Climate Feedbacks and Sensitivity

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Climate feedbacks

A climate feedback is an internal climate process that amplifies or dampens the initial climate response to a specific forcing (example: increase in atmospheric water vapor that is triggered by an initial warming due to rising carbon dioxide, which then acts to amplify the warming through the greenhouse properties of water vapour). If a process amplifies the climate response it is denoted as a *positive* feedback, if it dampens the response it's a *negative* feedback.

A useful starting point for looking at feedback is to picture it as a set of labelled system components and connection arrows so they form a closed loop. If an increase/reduction in A causes an increase/reduction in B then the arrow is assigned a +, if a reduction/increase in A causes an increase/reduction in B the arrow is assigned a -. The sign of the feedback can then be found by taking the product of the different signs. In the picture below we have (-)(-)(+)(-) which is (-), thus the loop is a negative feedback.

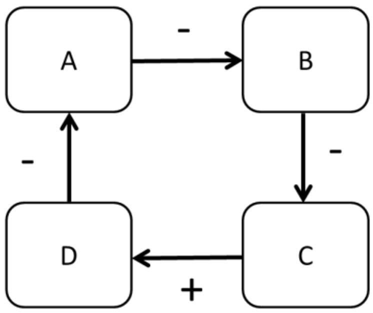


Figure: Schematic illustration of a feedback loop.

If we have no feedback the total effect equals the direct effect:

$$\text{Total effect}=\text{Direct effect}$$

If there is a feedback process with one pass around the loop the direct effect will be enhanced or dampened:

$$\text{Total effect}=\text{Direct effect} + \text{g} \cdot \text{Direct effect}$$

If g is positive the feedback is positive if g is negative the feedback is negative. In reality there will be infinitively many passes around the loop where each loop leads to another factor of g multiplying the input:

$$\text{Total effect}=\text{Direct effect} \cdot (1 + g + g^2 + g^3 + g^4 + \cdots.)$$

We now have three different cases:

- $0<g<1$: Total effect > Direct effect: Positive feedback
- $g<0$: Total effect < Direct effect: Negative feedback
- $g\geq 1$: $(1+g+g^2+g^3+\cdots.)$ goes to ∞ → Total effect goes to ∞: Instability ("runaway" feedback)

Assuming $g<<1$ (moderate feedbacks): $(1 + g + g^2 + g^3 + g^4 + \cdots.) = 1/(1-g)$ and we get:

$$\text{Total effect}=\text{Direct effect}\cdot\left(\frac{1}{1-g}\right)$$

If we translate the general feedback system into climate, the direct response Total effect is the total temperature change ΔT_s. The Direct effect is the temperature change in a system without feedbacks ΔT_s^0.

$$\Delta T_s = \Delta T_s^0 \left(\frac{1}{1-g}\right)$$

g is called the feedback factor and the quantity $\left(\frac{1}{1-g}\right)$ is called the gain G i.e. the ratio of the temperature change with feedbacks and the temperature change without feedbacks:

$$G = \frac{\Delta T_s}{\Delta T_s^0}$$

We can express the changes as a function of various factors x_i $F = F[x_1, x_2, x_3, \ldots.])$

$$g = \sum \frac{\partial F}{\partial x_i}\frac{\partial x_i}{\partial T_s} = \sum g_i$$

The direct response

The direct response of an object to increased radiation is that it heats up and efficiently radiates back energy to dampen the effect of the increased radiation. This is done rather efficient as the amount of radiation goes with the fourth power of the temperature.

$$F_{BB}^{\uparrow} = \sigma T^4$$

This is known as Stefan-Boltzmann's law and $F_{BB}^{\uparrow}$ is the blackbody radiation (W/m^2). This strong response in emitted radiation helps make the earth rather stable to changes in the radiative forcing.

Main Radiative Feedbacks

Water vapour feedback

Water vapour feedback is the coupling between water vapour and surface air temperature in which a change in radiative forcing perturbs the surface air temperature, leading to a change in water vapour, which could then amplify or weaken the initial temperature change.
Water vapour is the most dominant greenhouse gas and a major reason why temperature is so sensitive to changes in radiative forcing. Water vapour is brought into the atmosphere via evaporation and plant transpiration. The water holding capacity of the atmosphere (the amount of water vapour that the atmosphere can hold before it condenses and falls as rain or snow) is a function of temperature – the Clausius-Clapeyron relation which states that a 1 degree increase in temperature increases the water holding capacity of the atmosphere by 6-7%. However, most of the atmosphere is undersaturated with respect to water, so that local water-holding capacity is not the limiting factor determining atmospheric water vapour. The actual amount of water in the atmosphere is given by the water holding capacity times the relative humidity of the atmosphere

(range between 0 and 100%, where 100% indicate that the actual amount of water equals the water holding capacity). Thus, the strength of the water vapor feedback is given by both the changes in water holding capacity and atmospheric relative humidity.
Water vapour induces a strong positive feedback in the climate system. State of the art climate models indicates that the water vapour feedback alone approximately doubles the warming from what it would be for fixed water vapour.

