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1. Introduction

As the threat of climate catastrophe continues to loom, governments are working to prioritize green energy. Canada is not an outlier in this regard as the country has signed on to large-scale climate agreements and every major party had climate goals in their platform in the last election. On the provincial rather than federal level key goals have been made. Quebec is striving for carbon neutrality by 2050, with 5 billion dollars a year being allocated to the energy transition from 2022 to 2026 (Rolland, 2022). Nova Scotia has expanded its goals for offshore wind generation and is striving to be a leader in this field, similarly, this is to pursue the goal of carbon neutrality by 2050 (Communications Nova Scotia, 2018). With so much money and environmental stakes riding it is necessary that these farms are as optimal as possible, this includes where the wind farm is located but also where each turbine within the wind farm is placed. The layout is not always as simple as solely maximizing the number of wind turbines in a certain area. This problem is referred to as the Wind Farm Layout Optimization Problem (WFLOP) (Samorani, 2013).

The application optimization research in this field is often tied back to 1994 when Mosetti et al researched the optimal layout with a genetic algorithm (Wang, 2022). In this novel paper, the authors wanted to extract the maximum energy from a wind farm while minimizing the cost of installation for the farm. The authors also require key considerations including that the placed turbines will affect one another, and that the intensity and direction of wind changes (Mosetti, 1994). The influence of one wind turbine on another is called the wake effect. What the wake effect means is that after the wind connects with the first turbine it will be disrupted, if the wind is disrupted it connects with another turbine, then the power generated from this turbine will be decreased (Kiamchr, 2014). Further, as the turbines are not designed to receive this disrupted wind, the turbines receiving it will have increased maintenance costs (Charhouni, 2019). This wake effect is popularly visualized by the Jensen Model which is displayed below (Appendix A). While the Jensen Model is effective, if it is simple visualization, the Gaussian-Jensen wake model can be used for a deeper representation of the wake model, the Gaussian-Jensen model is shown below (Appendix B). The wake effect clearly makes the layout of the wind farm less intuitive than simply placing as many turbines as possible. This is the case regardless of whether wind direction and speed are constants or variable factors that require adjustment (Mosetti, 1994). It is also important to know that this is a necessary effect to consider for both onshore and offshore wind farms.

2. Problem Description and Formulation

2.1. Problem Description and Scope

The basic optimization model was built by maximizing the amount of energy output produced by wind turbines at wind farm locations. Currently, according to the Canadian Renewable Energy Association, Canada has 317 wind energy projects producing power across the country. To save on computation time as well as create a simplified yet realistic optimization problem, the Pubnico Point wind project located in Yarmouth County, Nova Scotia (Figure 1) has been chosen as the region of interest where wind turbine optimization would occur. Pubnico Point was chosen due to the fact that the borders are easily defined and laid out for problem-scoping purposes. In addition, the project site is relatively flat and there are no naturally occurring lakes or streams on the site, the topography condition at Pubnico Point makes it an ideal wind farm position, and the constraints imposed can be minimized to construct the problem.

