Does spoofing erode market confidence?

H. Duong [*] B. Taub[†‡]

November 11, 2024

Abstract

We present a model of strategic interactions between a spoofer and an anticipatory trader who can observe the signal of the fundamental informed investor's order in the second period. Learning the anticipatory trading strategy, the spoofer submits a spoofing order to mislead the anticipatory trader about the incoming order. The order anticipation HFT protects itself by reducing its market participation. A pure strategy spoofing equilibrium exists and both traders make positive profits. While spoofing delays price discovery in a short horizon, price divergence will be so brief as to have little economic efficiency implications. Moreover, spoofing improves market liquidity and fosters uninformed traders' welfare.

1 Introduction

Over the last decade, the US Commodity Futures Trading Commission (CFTC), the U.S. Securities and Exchange Commission (SEC), and the Department of Justice (DOJ) have stepped up their efforts to crack down on the type of disruptive trading called "spoofing". This emphasis coincides with a similarly increasing focus by the UK Financial Conduct Authority (FCA). To date, over 50 cases involving spoofing have been filed against individuals and companies by the US regulators while over 5 enforcement actions have been taken in the UK. One of the largest fines was JPMorgan Chase's case [1] in which they entered an agreement to pay regulators USD 920 million as part of a settlement admitting to spoofing precious metals futures and US government bonds. Spoofing is the practice of submitting big limit orders to the markets with the intention of avoiding their completion by canceling them before they are executed. Spoofing is considered illegal in many jurisdictions. On the topic concerning spoofing, Aitan Goelman [2], the CFTC's Director of Enforcement, commented: "Spoofing seriously threatens the integrity and stability of futures markets because

[*]University of Glasgow.
[†]University of Glasgow.
[‡]output/November 11, 2024
[1]Press release 8260-20,https://www.cftc.gov/PressRoom/PressReleases/8260-20
[2]Press release 7264-15, https://www.cftc.gov/PressRoom/PressReleases/7264-15

it discourages legitimate market participants from trading". The question of whether spoofing is harmful to market integrity and efficiency is a matter of debate among regulators and industry practitioners.

In this paper, we investigate the effects of spoofing from historical, regulatory, and market microstructure perspectives. We find that spoofing is not a disruptive practice, the spoofing resemblance strategies have existed for centuries. By looking at different legal cases on spoofing, we point out that the main victims of spoofing are HFTs that employ order anticipation strategies. Contrary to criticism expressed by regulators and industry practitioners, we show that while spoofing delays price discovery in a short horizon, price divergence will be so brief as to have no little economic efficiency implications. Furthermore, spoofing improves market liquidity and fosters uninformed traders' welfare.

Our investigation into this matter builds on a two-period Kyle model with two additional market participants: a spoofer and an anticipatory trader. The informed trader trades on proprietary information regarding the value of the traded asset, but his order is delayed by one period. The order anticipation HFT uses pattern recognition algorithms to detect the incoming order and trades on this signal. However, his trading strategy gets detected by the spoofer. The spoofer exploits the anticipatory trader by pretending to be a large trader and submitting two orders: one real order and another big spoofing order from the opposite side of the market then cancels the spoofing order. In this way, the spoofer can add more noise to the anticipatory trader's signal and give a false sense of market demand.

The irrationality of the anticipatory trader primarily drives our results. Order anticipators study trades and quotes to find traces of informed trades then trade ahead of such orders to profit from expected price changes. Computers play an important role in the successful implementation of order anticipation strategies because they can often perform pattern recognition faster and more accurately than humans do. However, these algorithmic trading strategies follow a rigid set of rules, thus making them vulnerable to other market participants in the ever-changing markets. Exploiting this feature, spoofers can trick those trading algorithms and make profits.

To study the effects of spoofing, we consider 4 different economies based on Kyle's model with: a spoofer and an order anticipator, only a spoofer, only an order anticipation HFT, and a standard Kyle model. We find that without an order anticipation algorithm, the spoofer falls victim to his own strategy and incurs a loss as he is uninformed and loses money to the informed trader. In the economy with both traders, a spoofing equilibrium exists and both the spoofer and the anticipatory trader make profits. When the spoofer increases his trading intensity, the signal of the anticipation HFT becomes noisier. The anticipatory trader strategically responds to the spoofer by reducing his participation. Therefore, he becomes less active in the market. As a result, the uninformed traders benefit from spoofing.

2 An overview of spoofing

This section provides a regulatory and market structure overview of spoofing. We will discuss the history of spoofing, how spoofing works, and how it is regulated under different jurisdictions.

2.1 What constitutes spoofing

While there is no universally accepted definition of what constitutes spoofing, some common practices are generally considered as spoofing. A simple spoofing scheme involves a trader placing one or more highly visible orders but has no **intention** of keeping. It is designed to create a false sense of investor demand in the market, thereby changing the behavior of other traders and allowing the spoofer to profit from these changes. Apart from simple spoofing, there are some other popular spoofing resemblance practices [3].

- **Layering**: A trader places a small order on the intent side of the market and orders at multiple price levels on the spoof side of the market to increase the depth of the spoof side.
- **Vacuuming**: A trader places a small order on one side of the market and a larger order on the same side of the market. The larger spoofing order is then canceled to entice market movement toward the smaller order.
- **Collapsing of layers**: A trader places a small order on one side of the market and several spoof orders at different price points on the other side of the market. The spoof orders are then changed into a single price point to give the appearance of a large volume.
- **Flipping**: A trader places orders on one side of the market with the intent of switching, or flipping, to the other side of the market.
- **Spread squeezing**: A trader places an order on the spoof side at successively higher or lower prices with the spread to squeeze it in one direction, enticing other market participants to join or beat the newly established top of the book. The trader then switches sides and executes against those participants.

Duong and Taub [2023] studies the dynamics of the limit order book and information value of order flow. By examining the limit orders of over 80 US stocks in February 2018, we find that over 90% of all limit orders got canceled eventually. Therefore it is challenging to distinguish between normal cancellation orders and spoofing orders. The most important aspect to classify whether it constitutes spoofing is the trader's intention to cancel the order before execution, which is hard to identify. Based on previous cases in the US, enforcement authorities may offer the following evidence.

[3]Automated spoofing ,https://library.tradingtechnologies.com/tt-score/inv-automated-spoofing.html

- For algorithmic trading, the contents of the algorithms are examined for evidence of intent.
- For manual trading, emails, instant messages, and phone recordings may help to establish intent. Witness testimony may be offered.
- For some cases, trading data is used to identify an individual's trading pattern, and then compare it to the market trend.

2.2 Who are victims of spoofing

Based on previous cases, "victims" of spoofing are principally high-frequency trading firms that used price quotes for their trading strategies. In the Igor B. Oystacher case (2016), CFTC claimed that Mr. Oystacher used spoofing to create false book pressure as he knew that algorithmic trading firms like CGTA and Citadel had programmed their algorithms to rely primarily upon book pressure when making trading decisions in particular markets. During the trial, The CFTC presented 2 victim witnesses [4], Richard May of Citadel and Matthew Wasko of HTG Capital Partners. Mr May testified that

"Specifically, Citadel's trading strategies tend to look at "three key factors": 1) relative value, 2) book pressure, and 3) trade flow...In 2013, Mr. May and his team observed what they believed was spoofing in the ES market. Around this time, they noticed a significant decline in Citadel's profitability...Immediately, Citadel **scaled back** its participation in the ES market by over fifty percent."

