Hồi Quy Tuyến Tính

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3/20/2020

Load thư viện

```r
library(car)
```

```
## Loading required package: carData
```

```r
library(Hmisc)
```

```
## Loading required package: lattice
```

```
## Loading required package: survival
```

```
## Loading required package: Formula
```

```
## Loading required package: ggplot2
```

```
##
## Attaching package: 'Hmisc'
```

```
## The following objects are masked from 'package:base':
##
##     format.pval, units
```

```r
library(psych)
```

```
##
## Attaching package: 'psych'
```

```
## The following object is masked from 'package:Hmisc':
##
##     describe
```

```
## The following objects are masked from 'package:ggplot2':
##
##     %+%, alpha
```

```
## The following object is masked from 'package:car':
##
##     logit
```

```r
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:Hmisc':
##
##     src, summarize

## The following object is masked from 'package:car':
##
##     recode

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

Đọc dữ liệu từ file .csv (Heart.csv)

```
data =
read.csv('https://raw.githubusercontent.com/pnhuy/datasets/master/heart_uci/h
eart.csv')
attach(data)
head(data)
```

```
##   X Age Sex    ChestPain RestBP Chol Fbs RestECG MaxHR ExAng Oldpeak Slope
Ca
## 1 1  63   1      typical    145  233   1       2   150     0     2.3     3
0
## 2 2  67   1 asymptomatic    160  286   0       2   108     1     1.5     2
3
## 3 3  67   1 asymptomatic    120  229   0       2   129     1     2.6     2
2
## 4 4  37   1   nonanginal    130  250   0       0   187     0     3.5     3
0
## 5 5  41   0   nontypical    130  204   0       2   172     0     1.4     1
0
## 6 6  56   1   nontypical    120  236   0       0   178     0     0.8     1
0
##        Thal AHD
## 1     fixed  No
## 2    normal Yes
## 3 reversable Yes
## 4    normal  No
## 5    normal  No
## 6    normal  No
```
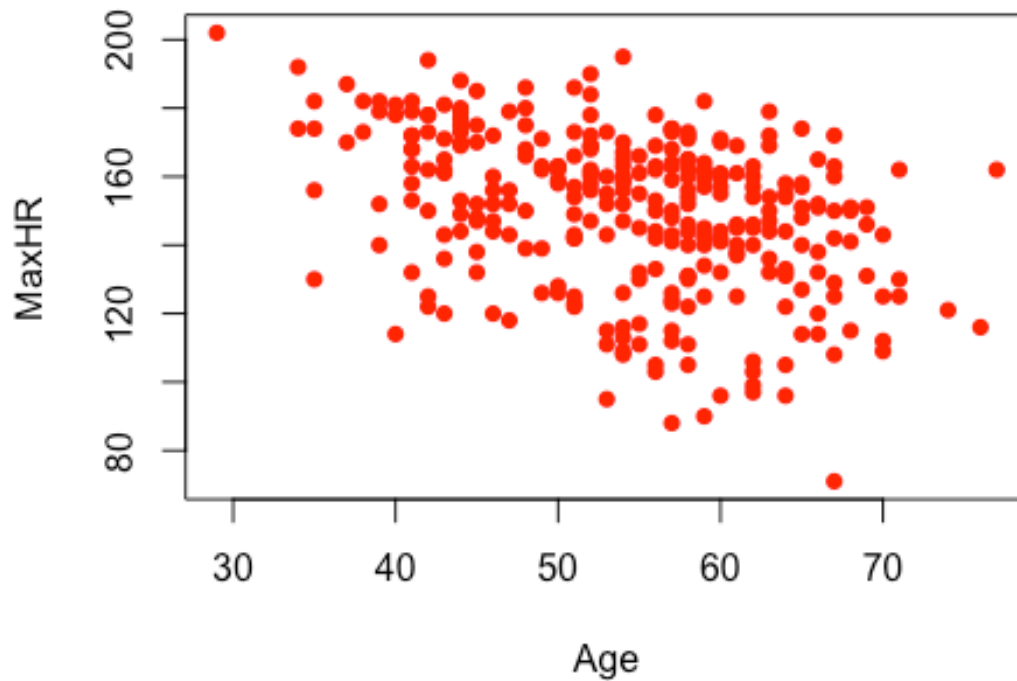
Tạo một biến Gender tương ứng với biến Sex (0 : Female , 1 : Male)

```
Gender = factor(Sex, levels = c(0,1), labels = c('Female','Male'))
```

Phân tích tương quan

Tương quan một biến

```
plot(MaxHR ~ Age, pch = 16 , col = 'red')
```



```
cor.test(MaxHR, Age)

##
##  Pearson's product-moment correlation
##
## data:  MaxHR and Age
## t = -7.4329, df = 301, p-value = 1.109e-12
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.4849644 -0.2941816
## sample estimates:
##        cor
## -0.3938058
```

```
cor.test(MaxHR, Age)

##
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```

```
## 
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## sample estimates:
##       cor
## -0.3938058
```

