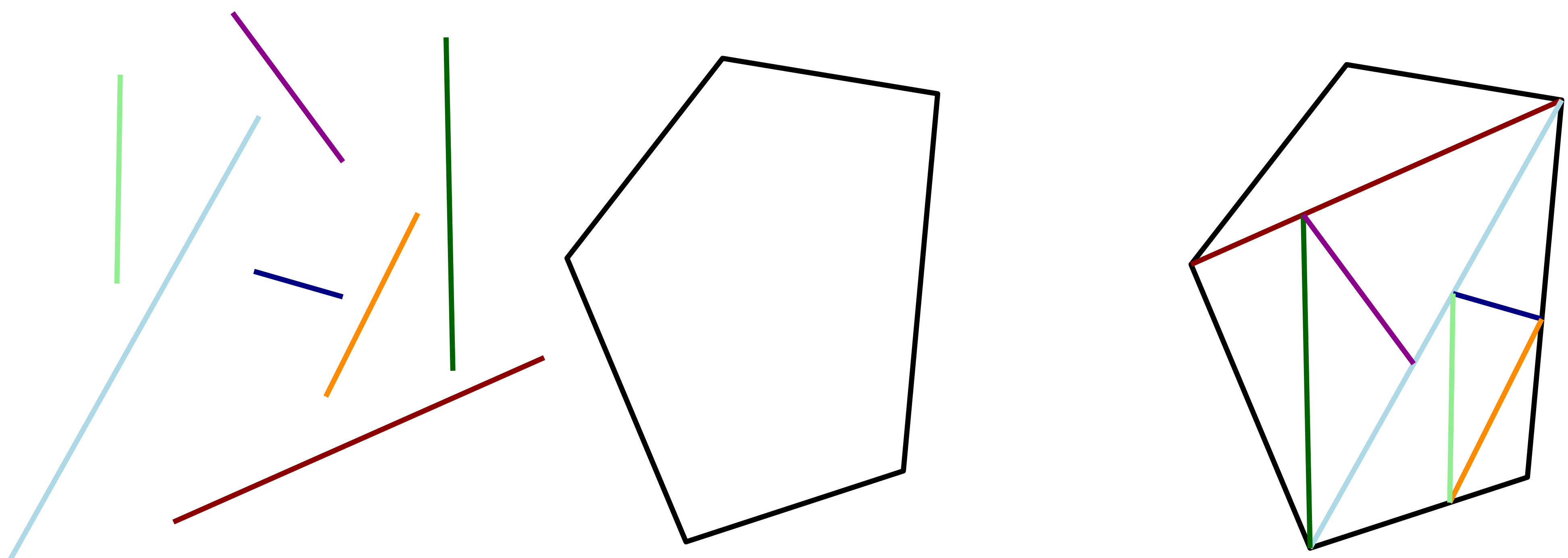
Packing Line Segments in a Convex 3-Polytope is NP-hard

