國立陽明交通大學 期貨與選擇權

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1. Standard Error and Number of Trials

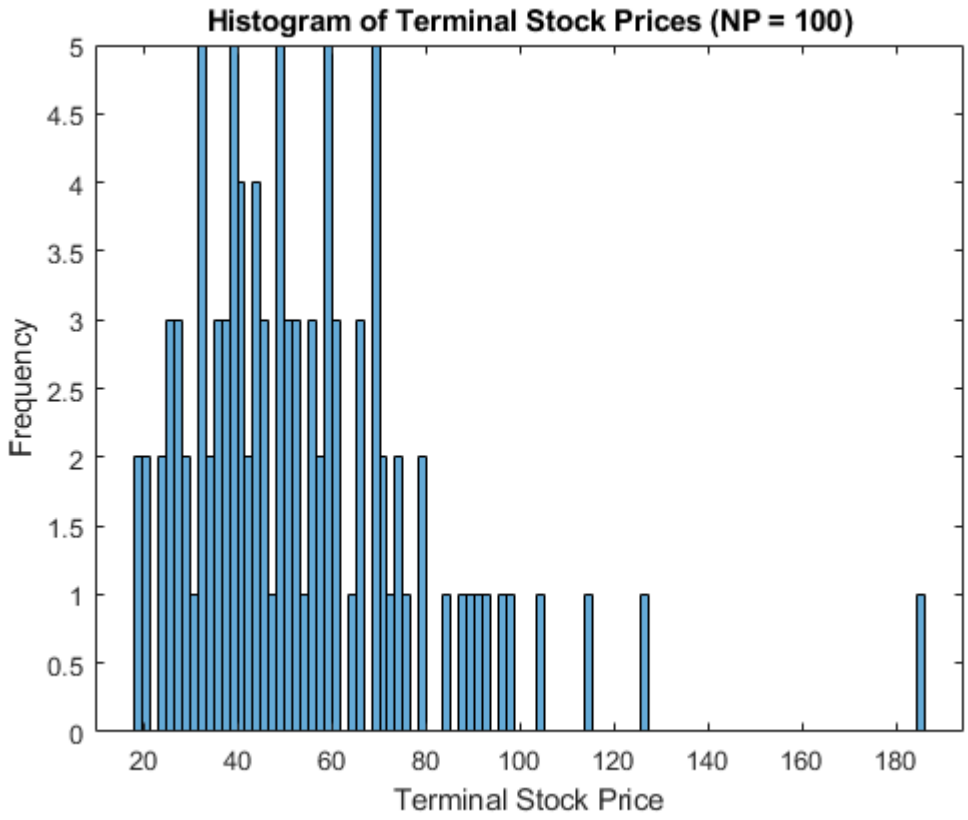
Findings

1. Black-Scholes Price
   The theoretical put option price calculated using the Black-Scholes formula is 9.7086.

2. Monte Carlo Simulation Results for Different NP Values
   As the number of paths
   NP increases, the estimated put option price from the Monte Carlo simulation converges closer to a specific value, and the confidence interval (CI) becomes narrower. This trend indicates that a higher number of trials reduces the uncertainty and standard error in the Monte Carlo estimate. The specific results for each NP are as follows:

- $NP = 100$
  Estimated Put Price = 7.6252
  Standard Error (SE) = 1.0170

Confidence Interval (CI) = [5.6318, 9.6186]