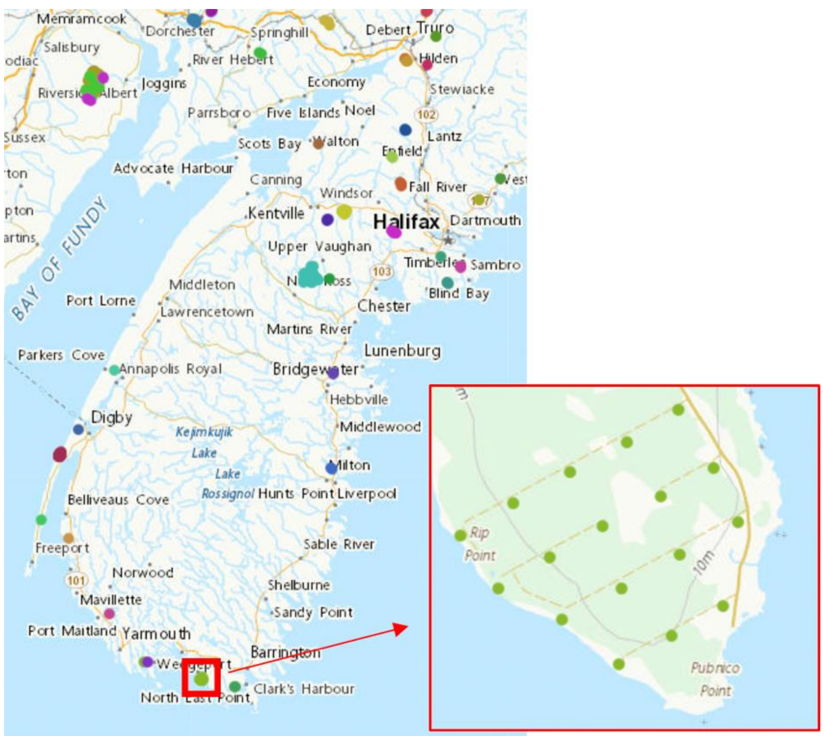


Figure 1: Wind farms in Nova Scotia

2.2. Variables and Parameters Description

2.2.1. Parameters

- P : Power capacity, kw per turbine

- L : The number of nodes horizontally in the grid

In the base model, we are making assumptions that the layout matrix of a wind farm is a square, therefore using the same number of grids for both horizontal and vertical.

2.2.2. Decision Variables

- $X_{i,j}$: A binary variable that indicates the wind turbine to be built at node (i, j) if $X_{i,j}$ = 1, and 0 otherwise.

2.3. Main Model Formulation

Our objective function focuses on maximizing the energy production of the allocated wind turbines, subtracting the amount of energy loss created by the wake effect of wind turbines. The amount of energy dissipated due to the wake effect is the product of a negative power multiplier, which is a pre-determined parameter assumed based on the make and model of the wind turbines, on the foundation of external research papers, the individual energy production of each turbine, and the 2 decision variables to determine whether there are other turbines diagonally located to the selected turbine.

Taking into consideration energy production from the wind, wind turbines are used. They are often defined by the wake effect where cones (wakes) of slower and more turbulent air are created behind them. To mitigate this phenomenon, it is recommended that wind turbines be installed with minimum distance between them. If the wind turbines that will be used are manufactured by the same vendor and the rotor diameter for each of them is R, the wind farmland can be divided into square cells, each of which possesses an edge of αR meters, where a is an integer number larger than 1.

For this paper, we will utilize the Vestas V80/1800 model Wind Turbine. This model is currently being used at the actual farm; hence the optimal results could be compared with the actual allocation of Pubnico Point wind farm.

For this paper, we will construct two models with two different objective functions, but the same set of constraints as follows:

- ***Model 1/Base model***: The objective function of Model 1 aims to maximize the energy output of all the allocated wind turbines:

$$Max\,(\sum_{i=0}^{L-1}\sum_{j=0}^{L-1} X_{i,j}.P)$$

- ***<u>Model 2</u>***: The objective function of Model 2 aims to maximize the energy output of all the allocated wind turbines but also take into consideration the amount of energy loss due to the wake effect if turbines are placed in the vicinity of one another. Since the wind turbines in the Pubnico Point are all structured facing west, when considering the wake effect in the problem, we will consider the affected turbines on the east of the point as the turbines at the back of the candidate point, which is j in the objective function, and i as the turbines on the left and right:

$$Max\left(\sum_{i=1}^{L-3}\sum_{j=0}^{L-3}X_{i,j}\ \cdot P-0.33\cdot P\cdot\left(X_{i+1,j+1}\cdot X_{i,j}+X_{i,j+2}\cdot X_{i,j}+X_{i-1,j+1}\cdot X_{i,j}\right)\right)$$

Subject to:

$$X_{i,j}\ \cdot\ \left(X_{i+1,j}+X_{i-1,j}+X_{i,j+1}+X_{i,j-1}\right)=0,\quad i\in\{1\ldots\ L-1\} \qquad (1.1)$$

$$X_{0,0}\ \cdot\ \left(X_{0,1}+X_{1,0}\right)\ =0 \qquad (1.2)$$

$$X_{0,L-1}\cdot\left(X_{0,L-2}+X_{1,L-1}\right)\ =\ 0 \qquad (1.3)$$

$$X_{L-1,0}\cdot\left(X_{L-1,1}+X_{L-2,0}\right)\ =\ 0 \qquad (1.4)$$

$$X_{L-1,L-1}\ \cdot\ \left(X_{L-1,L-2}+X_{L-2,L-1}\right)\ =\ 0 \qquad (1.5)$$

$$X_{0,j}\cdot\left(X_{0,j-1}+X_{0,j+1}+X_{1,j}\right)\ =\ 0,\quad j\ \in\ \{1,\ldots,\ L-2\} \qquad (1.6)$$

