EASY-TO-COMPUTE LOCAL CLIFFORD INVARIANTS FOR GRAPH STATES

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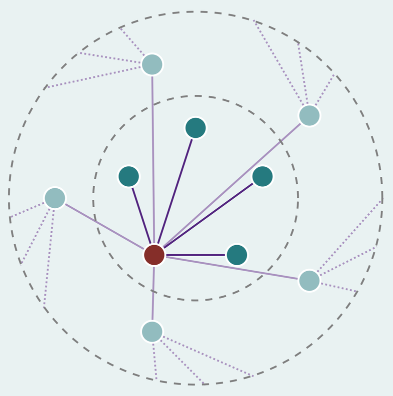
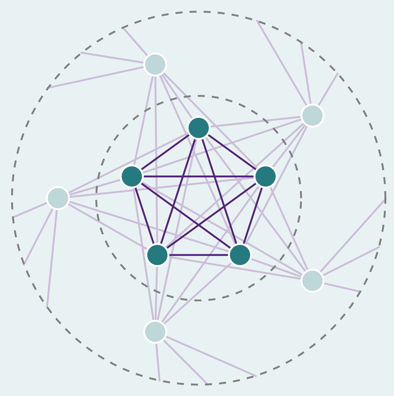
Abstract

We explore LC-invariants of graph states that are simple to compute. Previous research[1] has resulted in finite sets of LC-invariants that fully characterize the LC-equivalence classes of graph states, but they are computationally hard, as they require knowledge of the full stabilizer set, which grows exponentially with the number of qubits in the state.
Our paper[2] presents an LC-invariant of order $\mathcal{O}(n^3)$ that is straightforward to compute, without the need to calculate this entire stabilizer set. Our invariant is related to the foliage of a graph and has a clear graphical representation in terms of leaves, axils, and twins.

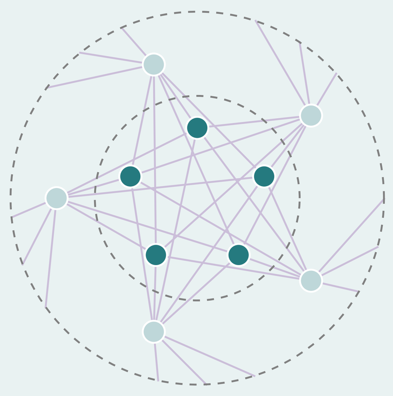
✧ Axil and leaves



✪ Connected twins



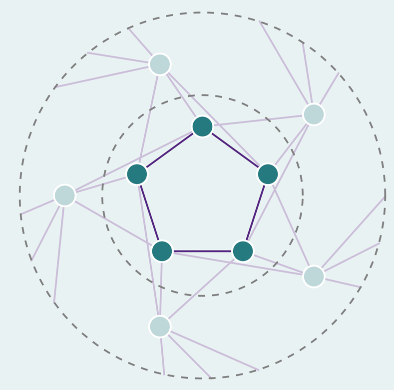
⁙ Disconnected twins



Foliage = leaves ∪ axils ∪ twins

A leaf is a vertex with degree one and an axil its unique neighbour. A twin is a vertex v that has the same neighbourhood as a second vertex $w \neq v$ in the sense that $N_v \backslash \{w\} = N_w \backslash \{v\}$. The set containing all the leaves, axils and twins of a graph is called the foliage of that graph.

