Assignment 3

DISC Nonlinear Control Systems 2017-2018

Due date: Monday 5 March 2018

Note

- Please send the solutions to `{b.besselink, b.jayawardhana}@rug.nl` with the email subject "DISC NCS: Assignment 3".

Exercises

1. Exercise 13.12 from [1]. Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_1 x_2,\\ \dot{x}_2 &= x_2 + x_3,\\ \dot{x}_3 &= \delta(x) + u,\\ y &= x_1 + x_2,\end{aligned}$$

   with $\delta : \mathbb{R}^3 \to \mathbb{R}$ a locally Lipschitz function of x.

   a) Is the system input-output linearizable?
   b) If yes, transform it into the normal form and specify the region of which the transformation is valid.
   c) Is the system minimum phase?
   d) Is the system feedback linearizable?
   e) If yes, find a feedback control law and a change of variables that linearize the state equation.

2. Exercise 13.17 from [1]. Show that the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2,\\ \dot{x}_2 &= x_1 - x_2 - x_1 x_3 + u,\\ \dot{x}_3 &= x_1 + x_1 x_2 - 2x_3\end{aligned}$$

   is feedback linearizable and design a state feedback control law to globally stabilize the origin.

3. Exercise 13.24 from [1]. Consider the system

$$\dot{\eta} = f_0(\eta, \xi), \tag{1}$$

$$\dot{\xi} = (A - BK)\xi + B\delta(z), \tag{2}$$

   where $A - BK$ is Hurwitz, the origin of (1) is asymptotically stable with a Lyapunov function V_0 such that

$$\frac{\partial V_0}{\partial \eta}(\eta) f_0(\eta, 0) \leq -W(\eta)$$

   for some positive definite function W. Suppose

$$\|\delta(z)\| \leq k\big(\|\xi\| + W(\eta)\big).$$

   Using a composite Lyapunov function of the form $V(\eta, \xi) = V_0(\eta) + \lambda\sqrt{\xi^{\mathrm{T}} P \xi}$, where P is the solution of

$$P(A - BK) +_(A - BK)^{\mathrm{T}} P = -I,$$

   show that, for sufficiently small k, the origin $z = 0$ is asymptotically stable.

References

[1] H.K. Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle River, USA, third edition, 2002.