

1.

$$d[E]/dt = (k_2 + k_3)[ES] - k_1[E][S]$$

$$d[S]/dt = k_2[ES] - k_1[E][S]$$

$$d[ES]/dt = k_1[E][S] - (k_2 + k_3)[ES]$$

$$d[P]/dt = k_3[ES]$$

2. Fail to get the answer.

3.

$$V = d[P]/dt = k_3[ES]$$

$$d[ES]/dt = k_1[E][S] - (k_2 + k_3)[ES]$$

Assuming that it's in a steady state which means that the concentration of ES remains constant.
E₀ means the initial amount of E.

$$k_1[E][S] = (k_2 + k_3)[ES]$$

$$\text{and } k_1[E][S] = k_1([E_0] - [ES])[S]$$

$$\text{Thus } k_1([E_0] - [ES])[S] = (k_2 + k_3)[ES]$$

$$[ES] = k_1[E_0][S] / (k_1[S] + k_2 + k_3)$$

$$V = d[P]/dt = k_3 k_1 [E_0][S] / (k_1[S] + k_2 + k_3) = k_3 [E_0][S] / ([S] + (k_2 + k_3)/k_1)$$

$$\text{Suppose } k_m = (k_2 + k_3)/k_1$$

$$V = k_3 [E_0][S] / ([S] + k_m) = k_3 [E_0] / (1 + k_m/[S])$$

When the concentrations of S are small, $[S] \ll k_m$, v increase approximately linearly, the gradient is about $k_3[E_0]/k_m$.

When concentrations of S become are very large, $[S] \gg k_m$

$$V_{\max} = k_3[E_0]$$

