Project 3 - Linear Programming

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Problem 1: Transshipment Model

PART A

Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

Note on variable names:

xijk: Number of refrigerator units shipped along a route defined by plant i, warehouse j and retailer k. i={1, 2, 3, 4}; j={1, 2, 3}; k={1, 2, 3, 4, 5, 6, 7}. Routes only involve 2 locations (either plant to warehouse or warehouse to retailer), so all route identifiers include a 0 in either the first or last position. For instance, the number of refrigerators shipped from plant 1 to warehouse 2 is x120 and the number of refrigerators shipped from warehouse 2 to retailer 3 is x023.

Goal: to minimize the total shipping cost = 10 x110 + 15 x120 + 11 x210 + 8 x220 + 13 x310 + 8 x320 + 9 x330 + 14 x420 + 8 x430 + 5 x011 + 6 x012 + 7 x013 + 10 x014 + 12 x023 + 8 x024 + 10 x025 + 14 x026 + 14 x034 + 12 x035 + 12 x036 + 6 x037

Constraints (“Subject To”):

The number of refrigerators shipped from a plant cannot exceed the plant’s production capacity.

x110 + x120 <= 150	(p1)
x210 + x220 <= 450	(p2)
x310 + x320 + x330 <= 250	(p3)
x420 + x430 <= 150	(p4)

The number of refrigerators received by a retailer cannot be less that the demand.

x011 >= 100

x012 >= 150

x013 + x023 > =100

x014 + x024 + x034 >= 200

x025 + x035 >= 200

x026 + x036 >= 150

x037 >= 100

Each warehouse ships as many or fewer refrigerators as it receives.

x011 + x012 + x013 + x014 <= x110 + x210 + x310

x023 + x024 + x025 + x026 <= x120 + x220 + x320 + x420

x034 + x035 + x036 + x037 <= x330 + x430

(The previous two groups of inequalities imply -- and therefore this is not included in the code -- that the number of refrigerators shipped across all plants cannot be less than the demand.

x110 + x120 + x210 + x220 + x310 + x320 + x330 + x420 + x430 >= 1000

)

Nonnegativity

Xijk>=0 for all i, j, k

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Software: LINDO

Code:

```
MIN 10 x110 + 15 x120 + 11 x210 + 8 x220 + 13 x310 + 8 x320 + 9 x330 + 14 x420 + 8 x430 + 5 x011 + 6 x012 + 7
x013 + 10 x014 + 12 x023 + 8 x024 + 10 x025 + 14 x026 + 14 x034 + 12 x035 + 12 x036 + 6 x037
ST
    x110 + x120 < 150
    x210 + x220 < 450
    x310 + x320 + x330 < 250
    x420 + x430 < 150
    x110 + x210 + x310 - x011 - x012 - x013 - x014 > 0
        x120 + x220 + x320 + x420 - x023 - x024 - x025 - x026 > 0
        x330 + x430 - x034 - x035 - x036 - x037 > 0
    x011 > 100
```

```
        x012 > 150
        x013 + x023 > 100
        x014 + x024 + x034 > 200
        x025 + x035 > 200
        x026 + x036 > 150
        x037 > 100
        x110 > 0
        x120 > 0
        x210 > 0
        x220 > 0
        x310 > 0
        x320 > 0
        x330 > 0
        x420 > 0
        x430 > 0
        x011 > 0
        x012 > 0
        x013 > 0
        x014 > 0
        x023 > 0
        x024 > 0
        x025 > 0
        x026 > 0
        x034 > 0
        x035 > 0
        x036 > 0
        x037 > 0
END
```

Results:

```
 LP OPTIMUM FOUND AT STEP     13

     OBJECTIVE FUNCTION VALUE

     1)      17100.00

 VARIABLE        VALUE          REDUCED COST
    X110       150.000000         0.000000
    X120         0.000000         8.000000
    X210       200.000000         0.000000
    X220       250.000000         0.000000
    X310         0.000000         2.000000
    X320       150.000000         0.000000
    X330       100.000000         0.000000
    X420         0.000000         7.000000
    X430       150.000000         0.000000
    X011       100.000000         0.000000
```

X012	150.000000	0.000000
X013	100.000000	0.000000
X014	0.000000	5.000000
X023	0.000000	2.000000
X024	200.000000	0.000000
X025	200.000000	0.000000
X026	0.000000	1.000000
X034	0.000000	7.000000
X035	0.000000	3.000000
X036	150.000000	0.000000
X037	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-11.000000
7)	0.000000	-8.000000
8)	0.000000	-9.000000
9)	0.000000	-16.000000
10)	0.000000	-17.000000
11)	0.000000	-18.000000
12)	0.000000	-16.000000
13)	0.000000	-18.000000
14)	0.000000	-21.000000
15)	0.000000	-15.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000

NO. ITERATIONS= 13

iii. What are the optimal shipping routes and minimum cost?

