LLR demodulation for 5G system

Jie Liu hahaliu2001@gmail.com

introduction

the document shows how to implement LLR(log-likelihood ratio) for pi/2-BPSK, BPSK, QPSK, 16QAM, 64QAM, 256QAM, 1024QAM for 5G system.

The LLR algorithm has implemented with Python and verified with Matlab 5G toolbox LLR function.

Some explanations

Why choose $LLR(b_k) = \ln\left(\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right)$ rather than$LLR(b_k) = \ln\left(\frac{P(b_k = 1|r)}{P(b_k = 0|r)}\right)$

LLR value represent the possibility of bit=0 or bit =1 which is used for LDPC decoder or pola decoder.

Which equation to be selected depends on how LDPC/polar decoder maps bit=0/1 to LLR value.

Starting from 2G era, $LLR(b_k) = \ln\left(\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right)$ has been chosen. The reason could be that for BPSK modulation, bit=0 maps to constellation value 1, bit=1 maps to constellation value -1.

When choosing LLR equation, it is expected that LLR>0 for bit=0, LLR <0 for bit=1

LLR demodulation input symbol average power should be 1

Input symbol: $r = s + n$

Expected that $E(rs^*) = E\big((s+n)s^*\big) = E(ss^*) = 1$

If using MMSE for channel equalization, $E(rs^*) \neq 1$, MMSE estimated symbol result need additional compensation before LLR demodulation.

LLR algorithm input noise variance is complex noise variance

Reference papers

[1] Juquan Mao; Mahmoud Alfa Abdullahi; Pei Xiao; Aijun Cao： “A low complexity 256QAM soft demapper for 5G mobile system”
https://www.researchgate.net/publication/307940673_A_low_complexity_256QAM_soft_demapper_for_5G_mobile_system
[2] Xianle Cao , Yi Liu, Dongfang Hu: “Simplified LLR algorithm for m-QAM demodulation”

Both papers have some errors, while the basic idea are correct.

complex normal distributions

normal distributions

$N \backsim \left(\mu, \sigma^2\right)$

probability density function（pdf）: $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

complex normal distributions

complex normal distributions $Z = X + iY$

where $X \backsim (\mu_x, \sigma_x^2), Y \backsim \left(\mu_y, \sigma_y^2\right), \mu = \mu_x = \mu_y, \sigma^2 = \sigma_x^2 = \sigma_y^2$

$\mu_z = \mu_x + i\mu_y = \mu + i\mu, \sigma_z^2 = 2\sigma^2$

probability density function（pdf）:

$$p(xy) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-\mu)^2+(y-\mu)^2}{2\sigma^2}}$$

$$p(z) = \frac{1}{2\pi\sigma^2} e^{-\frac{(z-\mu_z)^2}{2\sigma^2}} = \frac{1}{\pi\sigma_z^2} e^{-\frac{(z-\mu_z)^2}{\sigma_z 2}}$$

LLR demodulation

Input: $r = s + n$

$r_i + jr_q = s_i + js_q + n_i + jn_q$

probability density function(pdf) for real, imag, and complex:

$$p(r_i|s_i) = \frac{1}{\sqrt{\pi\sigma_z^2}} e^{-\frac{(r_i - s_i)^2}{\sigma_z^2}}$$

$$p(r_q|s_q) = \frac{1}{\sqrt{\pi\sigma_z^2}} e^{-\frac{(r_q - s_q)^2}{\sigma_z^2}}$$

$$p(r|s) = \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i - s_i)^2 + (r_q - s_q)^2}{\sigma_z^2}}$$

N bits are mapped to one constellation point. N value is related to different modulation. For 16QAM, 4 bits are mapped to one constellation point. Each bit value could be 0 or 1.

The possibility of $b_k = A(0 or\ 1)$:

$$\begin{aligned} p(r|b_k = A) &= \sum_{b:b_k=A} p(r|b) \\ &= \sum_{b:b_k=A} p\big(r - s(b)\big) \\ &= \sum_{b:b_k=A} \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i - s(b)_i)^2 + (r_q - s(b)_q)^2}{\sigma_z^2}} \end{aligned}$$

$$
\begin{aligned}
LLR(b_k) &= \ln\left(\frac{P(b_k=0|r)}{P(b_k=1|r)}\right) \\
&= \ln\left(\frac{\frac{p(r|b_k=0)P(b_k=0)}{p(r)}}{\frac{p(r|b_k=1)P(b_k=1)}{p(r)}}\right) \\
&= \ln\left(\frac{p(r|b_k=0)}{p(r|b_k=1)}\right) \\
&= \ln\left[\frac{\sum_{b:b_k=0}\frac{1}{\pi\sigma_z^2}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}{\sum_{b:b_k=1}\frac{1}{\pi\sigma_z^2}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}\right] \\
&= \ln\left[\frac{\sum_{b:b_k=0}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}{\sum_{b:b_k=1}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}\right]
\end{aligned}
$$

$$
LLR(b_k) \approx ln\left[\frac{max_{b:b_k=0}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}{max_{b:b_k=1}e^{-\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2}}}\right]
$$

$$
= -min_{b:b_k=0}\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2} + min_{b:b_k=1}\frac{(r_i-s(b)_i)^2+\left(r_q-s(b)_q\right)^2}{\sigma_z^2} \qquad (1)
$$

$$
LLR(b_k, k\ is\ even) = -min_{b:b_k=1}\frac{(r_i-s(b)_i)^2}{\sigma_z^2} + min_{b:b_k=0}\frac{(r_i-s(b)_i)^2}{\sigma_z^2} \qquad (2)
$$

$$
LLR(b_k, k\ is\ odd) = -min_{b:b_k=1}\frac{\left(r_q-s(b)_q\right)^2}{\sigma_z^2} + min_{b:b_k=0}\frac{\left(r_q-s(b)_q\right)^2}{\sigma_z^2} \qquad (3)
$$

Equation (1) is used to calculate π/2-BPSK 和 BPSK LLR

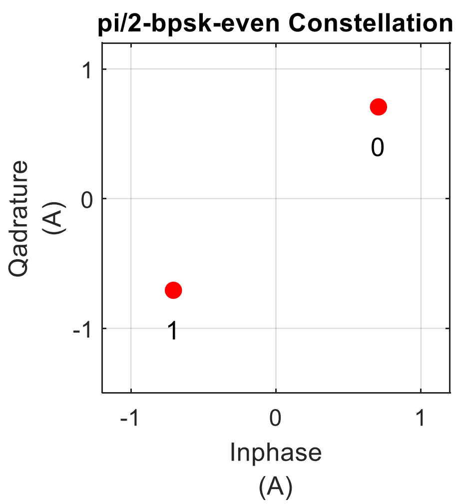
For QPSK/16QAM/64QAM/256QAM/1024QAM,   even bits maps to constellation real part, odds bits maps to constellation image part. Equation (2) is used to calculate even bits LLR. Equation (3) is used to calculate odd bits LLR

π/2-BPSK LLR

$$d(i) = \frac{e^{j\frac{\pi}{2}(i \bmod 2)}}{\sqrt{2}}\left[(1-2b(i)) + j(1-2b(i))\right]$$