Michael Gene Dobbins
POSTECH

Heuna Kim
FU Berlin

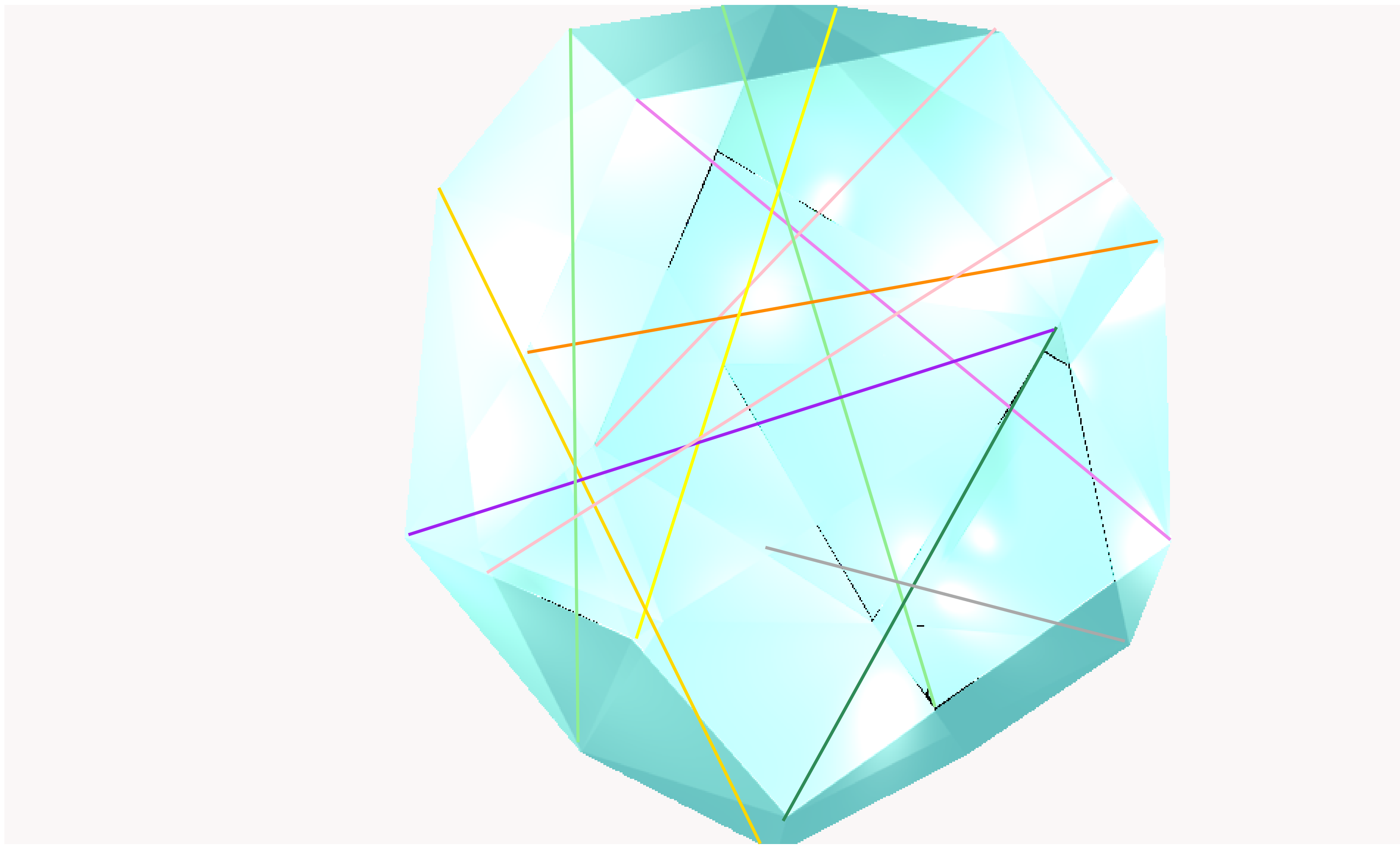
MaxSegPack$d$

Input : A collection of segments $\mathcal{S}$ and
a *bounded convex d-polytope* P



Output : The *maximum* number of segments in $\mathcal{S}$
that can be disjointly embedded in P by translation

MaxSegPack3 : *NP-hard*



Reduction from

the *Maximum Independent Set Problem*

for *bridgeless triangle-free cubic graphs* (MaxIndSet$G$)

Reduction
from MaxSegPack*3* to MaxIndSet$G$

A *vertex* v of G $\leftrightarrow$ A *line segment* l_v in $\mathcal{S}$

A *graph* G $\leftrightarrow$ The *intersection graph* of $\mathcal{S}$

Reduction
from MaxSegPack*3* to MaxIndSet*G*

A *vertex* v of G	$\leftrightarrow$	A *line segment* l_v in $\mathcal{S}$
A *graph* G	$\leftrightarrow$	The *intersection graph* of $\mathcal{S}$
Independent sets in G	$\leftrightarrow$	Sets of line segments that can be disjointly embedded
The *maximum independent set* in G	$\leftrightarrow$	The set of max.$\#$(line segments that can be disjointly embedded)