$$X_{L-1,j}\cdot\left(X_{L-1,j-1}+X_{L-1,j+1}+X_{L-2,j}\right)\ =\ 0,\quad j\ \in\ \{1,\ldots,\ L-2\} \qquad (1.7)$$

$$X_{i,L-1}\cdot\left(X_{i-1,L-1}+X_{i+1,L-1}+X_{i,L-2}\right)\ =\ 0,\quad i\ \in\ \{1,\ldots,\ L-2\} \qquad (1.8)$$

$$X_{i,0}\cdot\left(X_{i+1,0}+X_{i-1,0}+X_{i,1}\right)\ =\ 0,\quad i\ \in\ \{1,\ldots,\ L-2\} \qquad (1.9)$$

$$X_{i,j}\ \in\ \{0,1\} \qquad (2.0)$$

Constraint (1.1) ensures that for all the grids not on the outer edges, or so called the internal grid, there is no turbine placed right next to it in all four directions, $i+1$, $i-1$, $j+1$, $j-1$. Constraint (1.2) - (1.5) restrict the neighbor turbine for the top left, top right, bottom left, bottom right corner respectively. The key difference from (1.1) is that the corner points constraints are only limited to two directions. Constraint (1.6) - (1.9) ensures that for the top, bottom, right and left edges, there is no neighbor turbine placed for only three directions. For example, for top edges, since the node itself is on the top limit line, the constraints only limit the turbine places on its left, bottom, and right grid.

As for real-life projects, the assumption of perfect flat wind farm layout can hardly be true. There are some geological constraints like lakes, roads and forests which limit the construction location of the wind turbine. Therefore, the binary constraint (2.0) is a mandatory switch that allows us to adjust some points that are identified infeasible to build wind turbines to equal to 0, or equal to1 if a turbine must be built at a certain point.

3. Numerical Implementations and Results

3.1. Dataset Description

The dataset used to build the model, *Wind_Turbine_Database_FGP.xlsx*, was publicly shared on the government of Canada sites. It contains information of the wind turbine precise locations based on longitude and latitude, wind farms project names, total capacity, and turbine dimensions. For the scope of the project, only the turbines information in the Pubnico Point project will be included in the model.

3.2. Wind Farm Estimate

The limitation of the dataset is that there is no defined wind farm area. To get that information, we took an estimate of the wind farm area by calculating the distance from the top turbine to the bottom of the land as the width of the layout, and the distance from the left-most turbine to the right as the length. To do that, we plotted the exact location of the turbine on the graph using python and get the distance in meters. The estimated width and length are approximately 1540 and 1520 meters respectively, which is safe to assume the layout as a square grid.

We determined the approximate number of turbines to be placed in a row of the layout to be 6, which is based on the recommended distance between turbines, around 5 rotor diameters apart, and the diameter of the turbine in the dataset, 80 meters, to get the distance of 400m as the length of the diagonal and giving us the grid of 6x6 as the graph that we will use to model the optimal layout.

In order to portray real-world situations, the parameters were estimated based on real-time data. The power capacity of each turbine is assumed to be that of Vestas V80/1000 model, which is 1,800 kW.

3.3. Numerical Results

The model formulation was done using Gurobi and python. The model was made to maximize the total power generated from a wind farm by considering the minimum distance required between the turbines, and the wake effect of the wind. In the basic model, there is 1 decision variable for each turbine node of whether to build a turbine on that node.

The results of the final layout optimized by the model are shown in figure 2, and 3. We graphed our result on the map for easier comparison with the actual layout at Pubnico Point wind farm. Without adding any (2.0) constraint, the left layouts in figures 2 and 3 are the optimal results returned by model 1 and 2 respectively. However, we noted that some nodes that were identified as a candidate location for a wind turbine are in the infeasible area, so we manually force the nodes on the sea to be not available by adding a constraint (2.0) to be 0 for each and rerun the model.