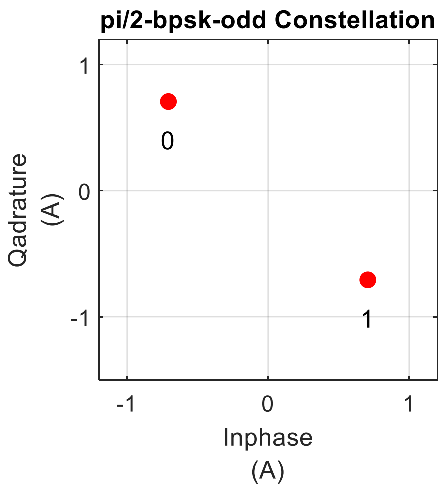
π/2-BPSK even bits and odd bits maps to different constellation and use different LLR equation

even bits π/2-BPSK



$$LLR = \frac{1}{\sigma_Z^2} 4(r_i + r_q)A \quad A = \frac{1}{\sqrt{2}}$$

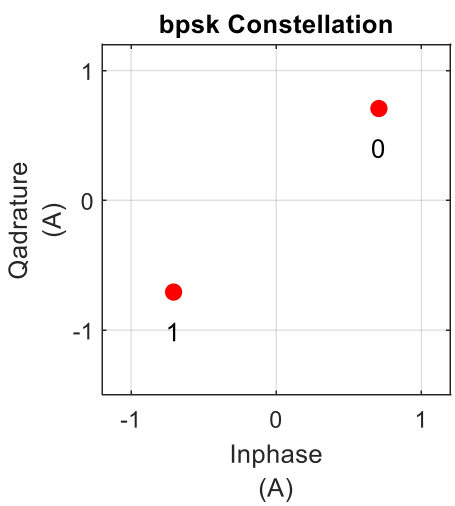
odd bits π/2-BPSK



$$LLR = \frac{1}{\sigma_Z^2}4(-r_i + r_q)A \quad A = \frac{1}{\sqrt{2}}$$

BPSK LLR

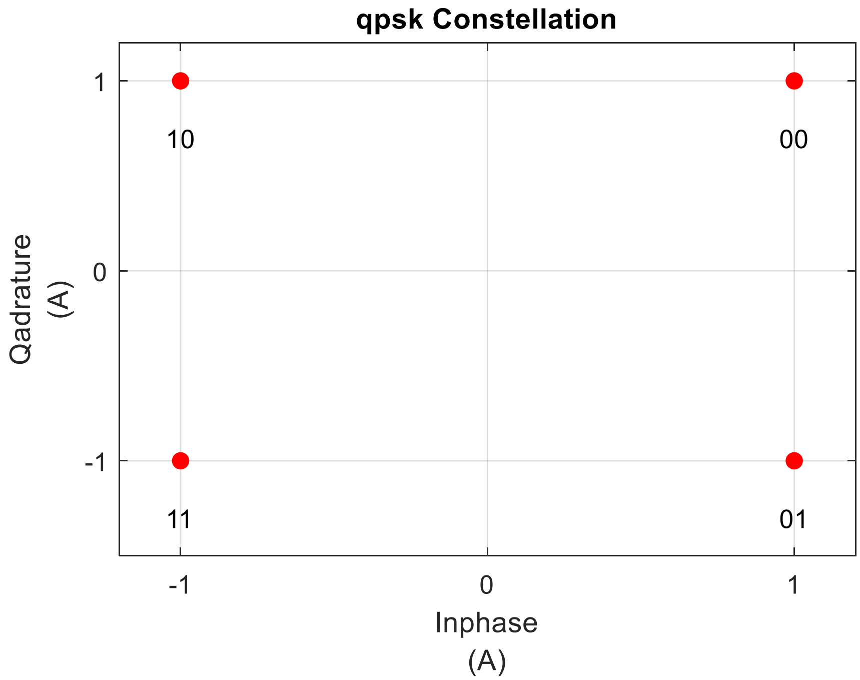
$$d(i) = \frac{1}{\sqrt{2}}\left[(1-2b(i)) + j(1-2b(i))\right]$$



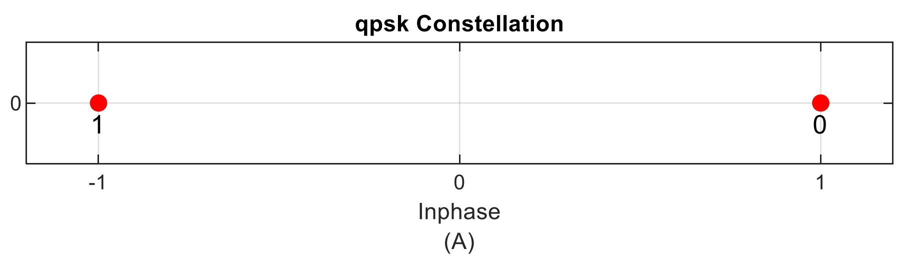
$$LLR = \frac{1}{\sigma_Z^2}4(r_i + r_q)A \quad A = \frac{1}{\sqrt{2}}$$

