Converse Lyapunov theorem and Input-to-State Stability

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Please send your solutions to
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by Friday, February 26, 2018 with email subject
"DISC Nonlinear Control System - Solutions to Assignment 2".
If you have problems in meeting the deadline please contact the instructors.
Note that you are allowed to write down your answers in paper and submit the scanned copies to us.

Exercise 1. (ISS & sector-bound nonlinearity)
Consider the following nonlinear system

$$\begin{aligned}\dot{x} &= \begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix} x + \begin{bmatrix}0\\ 1\end{bmatrix}\big(-f(y)+d\big)\\ y &= \begin{bmatrix}1 & 0\end{bmatrix} x,\end{aligned}$$

where f is a smooth function satisfying the following sector condition

$$c_1 y^2 < f(y)y < c_2 y^2,$$

where $0 < c_1 < c_2$.

a. Suppose that $c_1 = 2$ and $c_2 = 6$, show that the closed-loop system is ISS with respect to d and provide the corresponding ISS Lyapunov function. (Hint: you can consider a quadratic function as the ISS Lyapunov function.) **(Marks: 25)**

b. What is the lower-bound of c_1 such that the closed-loop system is ISS with respect to d? **(Marks: 25)**

Exercise 2. (Counter-examples)

a. Provide an example of state equation

$$\dot{x} = f(x, u)$$

that is globally asymptotically stable with zero input (*i.e.*, $u = 0$) but has a finite-escape time for some bounded signal u. **(Marks: 25)**

b. Provide an example of a state equation

$$\dot{x} = f(x, u, d),$$

where x is the state, u is the input and d is an external disturbance signal, which can be stabilized (when $d = 0$) by a state feedback $u = \alpha(x)$, but cannot be made ISS with respect to the disturbance signal d by any state-feedback law. **(Marks: 25)**