CS 677 Homework
Assignment 01

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Semtember 4, 2018

1. Arrange the list of functions in ascending order of growth rate:
   $f_1(n) = (n-1)!$
   $f_2(n) = 5lg(n+100)^n$
   $f_3(n) = 2^{2n}$
   $f_4(n) = 0.001n^4 + 3n^3 + 1$
   $f_5(n) = ln^2 n$
   $f_6(n) = \sqrt[3]{n}$
   $f_7(n) = 3^n$
   Ascending order: $f_5(n) < f_6(n) < f_2(n) < f_4(n) < f_7(n) < f_3(n) < f_1(n)$

2. Using the informal definition for the Θ notation, select the correct notation for the following expression:
   (a) $2(lgn)^2 + 4n + 3n^2 lgn = \Theta(n^2 lgn)$
   (b) $(6n^3 lgn + 4)(10 + n) = \Theta(n^4 lgn)$
   (c) $\frac{(n^2+lgn)(n+1)}{n+n^2} = \Theta(n)$
   (d) $2 + 4 + 8 + 16 + ... + 2^n = \Theta(2^n)$
   (e) $8^{lgn} = \Theta(n^3)$

3. Using mathematical induction, show that the following relations are true for every $n \geq 1$
   (a) $\sum_{i=1}^{n}(-1)^{(i+1)} i^2 = \frac{(-1)^{n+1} n(n+1)}{2}$

   Base case: $n = 1 : (-1)^2 \times 1^2 = \frac{(-1)^{1+1} 1(1+1)}{2}$ *(True)*
   Inductive case:

   Assume: $\sum_{i=1}^{n}(-1)^{(i+1)} i^2 = \frac{(-1)^{n+1} n(n+1)}{2}$ is True

   Prove: $\sum_{i=1}^{n+1}(-1)^{(i+1)} i^2 = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$

   $\rightarrow \sum_{i=1}^{n}(-1)^{(i+1)} i^2 + (-1)^{n+2}(n+1)^2 = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$

   $\rightarrow \frac{(-1)^{n+1} n(n+1)}{2} + (-1)^{n+2}(n+1)^2 = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$

   $\rightarrow (-1)^{n+1}(n+1)(\frac{n}{2} + (-1)(n+1)) = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$

   $\rightarrow (-1)^{n+1}(n+1)(\frac{n-2n-2}{2}) = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$

   $\rightarrow \frac{(-1)^{n+2}(n+1)(n+2)}{2} = \frac{(-1)^{n+2}(n+1)(n+2)}{2}$ *(True)*

   (b) $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$

   Base case: $n = 1 : \frac{1}{1 \times 3} = \frac{1}{2 \times 1 + 1}$ *(True)*
   Inductive case:

   Assume: $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$ is True

   Prove: $\sum_{i=1}^{n+1} \frac{1}{(2i-1)(2i+1)} = \frac{n+1}{2n+3}$

   $\rightarrow \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} + \frac{1}{(2n+1)(2n+3)} = \frac{n+1}{2n+3}$

$\rightarrow \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} = \frac{n+1}{2n+3}$

$\rightarrow \frac{n(2n+3)+1}{(2n+1)(2n+3)} = \frac{n+1}{2n+3}$

$\rightarrow \frac{(n+1)(2n+1)}{(2n+1)(2n+3)} = \frac{n+1}{2n+3}$

$\rightarrow \frac{n+1}{2n+3} = \frac{n+1}{2n+3}$ *(True)*

4. Using the formal definition of the asymptotic notations, prove the following statements:

   (a) $10n^2 + 1 \in O(n^3)$
   Prove with $\exists c, n_0, \forall n \geq n_0$:

   $$10n^2 + 1 \leq cn^3 \tag{1}$$

   We have:

   $$10n^2 + 1 \leq 10n^3 + n^3 = 11n^3 \tag{2}$$

   In order to satisfy Inequation 1, from Inequation 2 we can choose $c = 11, n_0 = 1$.

   (b) $5n^2 + 10 \in \Omega(n)$
   Prove with $\exists c, n_0, \forall n \geq n_0$:

   $$5n^2 + 1 \geq cn \tag{3}$$

   We have:

   $$5n^2 + 1 \geq 5n^2 \geq 5n \tag{4}$$

   In order to satisfy Inequation 3, from Inequation 4 we can choose $c = 5, n_0 = 1$.

5. Extra

   - Find the order of growth for the following sum: $\sum_{i=1}^{n-1}(i+2)^2$
   We have:

   $$\sum_{i=1}^{n-1}(i+2)^2 = \sum_{i=1}^{n-1}(i^2 + 4i + 4) = \sum_{i=1}^{n-1} i^2 + 4\sum_{i=1}^{n-1} i + 4(n-1) = \frac{(n-1)n(2n-1)}{6} + 4\frac{(n-1)n}{2} + 4(n-1) = \Theta(n^3)$$

   - Use the formal definition of the asymptotic notation to prove that:

   $$30n^2 + 100 \notin \Omega(n^3) \tag{5}$$

   Assume:

   $$30n^2 + 100 \in \Omega(n^3)$$

   It means $\exists c, n0$ so that with $\forall n \geq n0$:

   $$30n^2 + 100 \geq cn^3 \tag{6}$$

   We have:

   $$30n^2 + 100 \leq 30n^2 + 100n^2 = 130n^2 \tag{7}$$

From Inequation 6 and 7:

$$cn^3 \leq 130n^2 \rightarrow n \leq \frac{130}{c} \tag{8}$$

Inequation 8 cannot satisfy with $\forall n \geq n0$ so 5 holds True.