5G 系统下 LLR 软判决设计和实现

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前言

本文描述实现 pi/2-BPSK, BPSK, QPSK, 16QAM, 64QAM, 256QAM, 1024QAM 的 LLR(log-likelihood ratio)软判决。

本文描述的 LLR 软判决已经使用 python 实现，并且验证了结果与 Matlab 5G toolbox 的 LLR 判决结果一致。

几点说明

为什么选择$LLR(b_k) = \ln\left(\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right)$而不是$LLR(b_k) = \ln\left(\frac{P(b_k = 1|r)}{P(b_k = 0|r)}\right)$

LLR 代表的是取 1 或者取 0 的可能性，LLR 的结果用于 LDPC 或者 Polar 译码，选择哪一个公式取决于译码算法如何映射 0 或 1 到 LLR。

从 2G 时代开始$LLR(b_k) = \ln\left(\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right)$就用于 LLR 判决，已经是约定俗成的了。原因可能是调制的时候 bit=0 映射为 1，bit=1 映射为-1， 选择 LLR 的时候，希望 LLR>0 for bit=0, LLR <= for bit=1

软判决输入的符号的平均功率为 1

软判决输入数据：$r = s + n$

$$E(rs^*) = E\left((s + n)s^*\right) = E(ss^*) = 1$$

信道均衡如果采用 MMSE，输出结果不满足上面平均功率为 1 的条件，需要先做预补偿，然后再做 LLR 软判决

软判决输入的噪声方差是复数噪声方差，不是实部或者虚部噪声方差

参考资料

[1] Juquan Mao; Mahmoud Alfa Abdullahi; Pei Xiao; Aijun Cao： “A low complexity 256QAM soft demapper for 5G mobile system”
https://www.researchgate.net/publication/307940673_A_low_complexity_256QAM_soft_demapper_for_5G_mobile_system
[2] Xianle Cao , Yi Liu, Dongfang Hu: “Simplified LLR algorithm for m-QAM demodulation”

这两个参考资料里面都有一些错误，不过基本的思路是对的。

复高斯分布

高斯分布

$$N \backsim \left(\mu,\ \sigma^2\ \right)$$

概率密度函数： $p(x)\ =\ \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

复高斯分布

复高斯分布 $Z = X + iY$， 满足$X\ \backsim\ (\mu_x,\sigma_x^2), Y\ \backsim\ \left(\mu_y,\sigma_y^2\right), \mu\ =\ \mu_x\ =\ \mu_y, \sigma^2\ = \sigma_x^2 = \sigma_y^2$

则 $\mu_z = \mu_x + i\mu_y = \mu + i\mu, \sigma_z^2 = 2\sigma^2$

概率密度函数

$$p(xy) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-\mu)^2+(y-\mu)^2}{2\sigma^2}}$$

$$p(z) = \frac{1}{2\pi\sigma^2} e^{-\frac{(z-\mu_z)^2}{2\sigma^2}} = \frac{1}{\pi\sigma_z^2} e^{-\frac{(z-\mu_z)^2}{\sigma_z 2}}$$

LLR 软判决

输入数据：

$r = s + n$

$$r_i + jr_q = s_i + js_q + n_i + jn_q$$

实部，虚部和复数概率密度函数为：

$$p(r_i|s_i) = \frac{1}{\sqrt{\pi\sigma_z^2}} e^{-\frac{(r_i-s_i)^2}{\sigma_z^2}}$$

$$p(r_q|s_q) = \frac{1}{\sqrt{\pi\sigma_z^2}} e^{-\frac{(r_q-s_q)^2}{\sigma_z^2}}$$

$$p(r|s) = \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i-s_i)^2+(r_q-s_q)^2}{\sigma_z^2}}$$

针对不同的调制方式，N 个 bit 映射到一个星座点，比如 16QAM 下四个 bit 映射到一个星座点。每个 bit 取值为 0 或 1.

$b_k = A(0 or\ 1)$的概率：

$$p(r|b_k = A) = \sum_{b:b_k=A} p(r|b)$$

$$= \sum_{b:b_k=A} p\big(r - s(b)\big)$$

$$= \sum_{b:b_k=A} \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}$$

$$LLR(b_k) = \ln\left(\frac{P(b_k = 0|r)}{P(b_k = 1|r)}\right)$$

$$= \ln\left(\frac{\frac{p(r|b_k = 0)P(b_k = 0)}{p(r)}}{\frac{p(r|b_k = 1)P(b_k = 1)}{p(r)}}\right)$$

$$= \ln\left(\frac{p(r|b_k = 0)}{p(r|b_k = 1)}\right)$$

$$= \ln\left[\frac{\sum_{b:b_k=0} \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}{\sum_{b:b_k=1} \frac{1}{\pi\sigma_z^2} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}\right]$$

$$= \ln\left[\frac{\sum_{b:b_k=0} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}{\sum_{b:b_k=1} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}\right]$$

$$LLR(b_k) \approx ln\left[\frac{max_{b:b_k=0} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}{max_{b:b_k=1} e^{-\frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2}}}\right]$$

$$= -min_{b:b_k=0} \frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2} + min_{b:b_k=1} \frac{(r_i - s(b)_i)^2 + \left(r_q - s(b)_q\right)^2}{\sigma_z^2} \qquad (1)$$

$$LLR(b_k, k\ is\ even) = -min_{b:b_k=1}\frac{(r_i - s(b)_i)^2}{\sigma_z^2} + min_{b:b_k=0}\frac{(r_i - s(b)_i)^2}{\sigma_z^2} \quad (2)$$

$$LLR(b_k, k\ is\ odd) = -min_{b:b_k=1}\frac{\left(r_q - s(b)_q\right)^2}{\sigma_z^2} + min_{b:b_k=0}\frac{\left(r_q - s(b)_q\right)^2}{\sigma_z^2} \quad (3)$$

公式（1）可以用于计算 π/2-BPSK 和 BPSK LLR

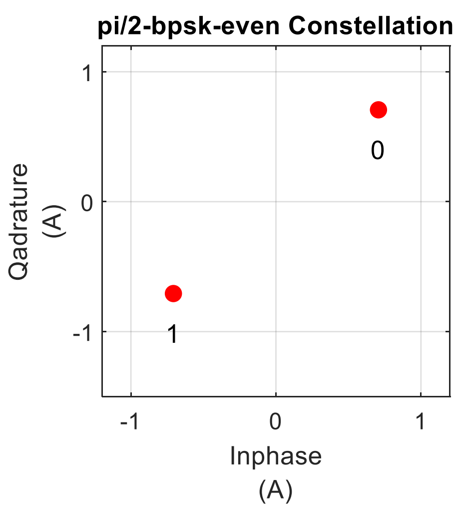
对 QPSK/16QAM/64QAM/256QAM/1024QAM，偶数 bits 映射到星座点实部，奇数 bits 映射到星座点虚部，公式（2）用于计算偶数 bits(b0,b2,b4...)的 LLR，公式(3)用于计算奇数 bits(b1,b3,b5,...)的 LLR

π/2-BPSK

$$d(i) = \frac{e^{j\frac{\pi}{2}(i \bmod 2)}}{\sqrt{2}}\left[(1-2b(i)) + j(1-2b(i))\right]$$