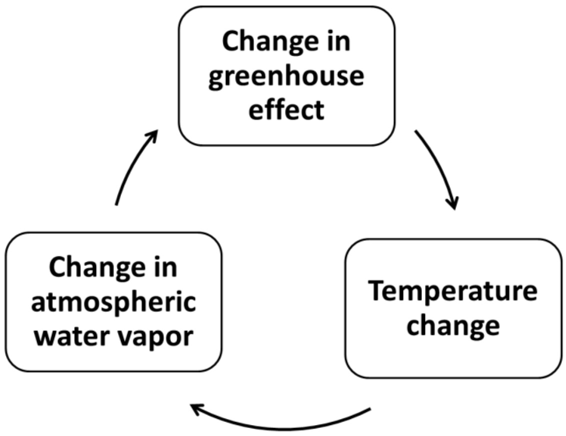


Figure: Schematic illustration of the water vapour feedback

Lapse rate feedback

The lapse rate feedback is the coupling between surface air temperature changes and the changes in the in the region that radiate out to space (upper troposphere) , leading to a change in how much the atmosphere cools with height which again affects the efficiency of the greenhouse effect.

For the greenhouse effect to work the atmospheric temperature has to be colder in the region that radiate out to space than at the surface. If this temperature difference is altered it will change the greenhouse effect. The lapse rate feedback is connected to the water vapour feedback. The radiative effect of the water vapour is roughly proportional to the logarithm of its concentration, so the influence of an increase in water-vapour content is larger in places which are dry, such as in the upper tropical troposphere (10-20 km above ground). This means that the warning in this height is higher than near the surface. Because of this the temperature difference between the surface and the upper troposphere is reduced and the greenhouse effect will become less efficient, providing a negative feedback.
The water-vapour feedback and the lapse-rate feedback are strongly connected as they both are related to changes in water vapour. They tend to partly compensate each other and are often combined.

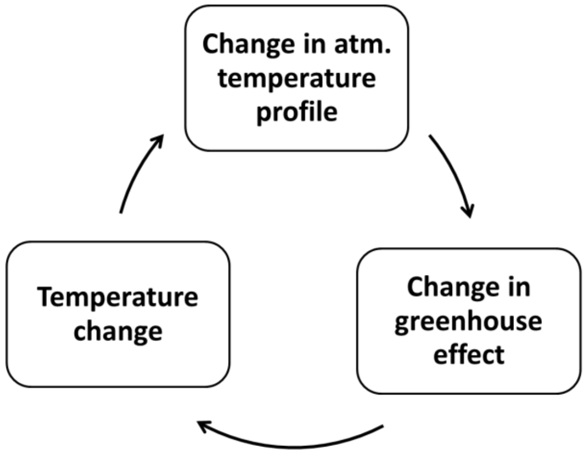


Figure: Schematic illustration of the water lapse rate feedback

Cloud feedbacks

Cloud feedback is the coupling between cloudiness and surface air temperature in which a change in surface air temperature, leads to a change in the temperature and optical properties of clouds, which could then amplify or weaken the surface temperature change.
Cloud feedbacks are many and complicated. The feedbacks can be grouped in two: Clouds are good absorbers of terrestrial radiation (radiation from the earth) and reduces the amount of radiation radiated back to space. This has a warming effect. At the same time clouds reflect solar radiation and thereby cooling the earth. State of the art climate models finds the total cloud feedback to be slightly positive.

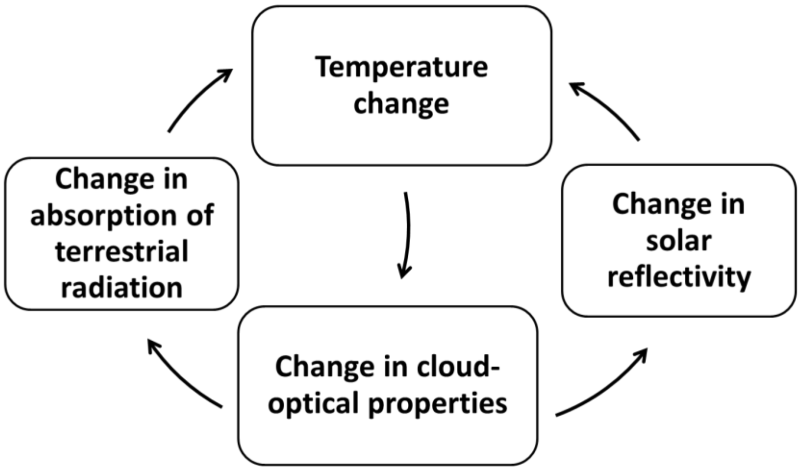


Figure: Schematic illustration of the two main cloud feedbacks

Albedo feedbacks

The albedo feedback is the coupling between changes in surface reflectivity and surface air temperature in which a change in surface air temperature may lead a change in the surface reflectivity (changes in snow cover, sea ice extent, vegetation) , which could then amplify or weaken the surface temperature change. State of the art climate models finds the albedo feedback to be positive due to the influence of changes in temperatures on snow and sea ice. It should be noted that the strength of the albedo feedback is dependent the strength of the solar radiation reaching the surface. For example the same change in snow cover over a region with strong solar radiation (for example a location with a lot of clear skies, or a location close to equator) would give a stronger feedback than a change in snow cover over a region with less solar radiation.

