Course Work 1

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1 Solution of question 1

1.1 Answer to question 1:

First, we define the state variables:

$$\text{Let } x_1 = \theta$$

$$\text{Let } x_2 = \dot{\theta}$$

Then the state vector is:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

We can write the state-space equations:

$$\dot{x} = f(x) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -kx_2 - \sin(x_1) \end{bmatrix}$$

Thus, the state-space of this system can be represented as follows:

$$\dot{x}(t) = \begin{bmatrix} x_2 \\ -kx_2 - \sin(x_1) \end{bmatrix}$$

1.2 Answer to question 2:

we can define the function named pendulum in Matlab.The specific description of the function is as follows:

```
function dx = pendulum(t, x, k)
    dx = zeros(2, 1);
    dx(1) = x(2);
    dx(2) = -k * x(2) - sin(x(1));
end
```

1.3 Answer to question 3:

We can define the function named simulatePendulum.And the values of k are $k = 0$ and $k = 1$.And select the initial condition with $[3; 0]$. And use the $ode23$ algorithm to get $[t, x]$. Then plot the figure of t and x.

The description of the function is as follows.

```
function simulatePendulum()
    k_values = [0, 1];
    initial_conditions = [3; 0]; % Initial condition [theta; angular velocity]

    for k = k_values
        tspan = [0, 50]; % Simulation time
        [t, x] = ode23(@(t, x) pendulumDynamics(x, k), tspan, initial_conditions);

        % Plot results
        figure;
        plot(t, x(:, 1), 'b', 'DisplayName', 'Theta (x_1)');
```

```
        hold on;
        plot(t, x(:, 2), 'r', 'DisplayName', 'Angular Velocity (x_2)');
        title(['Pendulum Simulation for $k = ', num2str(k), '$'], 'Interpreter', 'latex');
        xlabel('Time (s)', 'Interpreter', 'latex');
        ylabel('State Variables', 'Interpreter', 'latex');
        legend({'$\theta$ ($x_1$)', 'Angular Velocity ($x_2$)'}, 'Interpreter', 'latex');
        grid on;

    end
end
```

Run simulations for both $k = 0$ and $k = 1$. We can get the figures as follows.

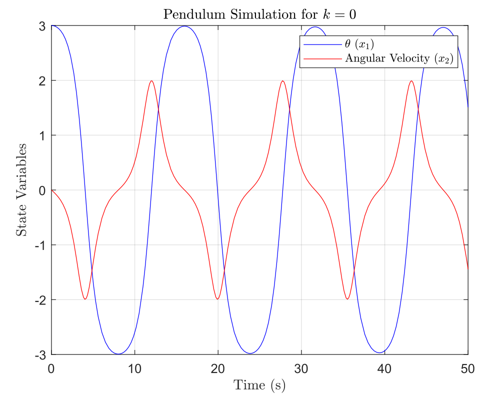

Figure 1: Pendulum Simulation for $k = 0$

It can be seen from Figure 1 that when $k = 0$, for system states $x1$ and $x2$, they both show obvious periodic characteristics.

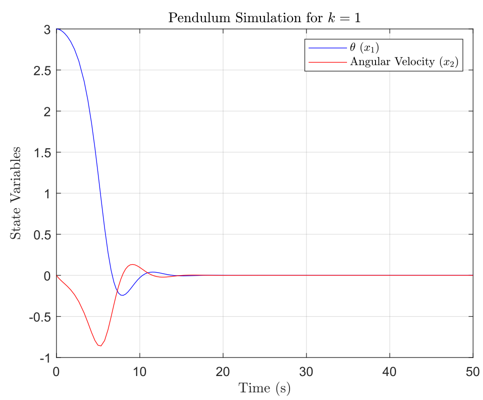

Figure 2: Pendulum Simulation for $k = 1$

It can be seen from Figure 2 that when $k = 1$, for the system states $x1$, $x2$, they gradually converge to 0 as time increases.we can find that the $\omega - limit$ is $[0, 0]$.

1.4 Answer to question 4:

```
k = 0;
[t, y] = ode23(@(t, y) pendulum(t, y, k), [0, 30], [3; 0]);
figure;
plot(y(:,1), y(:,2));
xlabel('x');
ylabel('y');
```

```
title('Phase Plane Plot');
grid on;
axis equal;
hold on;
if size(y, 1) > 1
    dx = y(2:end, 1) - y(1:end-1, 1);
    dy = y(2:end, 2) - y(1:end-1, 2);
    quiver(y(1:end-1, 1), y(1:end-1, 2), dx, dy, 0.5, 'r', 'LineWidth', 2);
end
xlabel('$x_1 (\theta)$', 'Interpreter', 'latex');
ylabel('$x_2 (\dot{\theta})$', 'Interpreter', 'latex');
title('Phase-Plane Portrait of the Pendulum', 'Interpreter', 'latex');
grid off;
hold off;
```

Two phase-plane diagrams are obtained by running the above MATLAB code, corresponding to the cases of $k = 0$ and $k = 1$.