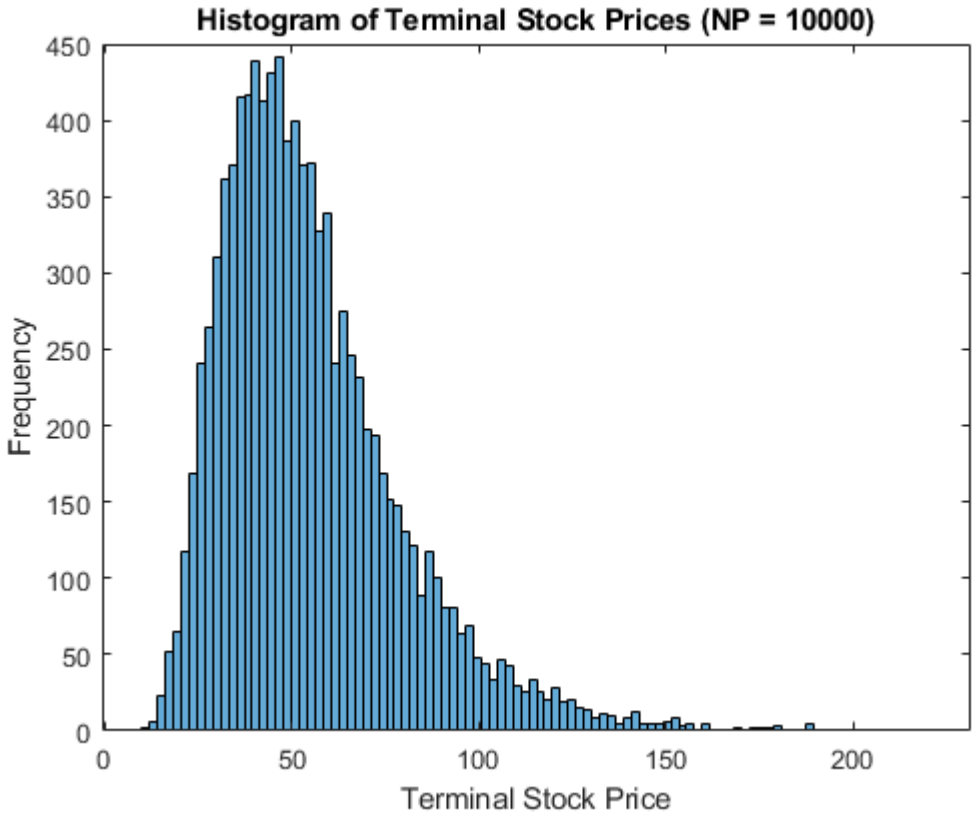


- $NP = 10,000$

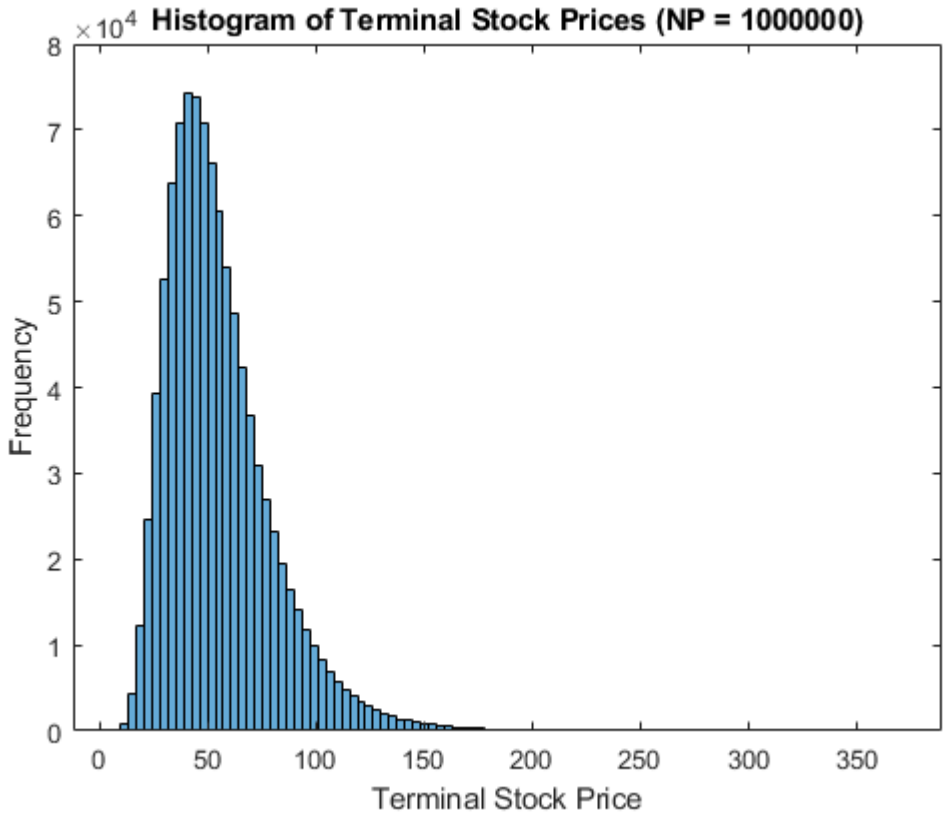
  Estimated Put Price = 6.7285

  SE = 0.0947

  CI = [6.5428, 6.9142]