• Outside the foliage

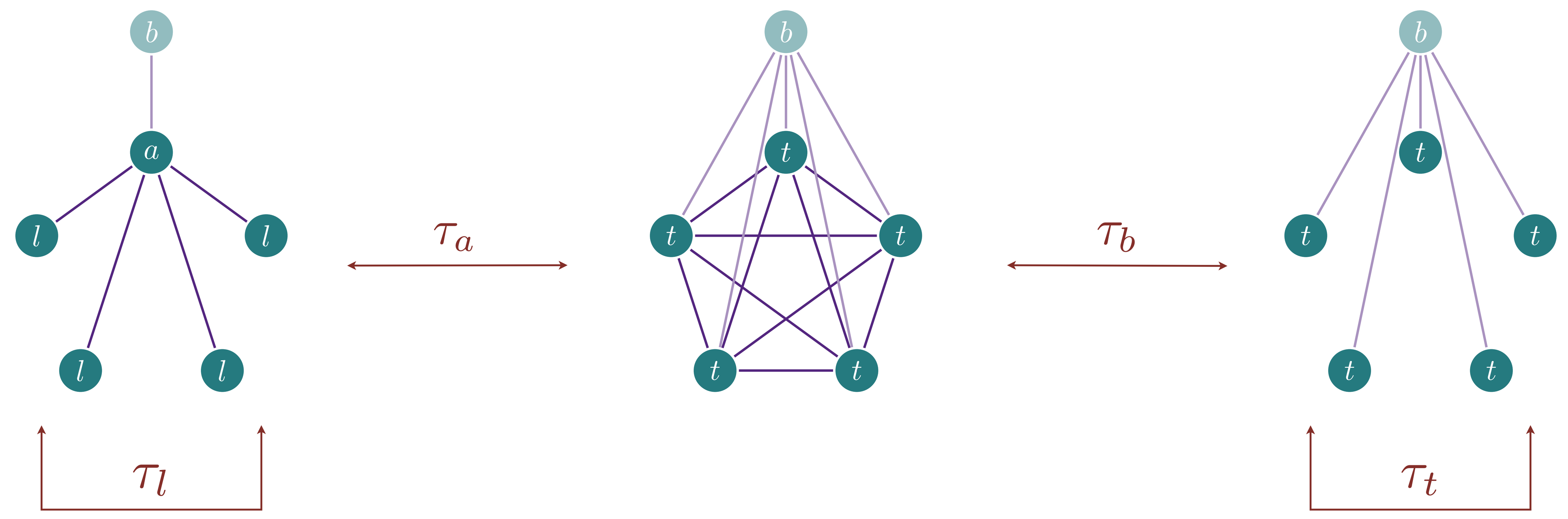


Local complementation $\tau_a(G)$

A graph $G = (V, E)$ and vertex $a \in V$ define a locally complemented graph $\tau_a(G)$ with adjacency matrix

$$\Gamma_{\tau_a(G)} := \Gamma_G + \Theta_a \mod 2$$

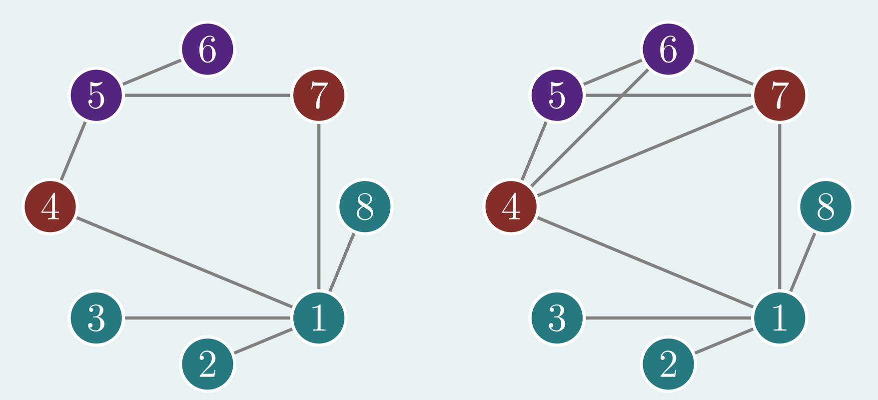
where Θ_a is the complete graph of the neighbourhood N_a.



The foliage relation

Consider a graph $G = (V, E)$ and its adjacency matrix $\Gamma_G = (\gamma_{ij})_{i,j \in V}$. We define an equivalence relation on the set of its vertices. Two vertices are related $v \sim w$ iff they are in the same connected component and for any other pair of vertices u_1, u_2, it holds that $\gamma_{v,u_1} \cdot \gamma_{w,u_2} = \gamma_{v,u_2} \cdot \gamma_{w,u_1}$.

Foliage partition example



The foliage partition $V_1, \dots, V_k \vdash V$

We define a partition of any graph's vertices based on the foliage relation and call it the graph's foliage partition. We demonstrate that these foliage partitions are invariant under local complementations of their graph, e.g. the foliage partition of both graphs to the left is $V_1, V_2, V_3 \vdash V$ with $V_1 = \{1, 2, 3, 8\}, V_2 = \{4, 7\}$ and $V_3 = \{5, 6\}$. As a result, foliage partitions represent simple local Clifford (LC)-invariants for graph states since there is a one-to-one relationship between LC-operations on a graph state and local complementations of its graph.