QPSK LLR

$$d(i) = \frac{1}{\sqrt{2}}\left[(1-2b(2i)) + j(1-2b(2i+1))\right]$$



Below figure shows even bit [b0] maps to real part



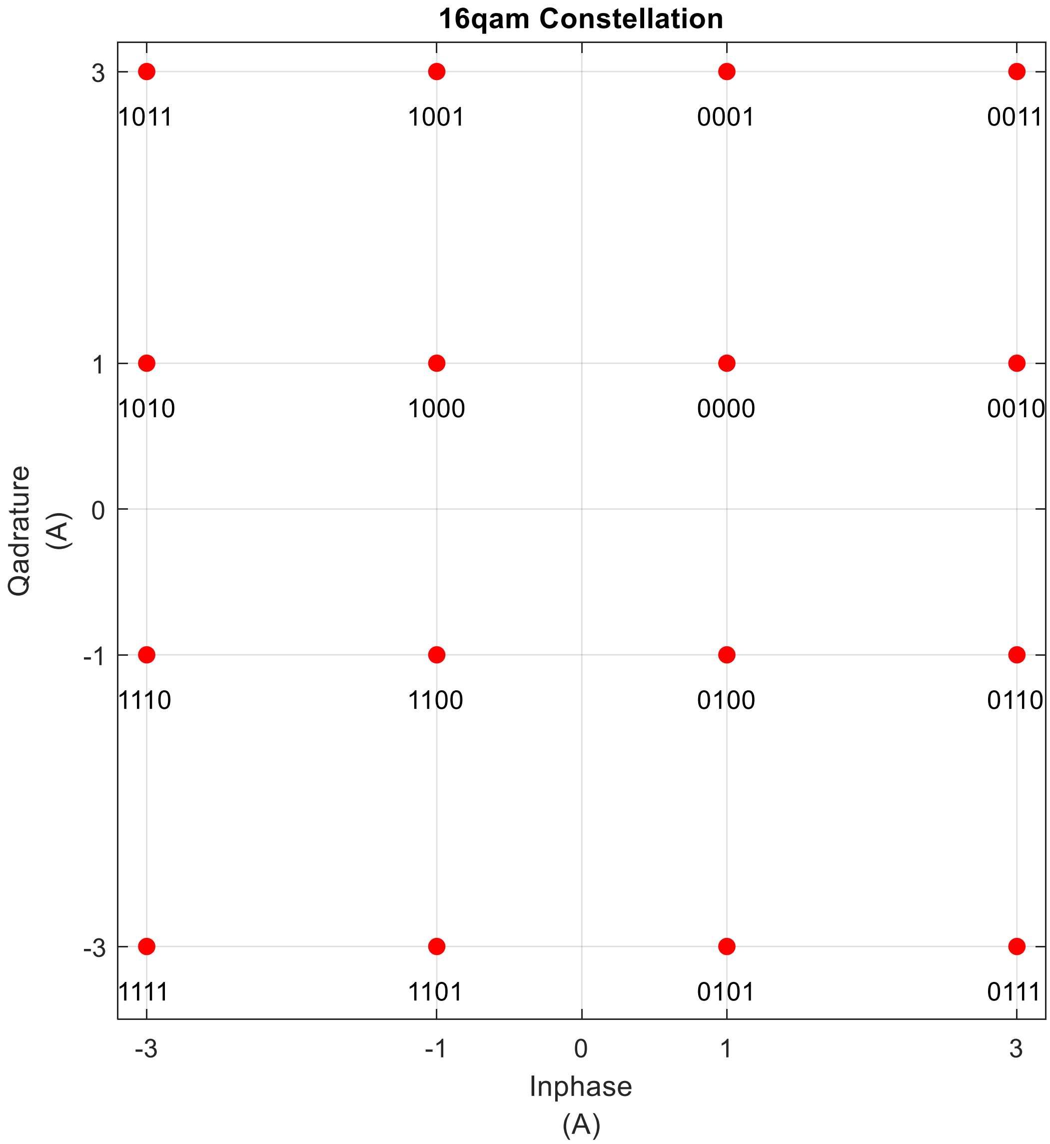
$$A = \frac{1}{\sqrt{2}}$$

$$LLR\_b0 = \frac{1}{\sigma_z^2} 4 r_i A$$

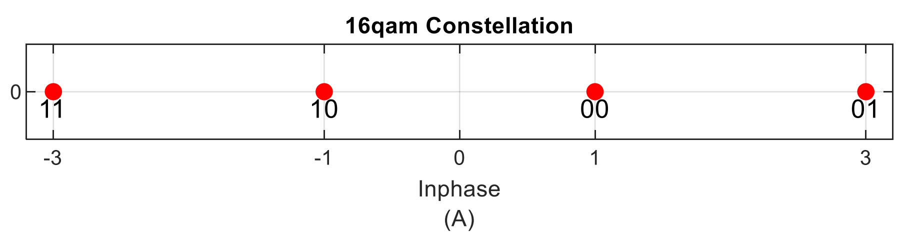
$$LLR\_b1 = \frac{1}{\sigma_z^2} 4 r_q A$$

16QAM LLR

$$d(i) = \frac{1}{\sqrt{10}} \left\{ \left(1-2b(4i)\right)\left[2-\left(1-2b(4i+2)\right)\right] + j\left(1-2b(4i+1)\right)\left[2-\left(1-2b(4i+3)\right)\right] \right\}$$



Below figure shows even bit [b0,b2] maps to real part



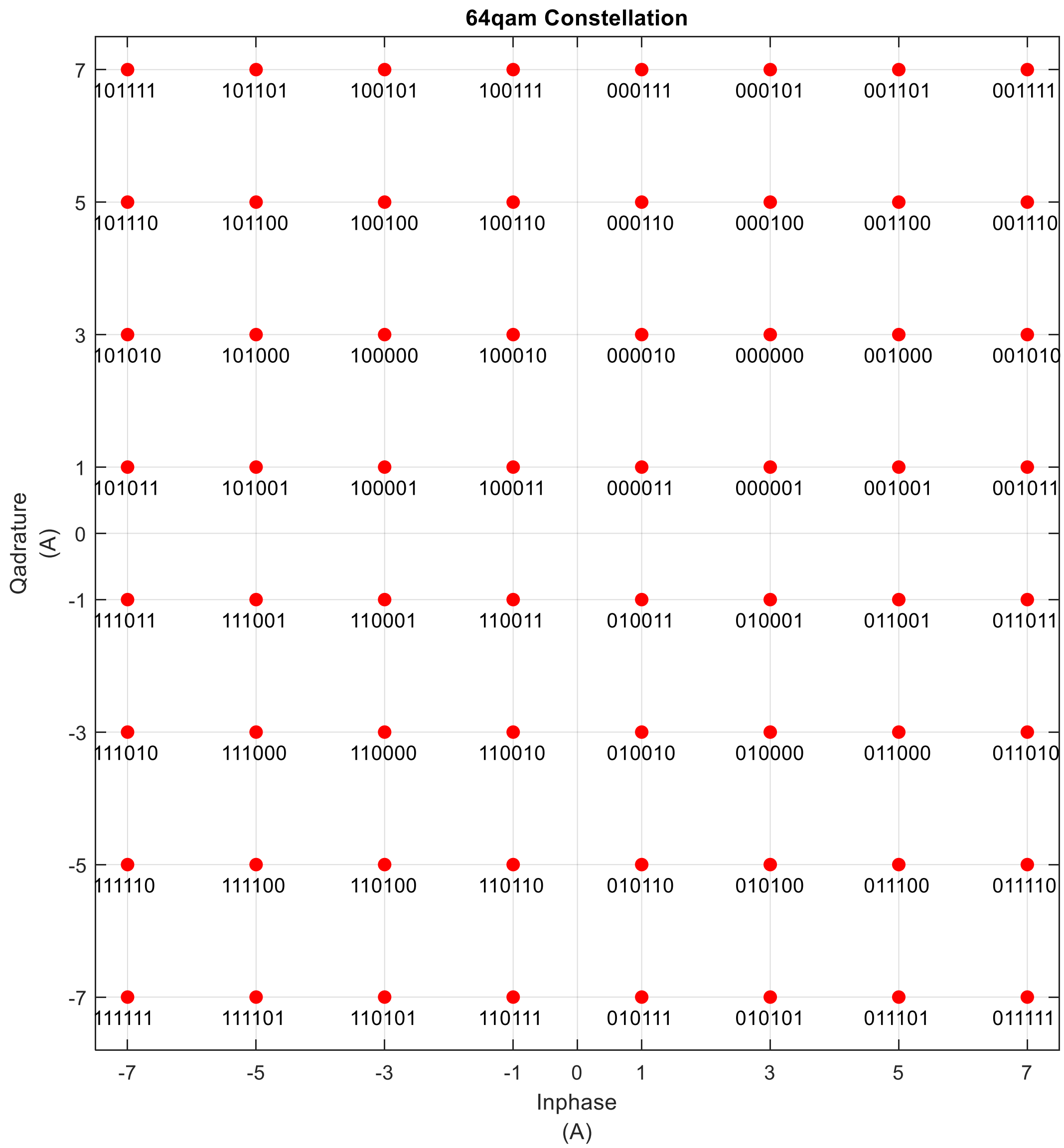
$$A = \frac{1}{\sqrt{10}}$$

$$LLR_{b0} = \begin{cases} \frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_z^2} 8A(r_i + A) & r_i < -2A \\ \frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 4Ar_i & -2A \le r_i < 2A \\ \frac{1}{\sigma_z^2}(-(r_i - 3A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 8A(r_i - A) & 2A \le r_i \end{cases}$$