When $k = 0$: The phase-plane diagram is shown in Figure 3.

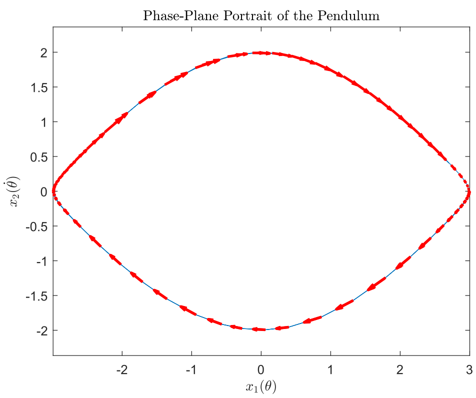

Figure 3: Phase-Plane Portrait of the Pendulum for $k = 0$

According to the figure, the phase-plane portrait displays closed, periodic trajectories that represent the undamped motion of the pendulum. These trajectories remain confined to the same region of the phase space without attenuation, indicating perpetual oscillations with a constant amplitude.

When $k = 1$: The phase-plane diagram is shown in Figure 4.

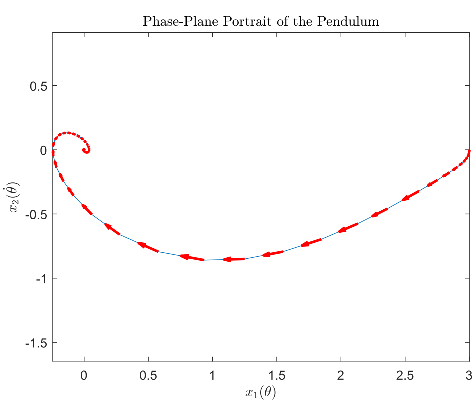

Figure 4: Phase-Plane Portrait of the Pendulum for $k = 1$

According to the figure, The system demonstrates damping, leading to trajectories that contract over time and converge at the origin. Higher values of k result in faster damping, causing the system to lose energy more quickly and reach a state of rest sooner. The trajectories transition from periodic to transient behavior as the pendulum gradually comes to a stop.

1.5 Answer to question 5:

In this system equation, k is a damping coefficient that controls the magnitude of the resistance related to the velocity. A larger value of k means the system will experience faster damping, and the amplitude will decrease more quickly.

2 Solution of question 2

The equations of the system can be written as follows.

$$\dot{x}_1 = \sigma(x_2 - x_1)$$
$$\dot{x}_2 = 28x_1 - x_2 - x_1x_3$$
$$\dot{x}_3 = x_1x_2 - \frac{8}{3}x_3$$

To describe this system equation, we can use MATLAB, the code is shown below.

```
sigma = sigma_values(i);
    % Define the system of differential equations
    dxdt = @(t, x) [
        sigma * (x(2) - x(1));
        28 * x(1) - x(2) - x(1) * x(3);
        x(1) * x(2) - 8/3 * x(3)
    ];
```

2.1 Answer to question 1:

The simulation of the system using Matlab is as follows, given the initial condition with $x_0 = [1, 1, 1]$.And hoose different σ.Then run the simulations for different values of the parameters σ.

```
%%
% Define the parameters for the equation
sigma_values = [0,5,10, 20, 30]; % Example different sigma values
initial_conditions = [1, 1, 1]; % Initial conditions
tspan = [0 100]
% Loop through different sigma values
for i = 1:length(sigma_values)
    sigma = sigma_values(i);
    % Define the system of differential equations
    dxdt = @(t, x) [
        sigma * (x(2) - x(1));
        28 * x(1) - x(2) - x(1) * x(3);
        x(1) * x(2) - 8/3 * x(3)
    ];
    % Solve the differential equations
    [t, x] = ode23(dxdt, tspan, initial_conditions);
    % Plot the 3D trajectory
    figure;
    plot3(x(:,1), x(:,2), x(:,3));
    % Add titles and labels with LaTeX formatting
    title(sprintf('3D Trajectory ($\\sigma = %d$)', sigma), 'Interpreter', 'latex');
    xlabel('$x_1$', 'Interpreter', 'latex');
    ylabel('$x_2$', 'Interpreter', 'latex');
    zlabel('$x_3$', 'Interpreter', 'latex');
    grid on;
end
```

We can know the Phase-Plan Portrait of the systems with parameters for $\sigma = 0, 5, 10, 20, 30$.The starting point and the end point of the system phase plane are shown in the figure.

The Phase-Plane Portrait of the system for $\sigma = 0$ is as follows.We can see that the ω-limit is $[1.00, 20.36, 7.64]$.