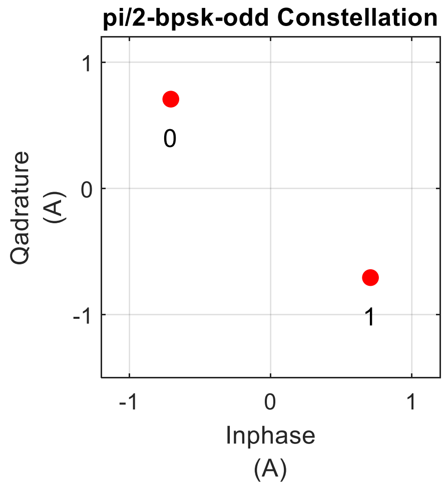
π/2-BPSK 偶数 bits 和奇数 bits 映射的星座点不同，LLR 公式也不同

偶数 bits π/2-BPSK



$$LLR = \frac{1}{\sigma_z^2}4\left(r_i + r_q\right)A \quad A = \frac{1}{\sqrt{2}}$$

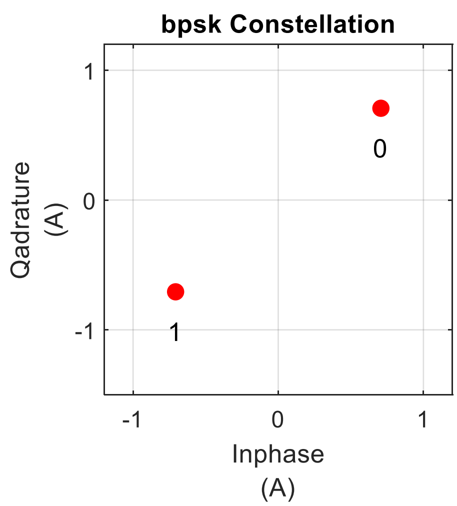
奇数 bits π/2-BPSK



$$LLR = \frac{1}{\sigma_Z^2} 4(-r_i + r_q)A \quad A = \frac{1}{\sqrt{2}}$$

BPSK

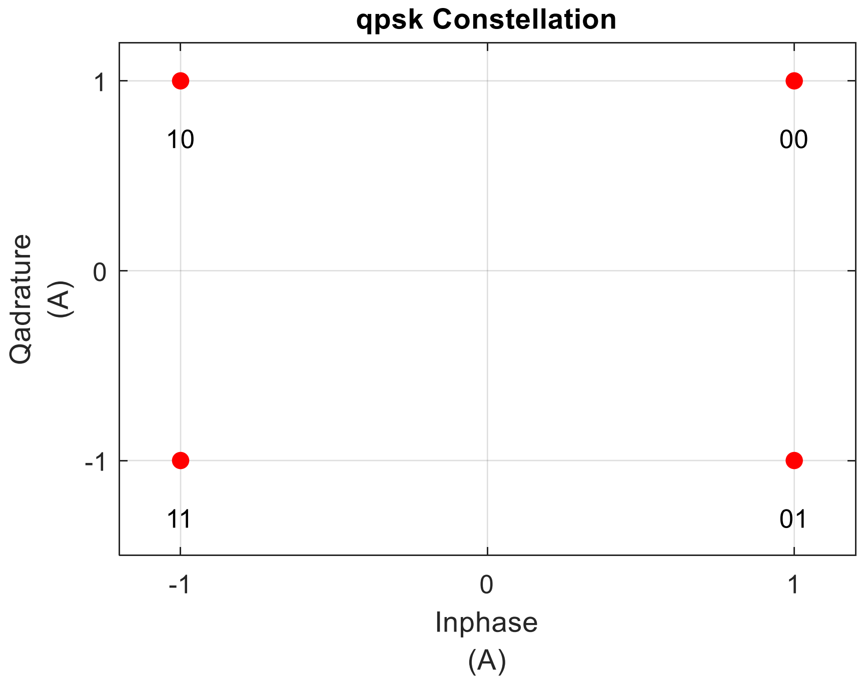
$$d(i) = \frac{1}{\sqrt{2}}\left[(1-2b(i)) + j(1-2b(i))\right]$$



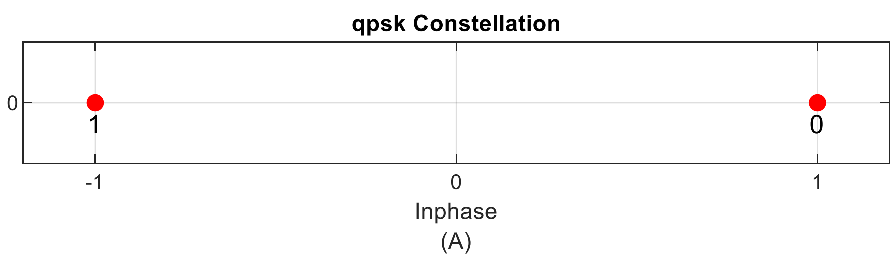
$$LLR = \frac{1}{\sigma_Z^2} 4\left(r_i + r_q\right)A \quad A = \frac{1}{\sqrt{2}}$$

QPSK

$$d(i)=\frac{1}{\sqrt{2}}\left[\left(1-2b(2i)\right)+j\left(1-2b(2i+1)\right)\right]$$