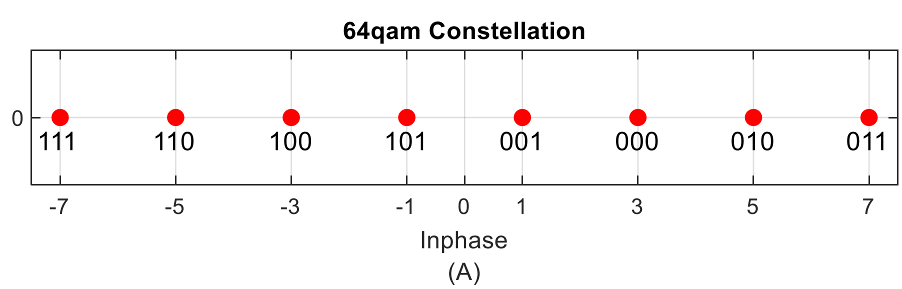
$$LLR_{b2} = \begin{cases} \frac{1}{\sigma_z^2}(-(r_i + A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_z^2} 4A(r_i + 2A) & r_i < 0 \\ \frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i - 3A)^2) = \frac{1}{\sigma_z^2} 4A(-r_i + 2A) & 0 \le r_i \end{cases}$$

64QAM LLR

$$d(i) = \frac{1}{\sqrt{42}}\left\{\left(1-2b(6i)\right)\left[4-\left(1-2b(6i+2)\right)\left[2-\left(1-2b(6i+4)\right)\right]\right] + j\left(1-2b(6i+1)\right)\left[4-\left(1-2b(6i+3)\right)\left[2-\left(1-2b(6i+5)\right)\right]\right]\right\}$$



Below figure shows even bit [b0,b2,b4] maps to real part



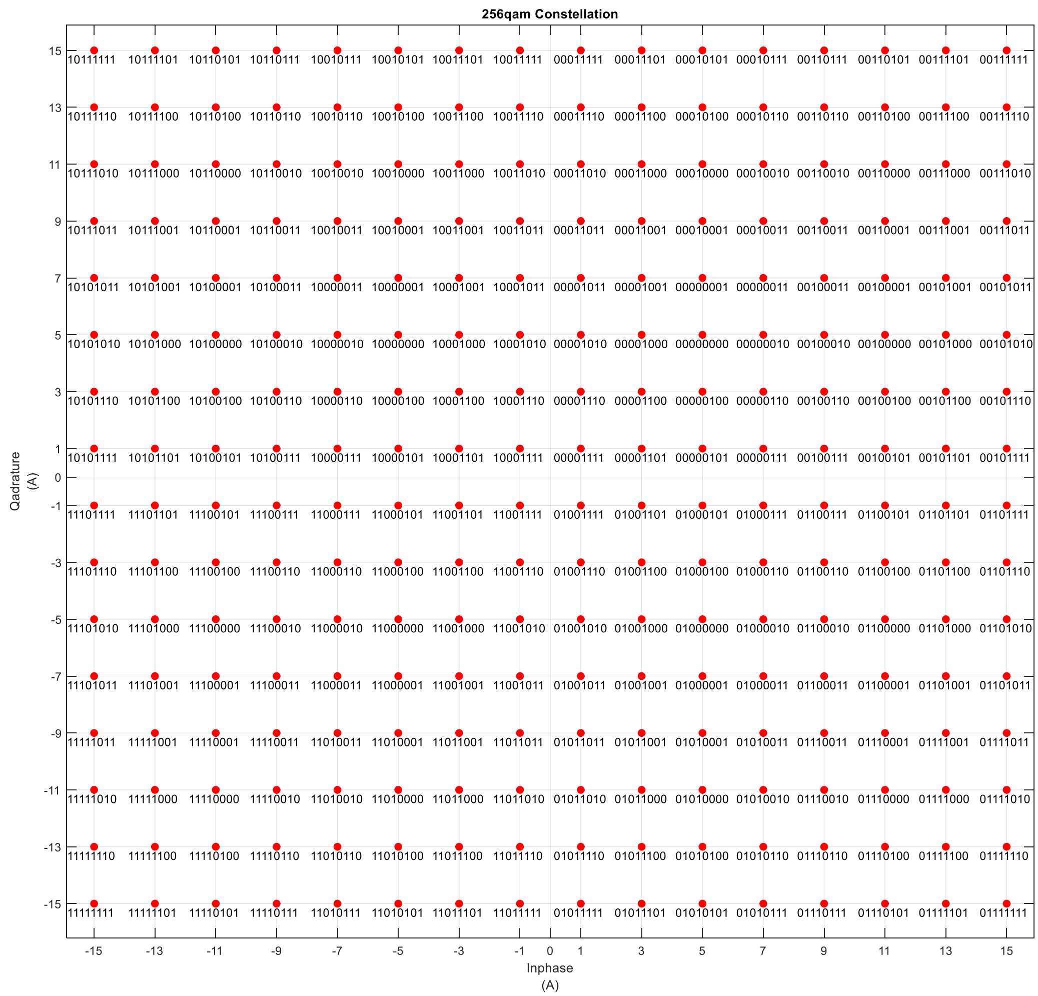
$$LLR_{b0}=\begin{cases}\frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i+7A)^2\right)=\frac{1}{\sigma_Z^2}16A(r_i+3A) & r_i<-6A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i+5A)^2\right)=\frac{1}{\sigma_Z^2}12A(r_i+2A) & -6A\le r_i<-4A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i+3A)^2\right)=\frac{1}{\sigma_Z^2}8A(r_i+A) & -4A\le r_i<-2A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_Z^2}4Ar_i & -2A\le r_i<2A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-3A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_Z^2}8A(r_i-A) & 2A\le r_i<4A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-5A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_Z^2}12A(r_i-2A) & 4A\le r_i<6A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-7A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_Z^2}16A(r_i-3A) & 6A\le r_i\end{cases}$$

$$LLR_{b2}=\begin{cases}\frac{1}{\sigma_Z^2}\left(-(r_i+3A)^2+(r_i+7A)^2\right)=\frac{1}{\sigma_Z^2}8A(r_i+5A) & r_i<-6A\\ \frac{1}{\sigma_Z^2}\left(-(r_i+3A)^2+(r_i+5A)^2\right)=\frac{1}{\sigma_Z^2}4A(r_i+4A) & -6A\le r_i<-2A\\ \frac{1}{\sigma_Z^2}\left(-(r_i+A)^2+(r_i+5A)^2\right)=\frac{1}{\sigma_Z^2}8A(r_i+3A) & -2A\le r_i<0\\ \frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i-5A)^2\right)=\frac{1}{\sigma_Z^2}8A(-r_i+3A) & 0\le r_i<2A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-3A)^2+(r_i-5A)^2\right)=\frac{1}{\sigma_Z^2}4A(-r_i+4A) & 2A\le r_i<6A\\ \frac{1}{\sigma_Z^2}\left(-(r_i-3A)^2+(r_i-7A)^2\right)=\frac{1}{\sigma_Z^2}8A(-r_i+5A) & 6A\le r_i\end{cases}$$