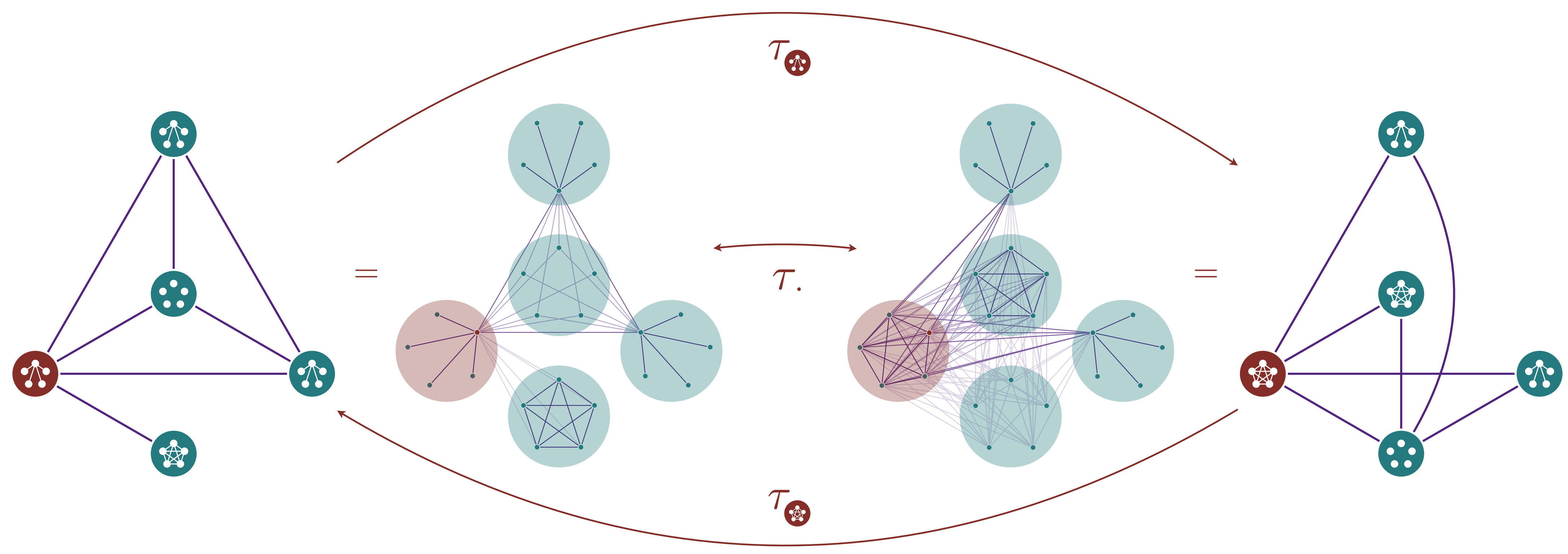
Lifted local complementation $\hat{\tau}_a(\hat{G})$

For any vertex $a \in V$, we define a lifted local complementation $\hat{\tau}_a(\hat{G}) := (\tau_{V_i}(\widetilde{G}), T', A')$, where $V_i \in \widetilde{V}$ such that $a \in V_i$, $\tau_{V_i}(\widetilde{G})$ is a local complementation of the foliage graph $\widetilde{G}$ with respect to its vertex $V_i \in \widetilde{V}$, and

$$T'(V_j) = \begin{cases} ✧ & \text{if } j = i \text{ and } T(V_i) = ✪, \\ ✪ & \text{if } j = i \text{ and } T(V_i) = ✧, \\ ⁙ & \text{if } \{V_i, V_j\} \in \widetilde{E} \text{ and } T(V_j) = ✪, \\ ✪ & \text{if } \{V_i, V_j\} \in \widetilde{E} \text{ and } T(V_j) = ⁙, \\ T(V_j) & \text{otherwise}, \end{cases}$$

where in the first case a is the axil and

$$A' = \begin{cases} A \cup \{a\} & \text{if } T(V_i) = ✪, \\ A \setminus \{a\} & \text{if } T(V_i) = ✧, \\ A & \text{otherwise}. \end{cases}$$



Type function T and axil set A

Let $A \subset V$ be the set of all axils of G. For every foliage partition $V_1, \dots, V_k \vdash V$ each V_i is of exactly one of the types ✧, ✪, ⁙, •, defining the type function $T : \{V_1, \dots, V_k\} \to \{✧, ✪, ⁙, •\}$.

Foliage graph $\widetilde{G}$ and foliage representation $\hat{G} = (\widetilde{G}, T, A)$

To every graph $G = (V, E)$ with foliage partition $V_1, \dots, V_k \vdash V$, we associate its foliage graph $\widetilde{G} = (\widetilde{V}, \widetilde{E})$ on vertices $\widetilde{V} = \{V_1, \dots, V_k\}$. Two vertices V_i, V_j are connected in $\widetilde{G}$ iff V_i, V_j are connected in G. The foliage representation is then a tuple $\hat{G} = (\widetilde{G}, T, A)$, where $\widetilde{G}$ is a foliage graph, and T and A are the corresponding type function and the axil set, respectively.

References

[1] Van den Nest, Dehaene & De Moor, Phys. Rev. A (2005).
[2] Hahn, Burchardt, in preparation (2023).

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