Reduction
from $\textsc{MaxSegPack}\mathit{3}$ to $\textsc{MaxIndSet}G$

Construct

Lines

$\Rightarrow \mathcal{L}$

Construct

a Convex Polyhedron

$\Rightarrow P$

Reduction
from $\textsc{MaxSegPack}3$ to $\textsc{MaxIndSet}G$

Construct
Lines
$\Rightarrow \mathcal{L}$

The intersection graph of $\mathcal{L}$ becomes G

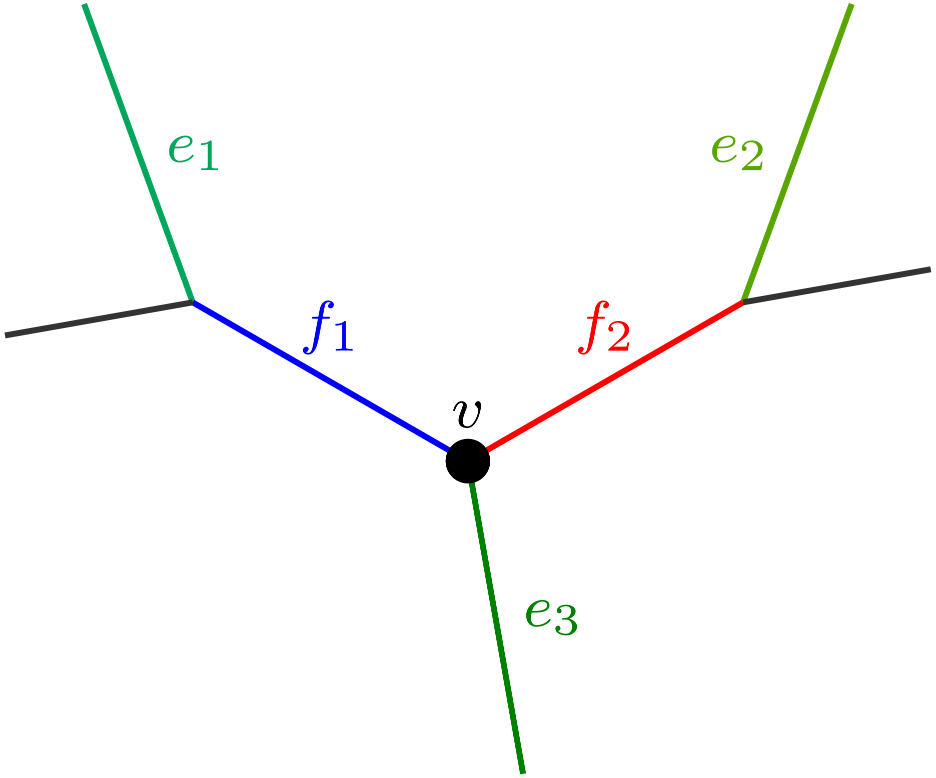
Construct
a Convex Polyhedron
$\Rightarrow P$