下图为偶数 bit b0 映射到实部



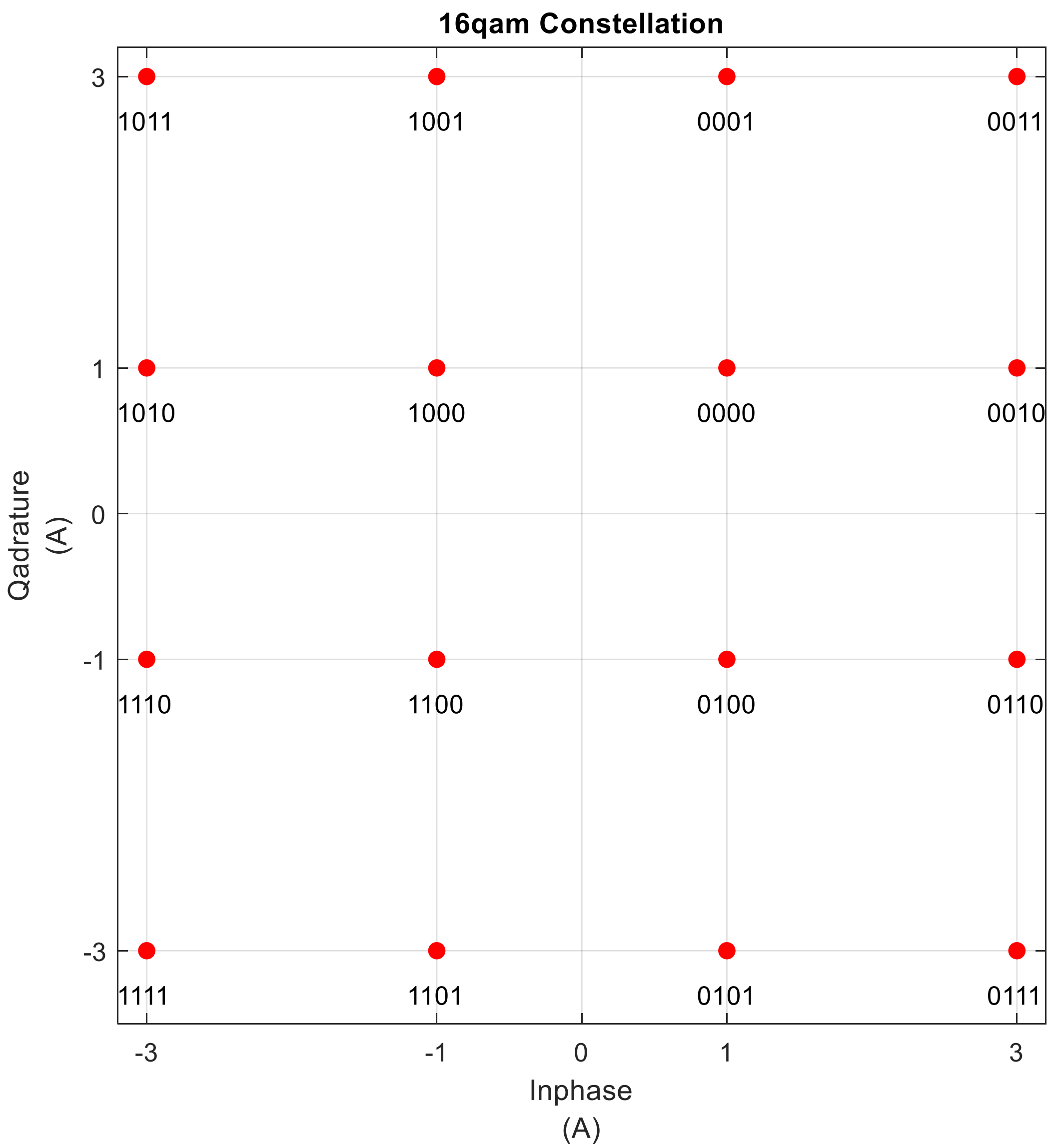
$$A=\frac{1}{\sqrt{2}}$$

$$LLR\_b0=\frac{1}{\sigma_z^2}4r_iA$$

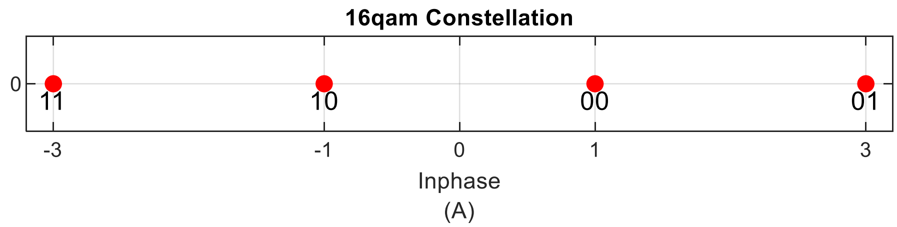
$$LLR\_b1=\frac{1}{\sigma_z^2}4r_qA$$

16QAM

$$d(i)=\frac{1}{\sqrt{10}}\left\{\left(1-2b(4i)\right)\left[2-\left(1-2b(4i+2)\right)\right]+j\left(1-2b(4i+1)\right)\left[2-\left(1-2b(4i+3)\right)\right]\right\}$$



下图为偶数 bits [b0,b2]映射到实部



$$A = \frac{1}{\sqrt{10}}$$

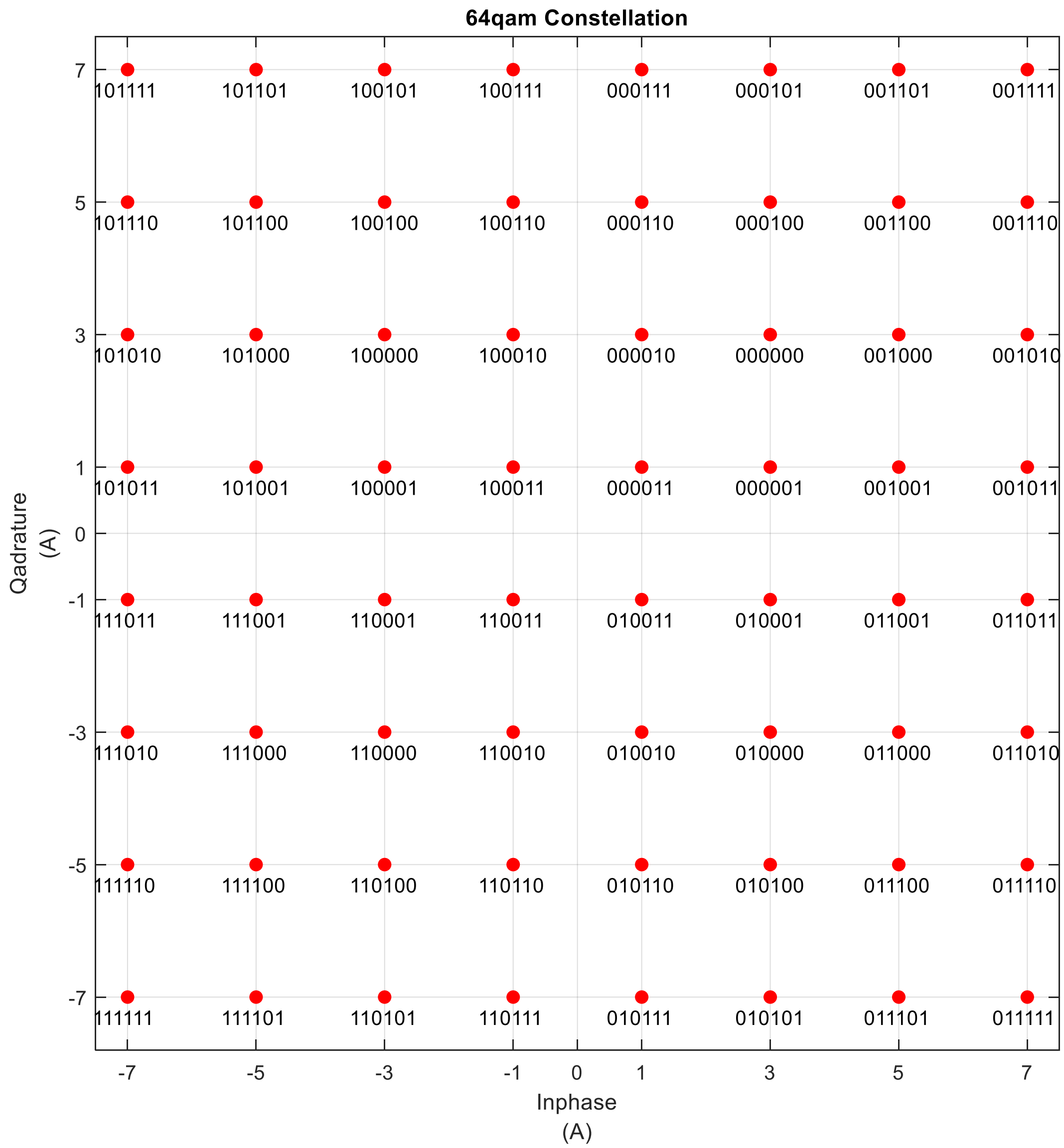
$$LLR_{b0} = \begin{cases} \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i + A) & r_i < -2A \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 4Ar_i & -2A \le r_i < 2A \\ \frac{1}{\sigma_Z^2}(-(r_i - 3A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i - A) & 2A \le r_i \end{cases}$$

$$LLR_{b2} = \begin{cases} \frac{1}{\sigma_Z^2}(-(r_i + A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_Z^2} 4A(r_i + 2A) & r_i < 0 \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i - 3A)^2) = \frac{1}{\sigma_Z^2} 4A(-r_i + 2A) & 0 \le r_i \end{cases}$$