Tương quan đa biến

```
vars = cbind(MaxHR,Age,Chol,RestBP,Oldpeak,Thal)
corr.test(vars)

## Call:corr.test(x = vars)
## Correlation matrix
##         MaxHR   Age Chol RestBP Oldpeak  Thal
## MaxHR    1.00 -0.39 0.00  -0.05   -0.34 -0.12
## Age     -0.39  1.00 0.21   0.28    0.20  0.06
## Chol     0.00  0.21 1.00   0.13    0.05  0.08
## RestBP  -0.05  0.28 0.13   1.00    0.19  0.06
## Oldpeak -0.34  0.20 0.05   0.19    1.00  0.21
## Thal    -0.12  0.06 0.08   0.06    0.21  1.00
## Sample Size
##         MaxHR Age Chol RestBP Oldpeak Thal
## MaxHR     303 303  303    303     303  301
## Age       303 303  303    303     303  301
## Chol      303 303  303    303     303  301
## RestBP    303 303  303    303     303  301
## Oldpeak   303 303  303    303     303  301
## Thal      301 301  301    301     301  301
## Probability values (Entries above the diagonal are adjusted for multiple
tests.)
##         MaxHR  Age Chol RestBP Oldpeak Thal
## MaxHR    0.00 0.00 1.00   1.00    0.00 0.29
## Age      0.00 0.00 0.00   0.00    0.00 1.00
## Chol     0.95 0.00 0.00   0.19    1.00 0.97
## RestBP   0.43 0.00 0.02   0.00    0.01 1.00
## Oldpeak  0.00 0.00 0.42   0.00    0.00 0.00
## Thal     0.04 0.29 0.16   0.32    0.00 0.00
## 
##  To see confidence intervals of the correlations, print with the
short=FALSE option

pairs.panels(vars)
```
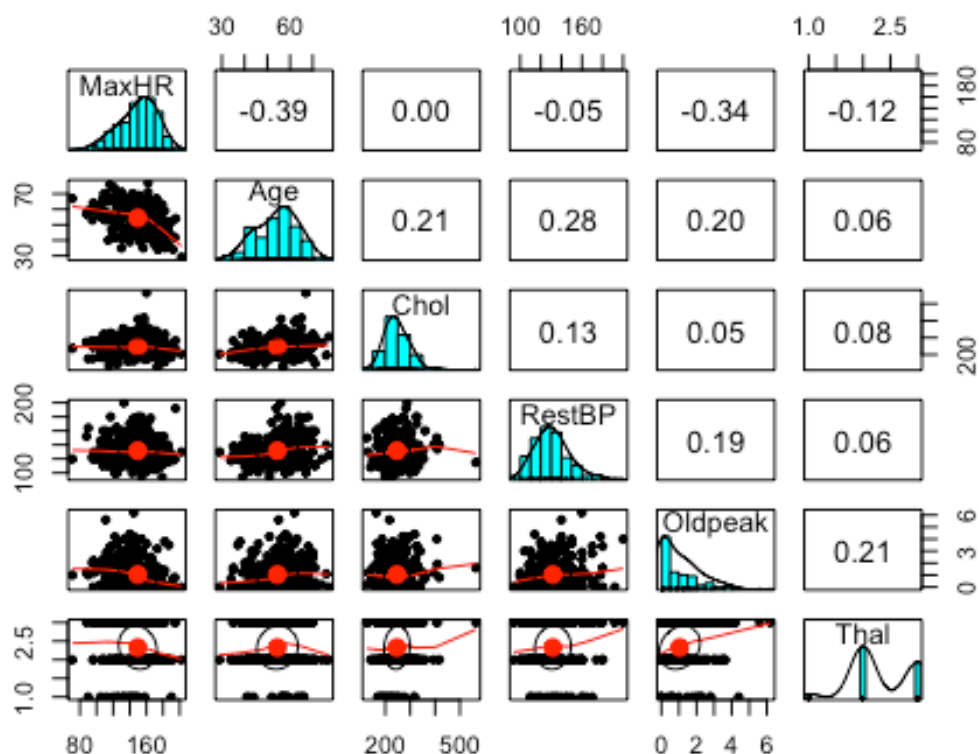
Hồi quy tuyến tính một biến

Hồi quy tuyến tính với biến liên tục

```
res = lm(MaxHR ~ Age)
summary(res)

##
## Call:
## lm(formula = MaxHR ~ Age)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -66.088 -12.040   3.965  15.937  44.955
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 203.8634     7.3991  27.553  < 2e-16 ***
## Age          -0.9966     0.1341  -7.433 1.11e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 21.06 on 301 degrees of freedom
## Multiple R-squared:  0.1551, Adjusted R-squared:  0.1523
## F-statistic: 55.25 on 1 and 301 DF,  p-value: 1.109e-12
```

Vẽ biểu đồ

```
plot(MaxHR ~ Age, col = 'blue', pch = 16)
abline(res, col = 2)
```
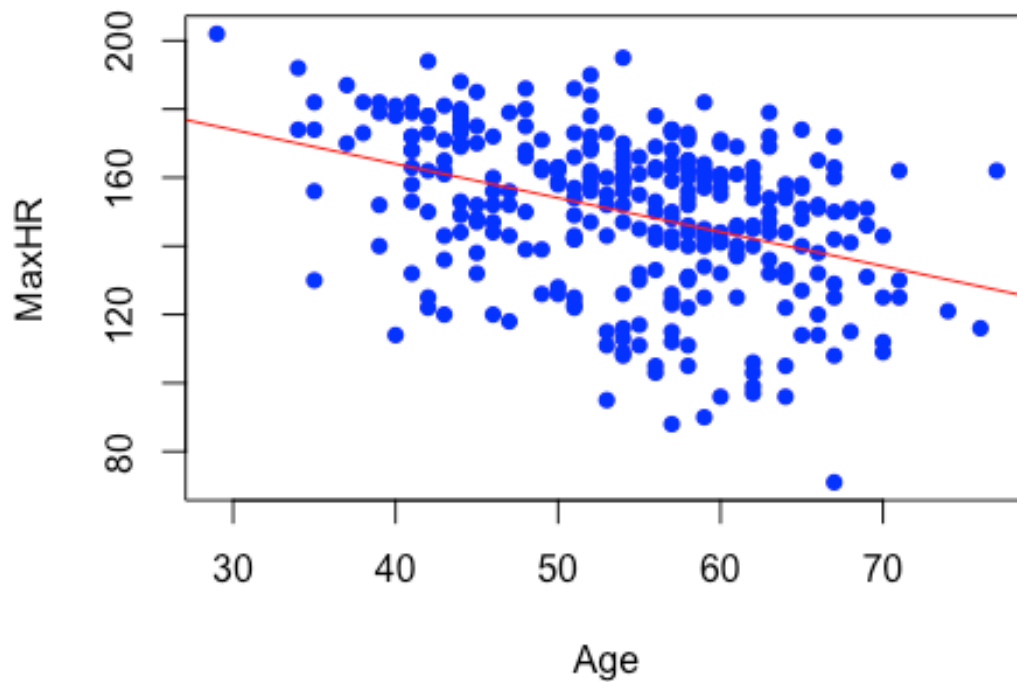