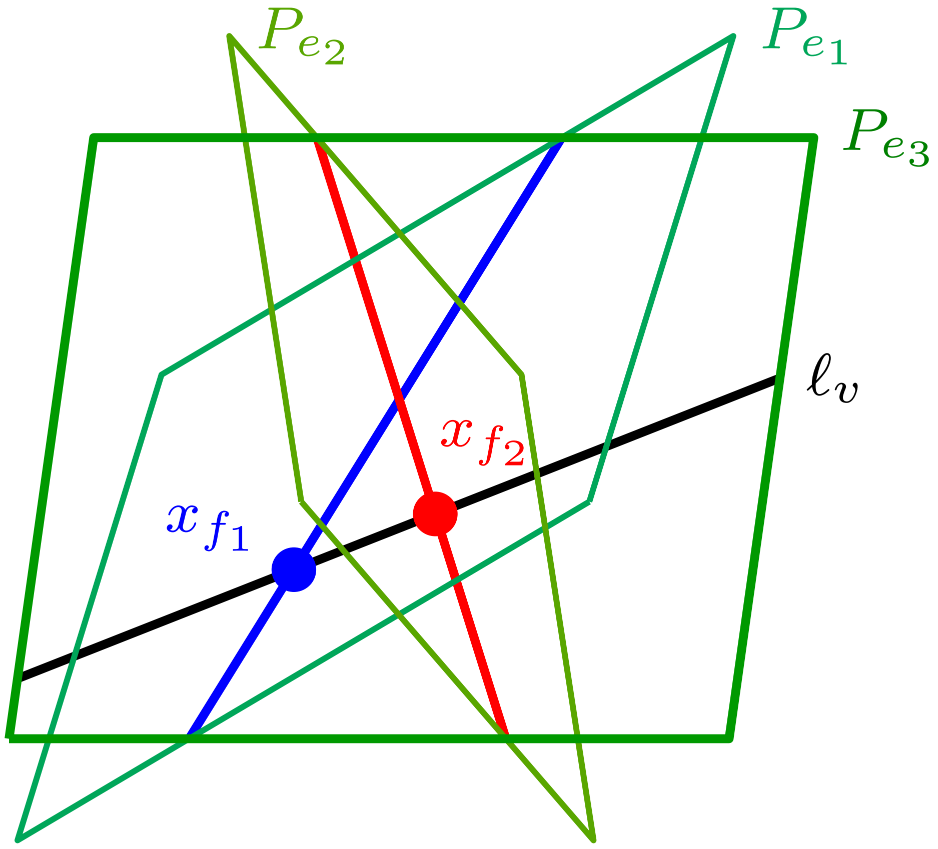
$\mathcal{S} = \mathcal{L} \cap P$

$\mathcal{S}$ becomes rigid in P

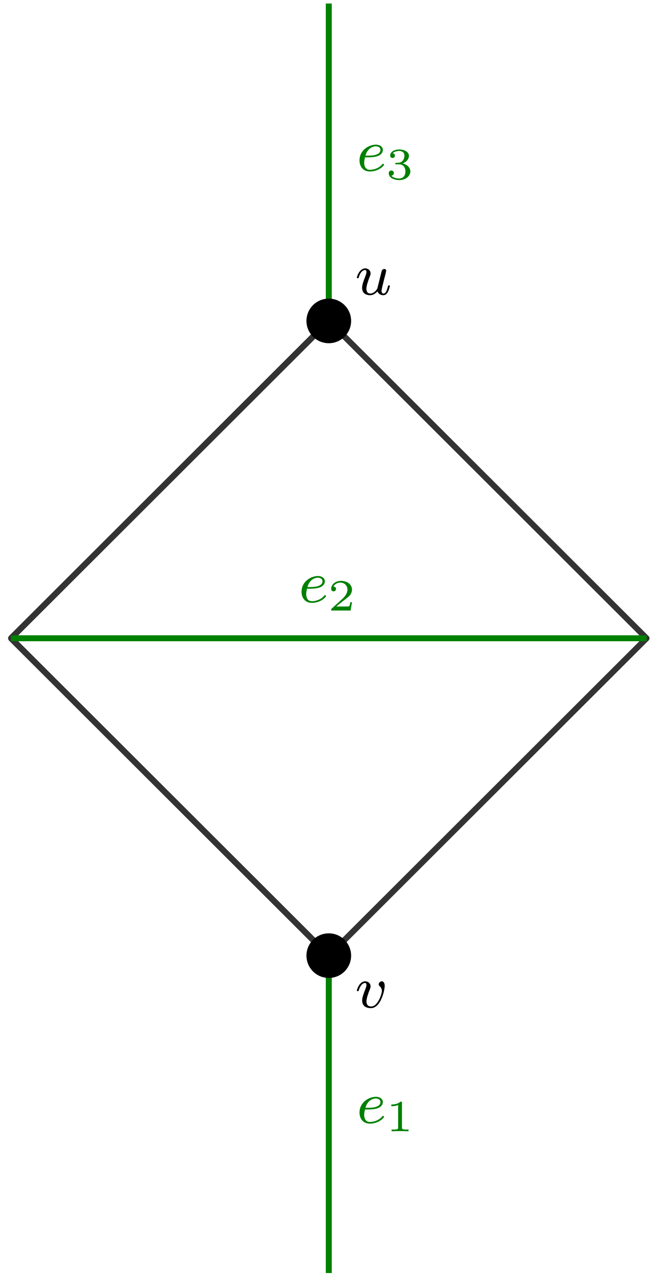
Construction of $\mathcal{L}$

Perfect Matching $M \Rightarrow$ Generic *Planes* $\mathcal{P} = \{P_e | e \in M\}$



