Limitations of nearest-neighbour quantum networks

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Abstract

Quantum communication research has shifted to include multi-partite networks for which questions of quantum network routing naturally emerge. To understand the potential for multi-partite routing, we focus on the most promising architectures for future quantum networks – those connecting nodes close to each other. Nearest-neighbour networks such as rings, lines, and grids, have been studied under different communication scenarios to facilitate the sharing of quantum resources, especially in the presence of bottlenecks. We analyze the potential of nearest-neighbour entangling gate quantum networks and identify some important limitations, by demonstrating that rings and lines cannot overcome bottleneck communication problems.[1]

Local complementation

A graph $G = (V, E)$ and vertex $v \in V$ define a locally complemented graph $\tau_v(G)$ with adjacency matrix

$$\Gamma_{\tau_v(G)} := \Gamma_G + \Theta_v \mod 2$$

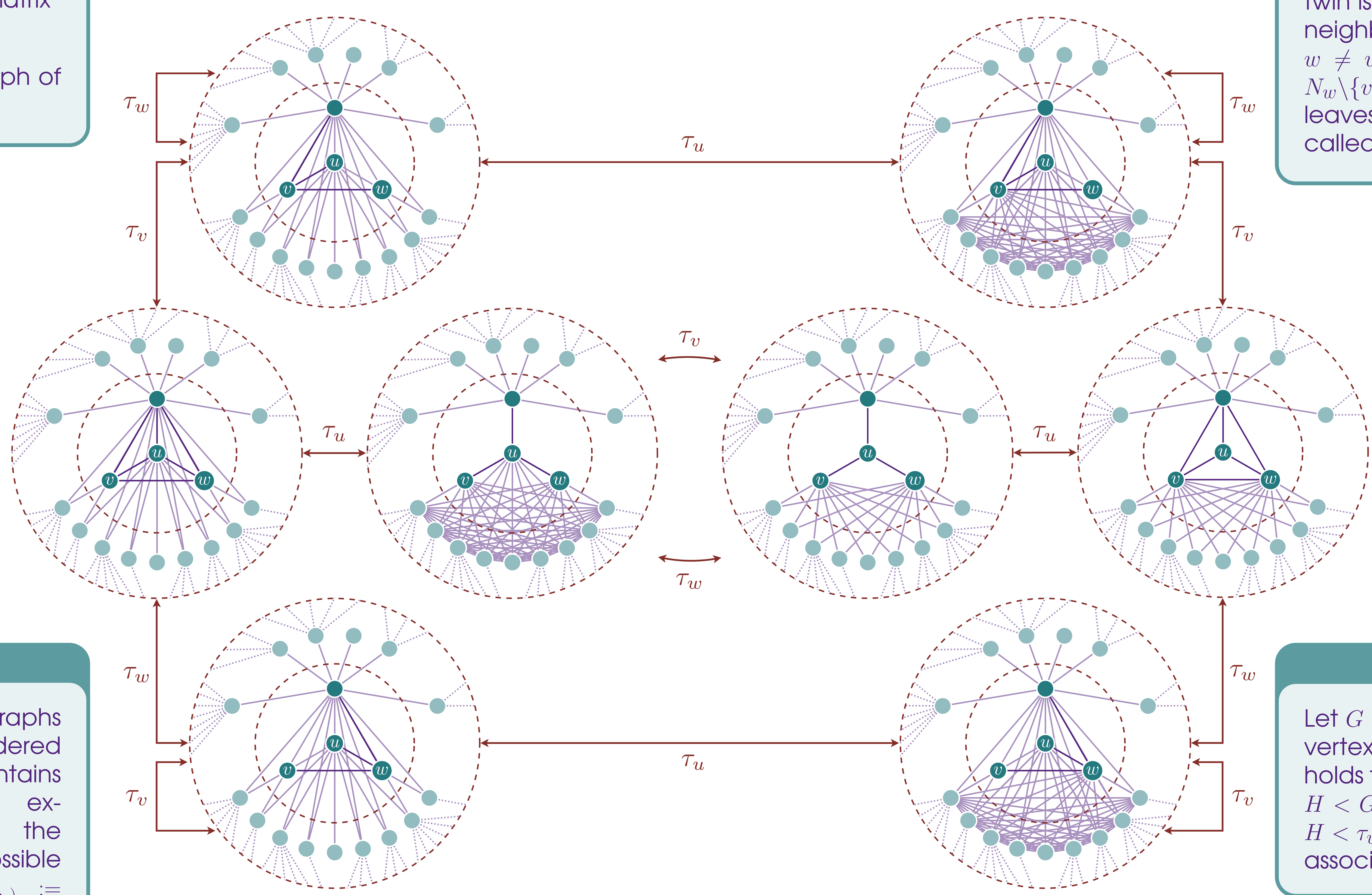
where Θ_v is the complete graph of the neighbourhood N_v.