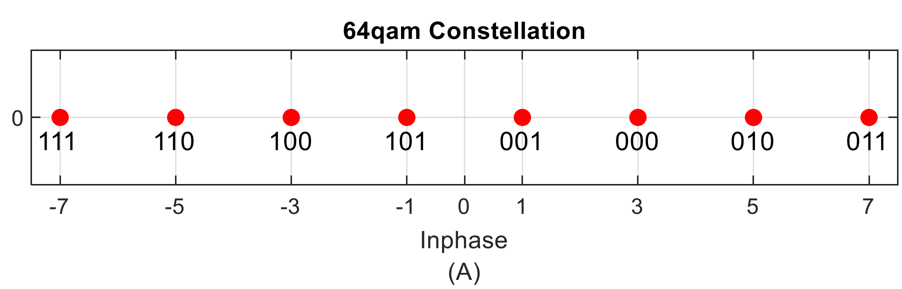
LLR_{b1}和LLR_{b3}公式与LLR_{b0}和LLR_{b1}类似，只是把r_i换为r_q

64QAM

$$d(i) = \frac{1}{\sqrt{42}}\left\{(1-2b(6i))\left[4-(1-2b(6i+2))\left[2-(1-2b(6i+4))\right]\right] + j(1-2b(6i+1))\left[4-(1-2b(6i+3))\left[2-(1-2b(6i+5))\right]\right]\right\}$$



下图为偶数 bits [b0,b2,b4]映射到实部



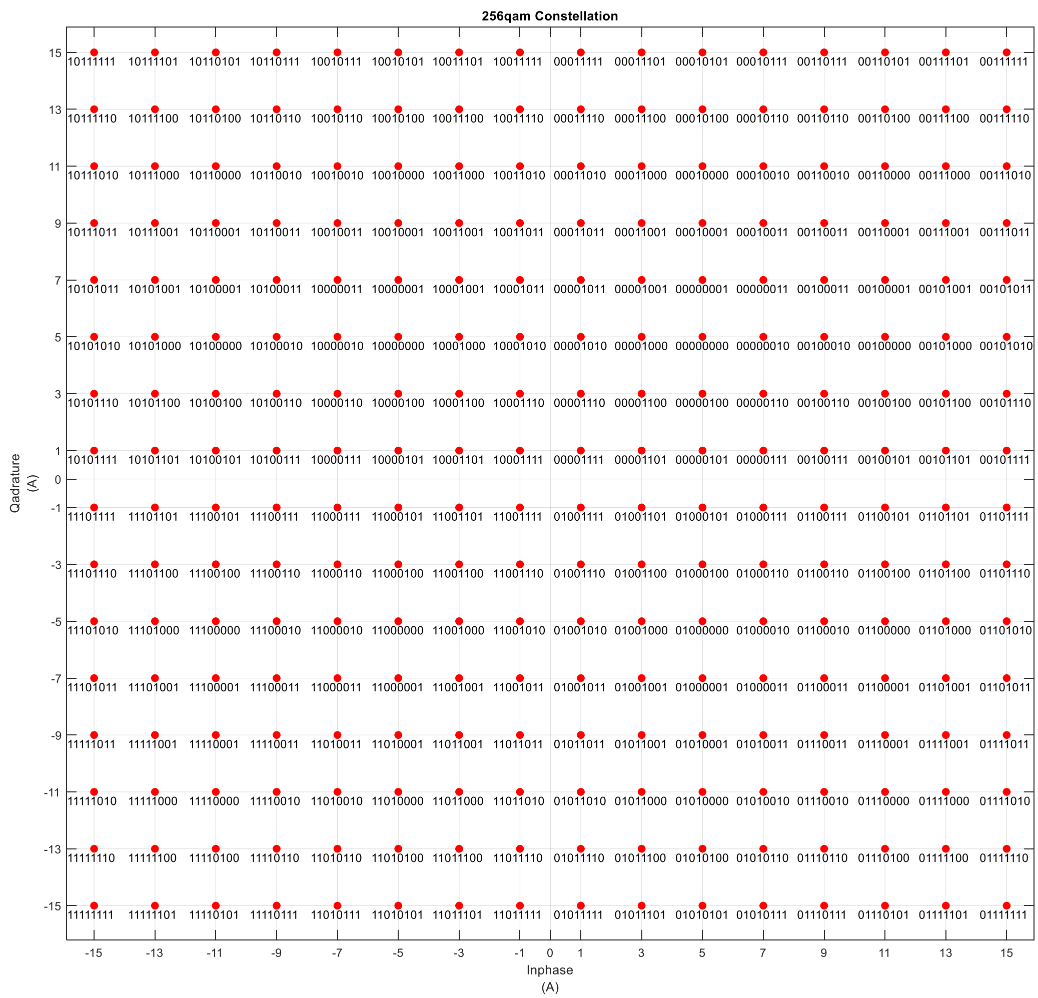
$$LLR_{b0} = \begin{cases} \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + 7A)^2) = \frac{1}{\sigma_Z^2} 16A(r_i + 3A) & r_i < -6A \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + 5A)^2) = \frac{1}{\sigma_Z^2} 12A(r_i + 2A) & -6A \le r_i < -4A \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i + A) & -4A \le r_i < -2A \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 4Ar_i & -2A \le r_i < 2A \\ \frac{1}{\sigma_Z^2}(-(r_i - 3A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i - A) & 2A \le r_i < 4A \\ \frac{1}{\sigma_Z^2}(-(r_i - 5A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 12A(r_i - 2A) & 4A \le r_i < 6A \\ \frac{1}{\sigma_Z^2}(-(r_i - 7A)^2 + (r_i + A)^2) = \frac{1}{\sigma_Z^2} 16A(r_i - 3A) & 6A \le r_i \end{cases}$$

$$LLR_{b2} = \begin{cases} \frac{1}{\sigma_Z^2}(-(r_i + 3A)^2 + (r_i + 7A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i + 5A) & r_i < -6A \\ \frac{1}{\sigma_Z^2}(-(r_i + 3A)^2 + (r_i + 5A)^2) = \frac{1}{\sigma_Z^2} 4A(r_i + 4A) & -6A \le r_i < -2A \\ \frac{1}{\sigma_Z^2}(-(r_i + A)^2 + (r_i + 5A)^2) = \frac{1}{\sigma_Z^2} 8A(r_i + 3A) & -2A \le r_i < 0 \\ \frac{1}{\sigma_Z^2}(-(r_i - A)^2 + (r_i - 5A)^2) = \frac{1}{\sigma_Z^2} 8A(-r_i + 3A) & 0 \le r_i < 2A \\ \frac{1}{\sigma_Z^2}(-(r_i - 3A)^2 + (r_i - 5A)^2) = \frac{1}{\sigma_Z^2} 4A(-r_i + 4A) & 2A \le r_i < 6A \\ \frac{1}{\sigma_Z^2}(-(r_i - 3A)^2 + (r_i - 7A)^2) = \frac{1}{\sigma_Z^2} 8A(-r_i + 5A) & 6A \le r_i \end{cases}$$

