國立陽明交通大學 期貨與選擇權

學生姓名:徐浩哲 學號:411551005

1. Basics ch15-15

a)

Since $N(x)$ is the cumulative probability that a variable with a standardized normal distribution will be less than x , $N'(x)$ is the probability density function for a standardized normal distribution, that is,

$$N'(x) = \frac{1}{2\pi}\exp^{\frac{x^2}{2}}$$

b)

$$N'(d1) = N'(d_2 + \sigma\sqrt{T-t})$$

$$= \frac{1}{2\pi}\exp[-\frac{d_2^2}{2} - \sigma d_2\sqrt{T-t} - \frac{1}{2}\sigma^2(T-t)]$$

$$= N'(d_2)\exp[-\sigma d_2\sqrt{T-t} - \frac{1}{2}\sigma^2(T-t)$$

Beacuse

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{(}T-t)}$$

it follows that

$$\exp[-\sigma d_2\sqrt{T-t} - \frac{1}{2}\sigma^2(T-t)] = \frac{Ke^{-r(T-t)}}{S}$$

As the results,

$$SN'(d_1) = Ke^{-r(T-t)}N'(d_2)$$

c)

$$d_1 = \frac{\ln S - \ln K + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

Hence,

$$\frac{\partial d_1}{\partial S} = \frac{1}{S\sigma\sqrt{T-t}}$$

Similarly,

$$d_1 = \frac{\ln S - \ln K + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

Hence,

$$\frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{T-t}}$$

Therefore,

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$$

d)

$$c = SN(d_1) - Ke^{-r(T-t)}N(d2)$$

$$\frac{\partial c}{\partial t} = SN'(d_1)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial t}$$

From (b),

$$SN'(d_1) = Ke^{-r(T-t)}N'(d_2)$$

Hence,

$$\frac{\partial c}{\partial t} = -Ke^{-r(T-t)}N(d_2) + SN'(d_1)(\frac{\partial d_1 - \partial d_2}{\partial t})$$

Since

$$d_1 - d_2 = \sigma\sqrt{T-t}$$

$$\frac{\partial d_1 - \partial d_2}{\partial t} = \frac{\partial}{\partial t}(\sigma\sqrt{T-t})$$

$$= -\frac{\sigma}{2\sqrt{T-t}}$$

Hence,

$$\frac{\partial c}{\partial t} = -rKe^{-r(T-t)}N(d_2) - SN'(d_1)\frac{\sigma}{2\sqrt{T-t}}$$

e)

From differentiating the Black–Scholes–Merton formula for a call price, we obtain

$$\frac{\partial c}{\partial s} = N(d_1) + SN'(d_1)\frac{\partial d_1}{\partial S} - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S}$$

From the results in b) and c), it follows that

$$\frac{\partial c}{\partial S} = N(d_1)$$

f)

Differentiating the result in e) and using the result in c), we obtain

$$\frac{\partial c}{\partial S} = N(d_1) + SN'(d_1)\frac{\partial d_1}{\partial S} - Ke^{-r(T-t)}N'(d_2)\frac{\partial d_2}{\partial S}$$

From the results in d) and e)

$$\frac{\partial c}{\partial S} = N(d_1)$$

This shows that the Black–Scholes–Merton formula for a call option does indeed satisfy the Black–Scholes–Merton differential equation.

g)

Consider what happens in the formula for c in part (d) as t approaches T . If $S > K$, d_1 and d_2 tend to infinity and $N(d1)$ and $N(d2)$ tend to 1. If $S < K$, d_1 and d_2 tend to minus infinity and $N(d1)$ and $N(d2)$ tend to zero. It follows that the formula for c tends to $max(S-K, 0)$.

2. Hedging Performance

a, b) Delta Hedging + Stop-Loss Strategy

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Interval : 5.00 Weeks
The performance of the Stop-Loss is : 1.02
The performance of the Delta Hedging is : 0.39
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Interval : 4.00 Weeks
The performance of the Stop-Loss is : 0.98
The performance of the Delta Hedging is : 0.36
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Interval : 2.00 Weeks
The performance of the Stop-Loss is : 0.89
The performance of the Delta Hedging is : 0.26
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Interval : 1.00 Weeks
The performance of the Stop-Loss is : 0.87
The performance of the Delta Hedging is : 0.19
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Interval : 0.50 Weeks
The performance of the Stop-Loss is : 0.81
The performance of the Delta Hedging is : 0.14
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Interval : 0.25 Weeks
The performance of the Stop-Loss is : 0.79
The performance of the Delta Hedging is : 0.10
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3. Greeks Letters

a) European put option

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delta = -0.3611
theta = -0.7454
gamma = 0.0177
vega = 26.4831
rho = -49.6351
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b) binomial model

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Binomial Greeks:
Delta: -0.361256
Gamma: 0.017682
Vega: 26.373242
Theta: -85.598369
Rho: -49.639643
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