Mr. Matthew Wasko gave his testimony that "Beginning around July 2012, Mr. Wasko and his team began to observe "suspicious trading" they believed was spoofing in the ES market. **After noticing a decline in profitability**, they began reviewing historical trading data from market replays of losing trades...This trading activity was detrimental to CGTA's trading because the initial large orders appeared to its algorithms as genuine interest from multiple market participants, leading CGTA to **join that movement and enter orders it intended to trade.**"

At the recent spoofing trial where two Two ex-Deutsche Bank traders were prosecuted for spoofing, the government testified on behalf of just two "victims" of the alleged spoofing by the two defendants on trial: one was a representative from Citadel Securities, the other was a company called Quantlab. Both of them are among the most secretive and highly profitable high-frequency trading firms.

Obviously, some HFTs are vulnerable to spoofers. However, HFT strategies vary considerably and only predatory HFTs can fall victim to spoofers easily. Following Harris [2013], there are three main types of HFTs.

- **Valuable HFT** High-frequency traders who use dealing and arbitrage strategies that make markets more liquid. Spoofers have little effect on

[4] Case: 1 15−cv−09196, U.S. District Court - Northern District of Illinois, https://www.govinfo.gov/content/pkg/USCOURTS−ilnd−_15−cv−09196/pdf/USCOURTS−ilnd−1_15−cv−09196−.pdf

this type of HFT trader as spreads across markets or exchanges are their concerns.

- **Harmful HFT** High-frequency traders use computers to monitor and interpret electronic news feeds. Obviously, with information acquisition, the HFTs become informed traders and know the true value of the stocks. Therefore, spoofers cannot influence their strategies.

- **Very Harmful HFT** A few high-frequency traders examine trades and quotes (book pressure and order flow) to detect when traders are using algorithms to split up large orders that will move the market. They then trade ahead of such orders to profit from expected price changes. Some market participants refer to this practice as "front-running". But this conduct is legal and different from "traditional front-running" which is defined as entering a trade with advance knowledge of a block transaction that will influence the price of the asset and the trader improperly obtain such information. In this case, HFTs just use public information, they are just faster and better in terms of transmitting and processing the data. To differentiate it from "traditional front-running", I refer to this practice as "order anticipation or anticipation strategy". This type of strategy is highly vulnerable to spoofers as spoofers can add more noise to quotes which are used as their main indicators.

From the witnesses' testimony and our reasonings, we can deduce that victims of spoofing are primarily HFTs which used order anticipation strategies. Their strategies were detected and exploited by spoofers. They only noticed that their strategies got exploited when they saw a significant decline in profitability and immediately scaled back their participation. This resonates with our results in the next section, as in equilibrium, with the spoofer, the anticipatory traders make less profit and scale back their trading activities when there is a spoofer.

2.3 Brief history of spoofing

Spoofing may have been occurring since the establishment of formal financial markets in Europe during the 1600s-1700s. The earliest recording of spoofing incidents was from Daniel Defoe in his essay "Anatomy of Exchange Alley." In a passage recounting the trading practice of Sir Josiah Child, an English economist, merchant, and governor of the East Indian Company, Daniel Defoe gave a glimpse into a spoofing-resemblance trick.

"If Sir Josiah had a mind to buy, the first thing he did was to commission his brokers to look sower, shake their heads, suggest bad news from India; and at the bottom, it followed, 'I have commission from Sir Josiah to sell out whatever I can,' and perhaps they would actually sell ten, perhaps twenty thousand pound. Immediately the Exchange was full of sellers; nobody would buy a shilling", "till perhaps the stock would fall six, seven, eight, ten per cent, sometimes more; then the cunning jobber had another set of men employed on purpose to buy".

Hundreds of years later, spoofing became a common practice in the trading pits during the twentieth century. MacKenzie [2022] interviewed different floor traders and recorded their recounts of spoofing "it sounds like a normal day in the pit. We spoofed all the time.". On the trading floor, traders and brokers communicated by shouting out bids and offers or using hand signals to indicate the prices and quantities. Their behaviors were under the scrutiny of all other traders. Many brokers wanted to hide their trading intentions. When they want to buy, they might shout or hand signal offers to sell without an intention of selling. Such practice would be labeled as spoofing nowadays, but well regarded as "good brokerage" among market participants. However, those employing this practice might face reputation risks. Even though verbal deals were not legally enforceable, constant spoofing might freeze the traders out of future trades as other traders knew who often regened their deals.

The adoption of electronic trading systems in the late 1990s and early 2000s created a perfect environment for spoofing to thrive. A computer-powered system was first introduced in the financial markets in 1969 but did not take off until the late 1990s. The new advent of technology brought in an anonymous trading mechanism, thus eradicating the need for social interactions and the reputation risks of spoofing in the pits. Zaloom [2003] documented the trading activities in the early 2000s "The most recurring character was called the "Spoofer." The Spoofer used large quantities of bids or offers to create the illusion that there was more demand to buy or pressure to sell than the "true" bids and offers represented". Most market participants considered this practice legitimate. "Although there would be nothing illegal about the Spoofer's maneuver of supplementing the numbers with the weight of his bid or off...". With the changes from face-to-face trading to electronic trading, there was a gradual shift in moral and regulatory treatments of spoofing. From a highly regarded practice, spoofing became a serious criminal conduct when New Jersey trader Michael Coscia became the first person to be convicted of "spoofing" and sentenced to 3 years in prison in 2016. Since then, regulators in many countries have intensified their crackdowns on this type of practice.

2.4 Regulations of spoofing

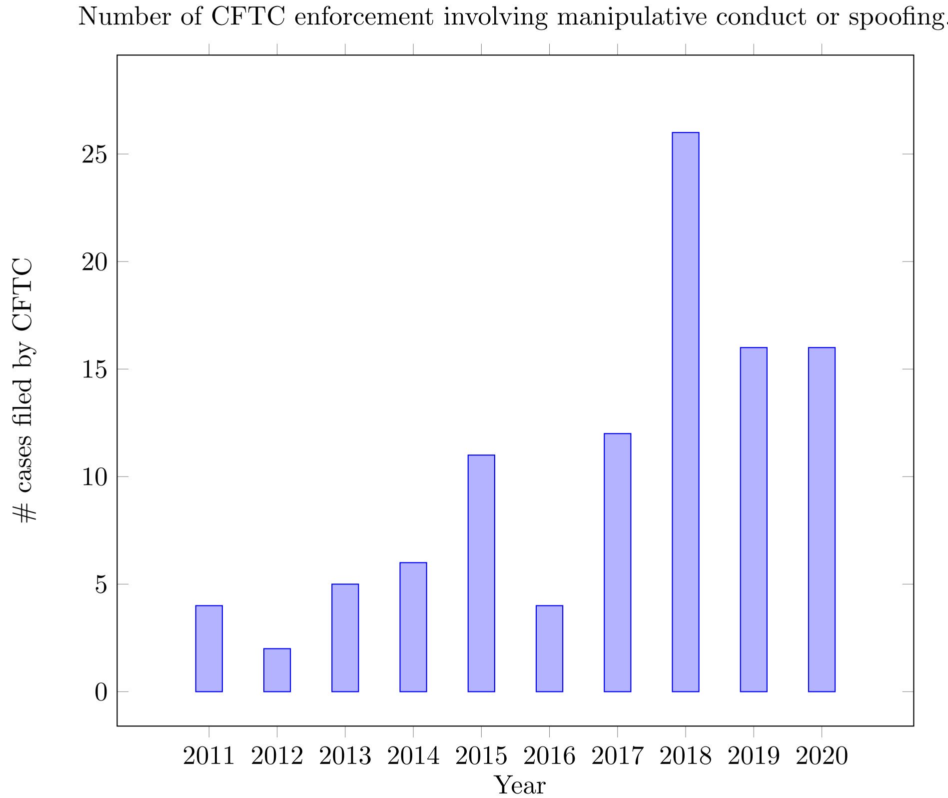
2.4.1 United States

Spoofing is prosecuted according to civil and criminal law in the US. Regulators are required to prove the traders' intention to spoof the market by canceling the orders before execution. Depending on markets and the severity of the cases, the SEC, CFTC, FINRA, and the DOJ (Department of Justice) enforce spoofing under different laws. In the US, regulators must provide evidence of traders' intention to cancel bids or ask before execution. Civil cases can brought in case of act with "with some degree of intent, or scienter, beyond recklessness." while criminal cases are for individuals who "knowingly" engage in Spoofing.