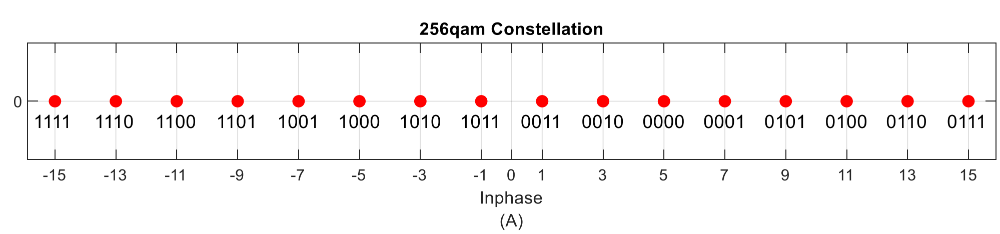
$$LLR_{b4} = \begin{cases} \frac{1}{\sigma_Z^2}\left(-(r_i+5A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_Z^2}4A(r_i+6A) & r_i < -4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i+3A)^2+(r_i+A)^2\right) = \frac{1}{\sigma_Z^2}4A(-r_i-2A) & -4A \le r_i < 0 \\ \frac{1}{\sigma_Z^2}\left(-(r_i-3A)^2+(r_i-A)^2\right) = \frac{1}{\sigma_Z^2}4A(r_i-2A) & 0 \le r_i < 4A \\ \frac{1}{\sigma_Z^2}\left(-(r_i-5A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_Z^2}4A(-r_i+6A) & 4A \le r_i \end{cases}$$

256QAM

$$d(i) = \frac{1}{\sqrt{170}}\Big\{\big(1-2b(8i)\big)\Big[8-\big(1-2b(8i+2)\big)\Big[4-\big(1-2b(8i+4)\big)\Big[2-\big(1-2b(8i+6)\big)\Big]\Big]\Big]$$
$$+j\big(1-2b(8i+1)\big)\Big[8-\big(1-2b(8i+3)\big)\Big[4-\big(1-2b(8i+5)\big)\Big[2-\big(1-2b(8i+7)\big)\Big]\Big]\Big]\Big\}$$



下图为偶数 bits [b0,b2,b4,b6]映射到实部



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LLR_{b0} = \begin{cases}
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 15A)^2\right) = \frac{1}{\sigma_Z^2} 32A(r_i + 7A) & r_i < -14A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 13A)^2\right) = \frac{1}{\sigma_Z^2} 28A(r_i + 6A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 11A)^2\right) = \frac{1}{\sigma_Z^2} 24A(r_i + 5A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 9A)^2\right) = \frac{1}{\sigma_Z^2} 20A(r_i + 4A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 7A)^2\right) = \frac{1}{\sigma_Z^2} 16A(r_i + 3A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 5A)^2\right) = \frac{1}{\sigma_Z^2} 12A(r_i + 2A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + 3A)^2\right) = \frac{1}{\sigma_Z^2} 8A(r_i + A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 4Ar_i & -2A \le r_i < 2A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 3A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 8A(r_i - A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 5A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 12A(r_i - 2A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 7A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 16A(r_i - 3A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 9A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 20A(r_i - 4A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 11A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 24A(r_i - 5A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 13A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 28A(r_i - 6A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_Z^2}\left(-(r_i - 15A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_Z^2} 32A(r_i - 7A) & 14A \le r_i
\end{cases}
$$

$$LLR_{b2} = \begin{cases} \frac{1}{\sigma_z^2}\left(-(r_i+7A)^2+(r_i+15A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+11A) & r_i < -14A \\ \frac{1}{\sigma_z^2}\left(-(r_i+7A)^2+(r_i+13A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+10A) & -14A \le r_i < -12A \\ \frac{1}{\sigma_z^2}\left(-(r_i+7A)^2+(r_i+11A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+9A) & -12A \le r_i < -10A \\ \frac{1}{\sigma_z^2}\left(-(r_i+7A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+8A) & -10A \le r_i < -6A \\ \frac{1}{\sigma_z^2}\left(-(r_i+5A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+7A) & -6A \le r_i < -4A \\ \frac{1}{\sigma_z^2}\left(-(r_i+3A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+6A) & -4A \le r_i < -2A \\ \frac{1}{\sigma_z^2}\left(-(r_i+A)^2+(r_i+9A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+5A) & -2A \le r_i < 0 \\ \frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+5A) & 0 \le r_i < 2A \\ \frac{1}{\sigma_z^2}\left(-(r_i-3A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+6A) & 2A \le r_i < 4A \\ \frac{1}{\sigma_z^2}\left(-(r_i-5A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+7A) & 4A \le r_i < 6A \\ \frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i-9A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+8A) & 6A \le r_i < 10A \\ \frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i-11A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+9A) & 10A \le r_i < 12A \\ \frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i-13A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+10A) & 12A \le r_i < 14A \\ \frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i-15A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+11A) & 14A \le r_i \end{cases}$$

$$
LLR_{b4} = \begin{cases}
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+15A)^2) = \frac{1}{\sigma_z^2}8A(r_i+13A) & r_i < -14A \\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+13A)^2) = \frac{1}{\sigma_z^2}4A(r_i+12A) & -14A \le r_i < -10A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+13A)^2) = \frac{1}{\sigma_z^2}8A(r_i+11A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_z^2}(-(r_i+7A)^2+(r_i+3A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-5A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+3A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-4A) & -6A \le r_i < -2A \\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-3A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-A)^2) = \frac{1}{\sigma_z^2}8A(r_i-3A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-3A)^2) = \frac{1}{\sigma_z^2}4A(r_i-4A) & 2A \le r_i < 6A \\
\frac{1}{\sigma_z^2}(-(r_i-7A)^2+(r_i-3A)^2) = \frac{1}{\sigma_z^2}8A(r_i-5A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-13A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+11A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-13A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+12A) & 10A \le r_i < 14A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-15A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+13A) & 14A \le r_i
\end{cases}
$$

$$
LLR_{b6} = \begin{cases}
\frac{1}{\sigma_z^2}\left(-(r_i + 13A)^2 + (r_i + 15A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i + 14A) & r_i < -12A \\
\frac{1}{\sigma_z^2}\left(-(r_i + 11A)^2 + (r_i + 9A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i - 10A) & -12A \le r_i < -8A \\
\frac{1}{\sigma_z^2}\left(-(r_i + 5A)^2 + (r_i + 7A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i + 6A) & -8A \le r_i < -4A \\
\frac{1}{\sigma_z^2}\left(-(r_i + 3A)^2 + (r_i + A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i - 2A) & -4A \le r_i < 0 \\
\frac{1}{\sigma_z^2}\left(-(r_i - 3A)^2 + (r_i - A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i - 2A) & 0 \le r_i < 4A \\
\frac{1}{\sigma_z^2}\left(-(r_i - 5A)^2 + (r_i - 7A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i + 6A) & 4A \le r_i < 8A \\
\frac{1}{\sigma_z^2}\left(-(r_i - 11A)^2 + (r_i - 9A)^2\right) = \frac{1}{\sigma_z^2} 4A(r_i - 10A) & 8A \le r_i < 12A \\
\frac{1}{\sigma_z^2}\left(-(r_i - 13A)^2 + (r_i - 15A)^2\right) = \frac{1}{\sigma_z^2} 4A(-r_i + 14A) & 12A \le r_i
\end{cases}
$$