Optimal shipping routes:
From plant to warehouse:

Route	Variable in Lindo	Number of Refrigerators
P1 --> W1	X110	150
P2 --> W1	X210	200
P2 --> W2	X220	250
P3 --> W2	X320	150
P3 --> W3	X330	100
P4 --> W3	X430	150
Total number of refrigerators shipped to warehouses		1000

From warehouse to retailer:

Route	Variable in Lindo	Number of Refrigerators
W1 à R1	X011	100
W1 à R2	X012	150
W1 à R3	X013	100
W2 à R4	X024	200
W2 à R5	X025	200
W3 à R6	X036	150
W3 à R7	X037	100
Total number of refrigerators shipped to retailers		1000

Minimum cost:
17100.00

PART B

Assume that W2 is not available.

Constraints: The following constraints are added to the problem formulation in Part A.

x120 = 0

x220 = 0

x320 = 0

x420 = 0

x023 = 0

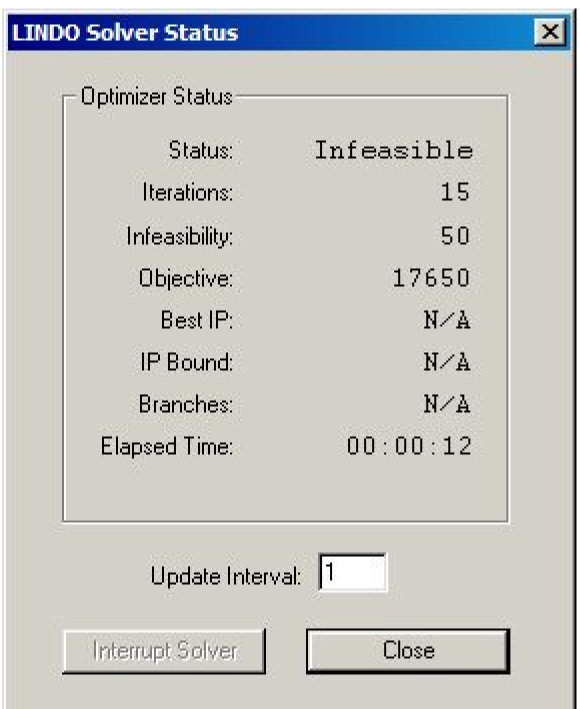
x024 = 0

x025 = 0

x026 = 0

Results and Interpretation:

Lindo's output reads as follows.



No solution was found. It is not feasible to ship all required units through warehouses 1 and 3 alone. One reason is that warehouse 3 is now the sole direct source of refrigerators for retailers 5, 6 and 7. These 3 dedicated retailers require a total of 450 refrigerators. Warehouse 3 receives at most 400: all of plat 4's output and (assuming no refrigerators are shipped to warehouse 1) all of plant 3's output. Therefore a sufficient number of refrigerators can never be sent to R5, R6, R7 combined.

PART C

Assume that W2 now has a limited capacity to store 100 refrigerators per week.

<u>Constraints</u>: The following constraints are added to the problem formulation in Part A. (The constraints from Part B are not used.)

x120 + x220 + x320 + x420 <= 100

x023 + x024 + x025 + x026 <= 100

<u>Results:</u>

```
LP OPTIMUM FOUND AT STEP     15

      OBJECTIVE FUNCTION VALUE
      1)      18300.00

 VARIABLE        VALUE          REDUCED COST
     X110      150.000000         0.000000
     X120        0.000000         8.000000
     X210      350.000000         0.000000
     X220      100.000000         0.000000
     X310        0.000000         4.000000
     X320        0.000000         2.000000
     X330      250.000000         0.000000
     X420        0.000000         9.000000
     X430      150.000000         0.000000
     X011      100.000000         0.000000
     X012      150.000000         0.000000
     X013      100.000000         0.000000
     X014      150.000000         0.000000
     X023        0.000000         7.000000
     X024       50.000000         0.000000
     X025       50.000000         0.000000
     X026        0.000000         4.000000
     X034        0.000000         4.000000
     X035      150.000000         0.000000
     X036      150.000000         0.000000
     X037      100.000000         0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)        0.000000          1.000000
       3)        0.000000          0.000000
       4)        0.000000          2.000000
       5)        0.000000          3.000000
       6)        0.000000        -11.000000
```

