p(g) p(z) \quad (7)$$

$$p(v \mid x) \approx \frac{1}{L} \sum_{l=1}^L p(v \mid x, g^l, z^l) \quad (8)$$

either way, we approximate the marginal posterior with a finite mixture. In case of sampling, of L components, for wich the covariance is given in the following form

$$C_{v|x} \approx \frac{1}{L} \sum_{l=1}^L C_{v|xgz}^l + (\mu_{v|xgz}^l - \frac{1}{L} \sum_{m=1}^L \mu_{v|xgz}^m)(\mu_{v|xgz}^l - \frac{1}{L} \sum_{m=1}^L \mu_{v|xgz}^m)^T \quad (9)$$

$$p_c(v \mid x, g, z) = \mathcal{N}(v; \mu_c, C_c) \quad (10)$$

$$C_c = \left(\frac{z^2}{\sigma_x} A^T A + \left[\sum_{k=1}^K g_k C_k \right]^{-1} \right)^{-1} \quad (11)$$

$$\mu_c = \frac{z}{\sigma_x} C_c A^T x \quad (12)$$

$$p_c(m \mid x, g, z) = \mathcal{N}(v; \mu_m, C_m) \quad (13)$$

$$C_m = \left(\frac{z^2}{\sigma_x} A^T A + \frac{1}{\sigma_v} I \right)^{-1} \quad (14)$$

$$\mu_m = C_m \left(\frac{z}{\sigma_x} A^T x + \frac{1}{\sigma_v} B g \right) \quad (15)$$