

Model selection for the gestalt model

1 The likelihood function - way 1

The full likelihood looks like this for a set of N images

$$p(X | V, G, Z, C_{1..k}) = \prod_{n=1}^N p(x_n | v_n, z_n) \quad (1)$$

Marginalising over all the latents will produce

$$p(X | C_{1..k}) = \iiint_{-\infty}^{\infty} \prod_{n=1}^N p(x_n | v_n, z_n) p(v_n | g_n) p(g_n) p(z_n) dV dG dZ \quad (2)$$

taking NL samples from the priors of all latents, we can approximate this as

$$p(X | C_{1..k}) \approx \frac{1}{L} \sum_{l=1}^L \prod_{n=1}^N p(x_n | v_{n,l}, z_{n,l}) \quad (3)$$

2 The likelihood function - way 2

$$p(X | C_{1..k}) = \prod_{n=1}^N p(x_n | C_{1..k}) \quad (4)$$

$$\begin{aligned} p(X | C_{1..k}) &= \prod_{n=1}^N \iiint_{-\infty}^{\infty} p(x_n | v_n, z_n) p(v_n | g_n) p(g_n) p(z_n) dv_n dg_n dz_n = \\ &= \prod_{n=1}^N \iiint_{-\infty}^{\infty} p(g_n) p(z_n) \int_{-\infty}^{\infty} p(x_n | v_n, z_n) p(v_n | g_n) dv_n dg_n dz_n \end{aligned} \quad (5)$$

the product of the two conditionals can be rewritten as a Gaussian over v times another Gaussian

$$p(x | v, z) p(v | g) = \frac{1}{\sqrt{\det(A^T A)}} \frac{1}{z^{D_v}} \mathcal{N}\left(\frac{1}{z} A^+ x; 0, \frac{\sigma_x}{z^2} (A^T A)^{-1} + C_v\right) \mathcal{N}(v; \mu_c, C_c) \quad (6)$$

where v only appears in the second Gaussian, so integrating this formula will set that term to 1 and leave everything else as it is.

$$p(X | C_{1..k}) = \det(A^T A)^{-\frac{N}{2}} \prod_{n=1}^N \int_0^\infty p(z) \frac{1}{z^{D_v}} \int_0^\infty p(g) \mathcal{N}\left(\frac{1}{z} A^+ x; 0, \frac{\sigma_x}{z^2} (A^T A)^{-1} + C_v\right) dg dz \quad (7)$$

3 Approximation with sampling

The integrals may be approximated by averaging over samples from $p(g)$ and $p(z)$, leaving out terms constant in g , z and x

$$p(X | C_{1..k}) \sim \prod_{n=1}^N \frac{1}{L^2} \sum_{l_1=1}^L \frac{1}{z_{l_1}^{D_v}} \sum_{l_2=1}^L \mathcal{N}\left(\frac{1}{z_{l_1}} A^+ x_n; 0, \frac{\sigma_x}{z_{l_1}^2} (A^T A)^{-1} + C_v^{l_2}\right) \quad (8)$$

as z and g are independent, we can choose a large enough L and merge the sums, and bring the division by L outside of the product

$$p(X | C_{1..k}) \sim \frac{1}{L^N} \prod_{n=1}^N \sum_{l=1}^L \frac{1}{z_{l_1}^{D_v}} \mathcal{N}\left(\frac{1}{z_l} A^+ x_n; 0, \frac{\sigma_x}{z_l^2} (A^T A)^{-1} + C_v^l\right) \quad (9)$$

taking the logarithm we get

$$\log p(X | C_{1..k}) \sim -N \log(L) + \sum_{n=1}^N \log \left[\sum_{l=1}^L \frac{1}{z_{l_1}^{D_v}} \mathcal{N}\left(\frac{1}{z_l} A^+ x_n; 0, \frac{\sigma_x}{z_l^2} (A^T A)^{-1} + C_v^l\right) \right] \quad (10)$$