7)	0.000000	-8.000000
8)	0.000000	-11.000000
9)	0.000000	-16.000000
10)	0.000000	-17.000000
11)	0.000000	-18.000000
12)	0.000000	-21.000000
13)	0.000000	-23.000000
14)	0.000000	-23.000000
15)	0.000000	-17.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	350.000000	0.000000
19)	100.000000	0.000000
20)	0.000000	0.000000
21)	0.000000	0.000000
22)	250.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	150.000000	0.000000
29)	0.000000	0.000000
30)	50.000000	0.000000
31)	50.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	150.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000
37)	0.000000	0.000000
38)	0.000000	5.000000

NO. ITERATIONS= 15

Interpretation:

A solution was found. The optimal total weekly cost of shipping is now 18300.00. This is 1200.00 (or 7%) more than the 17100.00 cost found in Part A. The capacity of Warehouse 2 therefore affects shipping costs. Management may want to contrast the cost reduction associated with a higher capacity warehouse 2 to the cost of upgrading the warehouse to reach this capacity. The minimal cost solution is shown in the tables below:

From plant to warehouse:

Route	Variable in Lindo	Number of Refrigerators
P1 à W1	x110	150
P2 à W1	x210	350
P2 à W2	X220	100
P3 à W3	x330	250
P4 à W3	X430	150
Total number of refrigerators shipped to warehouses		1000

From warehouse to retailer:

Route	Variable in Lindo	Number of Refrigerators
W1 à R1	X011	100
W1 à R2	X012	150
W1 à R3	X013	100
W1 à R4	X014	150
W2 à R4	X024	50
W2 à R5	X025	50
W3 à R5	X035	150
W3 à R6	X036	150
W3 à R7	X037	100
Total number of refrigerators shipped to retailers		1000

PART D

The goal is to find numbers x_{ij} and x_{jk} of amounts to ship from Plant P_i to Warehouse W_j and from Warehouse W_j to Retailer R_k to minimize the total transportation cost,

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}c_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} x_{jk}c_{jk} \qquad (1)$$

subject to the nonnegativity constraints, $x_{ij} \geq 0$ for all *i* and *j* and $x_{jk} \geq 0$ for all *j* and *k*,

the supply constraints,

$$\sum_{j=1}^{J} x_{ij} \leq s_i \quad for\ i = 1, ..., I \qquad (2)$$

the demand constraints,

$$\sum_{j=1}^{J} x_{jk} \geq r_k \quad for\ k = 1, ..., K \qquad (3)$$

the input-output congruence constraints (i.e. warehouses cannot ship more than they receive),

$$\sum_{i=1}^{I} x_{ij} \geq \sum_{k=1}^{K} x_{jk} \quad for\ j = 1, ..., J \qquad (4)$$

the warehousing capacity constraints,

$$\sum_{i=1}^{I} x_{ij} \leq w_j \quad for\ j = 1, ..., J \qquad (2)$$

and the path constraints,

$$\exists \left|E_{i\to j}\right| \quad for\ i = 1, ..., I\ and\ for\ j = 1, ..., J \qquad (6)$$

$$\exists \left|E_{j\to k}\right| \quad for\ j = 1, ..., J\ and\ for\ k = 1, ..., K \qquad (7)$$

where s_i, r_k, w_j, c_{ij} and c_{jk} are given nonnegative numbers representing supply, demand, warehouse capacity, and to- and from- warehouse shipping costs respectively.

Problem 2: A mixture problem

PART A

Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

Note on variable names:
Each ingredient in our salad combinations has been assigned a number as though the salad ingredients comprise an array of 8 variables. The optimal solution will consist of an optimal combination or subsequence of this array: { t=Tomato, l=Lettuce, s=Spinach, c=Carrot, ss=Sunflower Seeds, st=Smoked Tofu, cp=Chickpeas, o=Oil }. Each constraint or food restriction has been assigned a variable as well: P is Protein, Fat is F, Carbohydrates is C, Sodium is S, and Leafy Greens is LG.