Tính y^

```
predict(res)
```

```
##        1        2        3        4        5        6        7        8
## 141.0750 137.0884 137.0884 166.9876 163.0011 148.0514 142.0716 147.0548
##        9       10       11       12       13       14       15       16
## 141.0750 151.0414 147.0548 148.0514 148.0514 160.0111 152.0380 147.0548
##       17       18       19       20       21       22       23       24
## 156.0246 150.0447 156.0246 155.0279 140.0783 146.0582 146.0582 146.0582
##       25       26       27       28       29       30       31       32
## 144.0649 154.0313 146.0582 138.0850 161.0078 163.9977 135.0951 144.0649
##       33       34       35       36       37       38       39       40
## 140.0783 145.0615 160.0111 162.0044 161.0078 147.0548 149.0481 143.0682
##       41       42       43       44       45       46       47       48
## 139.0817 163.9977 133.1018 145.0615 143.0682 146.0582 153.0347 154.0313
```

```
##       49        50        51        52        53        54        55        56
## 139.0817 151.0414 163.0011 139.0817 160.0111 160.0111 144.0649 150.0447
##       57        58        59        60        61        62        63        64
## 154.0313 163.0011 150.0447 153.347  153.347  158.0179 146.0582 150.0447
##       65        66        67        68        69        70        71        72
## 150.0447 144.0649 144.0649 150.0447 145.0615 158.0179 139.0817 137.0884
##       73        74        75        76        77        78        79        80
## 142.0716 139.0817 160.0111 139.0817 144.0649 153.347  156.0246 146.0582
##       81        82        83        84        85        86        87        88
## 159.0145 151.0414 164.9944 136.0917 152.0380 160.0111 157.0212 151.0414
##       89        90        91        92        93        94        95        96
## 151.0414 153.347  138.0850 142.0716 142.0716 160.0111 141.0750 152.0380
##       97        98        99       100       101       102       103       104
## 145.0615 144.0649 152.0380 156.0246 159.0145 169.9776 147.0548 133.1018
##      105       106       107       108       109       110       111       112
## 155.0279 150.0447 145.0615 147.0548 143.0682 164.9944 143.0682 148.0514
##      113       114       115       116       117       118       119       120
## 152.0380 161.0078 142.0716 163.0011 146.0582 168.9809 141.0750 139.0817
##      121       122       123       124       125       126       127       128
## 156.0246 141.0750 153.347  149.0481 139.0817 159.0145 148.0514 150.0447
##      129       130       131       132       133       134       135       136
## 160.0111 142.0716 150.0447 153.347  174.9608 153.347  161.0078 149.0481
##      137       138       139       140       141       142       143       144
## 134.0985 142.0716 168.9809 153.347  145.0615 145.0615 152.0380 140.0783
##      145       146       147       148       149       150       151       152
## 146.0582 157.0212 147.0548 163.0011 159.0145 144.0649 152.0380 162.0044
##      153       154       155       156       157       158       159       160
## 137.0884 149.0481 140.0783 134.0985 153.347  146.0582 144.0649 136.0917
##      161       162       163       164       165       166       167       168
## 158.0179 127.1220 150.0447 146.0582 156.0246 147.0548 152.0380 150.0447
##      169       170       171       172       173       174       175       176
## 168.9809 159.0145 134.0985 151.0414 145.0615 142.0716 140.0783 147.0548
##      177       178       179       180       181       182       183       184
## 152.0380 148.0514 161.0078 151.0414 156.0246 148.0514 162.0044 145.0615
##      185       186       187       188       189       190       191       192
## 144.0649 141.0750 162.0044 138.0850 150.0447 135.0951 154.0313 153.347
##      193       194       195       196       197       198       199       200
## 161.0078 142.0716 136.0917 137.0884 135.0951 159.0145 154.0313 145.0615
##      201       202       203       204       205       206       207       208
## 154.0313 140.0783 147.0548 140.0783 161.0078 159.0145 146.0582 154.0313
##      209       210       211       212       213       214       215       216
## 149.0481 142.0716 166.9876 165.9910 163.0011 138.0850 152.0380 148.0514
##      217       218       219       220       221       222       223       224
## 158.0179 158.0179 140.0783 145.0615 163.0011 150.0447 164.9944 151.0414
##      225       226       227       228       229       230       231       232
## 141.0750 169.9776 157.0212 137.0884 150.0447 138.0850 152.0380 149.0481
##      233       234       235       236       237       238       239       240
## 155.0279 130.1119 150.0447 150.0447 148.0514 158.0179 155.0279 162.0044
##      241       242       243       244       245       246       247       248
## 163.0011 163.0011 155.0279 143.0682 144.0649 137.0884 146.0582 157.0212
```

```
##      249      250      251      252      253      254      255      256
## 152.0380 142.0716 147.0548 146.0582 140.0783 153.0347 161.0078 162.0044
##      257      258      259      260      261      262      263      264
## 137.0884 128.1186 134.0985 147.0548 160.0111 146.0582 144.0649 160.0111
##      265      266      267      268      269      270      271      272
## 143.0682 162.0044 152.0380 145.0615 163.9977 162.0044 143.0682 138.0850
##      273      274      275      276      277      278      279      280
## 158.0179 133.1018 145.0615 140.0783 138.0850 164.9944 147.0548 146.0582
##      281      282      283      284      285      286      287      288
## 147.0548 157.0212 149.0481 168.9809 143.0682 146.0582 146.0582 146.0582
##      289      290      291      292      293      294      295      296
## 148.0514 148.0514 137.0884 149.0481 160.0111 141.0750 141.0750 163.0011
##      297      298      299      300      301      302      303
## 145.0615 147.0548 159.0145 136.0917 147.0548 147.0548 165.9910
```