In the commodity markets, manipulative conduct is enforceable by the CFTC. Before the enactment of the Dodd-Frank Act in 2010, the CFTC's authority to regulate spoofing was limited to the Commodity Exchange Act (CEA) (Section 6c 9(a)(2)). Section 6(c) of the CEA gave the CFTC authority to take administrative enforcement action against traders who manipulated or attempted to manipulate the market price while Section 9(a)(2) made it unlawful to "manipulate or attempt to manipulate the price" of a commodity or future. This rule vaguely defines what"market manipulation" is, thus making it almost impossible to prove manipulation. Therefore, the agency is believed to have successfully brought only one market manipulation case to final judgment from 1975 to 2010. After the Dodd-Frank Act in 2010, Section 747 of the Dodd-Frank Act added Section 4c(a)(5)(C) to the Commodity Exchange Act (CEA) to ban three types of transactions labeled as "disruptive trading." One of those transactions is spoofing in commodity markets and this is the first time spoofing was expressly prohibited by a federal statute. Not until 2013, the CFTC took the first enforcement action under the amended CEA by settling with Panther Energy Trading, LLC (Panther). Since then the CFTC has stepped up enforcement against spoofing. The 2020 Division of Enforcement Annual Report [5] showed that the CFTC has intensified its crackdowns on spoofing. Nearly 10% number of cases filed by CFTC in 2020 involved spoofing.

Unlike the Commodity Exchange Act, the federal securities statutes do not expressly prohibit spoofing by name. Instead, the Securities and Exchange Commission (SEC) has taken action against spoofing by characterizing it as a manipulative practice. The SEC has been investigating and prosecuting alleged spoofing in the securities markets at least since the early 2000s. The full lists of civil and criminal cases against spoofing are in Appendix B.

[5] FY 2020 Division of Enforcement Annual Report https://www.cftc.gov/media/5321/DOE_FY2020_AnnualReport_120120/



2.4.2 The UK

UK law does not include any specific anti-spoofing provisions; rather, spoofing behavior is generally construed to be a form of market manipulation that may result in civil or criminal liability. The UK's MAR is modeled on the EU's Regulation No 596/2014 (Reg 596), which was passed on April 16, 2014. While the FCA made some changes to UK MAR as it adopted the regulation following Brexit, the UK regime is still primarily based on the EU Market Abuse Regulation.

In Europe and the UK, prosecutions can be made where the regulator deems that Spoofing took place and they don't need to prove that spoofers have the intention of canceling the orders. While there have not yet been any criminal spoofing cases in Britain, the Financial Conduct Authority (FCA) and Office of Gas and Electricity Markets (Ofgem)22 used their powers to impose stiff fines on those who engage in market manipulation practices in the UK. This intensified crackdown coincides with a similarly increasing focus by the US Commodity Futures Trading Commission and the US Department of Justice. In 2015, the Financial Conduct Authority (FCA) fined Da Vinci Invest Ltd, Mineworld Ltd, Mr Szabolcs Banya, Mr Gyorgy Szabolcs Brad and Mr Tamas Pornye £7,570,000 for spoofing. The case was instigated in 2011. The defendants were accused of using manipulative behavior which consisted of an abusive trading strategy known as "layering", involving the entering and trading of orders in relation to shares traded on the electronic trading platform of the London Stock Exchange ("LSE") and multi-lateral trading facilities ("MTFs")[2] in such a way as to create a false or misleading impression as to the supply and demand for those shares and enabling them to trade those shares at an artificial price.

2.4.3 Europe

Article 12 of EU MAR gives definitions of what constitutes market manipulations, including "entering into a transaction, placing an order to trade or any other behavior which gives, or is likely to give, false or misleading signals as to the supply of, demand for or price of a financial instrument; or secures, or is likely to secure, the price of one or several financial instruments at an abnormal or artificial level." Article 15, thereafter states that "a person shall not engage in or attempt to engage" in market manipulation such as those defined in Article 12.

While EU MAR doesn't provide any specific provisions regarding spoofing, the regulation clearly defines the indicators that firms should monitor and detect potential market manipulation. One of the indicators is for Spoofing. MAR Article 16 highlights that firms must have "effective arrangements, systems, and procedures" to prevent and detect insider dealing, market manipulation, and attempted insider dealing and market manipulation.

2.4.4 Asia

While regulations on spoofing take different forms across Asia, spoofing is deemed illegal in most countries. There is a big gap between emerging and developed markets in spoofing regulation and enforcement processes. While developed markets have clear regulations and streamlined enforcement processes, emerging markets lack clarity in spoofing definitions and strict enforcement actions against market manipulators.

We study the regulatory measure and enforcement system in 2 advanced markets, namely Japan and South Korea, and Hong Kong. In Japan, a market surveillance system has been implemented to oversight the market. Trading data from 2 exchanges, the spot market (Tokyo Stock Exchange) and derivatives market (Osaka Exchange), is analyzed daily to detect any abnormal activities. Any transactions that are suspected of spoofing are reported to the Securities and Exchange Surveillance Commission. Spoofing is classified as a market manipulation under Article 159, Paragraph 2, Item 1 of the Financial Instruments and Exchange Act[6] and the Securities and Exchange Surveillance Commission. Spoofing has issued administrative fines to some market participants for their alleged spoofing activity. In 2019, the market regulator fined Citi Group $ 1.2 million for spoofing in the future market. In 2022, an administrative penalty

[6] The Financial Instruments and Exchange Act,https://www.fsa.go.jp/common/law/fie01.pdf

order was issued to Atlantic Trading London Limited for their involvement in spoofing 10-year Japanese Government Bond Futures. In South Korea, the regulating authority is The Financial Service Commission (FSC), which is responsible for overseeing the securities and futures industry. Regulation on market disturbances was introduced on December 30, 2014, and went into effect on July 1, 2015, which is intended to enhance regulations on market manipulations. Article 178-2 of this provision bans market participants from engaging in "An act that adversely affects, or is likely to, adversely affect the market price by submitting a large volume of asking prices at which deals are unlikely to be concluded, or by repeatedly correcting or canceling asking prices after submitting them" [7]. Using these regulations, The Financial Service Commission has imposed a fine of 11.88 billion won on Citadel Securities for their distortion of stock prices by using immediate-or-cancel (IOC) buy market orders to exhaust the best ask prices and submitting buy limit orders on any remaining unfilled quantity, then cancel these orders.