Goal: *to "minimize" calories* but still meet nutritional requirements

Constraints ("Subject To"):
A salad at most can contain no more than 8 ingredients but at least 2 of the following ingredients (to form a combination), with each ingredient appearing only once in each combination: Tomato=t, Lettuce=l, Spinach=s, Carrot=c, Sunflower Seeds=ss, Smoked Tofu=st, Chickpeas=cp, and Oil=o.

Each salad (each combination of ingredients) must meet these food restrictions as a result of the optimal combination: 15 grams of protein (P >= 15.0g), between 2 and 8 grams of fat (2.0g <= F <= 8.0g), at least 4 grams of carbohydrates (C >= 4.0g), at most 200 milligrams of sodium (S <= .2g), at least 40% leafy greens by mass (LG = (s+l >= .4) of total salad weight). Figure 1 lists the caloric value (kcal) of each ingredient as well as the percentage of grams for each of these food restrictions.

Therefore, the constraints are:

P >= 15.0g
2.0g <= F <= 8.0g
C >= 4.0g
S <= .2g
LG >= 40% leafy greens by mass
In an optimal combination of at least 2 but no more than 8 ingredients with each variable only appearing once of: l, s, c, st, ss, cp, o.

Figure 1

The nutritional contents of these ingredients (per 100 grams) and cost are

Ingredient	Energy (kcal)	Protein (grams)	Fat (grams)	Carbohydrate (grams)	Sodium (mg)	Cost (100g)
Tomato	21	0.85	0.33	4.64	9.00	$1.00
Lettuce	16	1.62	0.20	2.37	28.00	$0.75
Spinach	40	2.86	0.39	3.63	65.00	$0.50
Carrot	41	0.93	0.24	9.58	69.00	$0.50
Sunflower Seeds	585	23.4	48.7	15.00	3.80	$0.45
Smoked Tofu	120	16.00	5.00	3.00	120.00	$2.15
Chickpeas	164	9.00	2.6	27.0	78.00	$0.95
Oil	884	0	100.00	0	0	$2.00

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Software: LINDO

Code: (to minimize calories)

```
MIN 21t + 16l + 40s + 41c + 585ss + 120st + 164cp + 884o
ST
0.85t + 1.62l + 2.86s + 0.93c + 23.4ss + 16st + 9cp >= 15
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o >= 2
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o <= 8
4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp >= 4
9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp <= 200
0.4l + 0.4s - 0.6t - 0.6c - 0.6ss - 0.6st - 0.6cp - 0.6o >= 0
t >= 0
l >= 0
s >= 0
c >= 0
ss >= 0
st >= 0
cp >= 0
o >= 0
END
```

Results:

```
LP OPTIMUM FOUND AT STEP      3

        OBJECTIVE FUNCTION VALUE
```

1) 117.2002

VARIABLE	VALUE	REDUCED COST
T	0.000000	19.372219
L	1.220836	0.000000
S	0.000000	14.311449
C	0.000000	38.747150
SS	0.000000	407.181244
ST	0.813890	0.000000
CP	0.000000	98.693436
O	0.000000	889.013550

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-7.813348
3)	2.313619	0.000000
4)	3.686381	0.000000
5)	1.335052	0.000000
6)	68.149757	0.000000
7)	0.000000	-8.355942
8)	0.000000	0.000000
9)	1.220836	0.000000
10)	0.000000	0.000000
11)	0.000000	0.000000
12)	0.000000	0.000000
13)	0.813890	0.000000
14)	0.000000	0.000000
15)	0.000000	0.000000

NO. ITERATIONS= 3

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
T	21.000000	INFINITY	19.372219
L	16.000000	12.999507	3.850000
S	40.000000	INFINITY	14.311449
C	41.000000	INFINITY	38.747150
SS	585.000000	INFINITY	407.181244
ST	120.000000	38.024693	144.000000
CP	164.000000	INFINITY	98.693436
O	884.000000	INFINITY	889.013550

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	15.000000	7.753087	3.753623
3	2.000000	2.313619	INFINITY
4	8.000000	INFINITY	3.686381
5	4.000000	1.335052	INFINITY
6	200.000000	INFINITY	68.149757
7	0.000000	1.981073	0.297701
8	0.000000	0.000000	INFINITY
9	0.000000	1.220836	INFINITY
10	0.000000	0.000000	INFINITY
11	0.000000	0.000000	INFINITY
12	0.000000	0.000000	INFINITY
13	0.000000	0.813890	INFINITY
14	0.000000	0.000000	INFINITY
15	0.000000	0.000000	INFINITY

iii. What is the cost of the low calorie salad?