Phân tích phương sai

```
aov = anova(res)
aov

## Analysis of Variance Table
##
## Response: MaxHR
##            Df Sum Sq Mean Sq F value    Pr(>F)
## Age         1  24507 24507.2  55.248 1.109e-12 ***
## Residuals 301 133519   443.6
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Phân tích residual

Phân phối chuẩn

- Tính y - y^

```
m = resid(res)
m

##            1            2            3            4            5
##   8.925045921 -29.088386747  -8.088386747  20.012358269   8.998925601
##            6            7            8            9           10
##  29.948553092  17.928404089  15.945194925   5.925045921   3.958627594
##           11           12           13           14           15
##   0.945194925   4.948553092  -6.051446908  12.988851099   9.961985761
##           16           17           18           19           20
##  26.945194925  11.975418430   9.955269427 -17.024581570  15.972060263
##           21           22           23           24           25
##   3.921687754  15.941836758  13.941836758  26.941836758 -12.064879577
##           26           27           28           29           30
##   3.968702095  25.941836758 -24.085028580   9.992209266 -49.997716232
##           31           32           33           34           35
##  15.904896918  15.935120423  17.921687754  15.938478590  18.988851099
##           36           37           38           39           40
```

```
##   15.995567433 -41.007790734 -35.054805075 -17.048088741  -6.068237744
##            41            42            43            44            45
## -25.081670413  14.002283768  28.898180584  11.938478590  25.931762256
##            46            47            48            49            50
##   18.941836758 -30.034656072 -26.031297905  17.918329587   0.958627594
##            51            52            53            54            55
##    4.998925601   0.918329587  -7.011148901  27.988851099  -0.064879577
##            56            57            58            59            60
## -41.044730573   8.968702095  -5.001074399   1.955269427 -28.034656072
##            61            62            63            64            65
## -11.034656072   1.982134764 -15.058163242  19.955269427 -37.044730573
##            66            67            68            69            70
##   -2.064879577  10.935120423  14.955269427  -5.061521410 -11.017865236
##            71            72            73            74            75
##    8.918329587  25.911613253 -43.071595911  18.918329587  16.988851099
##            76            77            78            79            80
##   11.918329587  -3.064879577 -11.034656072  23.975418430 -35.058163242
##            81            82            83            84            85
## -11.014507068  -8.041372406  17.005641935  13.908255085  19.961985761
##            86            87            88            89            90
##   19.988851099  -1.021223403 -36.041372406   8.958627594  -4.034656072
##            91            92            93            94            95
##   12.914971420   2.928404089   3.928404089  14.988851099  30.925045921
##            96            97            98            99           100
##    8.961985761  -3.061521410  12.935120423   5.961985761  29.975418430
##           101           102           103           104           105
##   25.985492932   4.022432771  11.945194925  -3.101819416 -16.027939737
##           106           107           108           109           110
##    5.955269427  16.938478590   2.945194925  -3.068237744 -24.994358065
##           111           112           113           114           115
##    2.931762256  -4.051446908  37.961985761 -25.007790734 -45.071595911
##           116           117           118           119           120
## -31.001074399  18.941836758  13.019074604  -9.074954079 -12.081670413
##           121           122           123           124           125
##   -6.024581570  12.925045921 -10.034656072 -38.048088741  34.918329587
##           126           127           128           129           130
##   15.985492932 -15.051446908 -24.044730573   9.988851099  20.928404089
##           131           132           133           134           135
##   -3.044730573   0.965343928  27.039223607  32.965343928   3.992209266
##           136           137           138           139           140
##   11.951911259  -9.098461249 -39.071595911 -38.980925396  12.965343928
##           141           142           143           144           145
##   18.938478590  13.938478590  31.961985761  -9.078312246   7.941836758
##           146           147           148           149           150
##   -5.021223403 -23.054805075  15.998925601  10.985492932  15.935120423
##           151           152           153           154           155
##   25.961985761 -40.004432567  22.911613253  -4.048088741 -44.078312246
##           156           157           158           159           160
## -25.098461249  19.965343928  24.941836758  25.935120423  14.908255085
##           161           162           163           164           165
```