Theorem

The foliage of a graph G is invariant under local complementation.[1]

Foliage = {leaves, axils, twins}

A leaf is a vertex with degree one and an axil its unique neighbour. A twin is a vertex v that has the same neighbourhood as a second vertex $w \neq v$ in the sense that $N_v \backslash \{w\} = N_w \backslash \{v\}$. The set containing all the leaves, axils and twins of a graph is called the foliage of that graph.

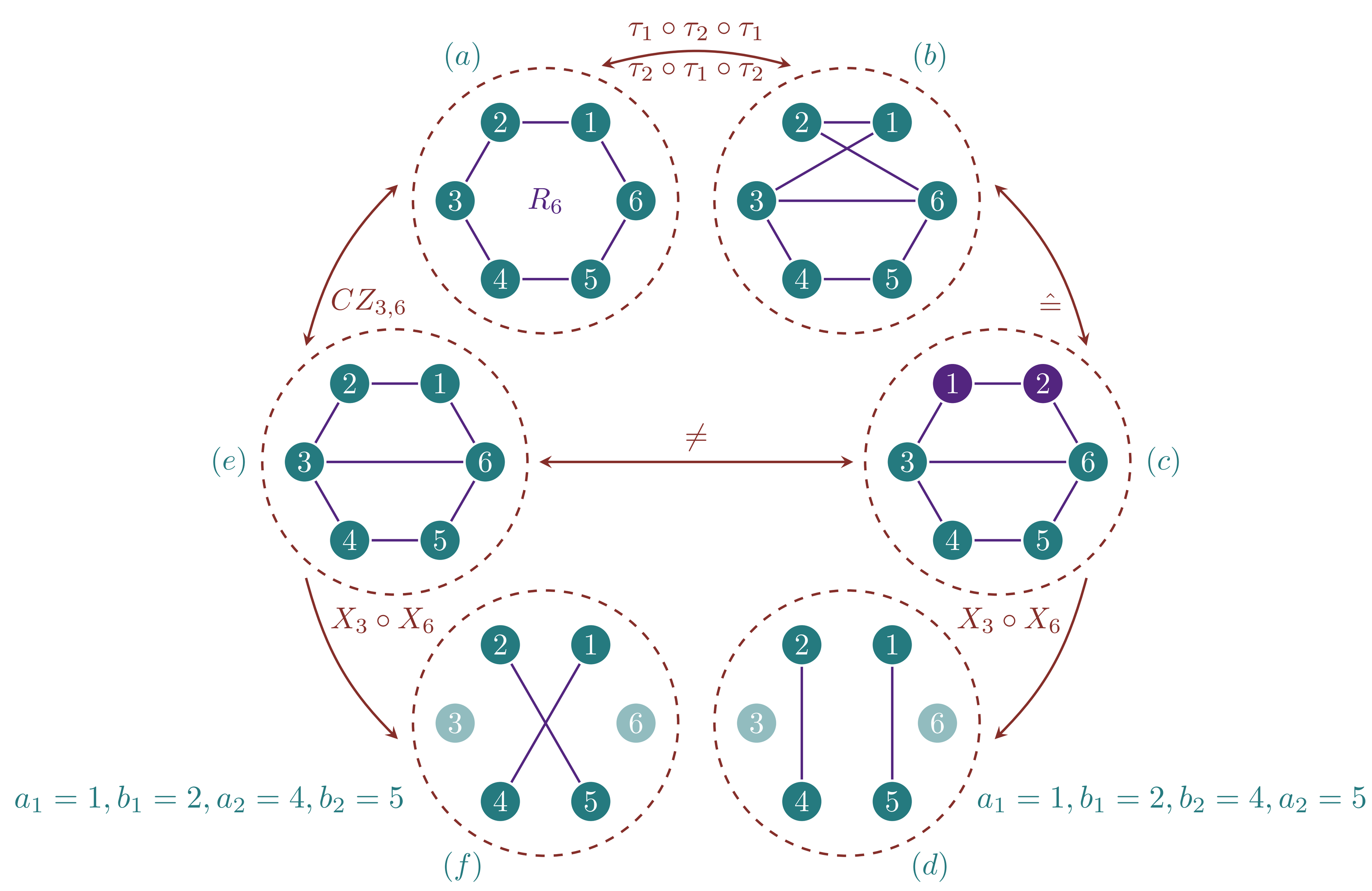


Lemma 1

Let G and H be two graphs and $(v_1, v_2, \ldots, v_k)$ be an ordered tuple of vertices that contains each element of $V_G \backslash V_H$ exactly once. We define the corresponding set of possible Pauli operations as $\mathcal{P}_{(v_1,v_2,\ldots,v_k)} := \{P_{v_k} \circ \cdots \circ P_{v_1} \mid P_v \in \{X_v, Y_v, Z_v\}\}$. Then H is a vertex-minor of G if and only if there exists an operation $P \in \mathcal{P}_{(v_1,v_2,\ldots,v_k)}$ such that H can be obtained from $P(G)$ via a sequence of local complementations.[2]

Lemma 2