"Low Calorie Salad" Solution:

Ingredient	Calories (kcal)	Cost ($)	Meeting Nutritional Requirements
1. Lettuce (1.220836)	20 kcal	$0.92	1.98g protein 0.24g fat 2.89g carbohydrates 0.034g sodium or 34 mg
2. Smoked Tofu (0.813890)	98 kcal	$1.75	13.02g protein 4.07g fat 2.44g carbohydrates 0.098g sodium or 98 mg
Optimal Combination of Ingredients	Total Calories **(Minimized)**	**Total Cost**	Requirements Met with at least 40% Leafy Greens by Mass
Lettuce and Smoked Tofu Salad	118 kcal	**$2.67**	Exactly 15g of protein 4.31g of fat (between 2g and 8g) 5.33g of carbohydrates (above 4g) 132 mg of sodium (less than 200 mg)

PART B

Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

Note on variable names:
Each ingredient in our salad combinations has been assigned a number as though the salad ingredients comprise an array of 8 variables. The optimal solution will consist of an optimal combination or subsequence of this array: { t=Tomato, l=Lettuce, s=Spinach, c=Carrot, ss=Sunflower Seeds, st=Smoked Tofu, cp=Chickpeas, o=Oil }. Each constraint or food restriction has been assigned a variable as well: P is Protein, Fat is F, Carbohydrates is C, Sodium is S, and Leafy Greens is LG.

Goal: *to "minimize" cost* of low calorie while still meeting nutritional requirements

Constraints ("Subject To"):
A salad at most can contain no more than 8 ingredients but at least 2 of the following ingredients (to form a combination), with each ingredient appearing only once in each combination: Tomato=t, Lettuce=l, Spinach=s, Carrot=c, Sunflower Seeds=ss, Smoked Tofu=st, Chickpeas=cp, and Oil=o.

Each salad (each combination of ingredients) must meet these food restrictions as a result of the optimal combination: 15 grams of protein (P >= 15.0g), between 2 and 8 grams of fat (2.0g <= F <= 8.0g), at least 4 grams of carbohydrates (C >= 4.0g), at most 200 milligrams of sodium (S <= .2g), at least 40% leafy greens by mass (LG = (s+l >= .4) of total salad weight). Figure 1 lists the caloric value (kcal) of each ingredient as well as the percentage of grams for each of these food restrictions.

Therefore, the constraints are:

P >= 15.0g
2.0g <= F <= 8.0g
C >= 4.0g
S <= .2g
LG >= 40% leafy greens by mass
In an optimal combination of at least 2 but no more than 8 ingredients with each variable only appearing once of: l, s, c, st, ss, cp, o.

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Software: LINDO

Code: (to minimize cost)

```
MIN 1t + 0.75l + 0.5s + 0.5c + 0.45ss + 2.15st + .95cp + 2o
ST
0.85t + 1.62l + 2.86s + 0.93c + 23.4ss + 16st + 9cp >= 15
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o >= 2
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o <= 8
4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp >= 4
9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp <= 200
0.4l + 0.4s - 0.6t - 0.6c - 0.6ss - 0.6st - 0.6cp - 0.6o >= 0
t >= 0
l >= 0
s >= 0
c >= 0
ss >= 0
st >= 0
cp >= 0
o >= 0
END
```

Results:

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      1.676833

 VARIABLE        VALUE          REDUCED COST
        T        0.000000         1.075116
        L        0.000000         0.414501
        S        1.525128         0.000000
        C        0.000000         0.558887
       SS        0.103289         0.000000
       ST        0.000000         0.344260
       CP        0.913462         0.000000
        O        0.000000         7.668215