- $NP = 1,000,000$
  Estimated Put Price = 6.7733
  SE = 0.0096
  CI = [6.7546, 6.7920]



3. Observation on CI Range and Accuracy
   With an increase in NP, the confidence interval range decreases significantly, indicating greater precision in the estimated put price. For instance, with $NP = 1,000,000$, the confidence interval is very narrow, demonstrating a high degree of certainty in the estimated price.

4. Discrepancy with Black-Scholes Price
   Even at higher NP values, the Monte Carlo estimated price (6.7733) is consistently lower than the Black-Scholes price (9.7086). This discrepancy could suggest that under the model assumptions and given the parameters, there might be a potential arbitrage opportunity if the actual market price aligns with the Black-Scholes valuation. This difference could be attributed to the approximations in the Monte Carlo method, or it could suggest that certain model assumptions, such as constant volatility, may not fully align with real-world conditions.

2.Standard Deviation of Estimated Option Values

$NP = 100$	$NP = 10000$	$NP = 1000000$
6.34574491275757	6.82152422986999	6.75879721475832
6.34057914890168	6.75989201416168	6.74605506531562
7.10709228348023	6.61553198061275	6.74795497169713
5.38043618753620	6.67630998713238	6.77008355305352
7.33485904198642	6.75108651170151	6.76922056518932
6.52433600325853	6.69789456373222	6.75424115427578
6.18125169683106	6.79263573995027	6.76380745243147
6.59456600652808	6.82712359299847	6.76459054723114
6.58245162413376	6.78727860034991	6.76435648619773
8.15817773972481	6.70810123352330	6.75935719793227
6.98374638214303	6.71768590442849	6.75119428324343
5.96752409268074	6.78733409446681	6.75964051623337
5.56538023948252	6.73112524100247	6.76262704306902
5.64938654847080	6.96039620310111	6.75813350811677
7.16024468176386	6.89395912466664	6.75863396845243
6.44000009472945	6.70331081066848	6.77023387311953
6.87161054333031	6.75432611680321	6.77143297788903
7.43036005743128	6.73234841086921	6.75492111234723
5.37270847653776	6.71169028391566	6.75757637197776
5.78241313902117	6.82718193433542	6.76765097024072
7.34055950190515	6.79282953517022	6.76844260238276
6.38234649774223	6.68627083752207	6.76453362468839
7.42978936001216	6.73935555021774	6.75536618672825

$NP = 100$	$NP = 10000$	$NP = 1000000$
5.25180574424020	6.79962237608108	6.76648851726982
7.18234661038956	6.73073046051044	6.75368955611223
6.85603248921101	6.77839838250161	6.75473226061558

- $NP = 100$
  Mean = 6.7343
  STD = 0.9105

- $NP = 10,000$
  Mean = 6.7555
  STD = 0.0873

- $NP = 1,000,000$
  Mean = 6.7596
  STD = 0.0088

1. Consistent Mean Across Experiments:The mean estimated put price across the 500 experiments is relatively consistent for different NP values, indicating that the Monte Carlo method converges to a stable estimate of the put price with a sufficient number of paths.

2. Decreasing Standard Deviation with Higher NP: As NP (the number of paths) increases, the standard deviation of the estimated put prices across the 500 experiments decreases significantly. This reduction in variability is due to the increased accuracy of each individual experiment as NP grows, demonstrating that a larger NP leads to more reliable and precise estimates with reduced random fluctuation.

3. Impact of NP on Estimate Precision: The decrease in standard deviation with higher NP illustrates that larger sample sizes in each Monte Carlo simulation lead to more concentrated and stable results around the mean. This trend confirms that the Monte Carlo method's accuracy and consistency are improved as NP increases, reducing the influence of random sampling error.