CS 477/677 Analysis of Algorithms

Fall 2018

Homework 1

Due date: September 11, 2018

1. **(U & G-required) [30 points]** Arrange the following list of functions in ascending order of growth rate. That is, if function g(n) immediately follows function f(n) in your list, then f(n) should be O(g(n)).

$f_1(n) = (n-2)!$

$f_2(n) = 5\lg(n+100)^n$

$f_3(n) = 2^{2n}$

$f_4(n) = 0.001n^4 + 3n^3 + 1$

$f_5(n) = \ln^2 n$

$f_6(n) = \sqrt[3]{n}$

$f_7(n) = 3^n$

2. **(U & G-required) [30 points]** Using the informal definition for the Θ notation, select the correct Θ notation for the following expressions:

(a) $2(\lg n)^2 + 4n + 3n^2\lg n$

(b) $(6n^3\lg n + 4)(10+n)$

(c) $\frac{(n^2+\lg n)(n+1)}{n+n^2}$

(d) $2 + 4 + 8 + 16 + \ldots + 2^n$

(e) $8^{\lg n}$

3. **(U & G-required) [40 points]** Using mathematical induction, show that the following relations are true for every $n \geq 1$:

a) $\sum_{i=1}^{n}(-1)^{i+1}i^2 = \frac{(-1)^{n+1}n(n+1)}{2}$

b) $\sum_{i=1}^{n}\frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$

4. (G-Required) [20 points] Using the formal definition of the asymptotic notations, prove the following statements:

a) $10n^2 + 1 \in O(n^3)$

b) $5n^2 + 10 \in \Omega(n)$

Extra credit:

5. [Extra credit - 20 points]

a) Find the order of growth for the following sum: $\sum_{i=0}^{n-1}(i+2)^2$

b) Use the formal definition of the asymptotic notation to prove that:

$$30n^2 + 100 \notin \Omega(n^3)$$