Homework I Mathematical Models of Systems, winter 2018

Hand out: January 29th

Hand in: February 12th

- It is allowed to discuss the problems with your colleagues.
- Cooperation in groups of two people of approximately the same level is allowed.
- Everybody hands in his or her *own* version.
- It is not allowed to copy from each other.
- The exercisenumbers refer to the Springer edition of the book. The pdf of the web site contains some additional exercises and is hence out of sync with the book.

  Between brackets the exercise numbers on the pdf are mentioned.

  In addition, a conversion table may be downloaded from the web site.

1. Exercise 1.9.
2. Exercises 2.3, 2.5 (Pdf: 2.6), 2.7 (Pdf: 2.8), 2.12 (Pdf: 2.13), 2.25 (Pdf: 2.26) notice that the coefficients in the linear combination are *polynomials*.
3. Exercises 3.1, 3.6, 3.20, 3.22, 3.36
4. Additional exercise. Let the trajectories

   $$w_1(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathrm{e}^{t} \quad w_2(t) = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \mathrm{e}^{-2t}$$

   be given.

   (a) Determine a polynomial matrix $P(\xi) \in \mathbb{R}^{2\times 2}[\xi]$ such that $P(\frac{d}{dt})w_i = 0$ for $i = 1, 2$ and moreover the behavior $\mathfrak{B}$ defined by $P(\xi)$ is as small as possible, i.e., $\mathfrak{B}$ consists of linear combination and shifts of w_1, w_2. Remark. The resulting behavior is called the *Most Powerful Unfalsified Model*, MPUM for short, of w_1, w_2. Notice that without loss of generality you can take $P(\xi)$ to be upper triangular.

   (b) Suppose now that a third trajectory is given:

   $$w_3(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathrm{e}^{2t}$$

   Determine the MPUM of w_1, w_2, w_3.
5. Simulation exercise A.3 (p388).