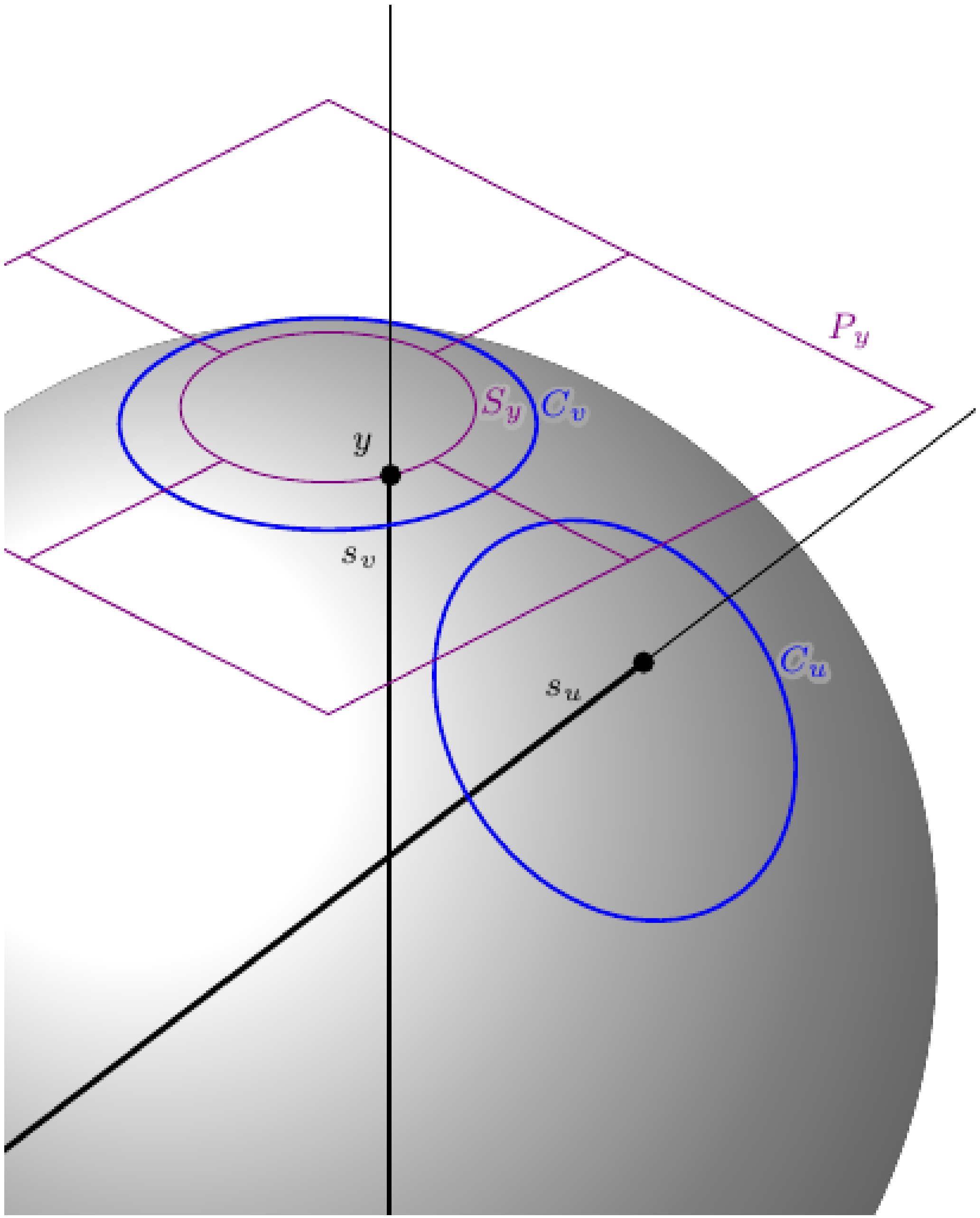


The Triangle-free Condition



l_u and l_v would intersect if e_1, e_2, e_3 are in the perfect matching.