For comparison, we examine the regulations governing spoofing in two Asian emerging markets, namely, India and China. In the case of India, the regulatory body which is in charge of the development and supervision of the Indian capital market is the Securities and Exchange Board of India. Section 12A of the Securities and Exchange Board of India Act, 1992 ("SEBI Act") bans market participants from engaging in fraudulent and unfair trade practices through the use of any manipulative device, insider trading. However, there was a lack of clarity regarding spoofing until SEBI issued a circular on "Order-based Surveillance Method-Persistent Noise Creators." [8] to address the issue of excessive cancellation of orders in 2021. The circular proposed a surveillance mechanism to deter excessive order modifications and cancellations with the intent to avoid execution. Various parameters, such as order-to-trade ratio and cancellation ratio, are examined daily to detect any potential market manipulation. In 2023, the Securities and Exchange Board of India issued an order [9] to investigate the trading activities of Nimi Enterprises for alleged engagement of spoofing. This was the first time, the term "spoofing" was introduced in a regulatory document in India to describe the actions undertaken by Nimi Enterprises. In the case of China, the stock market was closed in 1950 and reopened in December 1990. The official regulation of market manipulation began in 1993. Spoofing came into the spotlight in 2015 when China's securities regulator targeted high-frequency traders following the stock market turbulence. Article 55 of the Securities Law of the People's Republic of China (2019 Revision) [10] officially

[7] Disturbance of capital market, http://www.koreanlii.or.kr/w/index.php/Disturbance_of_capital_market?ckattempt=2

[8] Order Based Surveillance Measure: Persistent Noise Creators, https://www.bseindia.com/markets/MarketInfo /DispNewNoticesCirculars.aspx?page=20210326-55

[9] Order in the matter of trading activities of Nimi Enterprises, https://www.sebi.gov.in/enforcement/orders/apr-2023/order-in-the-matter-of-trading-activities-of-nimi-enterprises˙70718.html

[10] Securities Law of the People's Republic of China (2019 Revision), https //www.lawinfochina.com /display.aspx?id=31925&lib=law

banned any person from "placing and canceling orders frequently or in large numbers, not for the purpose of the consummation of trades."

3 Literature review

There is a paucity of social science literature on spoofing. To our best knowledge, our paper is the first paper to show that spoofing restricts the market participation of very harmful HFTs and doesn't impede price discovery. Our finding is in contrast to that of Williams and Skrzypacz [2020] who studies spoofing equilibrium under the Glosten-Milgron framework. They show that spoofing can occur in equilibrium, slowing price discovery and raising spreads and volatility. A novel prediction is that the prevalence of equilibrium spoofing is single-peaked in the measure of informed traders. However, they only allow spoofers to trade one unit of share in each period which deviates from the true sense of spoofing in which the traders trade high volume to give a false picture of supply and demand. In our model, spoofers use large orders to mislead other traders.

Our paper is closely related to papers that study front-running in the market. We adopt similar two-period settings to Bernhardt and Taub [2008], Xu and Cheng [2023]. However, our paper allows traders not to trade and cancel their orders. Compared to front-running papers, there is another type of trader added, a spoofer who can submit a big order to manipulate other HFTs' beliefs. Without a spoofer, our model collapses into a front-running model. Therefore, these papers' results and our results are complementary.

Most empirical papers have difficulties in identifying traders' intentions and spoofing activities. Lee and Park [2013] uses the complete intraday order and trade data of the Korea Exchange (KRX) (data with customer number) to study spoofing activities. They define a spoofing order as a bid/ask with a size at least twice the previous day's average order size and with an order price at least 6 ticks away from the market price, followed by an order on the opposite side of the market, and subsequently followed by the withdrawal of the first order. They show that price disclosure leads to a dramatic decrease in spoofing frequency.

4 Modelling spoofing

In this section, we present a variant of the dynamic Kyle model. As mentioned in the second section's analysis, spoofer and their "victims" are either HFTs or really fast traders who can take advantage of their speeds to capture short-term movements of the price. Sometimes, they are labeled as "scalpers" by other market participants. We modeled them as fast traders.

4.1 Traders and market

Similar to Kyle [1985], the model has two types of slow traders who can submit orders to the market with latency: i) noise traders whose trades are treated as entirely exogenous, that is, they do not react to observations about price in any way. Their trade is normally distributed $u \sim N(0, \sigma_u^2)$ and $\sigma_u > 0$; ii) one monopolistically informed trader who has private information of the stock $v \sim N(0, \sigma^2)$. The realized value of the security is privately observed by the informed trader who then exploits this information in his trade.

Apart from slow traders, there are three types of fast traders who employ different high-frequency strategies. First, a spoofer who can observe the trading activities with a low latency uses spoofing strategies. Second, an HFT who depends on pressure book to anticipate other traders' strategies is labeled as an "anticipatory trader". Third, competitive market makers set the price to absorb the order flow imbalance and make zero expected profit.

There are trading periods which denoted by $t = 1, 2$. In the period, the informed trader submits an order of volume x with a latency. The spoofer can submit an order of $-z_1$ with low latency to get an immediate execution and an order of z_2 with high latency with an option to cancel the order just before the period 2 execution. The anticipatory trader can observe a noisy private signal about the incoming big order flow in the second period $\tilde{i} + z_2 = x + z_2 + \epsilon$ with and he can trade m shares. In the second period, the spoofer can cancel the order of z_2 and submit an order of z_1. The order of the informed trader arrives in the market. The anticipatory trader trades an order of $-m$ to liquidate his all positions. We allow the possibility that spoofers and anticipatory traders choose not to trade.

The noise orders during the period 1 and 2 are respectively denoted by u_1 and u_2. Both of them have the same distribution as u and they are independent of each other and other random variables. The noise term ϵ is normally distributed, $\epsilon \sim N(0, \sigma_\epsilon^2)$ and independent of other random variables. z_2 is independent of each other and other random variables and $z_2 \sim N(0, \sigma_{z_2}^2)$. This assumption implies that the spoofer is an uninformed trader, he has no private signal and his spoofing order is uncorrelated with all other random variables.

4.2 Information sets and model discussion

No matter what type of spoofing strategies the spoofers employ, the main tenet is to give a false sense of supply and demand (false signal) to the anticipatory trader. In this way, spoofers can manipulate HFTs which use anticipatory algorithms. In this model, we model it as the signal $\tilde{i}$ which is analogous to "pressure book". One of interpretation of i is that the anticipatory trader can observe the limit order book. By spotting the big limit orders, the trader can deduce the incoming order flow from the big traders (informed traders) in the second section. Spoofers may have used trading data or "ping orders" to detect anticipatory traders. For example, Mizuho Bank[11] was fined a \$250,000 civil monetary penalty by the CFTC for using spoofing strategies to test the market's reaction to his spoof orders. When anticipatory traders' strategies get detected, the spoofer pretends to be an informed trader and sends a big order of z_2 to

[11] CFTC press release 7800-18,https://www.cftc.gov/PressRoom/PressReleases/7800-18

mislead the anticipatory trader. The big order is canceled immediately after the real order z_1 gets executed.

Both the spoofer and anticipatory trader hold no inventory at the end of the second period. The main reason for this assumption is that both of them are short-term traders and have no information about the fundamental value of the stock. Their strategy is to capture short-term movements of the price. Therefore, they are risk-averse to holding inventory and tend to hold no inventory at the end of the day. Recounting the Navinder Sarao's trading strategy, Liam Vaughan[12] gave a short description in his book "At the end of almost every session, he made sure he had no outstanding positions—that he was "flat," in the idiom of the trader. The next day he started afresh." This assumption is also consistent with empirical findings of Kirilenko, Kyle, Samadi, and Tuzun [2017].

Informed trader faces execution latency. There are several ways to interpret this assumption. First, the informed trader may have a private signal about the asset fundamentals but he is a slow trader. Second, informed trades tend to be a big order and can move the market equilibrium. The informed trader may chop his meta order into many small orders, thus slowing down his execution. Third, the informed trader may submit only limit orders and wait for a better price instead of market orders. Therefore, he faces a delay in execution.