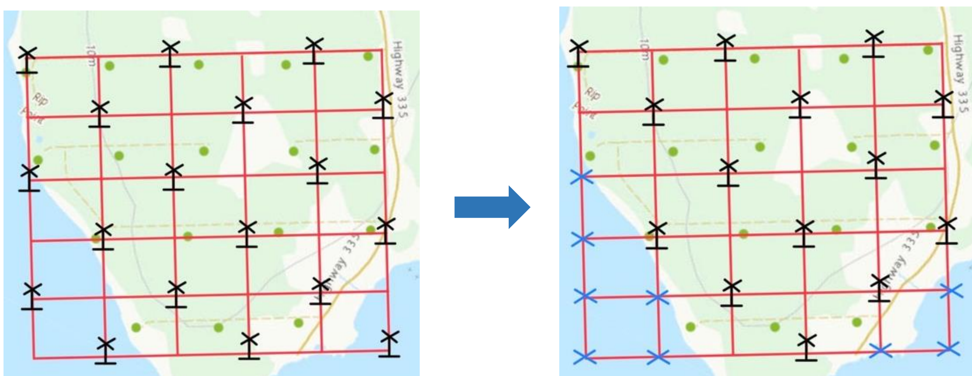


Figure 2: Layout plot on the actual farm without turning off any nodes (left) and after forcing some nodes to be off (right) in Model 1

Figure 3: Layout without turning off any nodes (left) and forcing some notes to be off (right) in Model 2

The optimal value of energy output of each of the model are as follows:

Model	Optimal energy output

Model 1 – No Wake effect	Without turned off nodes	32400 kW
	With turned off nodes	25200 kW
Model 2 – Wake effect	Without turned off nodes	16200 kW
	With turned off nodes	12600 kW

Table 1: Optimal result of each optimization model

3.4. Layout Comparison

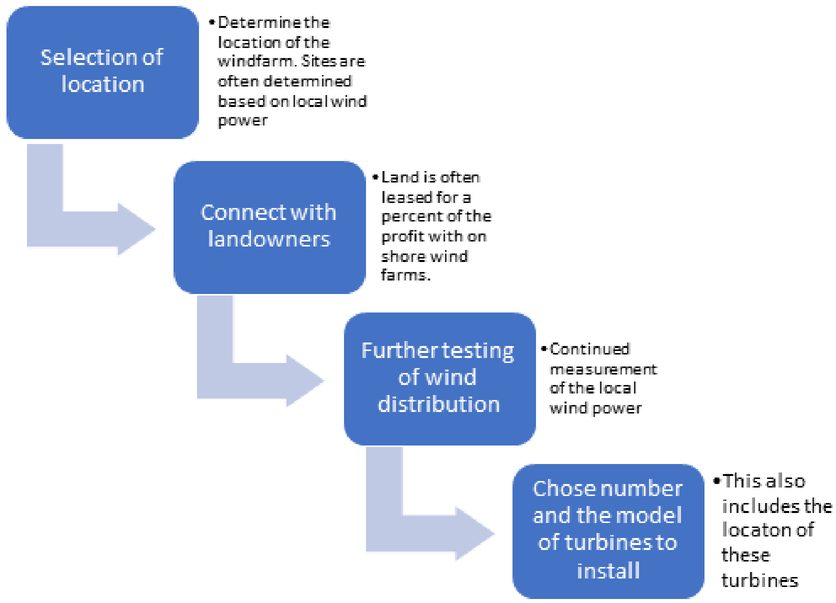
By comparing the result of our model with the actual layout of Pubnico Point Wind farm, we noticed the difference in terms of the number of turbines placed and the total project capacity. There are 17 turbines in the Pubnico Point Wind farm with a total capacity output of 30,600 kW. The possible justification of the variance could be:

1) The total project capacity calculation is likely calculated only using the number of turbines installed and does not indicate the 'actual power output' once wake is factored
2) Limited consideration of constraints (land conditions, cables etc.): The model in this paper only considers distances between turbines and the wake effect as the constraints. However, in reality, the setup of a wind farm involves a wide variety of external constraints, such as land conditions (whether the land is suitable to build the wind turbine), cables (the number of cables used to connect the turbines to an energy station), maintenance cost. As a result, the incorporation of these constraints would undoubtedly change the layout of the wind farm. A thorough examination of the area planned for the wind farm should be conducted prior to building the optimization model.
3) The minimum distance requirement may differ from the threshold in our model: In our model, we set the constraint for minimum distance requirement in order to mitigate the wake effect. In real life, the minimum distance requirement could be set by laws and regulations. The minimum distance could be smaller or larger than the one used in the model. Hence, this would affect the optimal layout of the final result.
4) Missing wind resource data: In the course of this report, data on wind resources (wind direction, wind strength) in the region is unavailable. Since the purpose of allocating wind turbines is to maximize the energy output using wind input, knowing where the wind blows would influence the location of a wind turbine. Hence, with this piece of information to be incorporated in the model, the result would prove to differ from that of our model.

