Homework 3 **Due: 23/04/2018 (Monday)**

Problem 1: Consider the following two-player, symmetric game where x can be 0, 1 or 2:

		Player B	
		X	Y
Player A	X	$1, 1$	$2, x$
	Y	$x, 2$	$3, 3$

a) For each of the possible values of x, find all pure strategy Nash equilibria and all evolutionarily stable strategies (ESS). (Hint: We have defined ESS for replicator dynamics in the class. To solve this problem, you need to check in the literature what the definition of ESS is for two-player games.)
b) Your answers to part a) should suggest that the difference between the predictions of evolutionary stability and Nash equilibrium arises when a Nash equilibrium uses a weakly dominated strategy. We say that a strategy s_i^* is weakly dominated if player i has another strategy s_i' with the property that: (i) No matter what the other player does, player i's payoff from s_i' is at least as large as the payoff from s_i* and (ii) There is some strategy for the other player so that player i's payoff from s_i' is strictly greater than the payoff from s_i^*.
Now consider the following claim that makes a connection between ESS and weakly dominated strategies:
Claim: Suppose that in the game below, $\{X, X\}$ is a Nash equilibrium and that strategy X is weakly dominated. Then X is not an evolutionary stable strategy.

		Player B	
		X	Y
Player A	X	a, a	b, c
	Y	c, b	d, d

Explain carefully why this claim is true.

Problem 2: Consider a symmetric two-player game with the following payoff matrix for player 1

$$A = \begin{pmatrix} 1 & 2 & 0 & -2 \\ 3 & 1 & 1 & 3 \\ 0 & 4 & 3 & -1 \\ 1 & 2 & -1 & 1 \end{pmatrix}.$$

Consider the first pure strategy of player 1.

a) Is it weakly dominated by a pure strategy?
b) Is it weakly dominated by a mixed strategy?
c) Is it strongly dominated by a strategy?
d) Is it possible for player 2 to play a strategy such that both players are at a Nash equilibrium?

Now assume both players can play any of their four strategies.
e) Calculate the set of Nash equilibrium analytically (Hint: One of the strategies 2, 3 or 4 may be strongly dominated after deleting the first column).
f) Is there any evolutionary stable strategy in this game? If yes, determine the strategy and explain why it is evolutionary stable. If no, explain why.

Problem 3: For replicator dynamics, show that a state $\hat{x}$ can be the ω-limit point of all orbits $x(t)$ in the interior of the simplex Δ without being Lyapunov stable. (Hint: try the matrix

$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.)

Problem 4: For replicator dynamics discussed in the lecture with the payoff matrix A, a closed nonempty subset $\mathcal{E}$ of the simplex Δ is said to be an *evolutionarily stable set* (or ES set) if for each $x \in \mathcal{E}$ there exists a neighborhood $\mathcal{W}$ such that

$$x^T Ay \geq y^T Ay$$

for all $y \in \mathcal{W}$, with strict inequality if $y \neq \mathcal{E}$. Show that every ES set is asymptotically stable (with the obvious definition for the asymptotically stability of a closed set.)