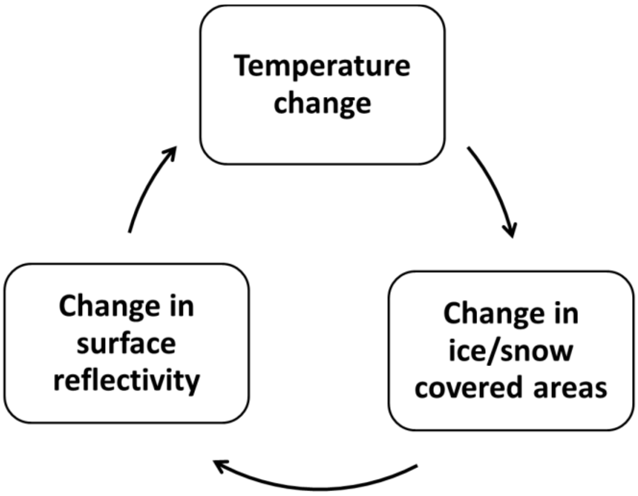
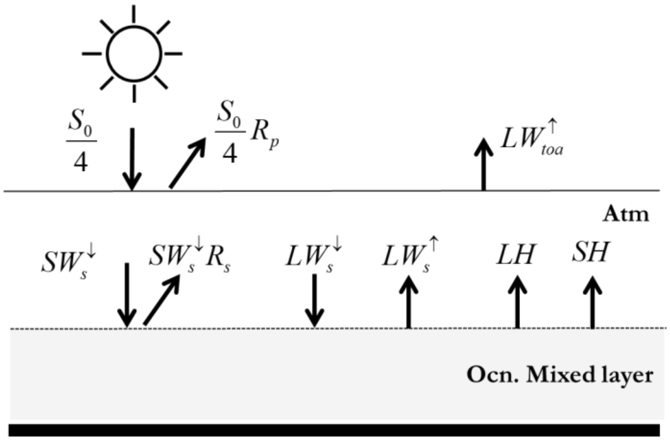


Figure: Schematic illustration of the albedo feedback

Climate sensitivity

The change in net radiation at the top-of-the-atmosphere (R_{TOA}) may depend on the radiative forcing, changes in near surface temperature T_s and changes due to relatively fast stratospheric T_{strato} and tropospheric temperature T_{tropo} adjustments (thermal adjustment in the stratosphere and fast adjustments in tropospheric humidity and clouds and hydrological cycle) . The stratospheric adjustment typically takes a few months while most of the tropospheric adjustments take a few weeks (the main one changing the clouds only takes a few days). Even though most adjustments are rapid, there is no fundamental time scale that separates rapid adjustments from feedback responses. The time scales of the two can overlap significantly for slowly responding parameters such as vegetation changes due to CO_2 changes.

The starting point is a very simple atmosphere- mixed layer ocean energy balance where no energy enters the deep ocean.



The energy change in the atmosphere is:

$$\frac{\partial E_a}{\partial t} = R_{TOA} - R_s + LH + SH$$

LH: Latent heat, *SH*: Sensible heat, E_a is the total energy of the atmosphere (internal + potential + latent + kinetic), R_{toa}: net radiation at top of the atmosphere $R_{TOA} = S_0/4\left(1 - R_p\right) - LW_{toa}^{\uparrow}$ and R_s: net radiation at the surface $R_s = SW_s^{\downarrow}(1 - R_s) + \left(LW_s^{\downarrow} - LW_s^{\uparrow}\right)$
The energy change of the ocean mixed-layer is given as:

$$\frac{\partial E_{mix}}{\partial t} = R_s - LH - SH$$

By combining the atmosphere and ocean mixed layer equation we are getting rid of the surface fluxes (R_s, LH and SH) . Noting that the heat capacity of the mixed layer is much larger than the atmosphere, assuming no change the mixed layer depth and ignoring changes in the kinetic energy and vertical structure of temperature (no change in potential energy) and finally assuming that the surface temperature change is in balance with the change in the mixed layer ocean ($\Delta T_s = \Delta T_{mix}$) we end up with a simple one layer model where the change in TOA net radiation is directly linked to the surface temperature changes:

$$c_{mix} \frac{d\Delta T_s}{dt} = \Delta R_{TOA}$$

$c_{mix} = \rho_O c_O h_{mix}$ is the effective heat capacity ($\rho_O$is the density of sea water, c_O the specific heat capacity of water and h_{mix} the mixed layer depth). ΔT_{mix} is the temperature change in the upper ocean mixed layer.

If we now introduce the radiative forcing concept we can divide the change in TOA net radiation ΔR_{TOA} into a change related to the radiative forcing Q and one due to resulting changes in radiative processes internal to the climate system: ΔR_{TOA}^i:

$$c_{mix} \frac{d\Delta T_s}{dt} = \Delta R_{TOA}^i + Q$$

We now have a budget equation that accounts for two kinds of changes to T_s

- One due to the radiative forcing: Q
- One due to resulting changes in radiative processes internal to the climate system: ΔR_{TOA}

The internal change in net radiation at TOA will be a function of changes in the surface, tropospheric and stratospheric changes including direct temperature changes and change in other climatic parameters x_i such has clouds, surface reflectivity and water vapour that affect the temperatures. If we group the variables into those that are connected with the relatively slow surface temperature change and those that are not, such as relatively fast changes related to changes in stratospheric and tropospheric temperatures we get:

$$c_{mix}\frac{d\Delta T_s}{dt} = \Delta R^i_{TOA}(\Delta T_s) + \Delta R^i_{TOA}(\Delta T_{tropo}, \Delta T_{strato}) + Q$$