```
##   -2.017865236  34.878031580   7.955269427 -24.058163242  18.975418430
##             166            167            168            169            170
##   20.945194925  16.961985761   8.955269427 -12.980925396 -21.014507068
##             171            172            173            174            175
##  -22.098461249 -40.041372406  -2.061521410  14.928404089  -8.078312246
##             176            177            178            179            180
##  -59.054805075  -5.038014239 -43.051446908   0.992209266  21.958627594
##             181            182            183            184            185
##    9.975418430   1.948553092  15.995567433  -0.061521410  16.935120423
##             186            187            188            189            190
##   37.925045921  31.995567433 -18.085028580  44.955269427  10.904896918
##             191            192            193            194            195
##    8.968702095 -31.034656072 -18.007790734 -36.071595911 -21.091744915
##             196            197            198            199            200
##  -12.088386747  -4.095103082  -7.014507068   7.968702095 -20.061521410
##             201            202            203            204            205
##    4.968702095  13.921687754  25.945194925  -7.078312246  -0.007790734
##             206            207            208            209            210
##  -12.014507068 -16.058163242 -28.031297905   5.951911259  11.928404089
##             211            212            213            214            215
##    3.012358269  16.009000102   4.998925601  26.914971420   7.961985761
##             216            217            218            219            220
##   13.948553092  13.982134764  -6.017865236 -18.078312246  36.938478590
##             221            222            223            224            225
##    8.998925601  16.955269427  14.005641935 -56.041372406  27.925045921
##             226            227            228            229            230
##   22.022432771 -14.021223403  34.911613253 -42.044730573  -6.085028580
##             231            232            233            234            235
##   16.961985761 -32.048088741 -29.027939737  -9.111893918  12.955269427
##             236            237            238            239            240
##  -34.044730573 -45.051446908 -14.017865236   6.972060263  -0.004432567
##             241            242            243            244            245
##  -10.001074399  -0.001074399   7.972060263   1.931762256 -48.064879577
##             246            247            248            249            250
##  -66.088386747   9.941836758 -39.021223403  15.961985761  -2.071595911
##             251            252            253            254            255
##  -21.054805075 -41.058163242 -35.078312246   3.965343928  19.992209266
##             256            257            258            259            260
##   10.995567433   4.911613253 -12.118610252   8.901538751  -6.054805075
##             261            262            263            264            265
##  -11.011148901   5.941836758  26.935120423   8.988851099 -18.068237744
##             266            267            268            269            270
##  -37.004432567   3.961985761 -11.061521410  17.002283768 -12.004432567
##             271            272            273            274            275
##   -5.068237744  -0.085028580 -38.017865236  -8.101819416  16.938478590
##             276            277            278            279            280
##   14.921687754  13.914971420 -12.994358065  16.945194925 -15.058163242
##             281            282            283            284            285
##   -4.054805075  21.978776597 -19.048088741   5.019074604  17.931762256
##             286            287            288            289            290
```

```
##   -6.058163242   -0.058163242   -2.058163242   14.948553092   20.948553092
##            291            292            293            294            295
##   12.911613253   16.951911259  -16.011148901    2.925045921   -5.074954079
##            296            297            298            299            300
##   18.998925601  -55.061521410  -24.054805075  -27.014507068    4.908255085
##            301            302            303
## -32.054805075   26.945194925    7.009000102
```

- Kiểm định phân phối chuẩn

```
hist(m)
```


Histogram of m
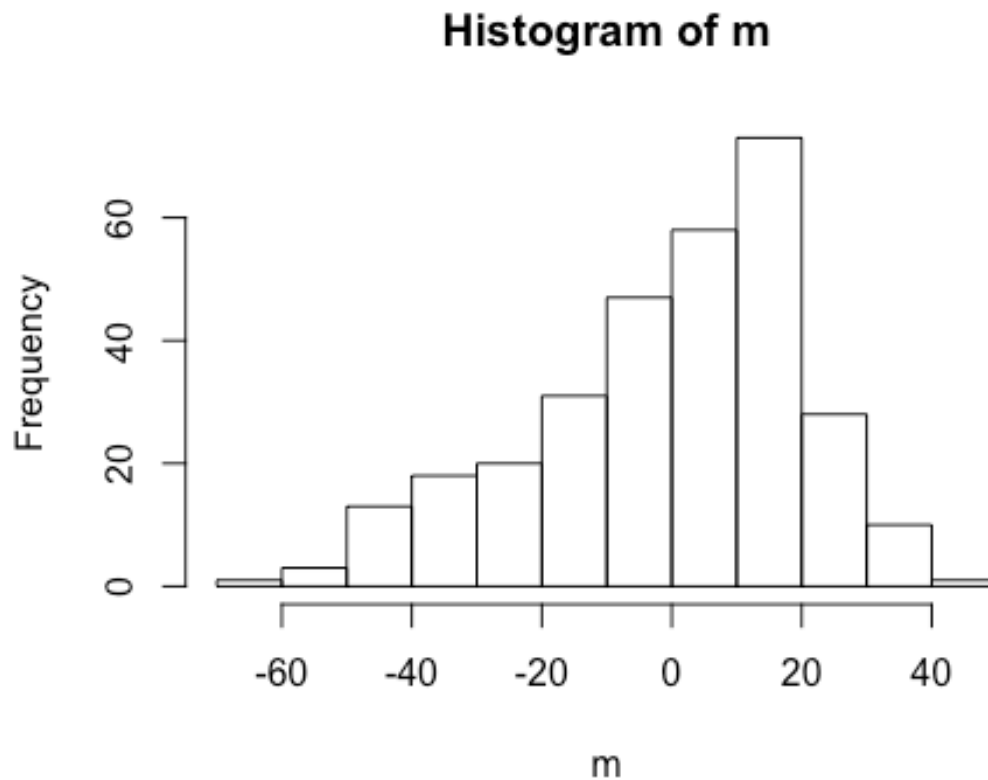
```
qqnorm(m, pch = 16)
qqline(m, col = 2, lwd = 3)
```

Normal Q-Q Plot
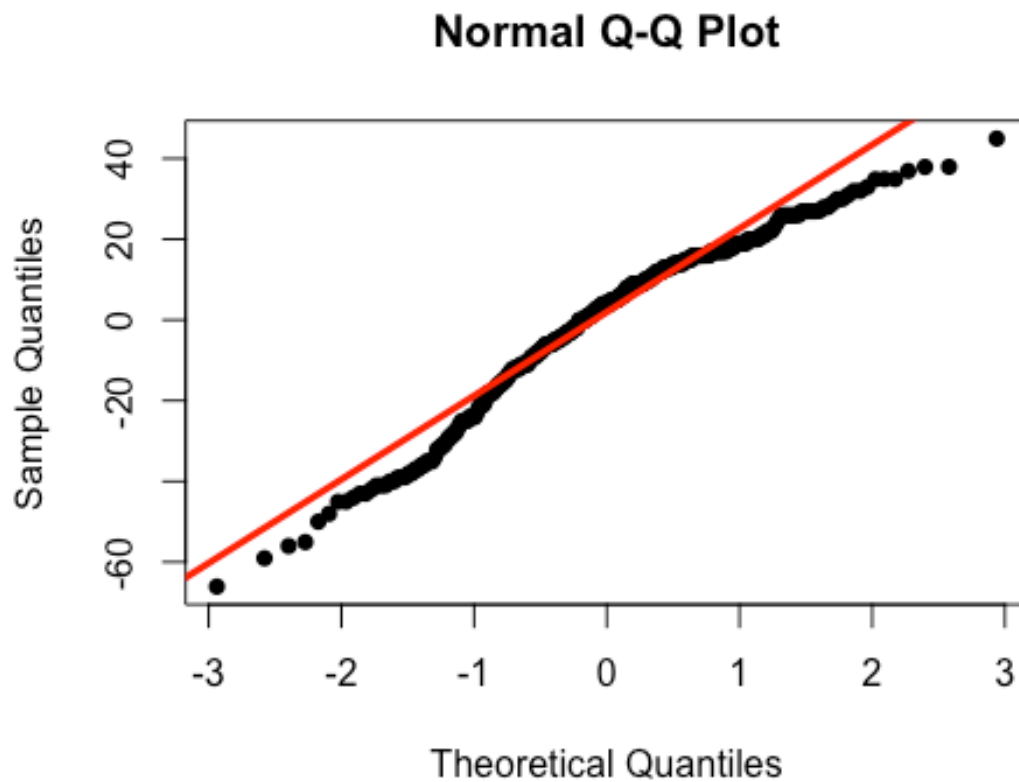