We capture spoofing by allowing spoofers to submit and cancel under the Kyle [1985] framework which in practice usually takes place with limit orders rather than market orders as we do not find any existing limit order book models that allow tractable modeling of spoofing. However, in Duong and Taub [2023], we draw an analogy between the limit order book and Kyle [1985] model. The limit order book has a visible structure: the set of resting orders forms a pattern, essentially a supply curve, with a slope that is driven by the underlying incentives created by the information possessed by some of the traders. A fundamental theory, the Kyle [1985] model, explains this structure, and predicts that the slope, λ, of the supply curve reflects the fundamental forces driving the stock value; λ reflects the marginal effect of trading on the price, and so is known as the price impact parameter. This analogy plays an important role in our model estimation in the next section.

Under the high-frequency setting, the time horizon is short. Therefore, we assume that once all market participants choose their trading strategies they are committed to their strategies at the beginning of the first periods. The timeline of the two-period model is as follows.

The information available to different market participants is as follows:

- **Informed trader** can observe a private signal of the true fundamental value of the asset v.
- **Spoofer** he's aware of the anticipatory HFT and his strategies but he doesn't know whether the anticipatory HFT will trade or not.

[12] Liam Vaughan, Flash Crash: a trading Savant, a Global Manhunt and the Most Mysterious Market crash in History

t=1	t=2
• Informed trader observes v and submits a slow order x. • The spoofer sells $-z_1$ shares and submits a slow spoofing order of z_2. • The anticipatory trader observes $\tilde{i} + z_2$ and submit order flow m. • Noise traders submit order flow u_1	• Order flow x arrives. • The spoofer cancels a slow spoofing order of z_2 and submits order flow z_1. • The anticipatory trader liquidates his position m. • Noise traders submit order flow u_2

Table 1: Model timeline

- **Anticipatory trader** can observe a private signal of $\tilde{i}$, but he's not able to distinguish between the true signal and the spoofed volume. One of the interpretations of $\tilde{i}$ is that the front runner can observe the limit order book. By spotting the big limit orders, the trader can deduce the incoming order flow from the big traders (informed traders). Knowingly, the spoofer submits a spoofing trader z_2 to manipulate the anticipatory trader's belief. Therefore, instead of observing the true signal of $\tilde{i}$, the anticipatory only observes $\tilde{i} + z_2$.
- **Market maker** is aware of the informed trader, spoofer, and anticipatory trader. He can observe the total order flow for each period but he doesn't know exactly how much informed trader, spoofer, or anticipatory trader trade.

5 Model equilibrium

We use the subscripts I, A, S, M, U for the variables or parameters of informed trader, anticipatory trader, spoofer, market maker, and uninformed traders. We denote the order flows of the first and second periods, y_1, y_2 respectively. Let the strategy functions of the spoofer, anticipatory trader, and informed trader be $S(.), A(.), I(.)$ and the market maker commits to a pricing function $P(.)$. The equilibrium is defined by four functions $S(.), A(.), I(.), P(.)$ such that the following conditions hold:

1. *Informed trader's profit maximization.* Given $I(.), A(.), P(.)$ and his signal about the true value of the assets v, he chooses x^* to maximize his expected

profit $\pi_I = x(v - p_2)$.

$$x^* = X(v, S(.), A(.), P(.)) = \arg\max_x E[\pi_I | v, S(.), A(.), P(.)] \tag{1}$$

Where p_2 is the execution price in the second period.

2. *The spoofer's profit maximization.* The optimal strategy of the spoofer, $S(.)$ is a set of real-valued functions $S(.) = \{(P_S(.)Z_1(.))\}$, P_S is the probability that the anticipatory trader chooses not to trade, $Z_1(.)$ is the optimal trading volume function if he decides to trade. The optimal strategy of the anticipatory trader. Given $I(.), A(.), P(.)$, his signal $\tilde{i}$ about the informed trades and spoofing order z_2 , he chooses (p_S^*, z_1^*) to maximize his expected profit $\pi_S = z_1(p_1 - p_2)$.

$$(p_S^*, z_1^*) = S(v, I(.), A(.), P(.)) = \arg\max_{p_S, z_1} E[\pi_S | I(.), A(.), P(.)] \tag{2}$$

3. *Anticipatory trader's profit maximization.* The optimal strategy of the anticipatory trader, $A(.)$ is a set of real-valued functions $A(.) = \{(P_A(.)M(.))\}$, P_A is the probability that the anticipatory trader chooses not to trade, $M(.)$ is the optimal trading volume function if he decides to trade. Given $I(.), P(.)$ and his signal $\tilde{i} + z_2$, he chooses (p_A^*, m^*) to maximize his expected profit $\pi_A = m(p_2 - p_1)$.

$$(p_A^*, m^*) = \arg\max_{p_A, m} E[\pi_A | \tilde{i} + z_2, I(.), P(.)] \tag{3}$$

4. *Market efficiency.* By the model's setting, the market maker observes only the total order flow at each period y_1, y_2. Given the strategies of the spoofer, anticipatory trader and inform trader, the market maker sets the price p_1, p_2, equal to the posterior expectations of v

$$p_1 = E[v | y_1, S(.), A(.), I(.)] \tag{4}$$

$$p_2 = E[v | y_1, y_2, S(.), A(.), I(.)] \tag{5}$$

Note that in the perfect Bayesian equilibrium, we allow mixed strategies, that is, in principle p_A, p_S are distributions over strategies of the anticipatory trader and the spoofer (Trade, Not trade) respectively. These 4 possibilities: (Trade, Trade), (Not Trade, Trade), (Trade, Not trade),(Not trade, Not trade) which represent different economies. The first economy corresponds to one in which the spoofer can trick the anticipatory trader into trading a big order. The second possibility represents one in which the anticipatory trader can extract signals about informed orders from the order book. The third one is the model in which the spoofer falls victim to his own strategy. The fourth possibility is the standard Kyle model. We use superscripts "AS", "0S", "A0" and "Kyle" to indicate these economies

Obviously, the strategy function $S(.), A(.), I(.), P(.)$ can take any forms. For the model's tractability, we will focus on linear equilibria, i.e., the trading strategies and pricing functions are linear. Formally, a *linear equilibrium* is defined as a perfect Bayesian equilibrium in which there exist constants (p_A, p_S) and 4 sets of $(\lambda_1, \lambda_{12}, \lambda_{22}, \beta, \beta_1, \beta_2)$ corresponding to 4 above possibilities, such that:

$$p_1 = \lambda_1 y_1 \tag{6}$$

$$p_2 = \lambda_{12} y_1 + \lambda_{22} y_2 \tag{7}$$

$$x = \beta v \tag{8}$$

$$m = \beta_1(\tilde{i} + z_2) \tag{9}$$

$$z_1 = \beta_2 z_2 \tag{10}$$

Following Bernhardt and Taub [2008]'s setting, we allows the possibility that $\lambda_1 \neq \lambda_{12}$. In this way, the market maker can reevaluate the information content of period-1 order flow in period-2 pricing.

The values of the net order flow depend primarily on the strategies of the spoofer and the anticipatory trader. We only allow these two types of traders not to trade. This assumption comes from the fact that the noise traders' trades are treated as entirely exogenous in the model and they always trade. At the same time, the private signal of informed traders is short-lived. His optimal is to trade on his information's advantages. He makes zero profit if no trade while he makes a positive profit if he chooses to trade.

If both the spoofer and anticipatory trader choose to not trade, the model turns into the standard Kyle [1985] model. In the first period, there are only noise traders. As there is no informed trade, the market maker sets the price $p_1 = v_0 = 0$. In the second period, there are only the informed trader and noise traders.