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)        0.000000        -0.141076
       3)        6.000000         0.000000
```

4)	0.000000	0.054913
5)	27.749039	0.000000
6)	29.224140	0.000000
7)	0.000000	-0.294848
8)	0.000000	0.000000
9)	0.000000	0.000000
10)	1.525128	0.000000
11)	0.000000	0.000000
12)	0.103289	0.000000
13)	0.000000	0.000000
14)	0.913462	0.000000
15)	0.000000	0.000000

NO. ITERATIONS= 0

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
T	1.000000	INFINITY	1.075116
L	0.750000	INFINITY	0.414501
S	0.500000	0.451467	0.177101
C	0.500000	INFINITY	0.558887
SS	0.450000	2.341986	17.027126
ST	2.150000	INFINITY	0.344261
CP	0.950000	0.230827	1.124052
O	2.000000	INFINITY	7.668215

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	15.000000	1.989268	10.253563
3	2.000000	6.000000	INFINITY
4	8.000000	18.698267	4.405192
5	4.000000	27.749039	INFINITY
6	200.000000	INFINITY	29.224140
7	0.000000	0.460197	0.916071
8	0.000000	0.000000	INFINITY
9	0.000000	0.000000	INFINITY
10	0.000000	1.525128	INFINITY
11	0.000000	0.000000	INFINITY
12	0.000000	0.103289	INFINITY
13	0.000000	0.000000	INFINITY
14	0.000000	0.913462	INFINITY

15 0.000000 0.000000 INFINITY

iii. How many calories are in the low cost salad?

"Low Cost Salad" Solution:

Ingredient	Calories (kcal)	Cost ($)	Meeting Nutritional Requirements
1. Spinach (1.525128)	61 kcal	$0.76	4.36g protein 0.60g fat 5.54g carbohydrates 0.099g sodium or 99 mg
2. Sunflower Seeds (0.103289)	60 kcal	$0.05	2.42g protein 5.02g fat 1.55g carbohydrates less than 1 mg sodium
3. Chickpeas (0.913462)	150 kcal	$0.87	8.22g protein 2.38g fat 24.66g carbohydrates 0.071g sodium or 71 mg
Optimal Combination of Ingredients	**Total Calories (total kcal)**	Total Cost **(Minimized)**	Requirements Met with at least 40% Leafy Greens by Mass
Spinach, Sunflower Seeds, and Chickpeas Salad	**271 kcal**	$1.68	Exactly 15g of protein 8g of fat (between 2g and 8g) 31.75g of carbohydrates (above 4g) 171 mg of sodium (less than 200 mg)

PART C

Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.

i. Suggest some possible ways that she select a combination of ingredients that is "near optimal" for both objectives. This is a type of multi-objective optimization.

Advertizing that the salad has under 250 calories will sell more salads. So the optimal solution would restrict calories for a 250 calorie maximum but also minimize the cost of ingredients.

<u>Software</u>: LINDO

<u>Code:</u> (to minimize cost with a cap of 250 calories)

```
MIN 1t + 0.75l + 0.5s + 0.5c + 0.45ss + 2.15st + .95cp + 2o
ST
21t + 16l + 40s + 41c + 585ss + 120st + 164cp + 884o <= 250
0.85t + 1.62l + 2.86s + 0.93c + 23.4ss + 16st + 9cp >= 15
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o >= 2
0.33t + 0.20l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o <= 8
4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp >= 4
9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp <= 200
0.4l + 0.4s - 0.6t - 0.6c - 0.6ss - 0.6st - 0.6cp - 0.6o >= 0
t >= 0
l >= 0
s >= 0
c >= 0
ss >= 0
st >= 0
cp >= 0
o >= 0
END
```

<u>Results:</u>

```
LP OPTIMUM FOUND AT STEP     10

       OBJECTIVE FUNCTION VALUE

       1)      1.721142

  VARIABLE        VALUE          REDUCED COST
         T        0.000000          1.066125
         L        0.000000          0.407098
         S        1.426983          0.000000
         C        0.000000          0.589763
        SS        0.101109          0.000000
        ST        0.128708          0.000000
        CP        0.721505          0.000000
         O        0.000000          8.618567

