Homework 4 **Due: 23/04/2018 (Monday)**

Problem 1: Consider the following replicator dynamics

$$\dot{x}_i = x_i((Ax)_i - x^T Ax), i = 1, 2, 3, 4,$$

which describe the evolution of the shares of players of four different strategies in a large population. Now the matrix A takes the form

$$A = \begin{pmatrix} 0 & 0 & -\mu & 1 \\ 1 & 0 & 0 & -\mu \\ -\mu & 1 & 0 & 0 \\ 0 & -\mu & 1 & 0 \end{pmatrix}.$$

a) Find all the equilibrium points of the replicator dynamics.
b) Compute the Jocobian matrix D of the interior equilibrium, i.e. the equilibrium not on the faces of the simplex $\{x | 0 \leq x_i \leq 1 \text{ and } \sum_{i=1}^{4} x_i = 1\}$. What are the eigenvalues of D?
c) Use Matlab to find the possible values of μ and plot the corresponding phase portraits of the replicator dynamics for the following three situations: (i) the interior equlibrium is stable; (ii) the interior equlibrium is unstable; and (3) there is a periodic attractor near the interior equlibrium.

Problem 2: In graph theory, a binary tree is a tree in which each vertex has at most two children. Consider a binary tree of 7 vertices, labeled by $1, \ldots, 7$, in three layers, for which vertex 1 is the root, vertices 4, 5, 6 and 7 are the leaves, and vertices 2 and 3 are in the middle layer. Suppose 7 agents are playing the evolutionary prisoner dilemma game on this binary tree, each vertex corresponding to a player and each edge corresponding to a possible 2-player game. Each player plays games using one of the two pure strategies, to cooperate or to defect, with all its neighbors and accumulates payoffs. The payoff matrix is

$$\begin{pmatrix} 0.8 & -0.2 \\ 1 & 0 \end{pmatrix},$$

and correspondingly to defect is the dominating strategy against to cooperate. Consider the control action that we can choose some controlled agents and fix them to be cooperative. Please find the minimum number of controlled agents and indicate who they will be in order to render *all* the 7 agents to be cooperative asymptotically for the following two cases:
a) if the uncontrolled agents update their strategies according to the best-response rule;
b) if the uncontrolled agents update their strategies according to the imitating rule.

Problem 3: Consider all networks of 6 agents under the linear threshold model, in which the agents are a mixture of coordinating and anti-coordinating agents.
a) Can the evolutionary dynamics on the network converge to an equilibrium? Justify your answer by an example or proof.
b) Can the evolution converge to a periodic solution of period 2? Justify your answer by an example or proof.
c) Can the evolution converges to a periodic solution of period 3? Justify your answer by an example or proof.
d) Can you find situations in which the evolution will *not* converge? Justify your answer by an example or proof.