Construction of P



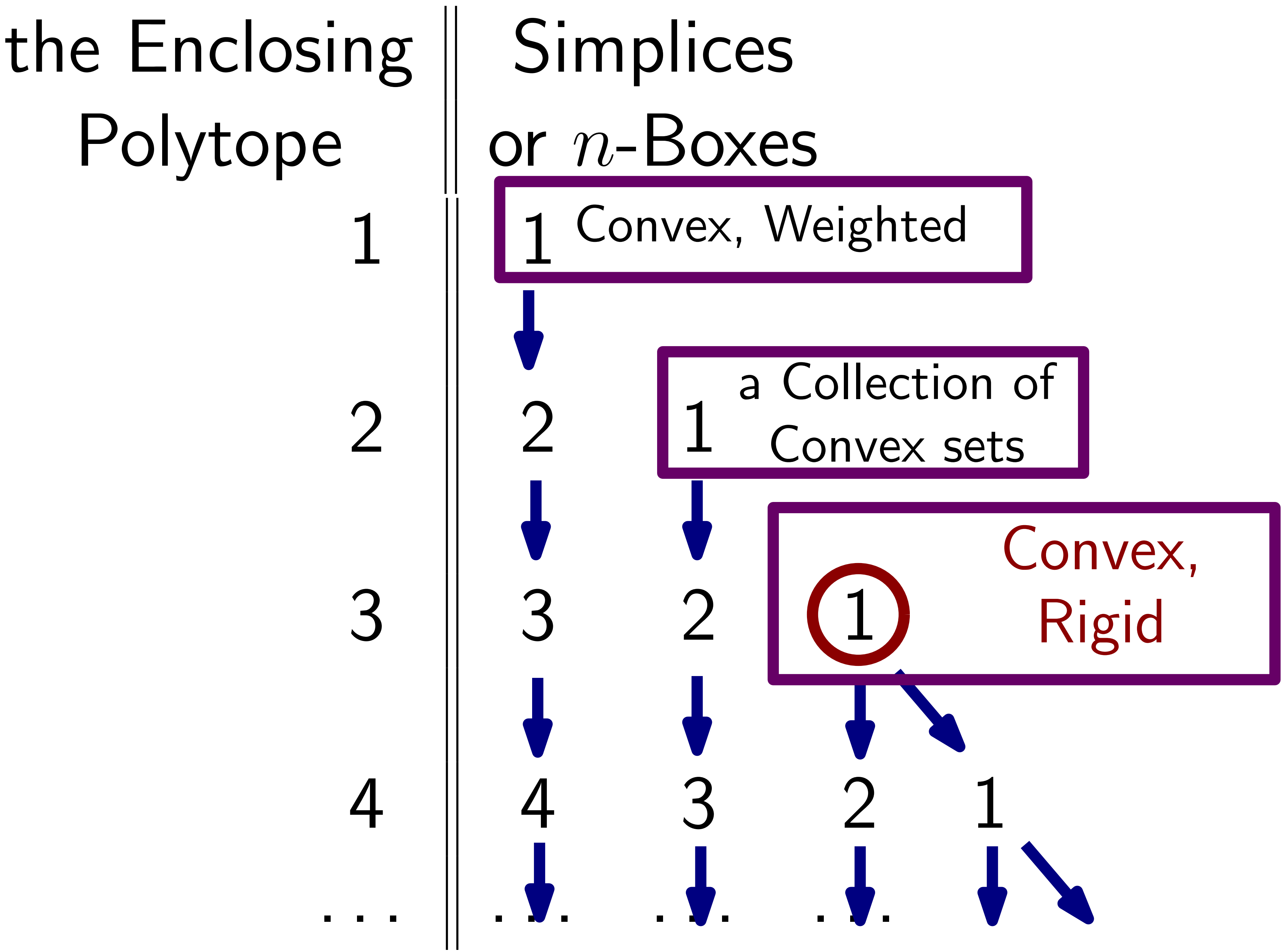
$\mathcal{S} = \mathcal{L} \cap P$ can rigidly fit into P

$$\mathcal{S} = \{l \cap B | l \in \mathcal{L}\}$$

$$P = \mathrm{conv}(\bigcup \mathcal{S})$$

Reduction in *polynomial* time

Find a *Perfect Matching*	✓
Generate *Planes*	✓
Choose polynomial#points	✓
Compute $\mathcal{L}$	✓
Compute a *Sphere*	✓
Compute $\mathcal{S}$	✓
Compute $conv(\bigcup \mathcal{S})$	✓


the Enclosing Polytope
Simplices or n-Boxes
1
1 Convex, Weighted
2
2
1 a Collection of Convex sets
3
3
2
1
Convex, Rigid
4
4
3
2
1
. . .
. . .
. . .
. . .

Open Problems

- Complexity for a Convex Polygon

- Approximation Algorithms