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SABARAGAMUWA UNIVERSITY OF SRI LANKA

FACULTY OF TECHNOLOGY

DEPARTMENT OF ENGINEERING TECHNOLOGY

BACHELOR OF ENGINEERING TECHNOLOGY HONORS DEGREE PROGRAMME

YEAR I SEMESTER II EXAMINATION-JUNE/JULY 2023

MATHEMATICS 2 - ET 12052

Time allowed: **Three (03) Hours**

INSTRUCTIONS TO CANDIDATES:

Answer only 5 questions including the first question.

Marks for each part is indicated within brackets.

Use of scientific calculators are allowed.

Write down required steps for every question.

The correct steps will be given marks accordingly.

1. a. Plot the points $A(-1,2)$ and $B(2,5)$ on the Cartesian plane (20 marks)

b. Find the distance and middle point coordinates between these two points. (20 marks)

c. Draw the complex number representing the coordinates $(3,-2)$ in an Argand Diagram. (10 marks)

d. Find the modulus of the above complex number. (10 marks)

e. Write down the order and the degree of the following differential equations. (10 marks)

(i). $2x\left(\frac{d^2y}{dx^2}\right)+y^2x=0.$

(ii). $\left(\frac{d^4y}{dx^4}\right)^3+y\left(\frac{d^2y}{dx^2}\right)^4+\left(\frac{dy}{dx}\right)^2=0.$

f. Form the differential equation by eliminating arbitrary constants. Where, A and B are arbitrary constants. (20 marks)

(i). $y=Ax^2+A^2.$

(ii). $y=A\cos(x)+B\sin(x).$

g. Evaluate the following expressions. (10 marks)

(i). $\int\frac{(3x^2+2x)}{(x^3+x^2)}\,dx.$

(ii). $\int\frac{3}{\sqrt{1-x^2}}\,dx.$

2. a. What is the referen

b. Find values of sine

c. Assuming the belo

d. Using the sum $\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$ and si

e. Find the exact val

3. a. Prove the follo trigonometric ide

(i). $\sin^2 A+$

(ii). $\sec^2 A=$

(iii). $\csc^2 A=$

b. Given that, sin A

Find the exact va

c. Given that sin θ

and 2π?

2. a. What is the reference angle of 120°? (10 marks)

b. Find values of sine, and cosine of the angle 120° using **symmetric relationship.** (20 marks)

c. Assuming the below sum formula of sine, obtain the difference formula of sine. (20 marks)

$$\sin(x+y) = \sin x \,.\, \cos y + \cos x \,.\, \sin y.$$

d. Using the sum and difference formulas of sine, calculate the values of $\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$. (30 marks)

e. Find the exact value of the $\sin 75^\circ$ by using the sum or difference formulas. (20 marks)

3. a. Prove the followings Pythagorean identities starting from the basic trigonometric identities, (30 marks)

(i). $\sin^2 A + \cos^2 A = 1.$

(ii). $\sec^2 A = 1 + \tan^2 A.$

(iii). $\csc^2 A = 1 + \cot^2 A.$

b. Given that, $\sin A = -\frac{4}{5}$ and $\tan A > 0$. What is the quadrant of angle A lies. (40 marks)

Find the exact values of all the basic trigonometric functions.

c. Given that $\sin\theta = \frac{1}{2}$ What are the possible values for the angle θ between 0 and 2π? (30 marks)

4. a. Solve $x^2 - 2x + 2 = 0$. (20 marks)

b. Express each of the following in the form $x + yi$. (20 marks)

(i). $(3+5i)+(2-3i)$.

(ii). $(3+5i)+6$.

c. Solve the following equations for real x and y. (30 marks)

(i). $3+5i+x-yi=6-2i$

(ii). $x+yi=(1-i)(2+8i)$.

d. The complex numbers u, v and w are related by, (30 marks)

$\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$.

Given that $v=3+4i, w=4-3i$, find u in the form $x+yi$.

5 a. Let $z_1=5+2i$, $z_2=1+3i$, $z_3=2-3i$, $z_4=-4-7i$. (20 marks)

Plot the complex numbers, z_1, z_2, z_3, z_4 on an Argand Diagram, and label them.

b. Plot the complex numbers z_1+z_2 and z_1-z_2 on the same Argand diagram. (30 marks)

Geometrically, how do the positions of the numbers z_1+z_2 and z_1-z_2 relate to z_1 and z_2?

c. Suppose you wish to combine two complex numbers of the form, (40 marks)

$$z_1=[r_1,\theta_1], \quad z_2=[r_2,\theta_2].$$

Note that,

$z_1=r_1\cos\theta_1+ir_1\sin\theta_1$ and $z_2=r_2\cos\theta_2+ir_2\sin\theta_2$.

Derive an expression for $z_1\times z_2$ in polar form.

d. Hence find $z_1\times z_2$, where $z_1=[2,\frac{\pi}{3}]$ and $z_2=[4,\frac{\pi}{2}]$. (10 marks)

6. a. Find an express

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c. Show that,

d. Simplify the fo

7. a. Define linear d
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b. Solve the follo

(i). $x\left(\frac{dy}{dx}\right.$

(ii). $(x+1)$

c. Find the soluti

6. a. Find an expression to represent z^2, z^3 where, $z = \cos\theta + i\sin\theta$. (20 marks)

b. Deduce an expression for z^n using the above pattern. (n is positive) (20 marks)

c. Show that, (30 marks)

$$\frac{1}{(\cos m\theta + i\sin m\theta)} = \cos m\theta - i\sin m\theta.$$

d. Simplify the following expression using the deduced expression in part (b), (30 marks)

$$\frac{(\cos 3\theta + i\sin 3\theta)}{(\cos 2\theta + i\sin 2\theta)}.$$

7. a. Define linear differential equation with its standard format and state the main three steps followed to find the solution of a linear differential equation. (20 marks)

b. Solve the following linear differential equations. (40 marks)

(i). $x\left(\frac{dy}{dx}\right) + y = x(x^3 - 3).$

(ii). $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2.$

c. Find the solution for Bernoulli's differential equation, (40 marks)

$$x^2dy + y(x+y)\,dx = 0.$$