Latent variables affecting mean and covariace

$$p(g) = \mathrm{Gam}(g; \alpha_g, \theta_g) \tag{1}$$
$$p(z) = \mathrm{Gam}(z; \alpha_z, \theta_z) \tag{2}$$
$$p(x \mid v, z) = \mathcal{N}(x; zAv, \sigma_x I) \tag{3}$$
$$p_c(v \mid g) = \mathcal{N}(v; 0, \sum_{k=1}^{K} g_k C_k) \tag{4}$$
$$p_m(v \mid g) = \mathcal{N}(v; Bg, \sigma_v I) \tag{5}$$

$$p(v \mid x) = \iint_{-\infty}^{\infty} p(v \mid x, g, z) p(g \mid x) p(z \mid x) \mathrm{d}g \mathrm{d}z \tag{6}$$

we can do this by either numerical integration or sampling

$$p(v \mid x) \approx \sum_{z \in [z_{min} z_{max}]} \sum_{g \in [g_{min} g_{max}]} p(v \mid x, g, z) p(g \mid x) p(z \mid x) \tag{7}$$
$$p(v \mid x) \approx \frac{1}{L} \sum_{l=1}^{L} p(v \mid x, g^l, z^l), \quad g^l \sim p(g \mid x),\ z^l \sim p(z \mid x) \tag{8}$$

either way, we approximate the marginal posterior with a finite mixture. In case of sampling, of L components, for wich the covariance is given in the following form

$$C_{v|x} \approx \frac{1}{L} \sum_{l=1}^{L} C^l_{v|xgz} + (\mu^l_{v|xgz} - \frac{1}{L} \sum_{m=1}^{L} \mu^m_{v|xgz})(\mu^l_{v|xgz} - \frac{1}{L} \sum_{m=1}^{L} \mu^m_{v|xgz})^T \tag{9}$$
$$C_{v|x} \approx \mathrm{E}\left[C^l_{v|xgz}\right]_l + \mathrm{Cov}\left[\mu^l_{v|xgz}\right]_l \tag{10}$$

$$p_c(v \mid x, g, z) = \mathcal{N}(v; \mu_c, C_c) \tag{11}$$

$$C_c = \left(\frac{z^2}{\sigma_x} A^T A + \left[\sum_{k=1}^{K} g_k C_k \right]^{-1} \right)^{-1} \tag{12}$$

$$\mu_c = \frac{z}{\sigma_x} C_c A^T x \tag{13}$$

$$p_c(m \mid x, g, z) = \mathcal{N}(v; \mu_m, C_m) \tag{14}$$

$$C_m = \left(\frac{z^2}{\sigma_x} A^T A + \frac{1}{\sigma_v} I \right)^{-1} \tag{15}$$

$$\mu_m = C_m \left(\frac{z}{\sigma_x} A^T x + \frac{1}{\sigma_v} B g \right) \tag{16}$$