```r
shapiro.test(m)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  m
## W = 0.96133, p-value = 3.296e-07
```

- Trung bình bằng 0

```r
mean(m)
```

```
## [1] -6.282132e-16
```

- Kiểm tra phương sai

```r
ncvTest(res)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.7229959, Df = 1, p = 0.39516
```

```r
spreadLevelPlot(res, pch = 16)
```

Spread-Level Plot for res
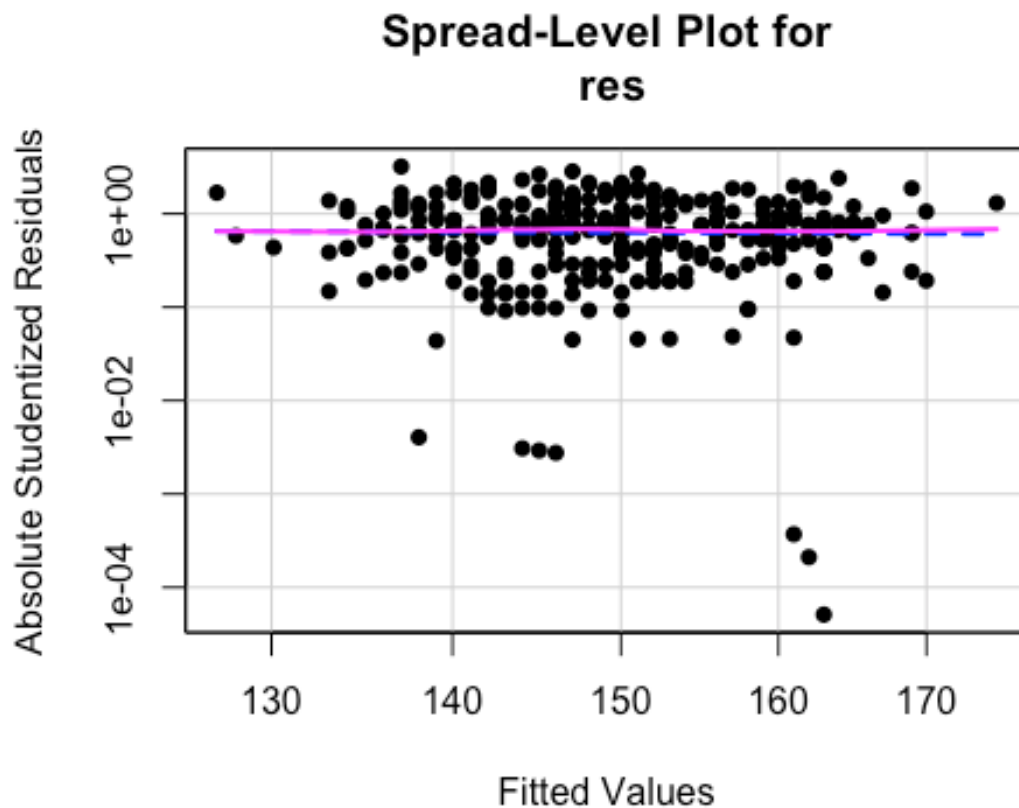
```
##
## Suggested power transformation:  1.217132
```

Hồi quy tuyến tính với biến phân nhóm

```
res1 = lm(MaxHR ~ Gender)
summary(res1)

##
## Call:
## lm(formula = MaxHR ~ Gender)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -77.845 -16.036   2.773  16.155  53.155
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  151.227      2.324  65.080   <2e-16 ***
## GenderMale    -2.382      2.818  -0.845    0.399
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.89 on 301 degrees of freedom
```

```
## Multiple R-squared:  0.002368,   Adjusted R-squared:  -0.0009463
## F-statistic: 0.7145 on 1 and 301 DF,  p-value: 0.3986
```