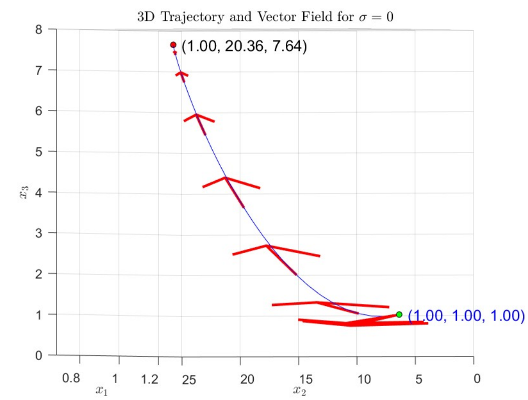

Figure 5: *Phase-Plane Portrait of the system for* $\sigma = 0$

The Phase-Plane Portrait of the system for $\sigma = 5$ is as follows.

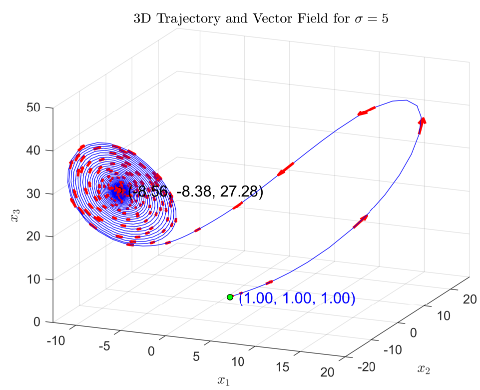

Figure 6: *Phase-Plane Portrait of the system for* $\sigma = 5$

The Phase-Plane Portrait of the system for $\sigma = 10$ is as follows.We can see that the ω-limit is $[-8.56, -8.38, 27.28]$.

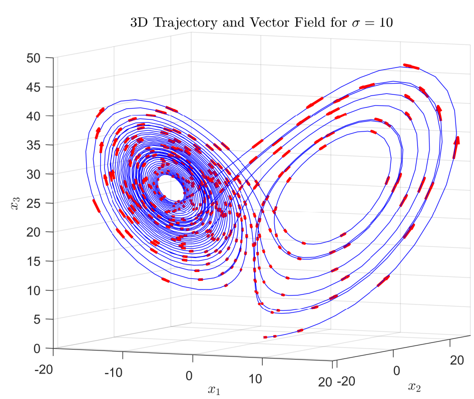

Figure 7: *Phase-Plane Portrait of the system for* $\sigma = 10$

The Phase-Plane Portrait of the system for $\sigma = 20$ is as follows.We can see that the ω-limit is $[-8.78, -8.81, 27.14]$.

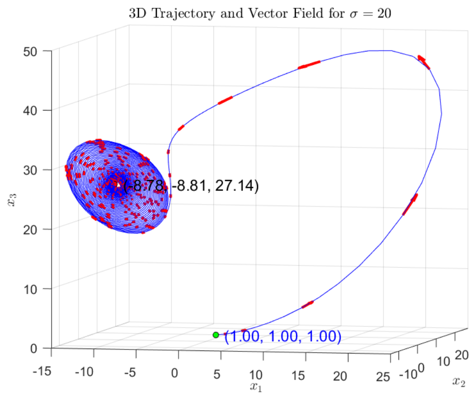

Figure 8: *Phase-Plane Portrait of the system for* $\sigma = 20$

The Phase-Plane Portrait of the system for $\sigma = 30$ is as follows.We can see that the ω-limit is $[-8.49, -8.49, 27.00]$.

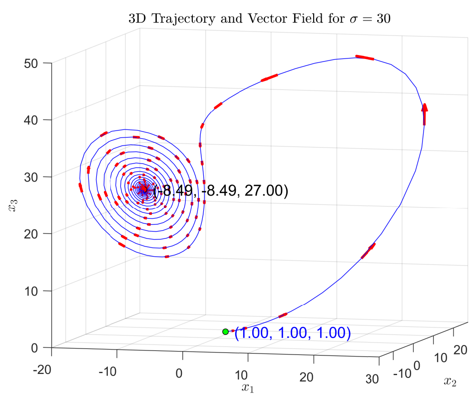

Figure 9: *Phase-Plane Portrait of the system for* $\sigma = 30$

2.2 Answer to question 2:

After running the simulations for different values of σ, we can find the following results:

- $\sigma = 0$ and $\sigma = 5$:
  In these cases, the phase trajectory eventually converges to a stable point, indicating that the system's ω-limit set is a stable equilibrium point. This implies that the system stabilizes over time and exhibits steady behavior.

- $\sigma = 10$:
  When $\sigma = 10$, the system displays chaotic behavior. The phase trajectory becomes complex and unpredictable, no longer converging to a single point or periodic orbit. In this case, the ω-limit set is a chaotic attractor, indicating high sensitivity to initial conditions and nonlinear complexity.

- $\sigma = 20$ and $\sigma = 30$:
  For these values, the system returns to stability. The phase trajectories converge to a stable point, demonstrating that the system has a stable equilibrium point as its ω-limit set.

2.3 Answer to question 3:

We can know that for the $\sigma = 10$, the $\omega - limit$ set is neither a closed-orbit nor an equilibrium.