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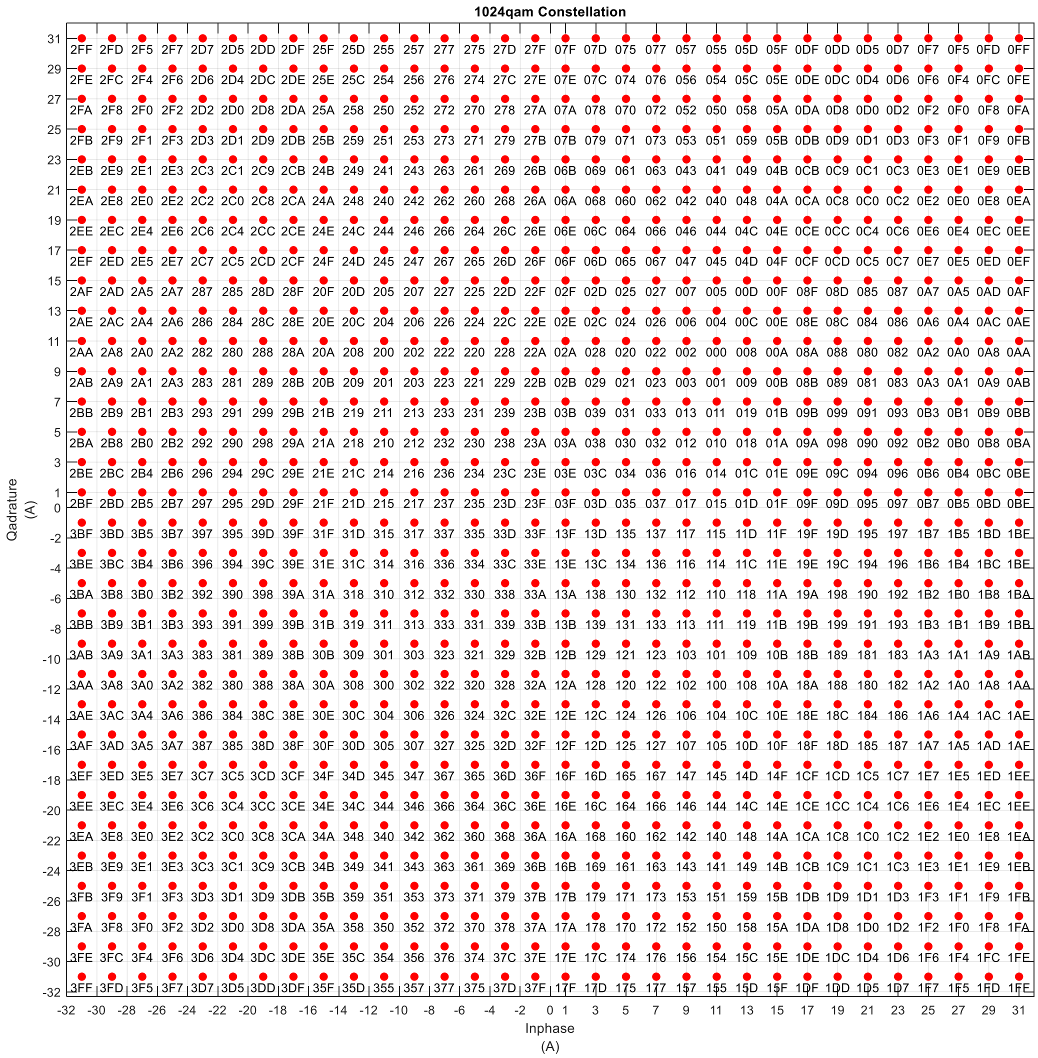
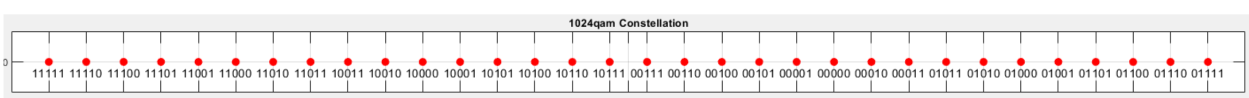
$$
\begin{aligned}
d(i) =& \\
&\frac{1}{\sqrt{682}}\big(1 - 2b(10i + 0)\big)\Big[16 - \big(1 - 2b(10i + 2)\big)\Big[8 - \big(1 - 2b(10i + 4)\big)\Big[4 - \big(1 - 2b(10i + 6)\big)\big[2 - \big(1 - 2b(10i + 8)\big)\big]\Big]\Big]\Big] \\
&+ j\frac{1}{\sqrt{682}}\big(1 - 2b(10i + 1)\big)\Big[16 - \big(1 - 2b(10i + 3)\big)\Big[8 - \big(1 - 2b(10i + 5)\big)\Big[4 - \big(1 - 2b(10i + 7)\big)\big[2 - \big(1 - 2b(10i + 9)\big)\big]\Big]\Big]\Big]
\end{aligned}
$$