If both spoofer and anticipatory trader decide to trade, they need to choose optimal trading volume to maximize their expected profits. The total net order flow y_1 and y_2 executed at t = 1 and t = 2 are

$$y_1 = -z_1 + m + u_1 \tag{11}$$

$$y_2 = z_1 - m + x + u_2 \tag{12}$$

If only the anticipatory trader chooses to trade, the model collapses into the Xu and Cheng [2023] model. The total net order flow y_1 and y_2 executed at t = 1 and t = 2 are

$$y_1 = m + u_1 \tag{13}$$

$$y_2 = -m + x + u_2 \tag{14}$$

If only the spoofer chooses to trade, the total net order flow y_1 and y_2 executed at t = 1 and t = 2 are

$$y_1 = -z_1 + u_1 \tag{15}$$

$$y_2 = z_1 + x + u_2 \tag{16}$$

5.1 Anticipatory trader's problem

The anticipatory trader is a short term fast trader who has speed advantages over other slow traders. Due to these advantages, he can trade twice in the model. He opens his position in the first period and liquidates it entirely in the second period. As a short term trader, he holds no inventory at the end. Unlike the informed trader whose the profit is determined by the difference between entry price and the fundamental value, the anticipatory trader profit depends on the difference his entry and exit prices. Therefore, his main focus is to predict the short-term price dynamics based on his signal, not the fundamental value of the asset.

If the anticipatory trader does not trade, his profit is zero. Therefore, we only need to consider his optimization problem if he decides to trade. Given the strategies of the informed trader, the spoofer, and the pricing rule of the market maker, the anticipatory trader chooses trading volume m to maximize his profit. In the anticipatory trader's belief, there may be only the informed trader and uninformed traders or he may be aware of the spoofer but do not know the spoofer's strategy. In this case, we assume that the order anticipation HFT misinterprets the spoofing order as the noise in his signal. This assumption is consistent with Richard May's testimony [13] "In 2013, Mr. May and his team observed what they believed was spoofing in the ES market. Around this time, they noticed a significant decline in Citadel's profitability...Immediately, Citadel scaled back its participation in the ES market by over fifty percent. Mr. May's team began investigating the market data to determine why Citadel was experiencing such a decline...Eventually, Mr. May's team developed a program to detect this behavior on a more automated basis in an effort to determine whether this was "a new phenomenon or something that had always been there that [they] hadn't previously seen". Anticipatory trader is only aware of the existence of spoofing when their profit declines.Therefore, in his optimization problem, the total net order flow y_1 and y_2 executed at t = 1 and t = 2 are

$$y_1 = m + u_1 \tag{17}$$

$$y_2 = -m + x + u_2 \tag{18}$$

If the anticipatory trader wants to trade, his optimization problem is to choose m to maximize his expected profit.

$$\max_m E[m(p_2 - p_1)|\tilde{i} + z_2, P(.), I(.)] \tag{19}$$

Plug equations (6), (7), (17), (18) into the equation (19) and simplify

$$\max_m E[m(\lambda_{22}x - (\lambda_1 + \lambda_{22} - \lambda_{12})m))|\tilde{i} + z_2, I(.)] \tag{20}$$

For tractability, we also conjecture that the informed trader's strategy admits a linear function of his signal $x = \beta v$. By using the projection formula, we can

[13] Case: 1 15−cv−09196, U.S. District Court - Northern District of Illinois, https://www.govinfo.gov/content/pkg/USCOURTS−ilnd−_15−cv−09196/pdf/USCOURTS −ilnd−1_15−cv−09196−.pdf

obtain $E[x|\tilde{i}+z_2] = \frac{\beta^2\sigma^2}{\beta^2\sigma^2+\sigma_\epsilon^2+\sigma_{z_2}^2}(\tilde{i}+z_2)$ .Therefore, the first-order condition for the anticipatory problem is

$$m = \frac{\lambda_{22}}{2(\lambda_1+\lambda_{22}-\lambda_{12})}\frac{\beta^2\sigma^2}{\beta^2\sigma^2+\sigma_\epsilon^2+\sigma_{z_2}^2}(x+\epsilon+z_2) \tag{21}$$

As the second order condition is $\lambda_1+\lambda_{22}-\lambda_{12} > 0$. Compare equation (21) and the conjectured strategy (9), we have

$$\beta_1 = \frac{\lambda_{22}}{2(\lambda_1+\lambda_{22}-\lambda_{12})}\frac{\beta^2\sigma^2}{\beta^2\sigma^2+\sigma_\epsilon^2+\sigma_{z_2}^2} \tag{22}$$

In the next section, we will also consider the problem in which the anticipatory trader can extract the informed trade perfectly from the noisy signal even when the spoofer adds more noise to the market.

5.2 Spoofer's problem

Similar to the anticipatory trader, the spoofer is a fast trader with a short-term trading horizon. He also has no inventory holding at the end of the second period. The main difference between the anticipatory and the spoofer lies in their trading strategies. While the anticipatory trader uses pattern recognition algorithms to examine trades and quotes to extract trading signals, the spoofer's focus is to add more noise to the limit order book by submitting a big spoofing order. In this way, the spoofer can mislead the anticipatory trader.

When the spoofer opts to trade, he needs to choose to submit a real order flow z_1 and a spoofing order z_2 to maximize his expected profit. For the model's simplicity, in this section, we only allow the spoofer to choose the optimal z_1 explicitly. Even though only z_1 is chosen optimally in the spoofer's optimization problem, z_2 implicitly faces some constraints. First, z_2 faces the upper bound of his inventory of the asset or his ability to borrow the asset. Second, the spoofer is a fast trader who does not want to hold inventory at the end. The higher the spoofing order z_2 is, the higher volume is exposed to the risk of execution. Even though, the spoofing order only rests for a short period in the limit order book, it faces the same risk of execution as other open orders when it is there. The execution risk can leave the spoofer holding the inventory at the end of the second period, thus deviating from his trading strategy. The spoofer's signal optimization problem is presented in the next section.

We only need to consider the spoofer's optimization problem when he opts to trade. Formally, the spoofer chooses z_1 to maximize his expected profit, given his signal, strategies of the anticipatory trader, the informed trader, and the market maker.

$$\max_{z_1} E[z_1(p_1-p_2)|z_2, I(.), A(.), P(.)] \tag{23}$$

Inserting (11), (12), (6), (7) into (23) yielding

$$\max_{z_1} E[z_1(\lambda_1(m-z_1)-\lambda_{12}(m-z_1)-\lambda_{22}(x+u_2-m+z_1))|z_2, I(.), A(.), P(.)] \tag{24}$$

By our assumption, $z_1 = \beta_2 z_2$ and z_2 is mutually independent of u_1, u_2, x. Therefore, $E[z_1 x] = 0, E[z_1 u_2] = 0, E[z_1 m] = \beta_2 z_2^2$. Substituting this expression into (24), we obtain

$$\max_{\beta_2} \beta_2(\lambda_1 + \lambda_{22} - \lambda_{12})(\beta_1 - \beta_2) z_2^2 \tag{25}$$

Taking the first-order-condition (FOC) results in the solution as follows:

$$\beta_2 = \frac{1}{2}\beta_1 \tag{26}$$

The second-order condition for the spoofer's problem is the same as the order anticipation HFT's problem $\lambda_1 + \lambda_{22} - \lambda_{12} > 0$.