$\Delta R^i_{TOA}(\Delta T_{tropo}, \Delta T_{strato})$ can be considered as part of forcings rather than feedbacks, thus we can introduce an adjusted forcing $Q_{adj} = Q + \Delta R^i_{TOA}(\Delta T_{tropo}, \Delta T_{strato})$ and we are left with a system where the temperature change is only dependent on an adjusted imposed forcing and the surface temperature change:

$$c_{mix}\frac{d\Delta T_s}{dt} = \Delta R^i_{TOA}(\Delta T_s) + Q_{adj}$$

Asuming that the changes are sufficiently small, we can linearize the budget about the equilibrium state using a Taylor expansion ($\Delta R^i_{TOA} \approx (\partial R_{TOA}/\partial T_s)\Delta T_s$):

$$c_{mix}\frac{d\Delta T_s}{dt} = \frac{\partial R_{TOA}}{\partial T_s}\Delta T_s + Q_{adj}$$

we then have system where the change in net radiation at TOA only depends on the change in surface temperature and the adjusted forcing.

$\frac{\partial R_{TOA}}{\partial T_s}$ $[\frac{W/m^2}{K}]$ tells us how strongly the net radiation at TOA reacts to changes in the near surface temperature (the rate at which the climate system returns the added forcing to space as longwave radiation or as reflected shortwave). In essence, this accounts for how feedbacks modify the surface temperature response to the forcing. This is called the climate *feedback parameter* and is often denoted with the letter α or λ. The climate *feedback parameter* is defined as:

$$\lambda \equiv -\frac{\partial R_{toa}}{\partial T_s}$$

The minus sign is introduced so a negative feedback will be associated with a negative feedback parameter. The change in surface temperature for a given adjusted forcing Q_{adj} can then be written as:

$$c_{mix}\frac{d\Delta T_s}{dt} = -\lambda\Delta T_s + Q_{adj}$$

We then have system where the internal change in net radiation at TOA is a function only of the change in surface temperature and the strength of the feedback parameter:

$$\Delta R_{TOA}^{i} = -\lambda \Delta T_s$$

Assuming that the surface temperature has had time to adjust to the climate forcing Q_{adj} we will have no change in surface temperature with time and we get a simple expression for the equilibrium temperature change $\Delta T_{s,eq}$.

$$\Delta T_{s,eq} = \frac{1}{\lambda_{eq}} Q_{adj}$$

Note that the change in net radiation at TOA ΔR_{toa} is closely connected to the radiative forcing, the change in surface temperature and the strength of the feedback parameter. If this is a case in reality it should be possible to estimate both the feedback parameter, the adjusted forcing and the equilibrium climate change from a transient simulation by plotting ΔR_{toa} vs ΔT_s:

$$\Delta R_{toa} = Q_{adj} - \lambda \Delta T_s$$

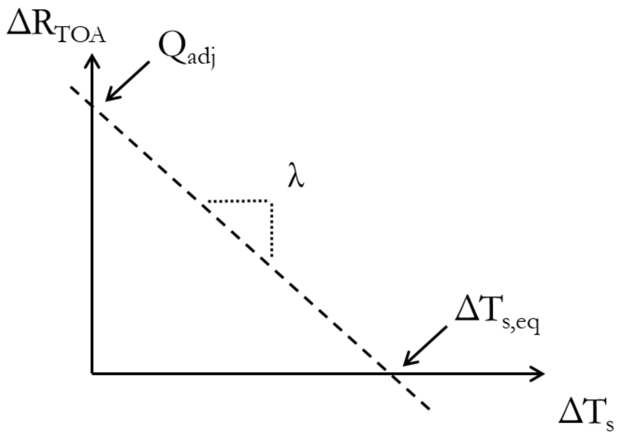


Figure: Finding the feedback parameter, the adjusted forcing and the equilibrium climate change from a ΔR_{toa} vs ΔT_s plot.

To gain insight into what the *climate feedback parameter* represent, we can divide the sensitivity of the net radiation at TOA to a change in surface temperature into:

- A direct response in net radiation at TOA from a surface temperature change if all other climatic parameters were constant
- Changes in other climatic parameters (x_i) that changes when the temperature changes and again affect the temperature (the feedbacks).

We neglect that the climatic parameters x_i will influence each other to get:

$$\frac{\partial R_{TOA}}{\partial T_s} = \left[\left. \frac{\partial R_{TOA}}{\partial T_s} \right|_{x_i=0} + \sum_{i=1}^{N} \frac{\partial R_{TOA}}{\partial x_i} \frac{\partial x_i}{\partial T_s} \right]$$

We now have an expression where the total change in the surface temperature is:

- The direct change in temperature due to the adjusted radiative forcing (first term on right hand side).
- The effect of changing all other possible parameters on ΔT_s (second term on right hand side). This latter term is the different feedbacks.