4. Problem Extensions

While an important one, the wake effect is not the sole consideration in the optimal layout of a windfarm, even when the wake effect is minimized with the maximization of power output, there is no guarantee that the farm is optimal. This of course cycles back to the key question of how to maximize the power output while minimizing the cost. A key component of the cost can be the turbines themselves, and the necessary investment in the cables for the windfarm. How the windfarm is laid out will determine these factors. The cost of the cables alone may be up to 9% of the total windfarm cost (Hou, 2019). This is in effect, money that cannot be spent on the turbines themselves and rather is just bringing the power generated to the grid. The truly optimal layout will need to consider this and ensure that the cost per each generated kilowatt is the best It can be. Further the layout question can be complicated by including different turbine types. Charhouni et al approached this problem and measured the best layout optimization with four different types of wind turbines (Charhouni, 2019). The different types were based on the rotor diameter of the turbine and the study found that a framing this as an optimization problem rather than a traditional approach did lead to higher production of energy, but that the size of the wind turbine did make a marked difference (Charhouni, 2019).

Finally, a key consideration in layout optimization is the shape of the windfarm. Many current approaches use rectangular layouts with identical rows and what is seen as a conveniently large distance between turbines, despite there being more effective approaches with irregular layouts (Samourani, 2013). While this is the current approach it is by no means guaranteed to be the most effective approach as it is based not on optimization but rather simplicity. The original WFLOP study by Mosetti et al used a square grid for the layout with discrete locations for the turbines (Mosetti, 1994). A later study by Şişbot et al determines that a rectangular grid can better be used rather than a square to limit wake model, however their model looked at the wind blowing from only one direction (Sisbot, 2010).



Moving forward, it would be ideal to study the effect of the windfarm layout through every step of the process. The above flowchart shows all the steps of the windfarm design process and some of the limitations of this report are evidenced. The direction of the wind that blows into the windfarm plays a dramatic role in the importance layout optimization. As does the model of turbine selected. For the next step of this research, it would be ideal to partner with an actual windfarm development plan and gather wind data at the proper height for their turbines and then optimize that layout. The turbines selected will also change the wake effect measure away from a proxy to an actual value. Finally, by not forcing a rectangular layout there may be further applications of use.

As a proof of concept, we tried modelling the same layout optimization problem at Pubnico wind farm by not enforcing the rectangular layout and instead imagining the actual wind turbine location as an irregular graph $G = (V, E)$ with V nodes and E edges in problem extension 3. We calculated the pairwise distance between each of the nodes to create all the edges of the graph and used the same constraints as the base model to come up with an optimal layout that maximizes power output. The optimized layout got rid of 3 nodes out of 17. Since we are assuming the actual wind turbine locations to be the candidate turbine locations, there is no way to reliably compare the results with the actual turbine location, but the results are conceptually correct. Our optimized layout gets rid of turbines that are clustered together to ensure the minimum turbine distance is maintained. However, we do need to enhance the model to make it more robust and realistic as we've taken a number of assumptions that might not hold in reality.

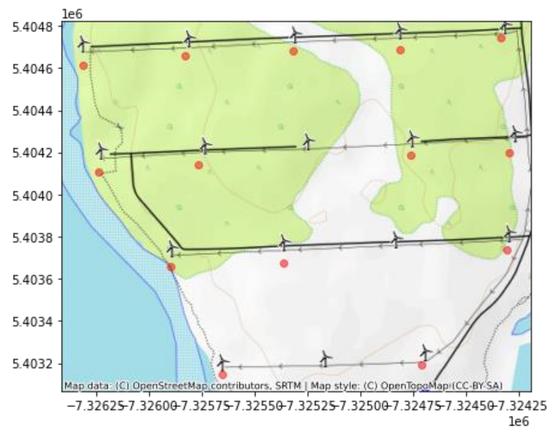


Figure 4: Results from the problem extension 3, red points are the optimized wind turbine locations, and the turbine icons represent actual turbine placements at Pubnico wind farm