下图为偶数 bits [b0,b2,b4,b6,b8]映射到实部



$$
LLR_{b0} = \begin{cases}
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 31A)^2) = \frac{1}{\sigma_z^2} 64A(r_i + 15A) & r_i < -30A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 29A)^2) = \frac{1}{\sigma_z^2} 60A(r_i + 14A) & -30A \le r_i < -28A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 27A)^2) = \frac{1}{\sigma_z^2} 56A(r_i + 13A) & -28A \le r_i < -26A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 25A)^2) = \frac{1}{\sigma_z^2} 52A(r_i + 12A) & -26A \le r_i < -24A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 23A)^2) = \frac{1}{\sigma_z^2} 48A(r_i + 11A) & -24A \le r_i < -22A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 21A)^2) = \frac{1}{\sigma_z^2} 44A(r_i + 10A) & -22A \le r_i < -20A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 19A)^2) = \frac{1}{\sigma_z^2} 40A(r_i + 9A) & -20A \le r_i < -18A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 17A)^2) = \frac{1}{\sigma_z^2} 36A(r_i + 8A) & -18A \le r_i < -16A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 15A)^2) = \frac{1}{\sigma_z^2} 32A(r_i + 7A) & -16A \le r_i < -14A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 13A)^2) = \frac{1}{\sigma_z^2} 28A(r_i + 6A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 11A)^2) = \frac{1}{\sigma_z^2} 24A(r_i + 5A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 9A)^2) = \frac{1}{\sigma_z^2} 20A(r_i + 4A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 7A)^2) = \frac{1}{\sigma_z^2} 16A(r_i + 3A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 5A)^2) = \frac{1}{\sigma_z^2} 12A(r_i + 2A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + 3A)^2) = \frac{1}{\sigma_z^2} 8A(r_i + A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_z^2}(-(r_i - A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 4Ar_i & -2A \le r_i < 2A \\
\frac{1}{\sigma_z^2}(-(r_i - 3A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 8A(r_i - A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_z^2}(-(r_i - 5A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 12A(r_i - 2A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_z^2}(-(r_i - 7A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 16A(r_i - 3A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_z^2}(-(r_i - 9A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 20A(r_i - 4A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_z^2}(-(r_i - 11A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 24A(r_i - 5A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_z^2}(-(r_i - 13A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 28A(r_i - 6A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_z^2}(-(r_i - 15A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 32A(r_i - 7A) & 14A \le r_i < 16A \\
\frac{1}{\sigma_z^2}(-(r_i - 17A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 36A(r_i - 8A) & 16A \le r_i < 18A \\
\frac{1}{\sigma_z^2}(-(r_i - 19A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 40A(r_i - 9A) & 18A \le r_i < 20A \\
\frac{1}{\sigma_z^2}(-(r_i - 21A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 44A(r_i - 10A) & 20A \le r_i < 22A \\
\frac{1}{\sigma_z^2}(-(r_i - 23A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 48A(r_i - 11A) & 22A \le r_i < 24A \\
\frac{1}{\sigma_z^2}(-(r_i - 25A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 52A(r_i - 12A) & 24A \le r_i < 26A \\
\frac{1}{\sigma_z^2}(-(r_i - 27A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 56A(r_i - 13A) & 26A \le r_i < 28A \\
\frac{1}{\sigma_z^2}(-(r_i - 29A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 60A(r_i - 14A) & 28A \le r_i < 30A \\
\frac{1}{\sigma_z^2}(-(r_i - 31A)^2 + (r_i + A)^2) = \frac{1}{\sigma_z^2} 64A(r_i - 15A) & 30A \le r_i
\end{cases}
$$

$$
LLR_{b2} = \begin{cases}
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+31A)^2\right) = \frac{1}{\sigma_z^2}32A(r_i+23A) & r_i < -30A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+29A)^2\right) = \frac{1}{\sigma_z^2}28A(r_i+22A) & -30A \le r_i < -28A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+27A)^2\right) = \frac{1}{\sigma_z^2}24A(r_i+21A) & -28A \le r_i < -26A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}20A(r_i+20A) & -26A \le r_i < -24A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+23A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+19A) & -24A \le r_i < -22A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+21A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+18A) & -22A \le r_i < -20A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+19A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+17A) & -20A \le r_i < -18A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+16A) & -18A \le r_i < -14A \\
\frac{1}{\sigma_z^2}\left(-(r_i+13A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+15A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_z^2}\left(-(r_i+11A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+14A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_z^2}\left(-(r_i+9A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+13A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_z^2}\left(-(r_i+7A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}20A(r_i+12A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_z^2}\left(-(r_i+5A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}24A(r_i+11A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_z^2}\left(-(r_i+3A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}28A(r_i+10A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_z^2}\left(-(r_i+A)^2+(r_i+17A)^2\right) = \frac{1}{\sigma_z^2}32A(r_i+9A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}\left(-(r_i-A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}32A(-r_i+9A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}\left(-(r_i-3A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}28A(-r_i+10A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_z^2}\left(-(r_i-5A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}24A(-r_i+11A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_z^2}\left(-(r_i-7A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}20A(-r_i+12A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+13A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_z^2}\left(-(r_i-11A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+14A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_z^2}\left(-(r_i-13A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+15A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-17A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+16A) & 14A \le r_i < 18A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-19A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+17A) & 18A \le r_i < 20A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-21A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+18A) & 20A \le r_i < 22A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-23A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+19A) & 22A \le r_i < 24A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}20A(-r_i+20A) & 24A \le r_i < 26A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-27A)^2\right) = \frac{1}{\sigma_z^2}24A(-r_i+21A) & 26A \le r_i < 28A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-29A)^2\right) = \frac{1}{\sigma_z^2}28A(-r_i+22A) & 28A \le r_i < 30A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-31A)^2\right) = \frac{1}{\sigma_z^2}32A(-r_i+23A) & 30A \le r_i
\end{cases}
$$

$$
LLR_{b4} = \begin{cases}
\frac{1}{\sigma_z^2}\left(-(r_i+23A)^2+(r_i+31A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+27A) & r_i < -30A \\
\frac{1}{\sigma_z^2}\left(-(r_i+23A)^2+(r_i+29A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+26A) & -30A \le r_i < -28A \\
\frac{1}{\sigma_z^2}\left(-(r_i+23A)^2+(r_i+27A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+25A) & -28A \le r_i < -26A \\
\frac{1}{\sigma_z^2}\left(-(r_i+23A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i+24A) & -26A \le r_i < -22A \\
\frac{1}{\sigma_z^2}\left(-(r_i+21A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i+23A) & -22A \le r_i < -20A \\
\frac{1}{\sigma_z^2}\left(-(r_i+19A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i+22A) & -20A \le r_i < -18A \\
\frac{1}{\sigma_z^2}\left(-(r_i+17A)^2+(r_i+25A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i+21A) & -18A \le r_i < -16A \\
\frac{1}{\sigma_z^2}\left(-(r_i+15A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i-11A) & -16A \le r_i < -14A \\
\frac{1}{\sigma_z^2}\left(-(r_i+13A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i-10A) & -14A \le r_i < -12A \\
\frac{1}{\sigma_z^2}\left(-(r_i+11A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i-9A) & -12A \le r_i < -10A \\
\frac{1}{\sigma_z^2}\left(-(r_i+9A)^2+(r_i+7A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i-8A) & -10A \le r_i < -6A \\
\frac{1}{\sigma_z^2}\left(-(r_i+9A)^2+(r_i+5A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i-7A) & -6A \le r_i < -4A \\
\frac{1}{\sigma_z^2}\left(-(r_i+9A)^2+(r_i+3A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i-6A) & -4A \le r_i < -2A \\
\frac{1}{\sigma_z^2}\left(-(r_i+9A)^2+(r_i+A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i-5A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i-A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i-5A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i-3A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i-6A) & 2A \le r_i < 4A \\
\frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i-5A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i-7A) & 4A \le r_i < 6A \\
\frac{1}{\sigma_z^2}\left(-(r_i-9A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}4A(r_i-8A) & 6A \le r_i < 10A \\
\frac{1}{\sigma_z^2}\left(-(r_i-11A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}8A(r_i-9A) & 10A \le r_i < 12A \\
\frac{1}{\sigma_z^2}\left(-(r_i-13A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}12A(r_i-10A) & 12A \le r_i < 14A \\
\frac{1}{\sigma_z^2}\left(-(r_i-15A)^2+(r_i-7A)^2\right) = \frac{1}{\sigma_z^2}16A(r_i-11A) & 14A \le r_i < 16A \\
\frac{1}{\sigma_z^2}\left(-(r_i-17A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+21A) & 16A \le r_i < 18A \\
\frac{1}{\sigma_z^2}\left(-(r_i-19A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+22A) & 18A \le r_i < 20A \\
\frac{1}{\sigma_z^2}\left(-(r_i-21A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+23A) & 20A \le r_i < 22A \\
\frac{1}{\sigma_z^2}\left(-(r_i-23A)^2+(r_i-25A)^2\right) = \frac{1}{\sigma_z^2}4A(-r_i+24A) & 22A \le r_i < 26A \\
\frac{1}{\sigma_z^2}\left(-(r_i-23A)^2+(r_i-27A)^2\right) = \frac{1}{\sigma_z^2}8A(-r_i+25A) & 26A \le r_i < 28A \\
\frac{1}{\sigma_z^2}\left(-(r_i-23A)^2+(r_i-29A)^2\right) = \frac{1}{\sigma_z^2}12A(-r_i+26A) & 28A \le r_i < 30A \\
\frac{1}{\sigma_z^2}\left(-(r_i-23A)^2+(r_i-31A)^2\right) = \frac{1}{\sigma_z^2}16A(-r_i+27A) & 30A \le r_i
\end{cases}
$$