We can link this ΔT_s expression to our definition of the ΔT_s feedback factor $\lambda \equiv -\partial R_{toa}/\partial T_s$to get

$$\lambda = -\left[\left.\frac{\partial R_{TOA}}{\partial T_s}\right|_{x_i=0} + \sum_{i=1}^{N} \frac{\partial R_{TOA}}{\partial x_i}\frac{\partial x_i}{\partial T_s} \right]$$

We see that the climate feedback factor in principle is depending on, climate change for a variety of parameters and the interaction between these parameters. Note that in this framework we treat the direct change in temperature due to a radiative forcing given by Stefan Boltzmann's law through the $\partial T_s/\partial R_{TOA}$ term, also as a part of the feedback parameter even if is not a feedback. If we had no feedbacks the sensitivity of the surface temperature to a 1 Wm^{-2} radiative forcing would be

$$\frac{\Delta T_s}{\Delta R_{TOA}} \approx \frac{\partial T_s}{\partial R_{TOA}} = \frac{1}{4\sigma T^3}$$

Using 255K this gives us a sensitivity of 0.27K/Wm^{-2} and a "feedback" parameter of -3.8 Wm^{-2}/K.

With the above assumption that the different climatic factors do not influence each other the feedback factors are additive, so we can divide them into different terms (feedback due to albedo, clouds, water vapour etc.).

$$\lambda = \lambda_{Planck} + \lambda_{water} + \lambda_{clouds} + \lambda_{albedo} + \ldots$$

where $\lambda_{planck} = \frac{1}{\left(\left.\frac{\partial T_s}{\partial R_{TOA}}\right|_{x_i=0}\right)}, \lambda_{water} = \frac{1}{\left(\frac{\partial T_s}{\partial q}\frac{\partial q}{\partial R_{TOA}}\right)}, \lambda_{cloud} = \frac{1}{\left(\frac{\partial T_s}{\partial c}\frac{\partial c}{\partial R_{TOA}}\right)}, \lambda_{albedo} = \frac{1}{\left(\frac{\partial T_s}{\partial \alpha}\frac{\partial \alpha}{\partial R_{TOA}}\right)}$ etc...

The $\partial T_s/\partial R_{toa}$ term may be looked upon as a "reference sensitivity" if no feedbacks are present (i.e. temperature change if no other climatic parameter changed).

Forcing Efficacy

As a complicating factor in understanding the equilibrium climate feedback factor concept it has been noted that the equilibrium climate sensitivity may depend on the mean climate state, the type of forcing applied to the climate system and their geographical and vertical distributions. Studies indicate that for forcings that is homogeneously distributed (for example: long lived greenhouse gases) $\boldsymbol{\lambda_{eq}}$ is a nearly invariant parameter for a variety of forcings. E.g. λ_{eq} calculated for a 2*CO_2 experiment is almost the same as the one calculated with a 4*CO_2 experiment. However, the climate effect of more short lived pollutants such as soot and ozone is complex, depending especially on their spatial distribution. To cope with the equilibrium climate sensitivity depending on the type of forcing. The term "Efficacy" (E) defined as the ratio of the climate feedback factor for CO2 ($\lambda_{eq_{CO2}}$) to the climate feedback factor for a given forcing agent (λ_{eq_i}):

$$E_i = \frac{\lambda_{eq_{CO2}}}{\lambda_{eq_i}}$$

Efficacy can be seen as a kind of *responsivity* of the equilibrium surface temperature to a change in a certain forcing compared to how the equilibrium temperature would have changed if the forcing was a CO_2 forcing that was equally strong. This can be used to define an effective radiative forcing (Q_{eff})

$$Q_{eff} = E_i Q_{adj}$$

For the effective radiative forcing, the climate feedback factor is independent of the mechanism used to perturb the model, so if the climate feedback factor is found by for example increasing CO_2 the equilibrium temperature may be calculated as:

$$\Delta T_{s_{eq}} = \frac{1}{\lambda_{eq,CO2}} E_i \Delta Q$$

Ocean heat uptake efficacy

It is shown that an additional process needs to be taken into account during the transient regime in order to represent the evolution of the radiative imbalance of the climate system. The transient regime geographical temperature change pattern may be different from the equilibrium pattern. Because the feedback strength varies geographically, the different pattern of surface temperature change may affect the global radiative imbalance (ΔR_{toa}) in the transient regime compared to the equilibrium case. Thus$\lambda \neq \lambda_{eq}$.

This evolving spatial pattern can be related to for example the oceanic heat uptake. The ocean heat uptake reduces the rate of warming and this effect occurs preferentially in some regions (for example in the sinking branches of the thermohaline circulation, in the North Atlantic Ocean and circumpolar ocean of the southern hemisphere).

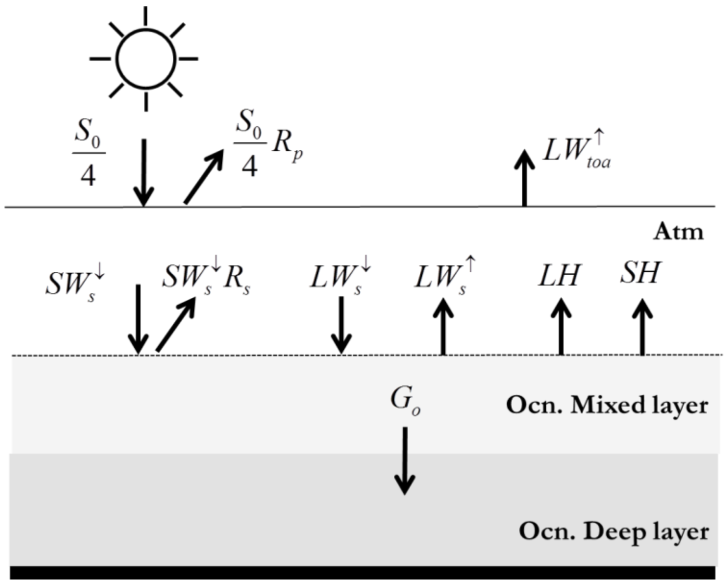
Dividing the ocean into two boxes (the mixed layer and deep ocean) introduced an efficacy factor for deep-ocean heat uptake:

$$\varepsilon = \frac{\lambda_{eq}}{\lambda}$$

Where λ is the climate feedback factor that varies with the evolving spatial pattern of the oceanic heat uptake.