```r
boxplot(MaxHR ~ Gender, col = c('red','blue'))
```
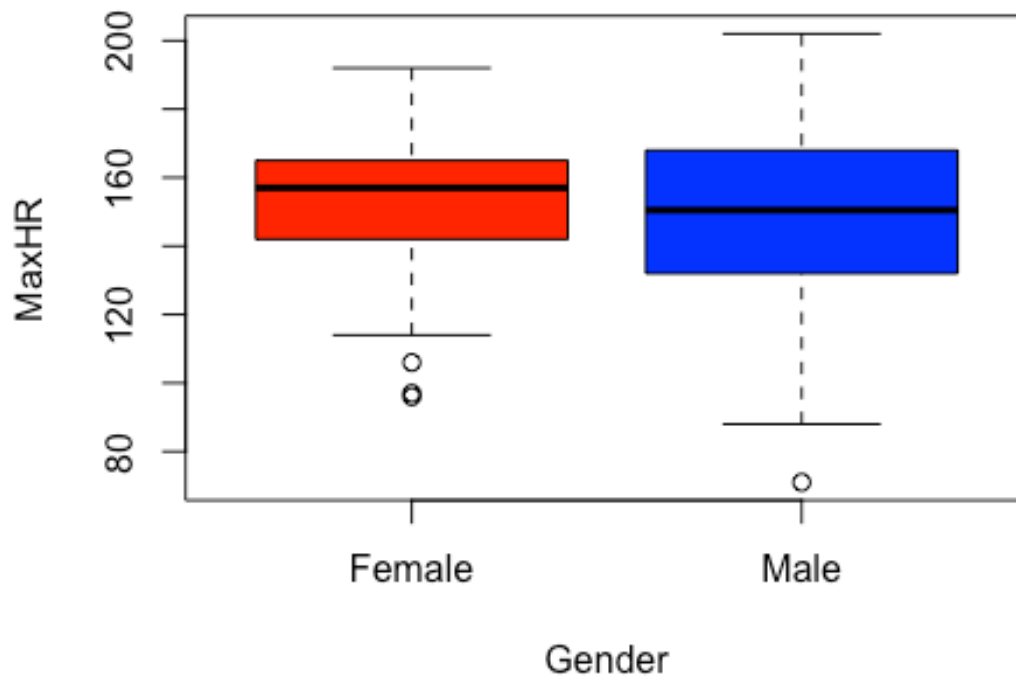

Hồi quy tuyến tính đa biến

```r
res2 = lm(MaxHR ~ RestBP + Age + Thal)
summary(res2)
```

```
##
## Call:
## lm(formula = MaxHR ~ RestBP + Age + Thal)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -70.046 -12.269   3.579  14.086  52.824
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 172.57393   11.48221  15.030  < 2e-16 ***
## RestBP        0.13314    0.06932   1.921 0.055720 .
## Age          -0.98287    0.13478  -7.292  2.8e-12 ***
```

```
## Thalnormal       18.34722     5.04476   3.637 0.000325 ***
## Thalreversable     7.69466     5.11709   1.504 0.133721
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.19 on 296 degrees of freedom
##   (2 observations deleted due to missingness)
## Multiple R-squared:  0.2301, Adjusted R-squared:  0.2197
## F-statistic: 22.12 on 4 and 296 DF,  p-value: 5.425e-16
```

Ảnh hưởng tương tác

```
res3 = lm(MaxHR ~ RestBP + Age + Thal  + Thal:Age)
summary(res3)

##
## Call:
## lm(formula = MaxHR ~ RestBP + Age + Thal + Thal:Age)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -67.300 -11.528   3.407  13.211  56.347
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       128.95595   36.95280   3.490 0.000558 ***
## RestBP              0.12410    0.06902   1.798 0.073191 .
## Age                -0.19128    0.63802  -0.300 0.764534
## Thalnormal         74.23500   37.37503   1.986 0.047937 *
## Thalreversable     32.78480   38.65078   0.848 0.396999
## Age:Thalnormal     -0.99950    0.65603  -1.524 0.128693
## Age:Thalreversable -0.43675    0.67762  -0.645 0.519725
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.07 on 294 degrees of freedom
##   (2 observations deleted due to missingness)
## Multiple R-squared:  0.2444, Adjusted R-squared:  0.2289
## F-statistic: 15.85 on 6 and 294 DF,  p-value: 8.885e-16
```

Xây dựng mô hình thống kê tối ưu

- Xử lý dữ liệu

```
dt = select(data, c(2:14))
head(dt)

##   Age Sex    ChestPain RestBP Chol Fbs RestECG MaxHR ExAng Oldpeak Slope
Ca
## 1  63   1      typical    145  233   1       2   150     0     2.3     3
```

```
0
## 2  67   1 asymptomatic    160  286   0        2   108    1    1.5    2
3
## 3  67   1 asymptomatic    120  229   0        2   129    1    2.6    2
2
## 4  37   1    nonanginal   130  250   0        0   187    0    3.5    3
0
## 5  41   0    nontypical   130  204   0        2   172    0    1.4    1
0
## 6  56   1    nontypical   120  236   0        0   178    0    0.8    1
0
##           Thal
## 1      fixed
## 2     normal
## 3 reversable
## 4     normal
## 5     normal
## 6     normal
```

- Xây dựng mô hình

```
model = lm(MaxHR ~ ., data = na.omit(dt))
```

- Chọn mô hình tối ưu

```
op = step(model)

## Start:  AIC=1740.99
## MaxHR ~ Age + Sex + ChestPain + RestBP + Chol + Fbs + RestECG +
##     ExAng + Oldpeak + Slope + Ca + Thal
##
##              Df Sum of Sq     RSS    AIC
## - Sex         1       0.5   93705 1739.0
## - Thal        2     714.5   94419 1739.2
## - RestECG     1     103.5   93808 1739.3
## - Fbs         1     130.9   93836 1739.4
## - Oldpeak     1     204.4   93909 1739.6
## - Ca          1     260.5   93965 1739.8
## <none>                      93705 1741.0
## - Chol        1     753.7   94458 1741.4
## - RestBP      1    1062.2   94767 1742.3
## - ChestPain   3    3261.9   96967 1745.2
## - Slope       1    4418.0   98123 1752.7
## - ExAng       1    4689.1   98394 1753.5
## - Age         1   13092.8  106798 1777.8
##
## Step:  AIC=1738.99
## MaxHR ~ Age + ChestPain + RestBP + Chol + Fbs + RestECG + ExAng +
##     Oldpeak + Slope + Ca + Thal
##
##              Df Sum of Sq     RSS    AIC
## - RestECG     1     105.5   93811 1737.3
```

