

Model selection for the gestalt model

1 The likelihood function

In order to evaluate the log-likelihood for cross validation steps, we have to use an approximation.

$$\begin{aligned} p(X | C_{1..k}) &= \prod_{n=1}^N \iiint_{-\infty}^{\infty} p(x_n | v_n, z_n) p(v_n | g_n) p(g_n) p(z_n) dv_n dg_n dz_n = \\ &= \prod_{n=1}^N \iint_{-\infty}^{\infty} p(g_n) p(z_n) \int_{-\infty}^{\infty} p(x_n | v_n, z_n) p(v_n | g_n) dv_n dg_n dz_n \end{aligned} \quad (1)$$

The Gaussian over x can be rewritten to a Gaussian over v in the following way

$$\mathcal{N}(x; zAv, \sigma_x I) = (2\pi)^{\frac{D_v - D_x}{2}} \sigma_x^{\frac{D_v - D_x}{2}} \frac{1}{z^{D_v}} \sqrt{\det(A^T A)} \mathcal{N}(v; \frac{1}{z} A^+ x, \frac{\sigma_x}{z^2} (A^T A)^{-1}) \quad (2)$$

and with $D_x = D_v$

$$\mathcal{N}(x; zAv, \sigma_x I) = \frac{1}{z^{D_v}} \sqrt{\det(A^T A)} \cdot \mathcal{N}(v; \frac{1}{z} A^+ x, \frac{\sigma_x}{z^2} (A^T A)^{-1}) \quad (3)$$

the product of the two conditionals can be rewritten as a Gaussian over v times another Gaussian

$$\begin{aligned} p(x | v, z) p(v | g) &= \frac{1}{z^{D_v}} \sqrt{\det(A^T A)} \cdot \mathcal{N}(v; \frac{1}{z} A^+ x, \frac{\sigma_x}{z^2} (A^T A)^{-1}) \mathcal{N}(v; 0, C_v) = \\ &= \frac{1}{z^{D_v}} \sqrt{\det(A^T A)} \cdot \mathcal{N}(\frac{1}{z} A^+ x; 0, \frac{\sigma_x}{z^2} (A^T A)^{-1} + C_v) \cdot \mathcal{N}(v; \mu_c, C_c) \end{aligned} \quad (4)$$

where v only appears in the second Gaussian, so integrating this formula will set that term to 1 and leave everything else as it is.

$$p(X | C_{1..k}) = \det(A^T A)^{\frac{N}{2}} \prod_{n=1}^N \int_0^\infty p(z) \frac{1}{z^{D_v}} \int_0^\infty p(g) \mathcal{N}(\frac{1}{z} A^+ x; 0, \frac{\sigma_x}{z^2} (A^T A)^{-1} + C_v) dg dz \quad (5)$$

2 Approximation with sampling

The integrals may be approximated by averaging over samples from $p(g)$ and $p(z)$, leaving out terms constant in g , z and x

$$p(X \mid C_{1..k}) \sim \prod_{n=1}^N \sum_{l_1=1}^L \frac{1}{z_{l_1}^{D_v}} \sum_{l_2=1}^L \mathcal{N}\left(\frac{1}{z_{l_1}} A^+ x_n; 0, \frac{\sigma_x}{z_{l_1}^2} (A^T A)^{-1} + C_v^{l_2}\right) \quad (6)$$