A simple two-box model using the climate feedback parameter concept

The starting point is a very simple atmosphere- mixed layer ocean –deep ocean energy balance.



Change in atmospheric energy:

$$\frac{\partial E_a}{\partial t} = R_{TOA} - R_s + LH + SH$$

LH: Latent heat, *SH*: Sensible heat, E_a is the total energy of the atmosphere (internal + potential + latent + kinetic), R_{toa}: net radiation at top of the atmosphere $R_{TOA} = S_0/4\left(1 - R_p\right) - LW_{toa}^{\uparrow}$ and R_s: net radiation at the surface $R_s = SW_s^{\downarrow}(1 - R_s) + \left(LW_s^{\downarrow}LW_s^{\uparrow}\right)$

Change in ocean mixed-layer energy:

$$\frac{\partial E_{mix}}{\partial t} = R_s - LH - SH - G_o$$

G_o: mixing mixed-layer/deeper ocean, E_o is the total energy of the mixed layer (internal + potential + kinetic) and R_s: net radiation at surface: $R_s = S_s(1 - \alpha_s) - F_s^{\uparrow} + F_s^{\downarrow}$

Change in deep ocean energy:

$$\frac{\partial E_o}{\partial t} = G_o$$

By combining the atmosphere and ocean mixed layer equations, we are getting rid of the surface fluxes (R_s, LH and SH):

$$\frac{\partial E_{mix}}{\partial t} + \frac{\partial E_a}{\partial t} = R_{TOA} - G_o$$

Noting that the heat capacity of the mixed layer is much larger than the atmosphere, assuming no change the mixed layer depth and ignoring changes in the kinetic energy and vertical structure of temperature (no change in potential energy) we end up with a simple two-box model, one box for the upper mixed layer ocean and one for the deep ocean. The two are connected through the mixing between the mixed layer and the deeper ocean G_o

$$c_{mix}\frac{d\Delta T_{mix}}{dt} = \Delta R_{TOA} - G_o$$

$$c_{deep}\frac{d\Delta T_o}{dt} = G_o$$

where $c_{mix} = \rho_O c_O h_{mix}$ is the effective heat capacity ($\rho_O$is the density of sea water, c_O the specific heat capacity of water and h_{mix} the mixed layer depth). ΔT_{mix} is the temperature change in the upper ocean mixed layer. The subscript *deep* denotes the same variables in the deep ocean ($h_{deep}$is the depth of the ocean minus the mixed layer). The deep ocean can only change its temperature by receiving more or less heat from the upper ocean. The deep layer is characterized by a heat capacity $c_{deep} >> c_{mix}$

The change in net radiation at TOA is given by substituting the $G_O$from the deep ocean equation into the mixed-layer equation

$$\Delta R_{TOA} = c_{mix}\frac{d\Delta T_{mix}}{dt} + c_{deep}\frac{d\Delta T_o}{dt}$$

We then assume that the heat flux exchanged between the two layers is given by a simple heat-exchange coefficientγ.

$$G_o = \gamma(\Delta T_s - \Delta T_o)$$

By assuming that the surface temperature change ΔT_s is in equilibrium with the temperature change in the upper ocean mixed layer ΔT_{mix} and that the effect of the surface temperature change on the top of the atmosphere net radiation change is a function of the climate forcing and the feedback factor λ ($\Delta R_{toa} = Q_{adj} - \lambda \Delta T_s$) we get:

Which leads us to the following two equations where the temperature changes for a given climate forcing Q_{adj} only depends on two parameters $\lambda$and γ.

$$c_{mix} \frac{d\Delta T_s}{dt} = Q_{adj} - \lambda \Delta T_s - \gamma(\Delta T_s - \Delta T_o)$$

$$c_{deep} \frac{d\Delta T_o}{dt} = \gamma(\Delta T_s - \Delta T_o)$$

where ΔT_S (K) is the temperature change at the surface, Q_{adj}(W/m^2) is the radiative forcing, λ(K/(W/m^2) is the sum of the feedback factors, ΔT_o is the deep ocean temperature change and γ K/(W/m^2) the heat transfer parameter that transfer heat from the upper to the deeper ocean. Note that the model we neglect land areas. Thus, our ΔT_s should be seen as being representative for SST changes.

We then have a system with two timescales. The fast relaxation time of the upper ocean and the slow relaxation time of the deeper ocean .

Some limiting cases

We start by assuming a constant forcing $Q_{adj} = Q_{adj}(t = 0)$ that is imposed on the climate system at time t=0 (e.g. 4*CO2 forcing).