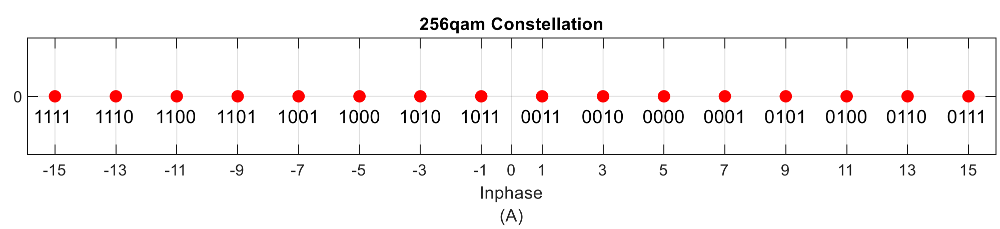
$$LLR_{b4} = \begin{cases} \frac{1}{\sigma_z^2}\left(-(r_i + 5A)^2 + (r_i + 7A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i + 6A) & r_i < -4A \\ \frac{1}{\sigma_z^2}\left(-(r_i + 3A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i - 2A) & -4A \leq r_i < 0 \\ \frac{1}{\sigma_z^2}\left(-(r_i - 3A)^2 + (r_i - A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i - 2A) & 0 \leq r_i < 4A \\ \frac{1}{\sigma_z^2}\left(-(r_i - 5A)^2 + (r_i - 7A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i + 6A) & 4A \leq r_i \end{cases}$$

256QAM LLR

$$d(i) = \frac{1}{\sqrt{170}}\left\{\left(1-2b(8i)\right)\left[8-\left(1-2b(8i+2)\right)\left[4-\left(1-2b(8i+4)\right)\left[2-\left(1-2b(8i+6)\right)\right]\right]\right]\right.$$
$$\left.+j\left(1-2b(8i+1)\right)\left[8-\left(1-2b(8i+3)\right)\left[4-\left(1-2b(8i+5)\right)\left[2-\left(1-2b(8i+7)\right)\right]\right]\right]\right\}$$



Below figure shows even bit [b0,b2,b4,b6] maps to real part



$$LLR_{b0} = \begin{cases} \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 15A)^2\right) = \frac{1}{\sigma_Z^2} 32A(r_i + 7A) & r_i < -14A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 13A)^2\right) = \frac{1}{\sigma_Z^2} 28A(r_i + 6A) & -14A \le r_i < -12A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 11A)^2\right) = \frac{1}{\sigma_Z^2} 24A(r_i + 5A) & -12A \le r_i < -10A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 9A)^2\right) = \frac{1}{\sigma_Z^2} 20A(r_i + 4A) & -10A \le r_i < -8A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 7A)^2\right) = \frac{1}{\sigma_Z^2} 16A(r_i + 3A) & -8A \le r_i < -6A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 5A)^2\right) = \frac{1}{\sigma_Z^2} 12A(r_i + 2A) & -6A \le r_i < -4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 3A)^2\right) = \frac{1}{\sigma_Z^2} 8A(r_i + A) & -4A \le r_i < -2A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 4Ar_i & -2A \le r_i < 2A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 3A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 8A(r_i - A) & 2A \le r_i < 4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 5A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 12A(r_i - 2A) & 4A \le r_i < 6A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 7A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 16A(r_i - 3A) & 6A \le r_i < 8A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 9A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 20A(r_i - 4A) & 8A \le r_i < 10A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 11A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 24A(r_i - 5A) & 10A \le r_i < 12A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 13A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 28A(r_i - 6A) & 12A \le r_i < 14A \\ \frac{1}{\sigma_Z^2}\left(-(r_i - 15A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 32A(r_i - 7A) & 14A \le r_i \end{cases}$$

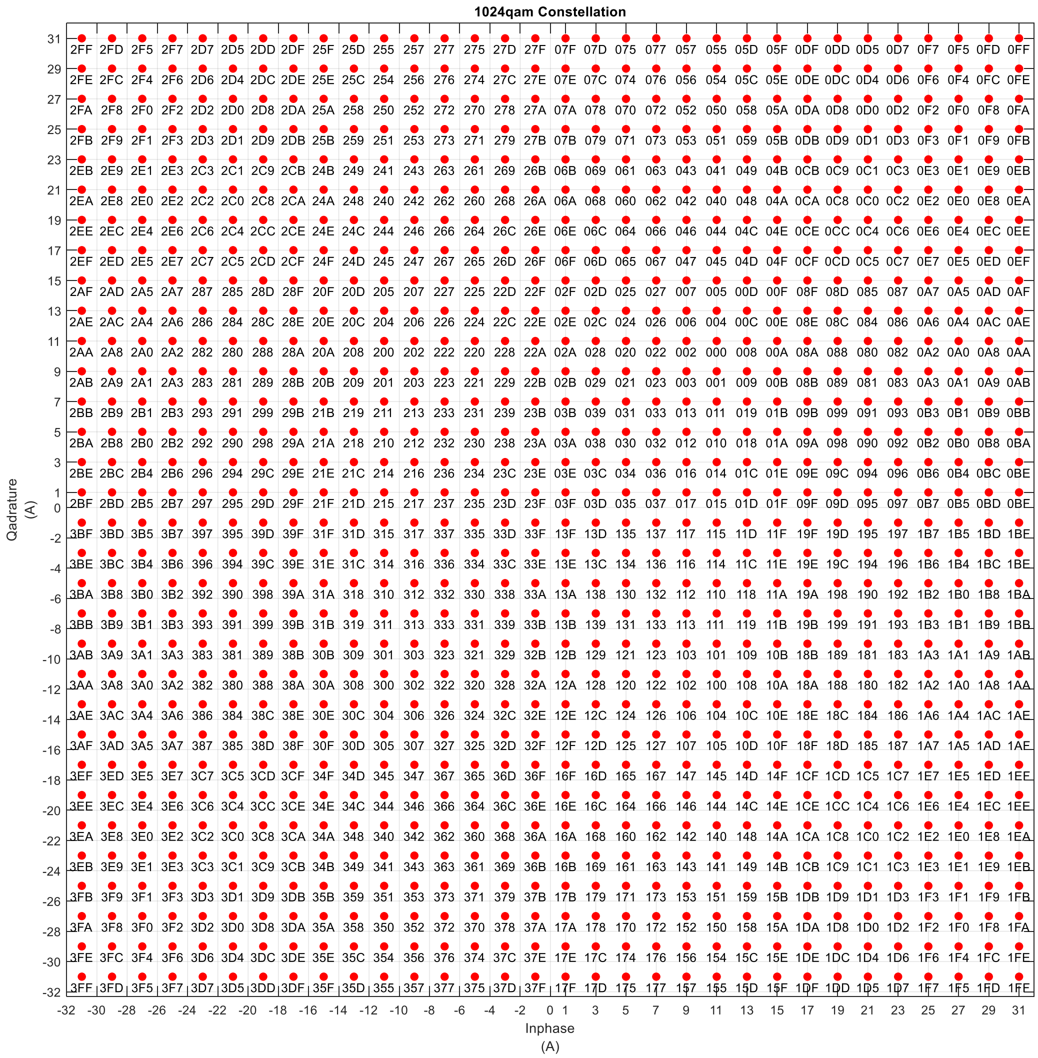
$$LLR_{b2} = \begin{cases} \frac{1}{\sigma_Z^2}\left(-(r_i+7A)^2+(r_i+15A)^2\right) = \frac{1}{\sigma_Z^2}16A(r_i+11A) & r_i < -14A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+7A)^2+(r_i+13A)^2\right) = \frac{1}{\sigma_Z^2}12A(r_i+10A) & -14A \le r_i < -12A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+7A)^2+(r_i+11A)^2\right) = \frac{1}{\sigma_Z^2}8A(r_i+9A) & -12A \le r_i < -10A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+7A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_Z^2}4A(r_i+8A) & -10A \le r_i < -6A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+5A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_Z^2}8A(r_i+7A) & -6A \le r_i < -4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+3A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_Z^2}12A(r_i+6A) & -4A \le r_i < -2A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_Z^2}16A(r_i+5A) & -2A \le r_i < 0 \\ \frac{1}{\sigma_Z^2}\left(-(r_i-A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_Z^2}16A(-r_i+5A) & 0 \le r_i < 2A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-3A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_Z^2}12A(-r_i+6A) & 2A \le r_i < 4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-5A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_Z^2}8A(-r_i+7A) & 4A \le r_i < 6A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-7A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_Z^2}4A(-r_i+8A) & 6A \le r_i < 10A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-7A)^2+(r_i-11A)^2\right) = \frac{1}{\sigma_Z^2}8A(-r_i+9A) & 10A \le r_i < 12A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-7A)^2+(r_i-13A)^2\right) = \frac{1}{\sigma_Z^2}12A(-r_i+10A) & 12A \le r_i < 14A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-7A)^2+(r_i-15A)^2\right) = \frac{1}{\sigma_Z^2}16A(-r_i+11A) & 14A \le r_i \end{cases}$$