Let G and H be graphs and v be a vertex in V_G but not in V_H. Then it holds that: (a) If v is a leaf: $H < G \Leftrightarrow H < G \backslash v$ (b) If v is an axil: $H < G \Leftrightarrow H < \tau_w \circ \tau_v(G) \backslash v$, where w is the leaf associated with v.[3]



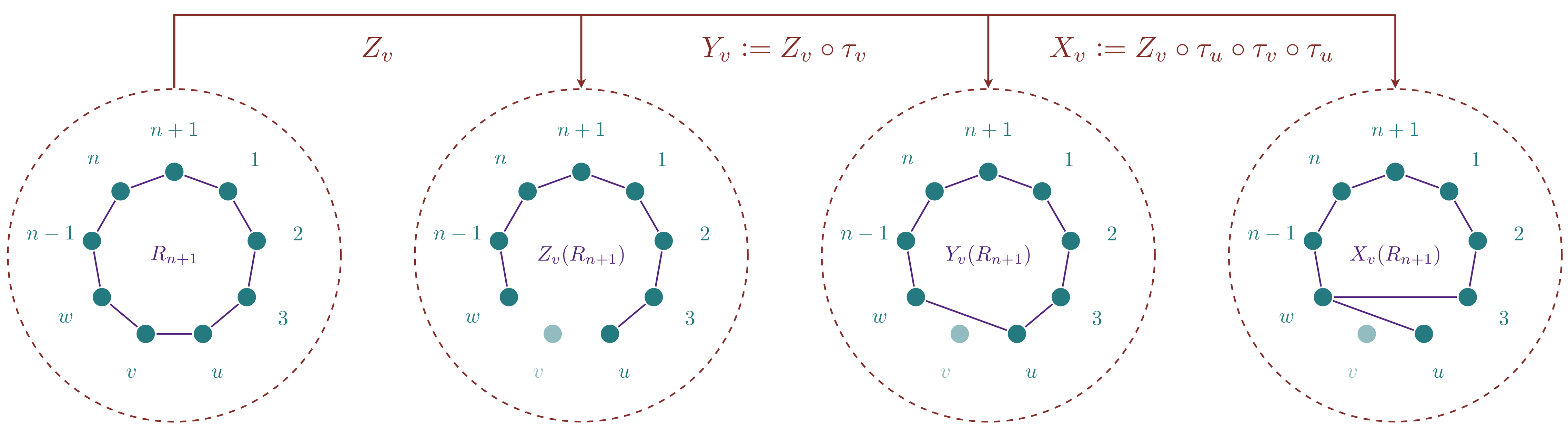
Theorem (No crossing on a ring)

It is not possible to extract two maximally entangled pairs from $|R_n\rangle$ if $a_1 = 1 < b_1 < a_2 < b_2$ for any $n \in \mathbb{N}$ with local Clifford operations, local Pauli measurements and classical communication.[1]

Theorem (No crossing on a line)

It is not possible to extract two maximally entangled pairs from $|L_n\rangle$ if $a_1 < b_1 < a_2 < b_2$ for any $n \in \mathbb{N}$ with local Clifford operations, local Pauli measurements and classical communication.[1]

Link to the paper



References

[1] F. Hahn, A. Dahlberg, J. Eisert and A. Pappa, "Limitations of nearest-neighbour quantum networks", PRA (2022).
[2] A. Dahlberg, J. Helsen, and S. Wehner, Quant. Sc. Tech. 5, 045016 (2020).
[3] A. Dahlberg and S. Wehner, Phil. Trans. Roy. Soc. A 376, 20170325 (2018)

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