```
## - Thal        2      747.6   94453 1737.3
## - Fbs         1      131.6   93837 1737.4
## - Oldpeak     1      203.9   93909 1737.6
## - Ca          1      260.0   93965 1737.8
## <none>                       93705 1739.0
## - Chol        1      774.6   94480 1739.4
## - RestBP      1     1070.6   94776 1740.4
## - ChestPain   3     3303.9   97009 1743.3
## - Slope       1     4477.9   98183 1750.8
## - ExAng       1     4704.6   98410 1751.5
## - Age         1    13253.5  106959 1776.3
##
## Step:  AIC=1737.32
## MaxHR ~ Age + ChestPain + RestBP + Chol + Fbs + ExAng + Oldpeak +
##     Slope + Ca + Thal
##
##               Df Sum of Sq    RSS    AIC
## - Thal        2      758.0   94569 1735.7
## - Fbs         1      140.8   93951 1735.8
## - Oldpeak     1      206.8   94017 1736.0
## - Ca          1      240.7   94051 1736.1
## <none>                       93811 1737.3
## - Chol        1      871.6   94682 1738.1
## - RestBP      1     1137.1   94948 1738.9
## - ChestPain   3     3251.5   97062 1741.4
## - Slope       1     4392.3   98203 1748.9
## - ExAng       1     4686.0   98497 1749.8
## - Age         1    13184.0  106995 1774.4
##
## Step:  AIC=1735.71
## MaxHR ~ Age + ChestPain + RestBP + Chol + Fbs + ExAng + Oldpeak +
##     Slope + Ca
##
##               Df Sum of Sq    RSS    AIC
## - Fbs         1       90.2   94659 1734.0
## - Oldpeak     1      223.5   94792 1734.4
## - Ca          1      331.9   94901 1734.8
## <none>                       94569 1735.7
## - RestBP      1     1020.2   95589 1736.9
## - Chol        1     1085.9   95655 1737.1
## - ChestPain   3     3653.3   98222 1741.0
## - ExAng       1     5008.0   99577 1749.0
## - Slope       1     5152.4   99721 1749.5
## - Age         1    13186.6  107755 1772.5
##
## Step:  AIC=1734
## MaxHR ~ Age + ChestPain + RestBP + Chol + ExAng + Oldpeak + Slope +
##     Ca
##
##               Df Sum of Sq    RSS    AIC
```

```
## - Oldpeak    1      251.0  94910 1732.8
## - Ca         1      285.6  94944 1732.9
## <none>                     94659 1734.0
## - Chol       1     1070.5  95729 1735.3
## - RestBP     1     1147.3  95806 1735.6
## - ChestPain  3     3830.4  98489 1739.8
## - ExAng      1     4969.9  99629 1747.2
## - Slope      1     5086.7  99746 1747.5
## - Age        1    13121.8 107781 1770.5
##
## Step:  AIC=1732.78
## MaxHR ~ Age + ChestPain + RestBP + Chol + ExAng + Slope + Ca
##
##             Df Sum of Sq    RSS    AIC
## - Ca         1      445.4  95355 1732.2
## <none>                     94910 1732.8
## - RestBP     1     1042.6  95952 1734.0
## - Chol       1     1065.2  95975 1734.1
## - ChestPain  3     3999.2  98909 1739.0
## - ExAng      1     5269.1 100179 1746.8
## - Slope      1     8707.8 103618 1756.9
## - Age        1    13067.3 107977 1769.1
##
## Step:  AIC=1732.17
## MaxHR ~ Age + ChestPain + RestBP + Chol + ExAng + Slope
##
##             Df Sum of Sq    RSS    AIC
## <none>                     95355 1732.2
## - Chol       1     1014.0  96369 1733.3
## - RestBP     1     1054.1  96409 1733.4
## - ChestPain  3     4757.4 100113 1740.6
## - ExAng      1     5310.0 100665 1746.3
## - Slope      1     8761.6 104117 1756.3
## - Age        1    16241.2 111597 1776.9

op

##
## Call:
## lm(formula = MaxHR ~ Age + ChestPain + RestBP + Chol + ExAng +
##     Slope, data = na.omit(dt))
##
## Coefficients:
##         (Intercept)                  Age  ChestPainnonanginal
##           187.70592             -0.88515              7.39669
## ChestPainnontypical      ChestPaintypical               RestBP
##             9.97711             11.14766              0.11303
##                Chol                ExAng                Slope
##             0.03665            -10.31114             -9.35208
```

```
summary(op)

##
## Call:
## lm(formula = MaxHR ~ Age + ChestPain + RestBP + Chol + ExAng +
##     Slope, data = na.omit(dt))
##
## Residuals:
##     Min     1Q  Median     3Q    Max
## -60.947 -12.047   2.099  11.722  44.007
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          187.70592   10.13348  18.523  < 2e-16 ***
## Age                   -0.88515    0.12638  -7.004 1.76e-11 ***
## ChestPainnonanginal    7.39669    2.73949   2.700  0.00734 **
## ChestPainnontypical    9.97711    3.31704   3.008  0.00286 **
## ChestPaintypical      11.14766    4.25312   2.621  0.00923 **
## RestBP                 0.11303    0.06335   1.784  0.07543 .
## Chol                   0.03665    0.02094   1.750  0.08118 .
## ExAng                -10.31114    2.57474  -4.005 7.91e-05 ***
## Slope                 -9.35208    1.81799  -5.144 4.98e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.2 on 288 degrees of freedom
## Multiple R-squared:  0.3879, Adjusted R-squared:  0.3709
## F-statistic: 22.82 on 8 and 288 DF,  p-value: < 2.2e-16
```