$$LLR_{b4} = \begin{cases} \frac{1}{\sigma_z^2}(-(r_i + 11A)^2 + (r_i + 15A)^2) = \frac{1}{\sigma_z^2} 8A(r_i + 13A) & r_i < -14A \\ \frac{1}{\sigma_z^2}(-(r_i + 11A)^2 + (r_i + 13A)^2) = \frac{1}{\sigma_z^2} 4A(r_i + 12A) & -14A \le r_i < -10A \\ \frac{1}{\sigma_z^2}(-(r_i + 9A)^2 + (r_i + 13A)^2) = \frac{1}{\sigma_z^2} 8A(r_i + 11A) & -10A \le r_i < -8A \\ \frac{1}{\sigma_z^2}(-(r_i + 7A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_z^2} 8A(-r_i - 5A) & -8A \le r_i < -6A \\ \frac{1}{\sigma_z^2}(-(r_i + 5A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_z^2} 4A(-r_i - 4A) & -6A \le r_i < -2A \\ \frac{1}{\sigma_z^2}(-(r_i + 5A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 8A(-r_i - 3A) & -2A \le r_i < 0 \\ \frac{1}{\sigma_z^2}(-(r_i - 5A)^2 + (r_i - A)^2) = \frac{1}{\sigma_z^2} 8A(r_i - 3A) & 0 \le r_i < 2A \\ \frac{1}{\sigma_z^2}(-(r_i - 5A)^2 + (r_i - 3A)^2) = \frac{1}{\sigma_z^2} 4A(r_i - 4A) & 2A \le r_i < 6A \\ \frac{1}{\sigma_z^2}(-(r_i - 7A)^2 + (r_i - 3A)^2) = \frac{1}{\sigma_z^2} 8A(r_i - 5A) & 6A \le r_i < 8A \\ \frac{1}{\sigma_z^2}(-(r_i - 9A)^2 + (r_i - 13A)^2) = \frac{1}{\sigma_z^2} 8A(-r_i + 11A) & 8A \le r_i < 10A \\ \frac{1}{\sigma_z^2}(-(r_i - 11A)^2 + (r_i - 13A)^2) = \frac{1}{\sigma_z^2} 4A(-r_i + 12A) & 10A \le r_i < 14A \\ \frac{1}{\sigma_z^2}(-(r_i - 11A)^2 + (r_i - 15A)^2) = \frac{1}{\sigma_z^2} 8A(-r_i + 13A) & 14A \le r_i \end{cases}$$

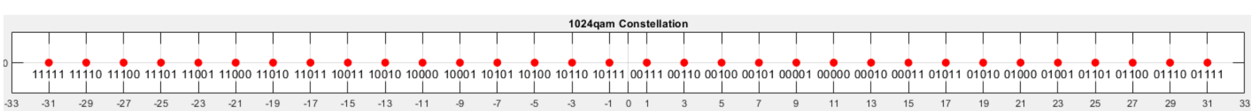
$$
LLR_{b6} = \begin{cases}
\frac{1}{\sigma_z^2}\left(-(r_i+13A)^2+(r_i+15A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+14A) & r_i < -12A \\
\frac{1}{\sigma_z^2}\left(-(r_i+11A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-10A) & -12A \le r_i < -8A \\
\frac{1}{\sigma_z^2}\left(-(r_i+5A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+6A) & -8A \le r_i < -4A \\
\frac{1}{\sigma_z^2}\left(-(r_i+3A)^2+(r_i+A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-2A) & -4A \le r_i < 0 \\
\frac{1}{\sigma_z^2}\left(-(r_i-3A)^2+(r_i-A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-2A) & 0 \le r_i < 4A \\
\frac{1}{\sigma_z^2}\left(-(r_i-5A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+6A) & 4A \le r_i < 8A \\
\frac{1}{\sigma_z^2}\left(-(r_i-11A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-10A) & 8A \le r_i < 12A \\
\frac{1}{\sigma_z^2}\left(-(r_i-13A)^2+(r_i-15A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+14A) & 12A \le r_i
\end{cases}
$$

1024QAM LLR

$$
\begin{aligned}
d(i) = \\
&\frac{1}{\sqrt{682}}\left(1-2b(10i+0)\right)\left[16-\left(1-2b(10i+2)\right)\left[8-\left(1-2b(10i+4)\right)\left[4-\left(1-2b(10i+6)\right)\left[2-\left(1-2b(10i+8)\right)\right]\right]\right]\right] \\
&+j\frac{1}{\sqrt{682}}\left(1-2b(10i+1)\right)\left[16-\left(1-2b(10i+3)\right)\left[8-\left(1-2b(10i+5)\right)\left[4-\left(1-2b(10i+7)\right)\left[2-\left(1-2b(10i+9)\right)\right]\right]\right]\right]
\end{aligned}
$$