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        0.000000          0.002086
        3)        0.000000         -0.174151
```

4)	6.000000	0.000000
5)	0.000000	0.046191
6)	22.563337	0.000000
7)	35.139565	0.000000
8)	0.000000	-0.258494
9)	0.000000	0.000000
10)	0.000000	0.000000
11)	1.426983	0.000000
12)	0.000000	0.000000
13)	0.101109	0.000000
14)	0.128708	0.000000
15)	0.721505	0.000000
16)	0.000000	0.000000

NO. ITERATIONS= 10

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
T	1.000000	INFINITY	1.066125
L	0.750000	INFINITY	0.407098
S	0.500000	0.451467	0.148102
C	0.500000	INFINITY	0.589763
SS	0.450000	2.006732	13.530115
ST	2.150000	1.823182	0.344260
CP	0.950000	0.230827	0.533147
O	2.000000	INFINITY	8.618567

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	250.000000	21.237278	79.823807
3	15.000000	3.419831	1.339644
4	2.000000	6.000000	INFINITY
5	8.000000	8.327758	4.392608
6	4.000000	22.563337	INFINITY
7	200.000000	INFINITY	35.139565
8	0.000000	0.514060	0.817576
9	0.000000	0.000000	INFINITY
10	0.000000	0.000000	INFINITY
11	0.000000	1.426983	INFINITY
12	0.000000	0.000000	INFINITY
13	0.000000	0.101109	INFINITY

14	0.000000	0.128708	INFINITY
15	0.000000	0.721505	INFINITY
16	0.000000	0.000000	INFINITY

ii. What combination of ingredient would you suggest and what is the associated cost and calorie?

<u>"250 Calorie Low Cost Salad" Solution:</u>

Ingredient	Calories (kcal)	Cost ($)	Meeting Nutritional Requirements
1. Spinach (1.426983)	57 kcal	$0.71	4.08g protein 0.56g fat 5.18g carbohydrates 0.093g sodium or 93 mg
2. Sunflower Seeds (0.101109)	59 kcal	$0.05	2.37g protein 4.92g fat 1.52g carbohydrates less than 1 mg sodium
3. Smoked Tofu (0.128708)	15 kcal	$0.28	2.06g protein 0.64g fat 0.39g carbohydrates 0.015g sodium or 15 mg
4. Chickpeas (0.721505)	118 kcal	$0.69	6.49g protein 1.88g fat 19.48g carbohydrates .056g sodium or 56 mg
Optimal Combination of Ingredients	**Total Calories (250 kcal or less)**	**Total Cost (Minimized)**	Requirements Met with at least 40% Leafy Greens by Mass
Spinach, Sunflower Seeds, Smoked Tofu, and Chickpeas Salad	**~249 kcal**	**$1.73**	Exactly 15g of protein 8g of fat (between 2g and 8g) 26.57g of carbohydrates (above 4g) 165 mg of sodium (less than 200 mg)

iii. Note: There is not one “right” answer. Discuss how you derived your solution.

By putting a cap on calories at 250, she will be able to sell more salads. So using an objective function which minimizes cost but keeps the calories at a cap of 250 calories, we are able to find the lowest cost ingredients which still meet her nutritional requirements that she can advertise as “250 calories or less.”

Problem 3: Solving shortest path problems using linear regression

a) What are the lengths of the shortest paths from vertex a to all other vertices?

Note on vertex names: All vertices (a, b, c, … , m) have been renamed to use numbers (1, 2, 3, … 13).

Software: MATLAB

Code:
DG = sparse([1 1 1 1 2 2 2 2 3 3 3 3 3 4 4 4 4 5 5 5 6 6 7 7 7 8 8 9 9 9 9 10 10 10 11 11 12],[2 3 4 8 1 3 5 6 4 3 7 9 6 1 7 10 6 8 3 9 9 7 4 10 11 9 11 1 11 10 13 9 11 12 8 13 13],[2 3 8 9 4 5 7 4 10 5 9 11 4 8 2 5 1 5 4 10 2 2 2 8 12 5 10 20 6 2 12 2 4 5 10 10 2],37,37)