5.3 Market maker's problem

From the equilibrium definition, the market maker sees the aggregate order flow in each period and sets the prices efficiently. However, these aggregate order flows vary according to the spoofer and the anticipatory's strategies. There are 4 possibilities:

1. *Both traders choose to trade.* By combining equations (9),(10), (11), (12), and (26), we can obtain the total net order flow y_1 and y_2 executed at t = 1 and t = 2 as follows:

$$y_1 = -\frac{1}{2}\beta_1 z_2 + \beta_1(\tilde{i} + z_2) + u_1 = \beta_1 \tilde{i} + \frac{1}{2}\beta_1 z_2 + u_1 \tag{27}$$

$$y_2 = x - \beta_1 \tilde{i} - \frac{1}{2}\beta_1 z_2 + u_2 \tag{28}$$

2. *Only the anticipatory trader trades.* In this case, the real order z_1 and spoofing order z_2 of the spoofer are both zero. By inserting the equation (9) into (13), (14), we arrive at the aggregate order flows

$$y_1 = \beta_1 \tilde{i} + u_1 \tag{29}$$

$$y_2 = x - \beta_1 \tilde{i} + u_2 \tag{30}$$

3. *Only the spoofer trader trades.* As the anticipatory opts out of the trade, his trading volume is zero. By using the equations (26) into (15), (16), we arrive at the aggregate order flows

$$y_1 = -\frac{1}{2}\beta_1 z_2 + u_1 = -\frac{1}{2}\beta_1 z_2 + u_1 \tag{31}$$

$$y_2 = x + \frac{1}{2}\beta_1 z_2 + u_2 \tag{32}$$

4. *If both traders do not trade.* In the first period, there are only noise traders. Therefore $p_1 = p_0 = 0$. In the second period, the economy collapses into the standard Kyle model.

We can rewrite these aggregate order flows under different circumstances into a unified general form

$$y_1 = a_1\tilde{i} + t + u_1 \tag{33}$$

$$y_2 = a_2x - a_1\epsilon - t + u_2 \tag{34}$$

Where we denote $a_1 = k\beta_1, a_2 = 1 - a_1$ and t is independent of u_1, u_2, v, ϵ. k and t for each case are as follows:

$$(k,t) = \begin{cases} (1, \frac{1}{2}\beta_1 z_2) & \text{Both trade} \\ (1,0) & \text{Only the anticipatory trades} \\ (0, -\frac{1}{2}\beta_1 z_2) & \text{Only the spoofer trades} \\ (0,0) & \text{Both do not trade} \end{cases}$$

In the first period, the market maker can observe the order flow y_1 and set the price $p_1 = E[v|y_1]$. By using the project theorem, we can derive λ_1

$$\lambda_1 = \frac{Cov(v,y_1)}{Var(y_1)} = \frac{a_1\beta\sigma^2}{a_1^2(\beta^2\sigma^2 + \sigma_\epsilon^2) + \frac{1}{4}\beta_1^2\sigma_{z_2}^2 + \sigma_u^2} \tag{35}$$

Similarly, the market maker see the aggregate order flow y_1, y_2 in the second period and set the price $p_2 = E[v|y_1, y_2]$. By combining equations (32), (33), (7) and applying the projection theorem, we obtain

$$\begin{aligned} \lambda_{12} &= \frac{Cov(y_1,v)Var(y_2) - Cov(y_1,y_2)Cov(v,y_2)}{Var(y_1)Var(y_2) - Cov^2(y_1,y_2)} \\ &= \frac{\sigma^2\beta(a_1^2\sigma_\epsilon^2 + \frac{1}{4}\beta_1^2\sigma_{z_2}^2 + a_1\sigma_u^2)}{\sigma_u^2(2a_1^2\sigma_\epsilon^2 + \frac{1}{2}\beta_1^2\sigma_{z_2}^2 + \sigma_u^2) + \sigma^2\beta^2(a_1^2\sigma_\epsilon^2 + \frac{1}{4}\beta_1^2\sigma_{z_2}^2 + (a_1^2 + a_2^2)\sigma_u^2)} \end{aligned} \tag{36}$$

$$\begin{aligned} \lambda_{22} &= \frac{Cov(y_2,v)Var(y_1) - Cov(y_1,y_2)Cov(v,y_1)}{Var(y_1)Var(y_2) - Cov^2(y_1,y_2)} \\ &= \frac{\sigma^2\beta(a_1^2\sigma_\epsilon^2 + \frac{1}{4}\beta_1^2\sigma_{z_2}^2 + a_2\sigma_u^2)}{\sigma_u^2(2a_1^2\sigma_\epsilon^2 + \frac{1}{2}\beta_1^2\sigma_{z_2}^2 + \sigma_u^2) + \sigma^2\beta^2(a_1^2\sigma_\epsilon^2 + \frac{1}{4}\beta_1^2\sigma_{z_2}^2 + (a_1^2 + a_2^2)\sigma_u^2)} \end{aligned} \tag{37}$$

5.4 Informed trader's problem

Unlike the anticipatory trader and the spoofer, the informed trader is a slow trader who can only trade in the second period. He submits his order in the first period and the order only arrives in the exchange in the second period. The latency allows the anticipatory trader to use pattern recognition algorithms to detect the informed trader. Even though our model only allows market orders, we can interpret the latency under the limit order settings in the following way. The informed trader may have submitted a big limit order and it takes time until the order gets executed. During that time, by using algorithms, the anticipatory trader can detect informed trading intentions.

Based on his signal about the true value of the assets v, The informed trader chooses x^* to maximize his expected profit. Using equations (33), (34), we can obtain his expected profit.

$$E[x(v-p_2)|v, I(.), A(.), P(.)] = x(v-(a_1\lambda_{12}+a_2\lambda_{22})x) \tag{38}$$

Taking the first-order-condition (FOC) results in the solution as follows:

$$x = \frac{1}{2(a_1\lambda_{12}+a_2\lambda_{22})}v \tag{39}$$

The second order condition is $2(a_1\lambda_{12}+a_2\lambda_{22}) > 0$. Combining with the conjectured strategy, we have

$$\beta = \frac{1}{2(a_1\lambda_{12}+a_2\lambda_{22})} \tag{40}$$

Inserting the equations (37), (36) into (40) to obtain:

$$\beta^2 = \frac{\sigma_u^2(2a_1^2\sigma_\epsilon^2+\frac{1}{2}\beta_1^2\sigma_{z_2}^2+\sigma_u^2)}{\sigma^2(a_1^2\sigma_\epsilon^2+\frac{1}{4}\beta_1^2\sigma_{z_2}^2+(a_1^2+a_2^2)\sigma_u^2)} \tag{41}$$

5.5 Equilibrium Characterization and Properties

We denote $\theta_\epsilon = \frac{\sigma_\epsilon^2}{\sigma_u^2}$ and $\theta_{z_2} = \frac{\sigma_{z_2}^2}{\sigma_u^2}$.From the above analysis, there are 4 possibilities depending on the strategies of the spoofer and the anticipatory trader. The following proposition formally specifies a linear equilibrium when both traders opt to trade.