For the first few years of the simulations we can assume a finite heat capacity of the upper ocean and a infinite heat capacity for the deep ocean leading to $\Delta T_o \approx 0$. This simplifies the upper ocean temperature change to:

$$c_{mix} \frac{d\Delta T_s}{dt} = Q_{adj}(\mathrm{t} = 0) - (\lambda + \gamma)\Delta T_s$$

This can easily be solved:

$$\Delta T_s(t) = \frac{Q_{adj}(\mathrm{t} = 0)}{\lambda + \gamma} \left\{ 1 - e^{-\frac{\lambda+\gamma}{c_{mix}} t} \right\}$$

The solution gives a temperature change that is increasing rapidly before it flattens out.

After 10–20 years, $c_{mix} d\Delta T_s/dt << Q_{adj}$ and the upper-ocean thermal inertia can be neglected. If we still assume that the deep layer has an infinite deep ocean heat capacity leading to $\Delta T_o \approx 0$ we get:

$$\Delta T_s(t) \approx \frac{1}{\lambda + \gamma} Q_{adj}(\mathrm{t} = 0)$$

The assumptions above means that this is the fast response which will be directly proportional to the forcing and dependent on both the strength of the feedback factor and the strength of the mixed-layer-deep ocean heat-exchange coefficient.

As times goes by the assumption of an infinite deep ocean heat capacity becomes less valid and a more appropriate assumption is that the upper layer ocean is in equilibrium with the forcing leading to $d\Delta T/dt = 0$ and the deep layer has a finite heat capacity.

$$0 = Q_{adj} - (\lambda + \gamma)\Delta T_s + \gamma \Delta T_o$$

$$c_{deep} \frac{d\Delta T_o}{dt} = \gamma(\Delta T_s - \Delta T_o)$$

Leading to:

$$\Delta T_s(t) = \frac{Q_{adj}(\mathrm{t} = 0)}{\lambda} \left\{ 1 - \frac{\gamma}{\lambda + \gamma} e^{-\frac{\lambda\gamma}{c_{deep}(\lambda+\gamma)}t} \right\}$$

and

$$\Delta T_o(t) = \frac{Q_{adj}(\mathrm{t} = 0)}{\lambda} \left\{ 1 - e^{-\frac{\lambda\gamma}{c_{deep}(\lambda+\gamma)}t} \right\}$$

Finally as $t \to \infty$ and climate has equilibrated to the new forcing $\Delta T_o = \Delta T_s$ we get:

$\Delta T_{s,eq} = \frac{1}{\lambda} Q_{adj}(t = 0)$

Timescales

The timescales associated with the time it takes for the upper ocean and slow deep ocean response to come in equilibrium with the forcing can be estimated using the concept of relaxation times. The relaxation time is defined as the time it takes to for the perturbed system to reach 1-e^{-1} (i.e. 63%) of the equilibrium response. For our system that has solutions on the form, $\Delta T_s(t) = a\{1 - e^{-bt}\}$ this means that the relaxation times become $\tau = 1/b$:

- Fast upper ocean response: $\tau_{mix} = \frac{c_{mix}}{\lambda+\gamma}$ which is 3-4 years for typical values of the three parameters

- Slow deep ocean response: $\tau_{deep} = \frac{c_{deep}(\lambda+\gamma)}{\lambda\gamma}$ which is 200-300 years for typical values of the three parameters

First few years	First 1-2 decades	Several decades	Centuries
Upper-ocean: Small infinite heat capacity	Upper-ocean: no heat capacity	Upper-ocean: no heat capacity	Upper-ocean: Small infinite heat capacity
Deep ocean: infinite heat capacity	Deep ocean: infinite heat capacity	Deep ocean: large finite heat capacity	Deep ocean: large finite heat capacity
$\Delta T_s(t) = \frac{Q_{adj}(\mathrm{t}=0)}{\lambda+\gamma}\left\{1 - e^{-\frac{\lambda+\gamma}{c_{mix}}t}\right\}$	$\Delta T_s(t) \approx \frac{1}{\lambda+\gamma} Q_{adj}(\mathrm{t} = 0)$	$\Delta T_s(t) = \frac{Q_{adj}(\mathrm{t}=0)}{\lambda}\left\{1 - \frac{\gamma}{\lambda+\gamma} e^{-\frac{\lambda\gamma}{c_{deep}(\lambda+\gamma)}t}\right\}$	$\Delta T_{s,eq} = \frac{Q_{adj}(\mathrm{t}=0)}{\lambda}$
$\Delta T_o(t) = 0$	$\Delta T_o(t) = 0$	$\Delta T_o(t) = \frac{Q_{adj}(\mathrm{t}=0)}{\lambda}\left\{1 - e^{-\frac{\lambda\gamma}{c_{deep}(\lambda+\gamma)}t}\right\}$	$\Delta T_{o,eq} = \frac{Q_{adj}(\mathrm{t}=0)}{\lambda}$
$\tau_{mix} = \frac{c_{mix}}{\lambda+\gamma}$		$\tau_{deep} = \frac{c_{deep}(\lambda+\gamma)}{\lambda\gamma}$	

Table: Response of the upper and deep-layer temperatures for a constant forcing $Q_{adj} = Q_{adj}(t=0)$ that is imposed on the climate system at time t=0 using four different approximations to the two-layer model.