Below figure shows even bit [b0,b2,b4,b6,b8] maps to real part



$$LLR_{b0}=\begin{cases}\frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+31A)^2\right)=\frac{1}{\sigma_z^2}64A(r_i+15A) & r_i<-30A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+29A)^2\right)=\frac{1}{\sigma_z^2}60A(r_i+14A) & -30A\le r_i<-28A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+27A)^2\right)=\frac{1}{\sigma_z^2}56A(r_i+13A) & -28A\le r_i<-26A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+25A)^2\right)=\frac{1}{\sigma_z^2}52A(r_i+12A) & -26A\le r_i<-24A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+23A)^2\right)=\frac{1}{\sigma_z^2}48A(r_i+11A) & -24A\le r_i<-22A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+21A)^2\right)=\frac{1}{\sigma_z^2}44A(r_i+10A) & -22A\le r_i<-20A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+19A)^2\right)=\frac{1}{\sigma_z^2}40A(r_i+9A) & -20A\le r_i<-18A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+17A)^2\right)=\frac{1}{\sigma_z^2}36A(r_i+8A) & -18A\le r_i<-16A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+15A)^2\right)=\frac{1}{\sigma_z^2}32A(r_i+7A) & -16A\le r_i<-14A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+13A)^2\right)=\frac{1}{\sigma_z^2}28A(r_i+6A) & -14A\le r_i<-12A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+11A)^2\right)=\frac{1}{\sigma_z^2}24A(r_i+5A) & -12A\le r_i<-10A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+9A)^2\right)=\frac{1}{\sigma_z^2}20A(r_i+4A) & -10A\le r_i<-8A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+7A)^2\right)=\frac{1}{\sigma_z^2}16A(r_i+3A) & -8A\le r_i<-6A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+5A)^2\right)=\frac{1}{\sigma_z^2}12A(r_i+2A) & -6A\le r_i<-4A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+3A)^2\right)=\frac{1}{\sigma_z^2}8A(r_i+A) & -4A\le r_i<-2A\\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}4Ar_i & -2A\le r_i<2A\\ \frac{1}{\sigma_z^2}\left(-(r_i-3A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}8A(r_i-A) & 2A\le r_i<4A\\ \frac{1}{\sigma_z^2}\left(-(r_i-5A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}12A(r_i-2A) & 4A\le r_i<6A\\ \frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}16A(r_i-3A) & 6A\le r_i<8A\\ \frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}20A(r_i-4A) & 8A\le r_i<10A\\ \frac{1}{\sigma_z^2}\left(-(r_i-11A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}24A(r_i-5A) & 10A\le r_i<12A\\ \frac{1}{\sigma_z^2}\left(-(r_i-13A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}28A(r_i-6A) & 12A\le r_i<14A\\ \frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}32A(r_i-7A) & 14A\le r_i<16A\\ \frac{1}{\sigma_z^2}\left(-(r_i-17A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}36A(r_i-8A) & 16A\le r_i<18A\\ \frac{1}{\sigma_z^2}\left(-(r_i-19A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}40A(r_i-9A) & 18A\le r_i<20A\\ \frac{1}{\sigma_z^2}\left(-(r_i-21A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}44A(r_i-10A) & 20A\le r_i<22A\\ \frac{1}{\sigma_z^2}\left(-(r_i-23A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}48A(r_i-11A) & 22A\le r_i<24A\\ \frac{1}{\sigma_z^2}\left(-(r_i-25A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}52A(r_i-12A) & 24A\le r_i<26A\\ \frac{1}{\sigma_z^2}\left(-(r_i-27A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}56A(r_i-13A) & 26A\le r_i<28A\\ \frac{1}{\sigma_z^2}\left(-(r_i-29A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}60A(r_i-14A) & 28A\le r_i<30A\\ \frac{1}{\sigma_z^2}\left(-(r_i-31A)^2+(r_i+A)^2\right)=\frac{1}{\sigma_z^2}64A(r_i-15A) & 30A\le r_i\end{cases}$$

$$LLR_{b2} = \begin{cases}
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+31A)^2) = \frac{1}{\sigma_z^2}32A(r_i+23A) & r_i < -30A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+29A)^2) = \frac{1}{\sigma_z^2}28A(r_i+22A) & -30A \le r_i < -28A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+27A)^2) = \frac{1}{\sigma_z^2}24A(r_i+21A) & -28A \le r_i < -26A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}20A(r_i+20A) & -26A \le r_i < -24A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+23A)^2) = \frac{1}{\sigma_z^2}16A(r_i+19A) & -24A \le r_i < -22A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+21A)^2) = \frac{1}{\sigma_z^2}12A(r_i+18A) & -22A \le r_i < -20A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+19A)^2) = \frac{1}{\sigma_z^2}8A(r_i+17A) & -20A \le r_i < -18A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}4A(r_i+16A) & -18A \le r_i < -14A \\
\frac{1}{\sigma_z^2}(-(r_i+13A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}8A(r_i+15A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}12A(r_i+14A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}16A(r_i+13A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_z^2}(-(r_i+7A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}20A(r_i+12A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}24A(r_i+11A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_z^2}(-(r_i+3A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}28A(r_i+10A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_z^2}(-(r_i+A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}32A(r_i+9A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}(-(r_i-A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}32A(-r_i+9A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}(-(r_i-3A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}28A(-r_i+10A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}24A(-r_i+11A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_z^2}(-(r_i-7A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}20A(-r_i+12A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}16A(-r_i+13A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}12A(-r_i+14A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_z^2}(-(r_i-13A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+15A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+16A) & 14A \le r_i < 18A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-19A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+17A) & 18A \le r_i < 20A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-21A)^2) = \frac{1}{\sigma_z^2}12A(-r_i+18A) & 20A \le r_i < 22A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-23A)^2) = \frac{1}{\sigma_z^2}16A(-r_i+19A) & 22A \le r_i < 24A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}20A(-r_i+20A) & 24A \le r_i < 26A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-27A)^2) = \frac{1}{\sigma_z^2}24A(-r_i+21A) & 26A \le r_i < 28A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-29A)^2) = \frac{1}{\sigma_z^2}28A(-r_i+22A) & 28A \le r_i < 30A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-31A)^2) = \frac{1}{\sigma_z^2}32A(-r_i+23A) & 30A \le r_i
\end{cases}$$

$$
LLR_{b4} = \begin{cases}
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+31A)^2) = \frac{1}{\sigma_z^2}16A(r_i+27A) & r_i < -30A \\
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+29A)^2) = \frac{1}{\sigma_z^2}12A(r_i+26A) & -30A \le r_i < -28A \\
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+27A)^2) = \frac{1}{\sigma_z^2}8A(r_i+25A) & -28A \le r_i < -26A \\
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}4A(r_i+24A) & -26A \le r_i < -22A \\
\frac{1}{\sigma_z^2}(-(r_i+21A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}8A(r_i+23A) & -22A \le r_i < -20A \\
\frac{1}{\sigma_z^2}(-(r_i+19A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}12A(r_i+22A) & -20A \le r_i < -18A \\
\frac{1}{\sigma_z^2}(-(r_i+17A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}16A(r_i+21A) & -18A \le r_i < -16A \\
\frac{1}{\sigma_z^2}(-(r_i+15A)^2+(r_i+7A)^2) = \frac{1}{\sigma_z^2}16A(-r_i-11A) & -16A \le r_i < -14A \\
\frac{1}{\sigma_z^2}(-(r_i+13A)^2+(r_i+7A)^2) = \frac{1}{\sigma_z^2}12A(-r_i-10A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+7A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-9A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+7A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-8A) & -10A \le r_i < -6A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+5A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-7A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+3A)^2) = \frac{1}{\sigma_z^2}12A(-r_i-6A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+A)^2) = \frac{1}{\sigma_z^2}16A(-r_i-5A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-A)^2) = \frac{1}{\sigma_z^2}16A(r_i-5A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-3A)^2) = \frac{1}{\sigma_z^2}12A(r_i-6A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-5A)^2) = \frac{1}{\sigma_z^2}8A(r_i-7A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-7A)^2) = \frac{1}{\sigma_z^2}4A(r_i-8A) & 6A \le r_i < 10A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-7A)^2) = \frac{1}{\sigma_z^2}8A(r_i-9A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_z^2}(-(r_i-13A)^2+(r_i-7A)^2) = \frac{1}{\sigma_z^2}12A(r_i-10A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_z^2}(-(r_i-15A)^2+(r_i-7A)^2) = \frac{1}{\sigma_z^2}16A(r_i-11A) & 14A \le r_i < 16A \\
\frac{1}{\sigma_z^2}(-(r_i-17A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}16A(-r_i+21A) & 16A \le r_i < 18A \\
\frac{1}{\sigma_z^2}(-(r_i-19A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}12A(-r_i+22A) & 18A \le r_i < 20A \\
\frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+23A) & 20A \le r_i < 22A \\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+24A) & 22A \le r_i < 26A \\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-27A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+25A) & 26A \le r_i < 28A \\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-29A)^2) = \frac{1}{\sigma_z^2}12A(-r_i+26A) & 28A \le r_i < 30A \\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-31A)^2) = \frac{1}{\sigma_z^2}16A(-r_i+27A) & 30A \le r_i
\end{cases}
$$