5. Conclusion

Clearly there are many factors that are required to determine the optimal layout of a windfarm, and even placing the most turbines does not necessarily mean that the farm will have the best cost per unit of energy generated outcome. There is also a fair amount of literature about this optimization problem. It is worth noting that the WFLOP has been considered a NP-Hard Problem (Hou, 2019). This report outlines an approach for non-rectangular shaped wind farm layouts, as well as a scalable and simplistic approach for the modeling and layout of the WFLOP. This report does work to give a replicable example and visualization of the challenges in placing turbines and designing layouts. It includes a proxy for the wake turbine based on the Jensen model, and looks sot solve challenges in placing turbines when some locations are forced off. This research and optimization are highly relevant to the changing energy demands and how governments are striving to fill this energy demand. Should we reach more sustainable practices, optimizations like this will continue to be more and more pertinent.

6. Appendix & Bibliography

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Appendix A

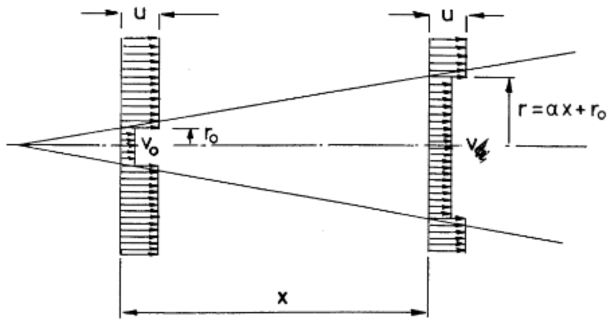

Figure 1 N.O. Jensen, "A Note on Wind Generator Interaction," Report, A Note on Wind Generator Interaction (Roskilde: Risø National Laboratory, 1983).

Appendix B

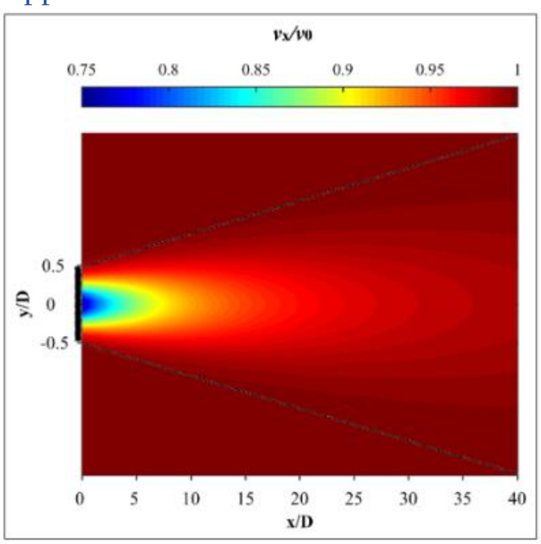

Figure 2 Zilu Wang et al., "Increase a Real Wind Farm Productivity through Optimizing Wind Turbines Layout," Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy 236, no. 8 (December 1, 2022): 1593–1607, https://doi.org/10.

Appendix C

Model	Description

Model 1	Without wake effect	Imagining the wind farm as a square grid with equidistant nodes, optimizing the turbine layout based on node distance and forcing certain nodes to be on/off based on geographical features or wind resource availability
Model 2	With Wake effect	Imagining the wind farm as a square grid with equidistant nodes and factoring in the power loss due to wake effect in the objective function, optimizing the turbine layout based on node distance and forcing certain nodes to be on/off based on geographical features or wind resource availability
Problem Extension 1	No wake effect, regular graph	Imagining the wind farm as a regular grid graph G = (V, E) with V the pairwise distance to each immediate neighbor and optimizing the turbine layout based on the wake effect and forcing certain nodes to be on/off based on geographical features or wind resource availability
Problem Extension 2&3	No wake effect, irregular graph	Imagining the wind farm as an asymmetric graph G = (V, E) with V vertices and E edges to closely resemble real word scenarios with topological and geographical restrictions. Optimizing the wind turbine layout based on the pairwise distance between nodes and the minimum distance required to maximize output

Table 2: Summary of all the models and extensions built