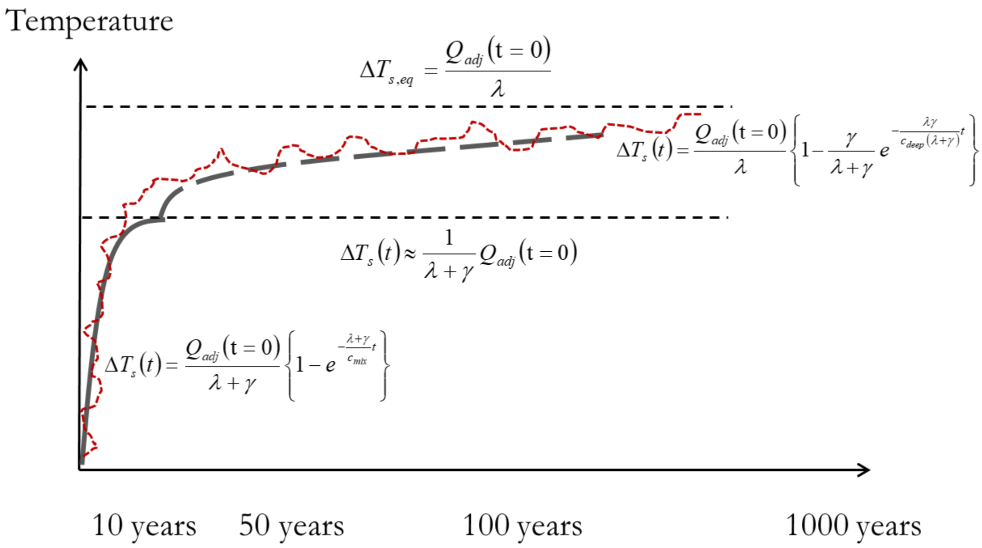


Figure: Schematic picture showing different solutions to the two-layer model forced with a constant forcing.

Finding the feedback factor λ

The concept of "radiative forcing" and "feedback factors" provides a way to quantify and compare the contributions of different agents that affect surface temperature and provides a mechanism for conceptualizing the Earth's climate as a closed system with one detectable metric of change: global mean surface temperature.

The implication of $\Delta T_{s,eq} \approx Q_{adj}/\lambda_{eq}$ with a fixed λ_{eq} is that surface temperature change at TOA for a climate in equilibrium with the climate forcing is uniquely determined by the radiative forcing at the tropopause (or at the top of the atmosphere if the stratosphere is adjusted for radiative equilibrium).

The determination of the equilibrium temperature change $\Delta T_{s,eq}$ requires very long simulations (thousands of years) which are computationally expensive and alternative methods have been proposed.

- It can be evaluated by coupling the atmospheric general circulation model to a mixed-layer ocean. However, the estimates of the equilibrium climate sensitivity in this way may differ from a fully coupled simulation as the ocean circulation redistributes the energy and impacts the Earth's energy balance through its interaction with atmospheric processes.
- Another type of methods consists in extrapolating the result from a transient simulation to equilibrium using a fully coupled model. This relies on the linear assumption between the TOA radiative imbalance ΔR_{toa} and the mean surface temperature response: $\Delta R_{toa} = Q_{adj} - \lambda\Delta T_s$. Using a simulation where Q is a constant (for example abrupt 4 times CO_2) and treating λ as a constant, λmay be calculated using linear regression between ΔR_{toa} and ΔT_s. Such a fit gives the *effective forcing* Q_{eff} (intercept), the *effective radiative feedback parameter* λ_{eff} (slope) and the *effective equilibrium climate sensitivity* $\Delta T_{s,eq,eff}$ (x-axis intersection).

Several papers question the validity of the linear assumption between ΔR_{toa} and ΔT_s . However some of the non linearities found may be due to biases in the estimation of the radiative forcing.

Several methods have been used to diagnose individual climate feedbacks in GCMs, whose strengths and weaknesses are reviewed in several papers. These methods include the "partial radiative perturbation" approach and its variants, the "radiative kernel method" and the "cloud radiative forcing" method.

Finding the ocean the heat-exchange coefficient γ

Assuming a forcing that increases regularly with time after 10–20 years the upper ocean mixed layer will be in approximate balance with the net radiation at the TOA ($d\Delta T_{mix}/dt \approx 0$). This implies that the change in net radiation at the TOA must equal the ocean heat uptake efficiencyκ:

$$\kappa = \frac{\Delta R_{TOA}}{\Delta T_s}$$

From the two-layer model we have
$c_{deep}\frac{d\Delta T_o}{dt} = \gamma(\Delta T_s - \Delta T_o)$ remembering that $\Delta R_{TOA} = c_{mix}\frac{d\Delta T_{mix}}{dt} + c_{deep}\frac{d\Delta T_o}{dt}$we get:

$$\frac{\Delta R_{TOA}}{\Delta T_s} = \gamma_o\left(1 - \frac{\Delta T_o}{\Delta T_s}\right)$$

To retain the balance of the upper ocean the ocean heat uptake must be transported to the deep ocean. Thus, we see that the heat-exchange coefficient γ is strongly connected to the ocean heat uptake efficiency:

$$\kappa = \gamma\left(1 - \frac{\Delta T_o}{\Delta T_s}\right)$$

The above equation implies that ocean heat uptake efficiency κ is decreasing as the deeper ocean start warming relative to the upper ocean.
For next century warming a good approximation would be:

$$\bar{\gamma} \approx 0.9\overline{\kappa}$$

The κ - γ connection has limited validity. It is not valid when the climate tends toward equilibrium nor immediately (first 10-20 years) after applying an abrupt forcing.