$$LLR_{b6}=\begin{cases}
\frac{1}{\sigma_z^2}(-(r_i+27A)^2+(r_i+31A)^2)=\frac{1}{\sigma_z^2}8A(r_i+29A) & r_i<-30A\\
\frac{1}{\sigma_z^2}(-(r_i+27A)^2+(r_i+29A)^2)=\frac{1}{\sigma_z^2}4A(r_i+28A) & -30A\le r_i<-26A\\
\frac{1}{\sigma_z^2}(-(r_i+25A)^2+(r_i+29A)^2)=\frac{1}{\sigma_z^2}8A(r_i+27A) & -26A\le r_i<-24A\\
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+19A)^2)=\frac{1}{\sigma_z^2}8A(-r_i-21A) & -24A\le r_i<-22A\\
\frac{1}{\sigma_z^2}(-(r_i+21A)^2+(r_i+19A)^2)=\frac{1}{\sigma_z^2}4A(-r_i-20A) & -22A\le r_i<-18A\\
\frac{1}{\sigma_z^2}(-(r_i+21A)^2+(r_i+17A)^2)=\frac{1}{\sigma_z^2}8A(-r_i-19A) & -18A\le r_i<-16A\\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+15A)^2)=\frac{1}{\sigma_z^2}8A(r_i+13A) & -16A\le r_i<-14A\\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+13A)^2)=\frac{1}{\sigma_z^2}4A(r_i+12A) & -14A\le r_i<-10A\\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+13A)^2)=\frac{1}{\sigma_z^2}8A(r_i+11A) & -10A\le r_i<-8A\\
\frac{1}{\sigma_z^2}(-(r_i+7A)^2+(r_i+3A)^2)=\frac{1}{\sigma_z^2}8A(-r_i-5A) & -8A\le r_i<-6A\\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+3A)^2)=\frac{1}{\sigma_z^2}4A(-r_i-4A) & -6A\le r_i<-2A\\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+A)^2)=\frac{1}{\sigma_z^2}8A(-r_i-3A) & -2A\le r_i<0\\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-A)^2)=\frac{1}{\sigma_z^2}8A(r_i-3A) & 0\le r_i<2A\\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-3A)^2)=\frac{1}{\sigma_z^2}4A(r_i-4A) & 2A\le r_i<6A\\
\frac{1}{\sigma_z^2}(-(r_i-7A)^2+(r_i-3A)^2)=\frac{1}{\sigma_z^2}8A(r_i-5A) & 6A\le r_i<8A\\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-13A)^2)=\frac{1}{\sigma_z^2}8A(-r_i+11A) & 8A\le r_i<10A\\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-13A)^2)=\frac{1}{\sigma_z^2}4A(-r_i+12A) & 10A\le r_i<14A\\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-15A)^2)=\frac{1}{\sigma_z^2}8A(-r_i+13A) & 14A\le r_i<16A\\
\frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-17A)^2)=\frac{1}{\sigma_z^2}8A(r_i-19A) & 16A\le r_i<18A\\
\frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-19A)^2)=\frac{1}{\sigma_z^2}4A(r_i-20A) & 18A\le r_i<22A\\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-19A)^2)=\frac{1}{\sigma_z^2}8A(r_i-21A) & 22A\le r_i<24A\\
\frac{1}{\sigma_z^2}(-(r_i-25A)^2+(r_i-29A)^2)=\frac{1}{\sigma_z^2}8A(-r_i+27A) & 24A\le r_i<26A\\
\frac{1}{\sigma_z^2}(-(r_i-27A)^2+(r_i-29A)^2)=\frac{1}{\sigma_z^2}4A(-r_i+28A) & 26A\le r_i<30A\\
\frac{1}{\sigma_z^2}(-(r_i-27A)^2+(r_i-31A)^2)=\frac{1}{\sigma_z^2}8A(-r_i+29A) & 30A\le r_i
\end{cases}$$

$$
LLR_{b8} = \begin{cases}
\frac{1}{\sigma_z^2}\left(-(r_i+29A)^2+(r_i+31A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+30A) & r_i < -28A \\
\frac{1}{\sigma_z^2}\left(-(r_i+27A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-26A) & -28A \le r_i < -24A \\
\frac{1}{\sigma_z^2}\left(-(r_i+21)^2+(r_i+23A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+22A) & -24A \le r_i < -20A \\
\frac{1}{\sigma_z^2}\left(-(r_i+19A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-18A) & -20A \le r_i < -16A \\
\frac{1}{\sigma_z^2}\left(-(r_i+13A)^2+(r_i+15A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+14A) & -16A \le r_i < -12A \\
\frac{1}{\sigma_z^2}\left(-(r_i+11A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-10A) & -12A \le r_i < -8A \\
\frac{1}{\sigma_z^2}\left(-(r_i+5A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+6A) & -8A \le r_i < -4A \\
\frac{1}{\sigma_z^2}\left(-(r_i+3A)^2+(r_i+A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-2A) & -4A \le r_i < 0 \\
\frac{1}{\sigma_z^2}\left(-(r_i-3A)^2+(r_i-A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-2A) & 0A \le r_i < 4A \\
\frac{1}{\sigma_z^2}\left(-(r_i-5A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+6A) & 4A \le r_i < 8A \\
\frac{1}{\sigma_z^2}\left(-(r_i-11A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-10A) & 8A \le r_i < 12A \\
\frac{1}{\sigma_z^2}\left(-(r_i-13A)^2+(r_i-15A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+14A) & 12A \le r_i < 16A \\
\frac{1}{\sigma_z^2}\left(-(r_i-19A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-18A) & 16A \le r_i < 20A \\
\frac{1}{\sigma_z^2}\left(-(r_i-21A)^2+(r_i-23A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+22A) & 20A \le r_i < 24A \\
\frac{1}{\sigma_z^2}\left(-(r_i-27A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-26A) & 24A \le r_i < 28A \\
\frac{1}{\sigma_z^2}\left(-(r_i-29A)^2+(r_i-31A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+30A) & 28A \le r_i
\end{cases}
$$