$$
LLR_{b6} = \begin{cases}
\frac{1}{\sigma_z^2}(-(r_i+27A)^2+(r_i+31A)^2) = \frac{1}{\sigma_z^2}8A(r_i+29A) & r_i < -30A \\
\frac{1}{\sigma_z^2}(-(r_i+27A)^2+(r_i+29A)^2) = \frac{1}{\sigma_z^2}4A(r_i+28A) & -30A \le r_i < -26A \\
\frac{1}{\sigma_z^2}(-(r_i+25A)^2+(r_i+29A)^2) = \frac{1}{\sigma_z^2}8A(r_i+27A) & -26A \le r_i < -24A \\
\frac{1}{\sigma_z^2}(-(r_i+23A)^2+(r_i+19A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-21A) & -24A \le r_i < -22A \\
\frac{1}{\sigma_z^2}(-(r_i+21A)^2+(r_i+19A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-20A) & -22A \le r_i < -18A \\
\frac{1}{\sigma_z^2}(-(r_i+21A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-19A) & -18A \le r_i < -16A \\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+15A)^2) = \frac{1}{\sigma_z^2}8A(r_i+13A) & -16A \le r_i < -14A \\
\frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+13A)^2) = \frac{1}{\sigma_z^2}4A(r_i+12A) & -14A \le r_i < -10A \\
\frac{1}{\sigma_z^2}(-(r_i+9A)^2+(r_i+13A)^2) = \frac{1}{\sigma_z^2}8A(r_i+11A) & -10A \le r_i < -8A \\
\frac{1}{\sigma_z^2}(-(r_i+7A)^2+(r_i+3A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-5A) & -8A \le r_i < -6A \\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+3A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-4A) & -6A \le r_i < -2A \\
\frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+A)^2) = \frac{1}{\sigma_z^2}8A(-r_i-3A) & -2A \le r_i < 0 \\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-A)^2) = \frac{1}{\sigma_z^2}8A(r_i-3A) & 0 \le r_i < 2A \\
\frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-3A)^2) = \frac{1}{\sigma_z^2}4A(r_i-4A) & 2A \le r_i < 6A \\
\frac{1}{\sigma_z^2}(-(r_i-7A)^2+(r_i-3A)^2) = \frac{1}{\sigma_z^2}8A(r_i-5A) & 6A \le r_i < 8A \\
\frac{1}{\sigma_z^2}(-(r_i-9A)^2+(r_i-13A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+11A) & 8A \le r_i < 10A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-13A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+12A) & 10A \le r_i < 14A \\
\frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-15A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+13A) & 14A \le r_i < 16A \\
\frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}8A(r_i-19A) & 16A \le r_i < 18A \\
\frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-19A)^2) = \frac{1}{\sigma_z^2}4A(r_i-20A) & 18A \le r_i < 22A \\
\frac{1}{\sigma_z^2}(-(r_i-23A)^2+(r_i-19A)^2) = \frac{1}{\sigma_z^2}8A(r_i-21A) & 22A \le r_i < 24A \\
\frac{1}{\sigma_z^2}(-(r_i-25A)^2+(r_i-29A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+27A) & 24A \le r_i < 26A \\
\frac{1}{\sigma_z^2}(-(r_i-27A)^2+(r_i-29A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+28A) & 26A \le r_i < 30A \\
\frac{1}{\sigma_z^2}(-(r_i-27A)^2+(r_i-31A)^2) = \frac{1}{\sigma_z^2}8A(-r_i+29A) & 30A \le r_i
\end{cases}
$$

$$LLR_{b8} = \begin{cases} \frac{1}{\sigma_z^2}(-(r_i+29A)^2+(r_i+31A)^2) = \frac{1}{\sigma_z^2}4A(r_i+30A) & r_i < -28A \\ \frac{1}{\sigma_z^2}(-(r_i+27A)^2+(r_i+25A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-26A) & -28A \le r_i < -24A \\ \frac{1}{\sigma_z^2}(-(r_i+21)^2+(r_i+23A)^2) = \frac{1}{\sigma_z^2}4A(r_i+22A) & -24A \le r_i < -20A \\ \frac{1}{\sigma_z^2}(-(r_i+19A)^2+(r_i+17A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-18A) & -20A \le r_i < -16A \\ \frac{1}{\sigma_z^2}(-(r_i+13A)^2+(r_i+15A)^2) = \frac{1}{\sigma_z^2}4A(r_i+14A) & -16A \le r_i < -12A \\ \frac{1}{\sigma_z^2}(-(r_i+11A)^2+(r_i+9A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-10A) & -12A \le r_i < -8A \\ \frac{1}{\sigma_z^2}(-(r_i+5A)^2+(r_i+7A)^2) = \frac{1}{\sigma_z^2}4A(r_i+6A) & -8A \le r_i < -4A \\ \frac{1}{\sigma_z^2}(-(r_i+3A)^2+(r_i+A)^2) = \frac{1}{\sigma_z^2}4A(-r_i-2A) & -4A \le r_i < 0 \\ \frac{1}{\sigma_z^2}(-(r_i-3A)^2+(r_i-A)^2) = \frac{1}{\sigma_z^2}4A(r_i-2A) & 0A \le r_i < 4A \\ \frac{1}{\sigma_z^2}(-(r_i-5A)^2+(r_i-7A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+6A) & 4A \le r_i < 8A \\ \frac{1}{\sigma_z^2}(-(r_i-11A)^2+(r_i-9A)^2) = \frac{1}{\sigma_z^2}4A(r_i-10A) & 8A \le r_i < 12A \\ \frac{1}{\sigma_z^2}(-(r_i-13A)^2+(r_i-15A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+14A) & 12A \le r_i < 16A \\ \frac{1}{\sigma_z^2}(-(r_i-19A)^2+(r_i-17A)^2) = \frac{1}{\sigma_z^2}4A(r_i-18A) & 16A \le r_i < 20A \\ \frac{1}{\sigma_z^2}(-(r_i-21A)^2+(r_i-23A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+22A) & 20A \le r_i < 24A \\ \frac{1}{\sigma_z^2}(-(r_i-27A)^2+(r_i-25A)^2) = \frac{1}{\sigma_z^2}4A(r_i-26A) & 24A \le r_i < 28A \\ \frac{1}{\sigma_z^2}(-(r_i-29A)^2+(r_i-31A)^2) = \frac{1}{\sigma_z^2}4A(-r_i+30A) & 28A \le r_i \end{cases}$$