```
DG =

   (2,1)         4
   (4,1)         8
   (9,1)        20
   (1,2)         2
   (1,3)         3
   (2,3)         5
   (3,3)         5
   (5,3)         4
   (1,4)         8
   (3,4)        10
   (7,4)         2
   (2,5)         7
   (2,6)         4
   (3,6)         4
   (4,6)         1
   (3,7)         9
   (4,7)         2
   (6,7)         2
   (1,8)         9
   (5,8)         5
  (11,8)        10
   (3,9)        11
   (5,9)        10
   (6,9)         2
   (8,9)         5
  (10,9)         2
   (4,10)        5
   (7,10)        8
   (9,10)        2
   (7,11)       12
   (8,11)       10
   (9,11)        6
  (10,11)        4
  (10,12)        5
   (9,13)       12
  (11,13)       10
  (12,13)        2
```

Getting shortest distances between two vertices (from a to b):
[dist,path] = graphshortestpath(DG,1,2)

Solution:

destination vertex	**distance**	**shortest path from source vertex a**
a	0	{ a }
b	2	{ a, b }
c	3	{ a, c }
d	8	{ a, d }
e	9	{ a, b, e }
f	6	{ a, b, f }
g	8	{ a, b, f, g }
h	9	{ a, h }
i	8	{ a, b, f, i }
j	10	{ a, b, f, i, j }
k	14	{ a, b, f, i, k }
l	15	{ a, b, f, i, j, l }
m	17	{ a, b, f, i, j, l, m }

b) If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a?

There would be two possible results: 1) The length would reach infinity as the path continues on an endless loop, or 2) There would be some maximum length which would eventually occur as the path reaches a dead-end.

c) What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

<u>Solution:</u>

source vertex	**distance**	**shortest path to destination vertex m**
a	17	{ a, b, f, i, j, l, m }
b	15	{ b, f, i, j, l, m }
c	15	{ c, 6, i, j, l, m }
d	12	{ d, j, l, m }
e	19	{ e, i, j, l, m }
f	11	{ f, i, j, l, m }
g	14	{ g, d, j, l, m }
h	14	{ h, i, j, l, m }
i	9	{ i, j, l, m }
j	7	{ j, l, m }
k	10	{ k, m }
l	2	{ l, m }
m	0	{ m }

d) Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

You can calculate the length from the source vertex to vertex i, then add this value to the length from i to the destination vertex.

source vertex	**destination vertex**	**distance (n/a if not possible)**
a	a	28

a	b	30
a	c	31
a	d	36
a	e	37
a	f	34
a	g	36
a	h	24
a	i	8
a	j	10
a	k	14
a	l	15
a	m	17
b	a	26
b	b	0
b	c	29
b	d	34
b	e	35
b	f	32
b	g	34
b	h	22
b	i	6
b	j	8
b	k	12
b	l	13
b	m	15
c	a	26

c	b	28
c	c	0
c	d	34
c	e	35
c	f	32
c	g	34
c	h	22
c	i	6
c	j	8
c	k	12
c	l	13
c	m	15
d	a	23
d	b	25
d	c	26
d	d	0
d	e	32
d	f	29
d	g	31
d	h	19
d	i	3
d	j	5
d	k	9
d	l	10
d	m	12
e	a	30

e	b	32
e	c	33
e	d	38
e	e	0
e	f	36
e	g	38
e	h	26
e	i	10
e	j	12
e	k	16
e	l	17
e	m	19
f	a	22
f	b	24
f	c	25
f	d	30
f	e	31
f	f	0
f	g	30
f	h	18
f	i	2
f	j	4
f	k	8
f	l	9
f	m	11
g	a	25

g	b	27
g	c	28
g	d	33
g	e	34
g	f	31
g	g	0
g	h	21
g	i	5
g	j	7
g	k	11
g	l	12
g	m	14
h	a	25
h	b	27
h	c	28
h	d	33
h	e	34
h	f	31
h	g	33
h	h	0
h	i	5
h	j	7
h	k	11
h	l	12
h	m	14
i	a	20

i	b	22
i	c	23
i	d	28
i	e	29
i	f	26
i	g	28
i	h	16
i	i	0
i	j	2
i	k	6
i	l	7
i	m	9
j	a	22
j	b	24
j	c	25
j	d	30
j	e	31
j	f	28
j	g	30
j	h	18
j	i	2
j	j	0
j	k	8
j	l	9
j	m	11
k	a	35

k	b	37
k	c	38
k	d	43
k	e	44
k	f	41
k	g	43
k	h	31
k	i	15
k	j	17
k	k	0
k	l	22
k	m	24
l	a	n/a
l	b	n/a
l	c	n/a
l	d	n/a
l	e	n/a
l	f	n/a
l	g	n/a
l	h	n/a
l	i	n/a
l	j	n/a
l	k	n/a
l	l	0
l	m	n/a
m	a	n/a

m	b	n/a
m	c	n/a
m	d	n/a
m	e	n/a
m	f	n/a
m	g	n/a
m	h	n/a
m	i	n/a
m	j	n/a
m	k	n/a
m	l	n/a
m	m	0