Proposition 5.1 *In the economy where both the spoofer and the anticipatory trader choose to trade, there exists a unique linear strategy equilibrium. The equilibrium is characterized by a tuple of* $(\lambda_1, \lambda_{12}, \lambda_{22}, \beta, \beta_1, \beta_2)$ *through the system of equations:*

$$\lambda_{12} = \frac{\sigma^2\beta(a_1^2\theta_\epsilon+\frac{1}{4}\beta_1^2\theta_{z_2}+a_1)}{2\sigma_u^2(2a_1^2\theta_\epsilon+\frac{1}{2}\beta_1^2\theta_{z_2}+1)} \tag{42}$$

$$\lambda_{22} = \frac{\sigma^2\beta(a_1^2\theta_\epsilon+\frac{1}{4}\beta_1^2\theta_{z_2}+a_2)}{2\sigma_u^2(2a_1^2\theta_\epsilon+\frac{1}{2}\beta_1^2\theta_{z_2}+1)} \tag{43}$$

$$\lambda_1 = \frac{a_1\beta\sigma^2}{a_1^2(\beta^2\sigma^2+\sigma_\epsilon^2)+\frac{1}{4}\beta_1^2\sigma_{z_2}^2+\sigma_u^2} \tag{44}$$

$$\beta_2 = \frac{1}{2}\beta_1 \tag{45}$$

$$\beta^2 = \frac{\sigma_u^2(2a_1^2\theta_\epsilon+\frac{1}{2}\beta_1^2\theta_{z_2}+1)}{\sigma^2(a_1^2\theta_\epsilon+\frac{1}{4}\beta_1^2\theta_{z_2}+(a_1^2+a_2^2))} \tag{46}$$

$$\beta_1 = \frac{\lambda_{22}}{2(\lambda_1 + \lambda_{22} - \lambda_{12})} \frac{\beta^2\sigma^2}{\beta^2\sigma^2 + \sigma_\epsilon^2 + \sigma_{z_2}^2} \tag{47}$$

Where $a_1 = \beta_1, a_2 = 1 - a_1$*. Thus, the profit of each trader is given by*

$$e[\pi_A^{AS}] = E[m(p_2 - p_1)] = (\lambda_1^{AS} + \lambda_{22}^{AS} - \lambda_{12}^{AS})(\beta_1^{AS})^2(\frac{3}{2}\sigma_{z_2}^2 + (\beta^{AS})^2\sigma^2 + \sigma_\epsilon^2) \tag{48}$$

$$E[\pi_S^{AS}] = E[z_1(p_1 - p_2)] = \frac{\lambda_1^{AS} + \lambda_{22}^{AS} - \lambda_{12}^{AS}}{4}(\beta_1^{AS})^2\sigma_{z_2}^2 \tag{49}$$

$$E[\pi_I^{AS}] = E[x(v - p_2)] = \frac{\beta^{AS}}{2}\sigma^2 \tag{50}$$

In the cases of both traders trading, the second order condition from the optimization problem indicates that $\lambda_1^{AS} + \lambda_{22}^{AS} - \lambda_{12}^{AS} > 0$. Therefore, $E[\pi_A^{AS}] > 0$ and $E[\pi_A^{AS}] > 0$. In other words, in the economy where both the spoofer and the anticipatory trader choose to trade, both of them make positive profits.

Proposition 5.1 reveals that in equilibrium, the spoofer only uses part of the signal that he sent to the anticipatory trader. The anticipatory trader loses money from trading against the spoofer but makes a positive profit from anticipating the informed order. As the anticipatory trader can deduce the informed order, he protects himself from the spoofer by reducing the trading intensity when there is the spoofer who adds more noise to the anticipatory trader's signal. On average, the anticipatory trader still makes a positive profit as the loss from the spoofer is compensated by profit from exploiting the informed trader.

Now, we consider the economy where only anticipatory trader trades. Similarly, the equilibrium of this economy is characterized by a tuple of $(\lambda_1, \lambda_{12}, \lambda_{22}, \beta, \beta_1, \beta_2)$. As the spoofer opts out of the market, his real and spoofing orders are zero. Therefore, we do not need to consider the spoofer problem or $\beta_2 = 0$. The following proposition formally specifies a linear strategy equilibrium of this economy.

Proposition 5.2 *In an economy where only the anticipatory trader trades, there exists a unique linear strategy equilibrium. The equilibrium is characterized by a tuple of* $(\lambda_1, \lambda_{12}, \lambda_{22}, \beta, \beta_1, \beta_2)$ *through the system of equations:*

$$\lambda_{12} = \frac{\sigma^2\beta(a_1^2\theta_\epsilon + a_1)}{2\sigma_u^2(2a_1^2\theta_\epsilon + 1)} \tag{51}$$

$$\lambda_{22} = \frac{\sigma^2\beta(a_1^2\theta_\epsilon + a_2)}{2\sigma_u^2(2a_1^2\theta_\epsilon + 1)} \tag{52}$$

$$\lambda_1 = \frac{a_1\sigma^2\beta(a_1^2\theta_\epsilon + (a_1^2 + a_2^2))}{a_1^2\sigma_u^2(2a_1^2\theta_\epsilon + 1) + \sigma_u^2(a_1^2\theta_\epsilon + 1)(a_1^2\theta_\epsilon + (a_1^2 + a_2^2))} \tag{53}$$

$$\beta_2 = 0 \tag{54}$$

$$\beta^2 = \frac{\sigma_u^2(2a_1^2\theta_\epsilon + 1)}{\sigma^2(a_1^2\theta_\epsilon + (a_1^2 + a_2^2))} \tag{55}$$

$$\beta_1 = \frac{\lambda_{22}}{2(\lambda_1 + \lambda_{22} - \lambda_{12})} \frac{\beta^2\sigma^2}{\beta^2\sigma^2 + \sigma_\epsilon^2} \tag{56}$$

Where $a_1 = \beta_1, a_2 = 1 - a_1$. Thus, the profit of each trader is given by

$$E[\pi_A^{A0}] = E[m(p_2 - p_1)] = (\lambda_1 + \lambda_{22} - \lambda_{12})(\beta_1^{A0})^2((\beta^{A0})^2\sigma^2 + \sigma_\epsilon^2) \tag{57}$$

$$E[\pi_S^{A0}] = 0 \tag{58}$$

$$E[\pi_I^{A0}] = E[x(v - p_2)] = \frac{\beta^{A0}}{2}\sigma^2 \tag{59}$$

The result is immediate using the proof of the proposition 5.1. The proposition 5.2 is the special case of the the proposition 5.1 with $\theta_{z_2} = 0$. Similarly the expected profit $E[\pi_A^{A0}]$ of the anticipatory trader is positive.

If the order anticipation HFT does not participate in the market, his trading volume is zero or $\beta_1 = 0$. However, the spoofer still expects the HFT's order. We denote $\tilde{\beta_1}$ the trading intensity of the HFT under the spoofer's belief. As a result of the false anticipation of HFT's strategy, the spoofer loses money. The following proposition formally specifies a linear strategy equilibrium of this economy.

Proposition 5.3 *When only spoofer opts to trade, there exists a unique linear strategy equilibrium $(\lambda_1, \lambda_{12}, \lambda_{22}, \beta, \beta_1, \beta_2)$ specified by the system of equations:*

$$\lambda_{12} = \frac{\sigma^2\beta\frac{1}{4}\tilde{\beta_1}^2\theta_{z_2}}{2\sigma_u^2(\frac{1}{2}\tilde{\beta_1}^2\theta_{z_2} + 1)} \tag{60}$$

$$\lambda_{22} = \frac{\sigma^2\beta(\frac{1}{4}\tilde{\beta_1}^2\theta_{z_2} + 1)}{2\sigma_u^2(\frac{1}{2}\tilde{\beta_1}^2\theta_{z_2} + 1)} \tag{61}$$

$$\lambda_1 = 0 \tag{62}$$

$$\beta_2 = \frac{1}{2}\tilde{\beta_1} \tag{63}$$

$$\beta^2 = \frac{\sigma_u^2(\frac{1}{2}\tilde{\beta_1}^2\theta_{z_2} + 1)}{\sigma^2(\frac{1}{4}\tilde{\beta_1}^2\theta_{z_2} + 1)} \tag{64}$$

$$\tilde{\beta_1} = \frac{\lambda_{22}}{2(\lambda_1 + \lambda_{22} - \lambda_{12})} \frac{\beta^2\sigma^2}{\beta^2\sigma^2 + \sigma_\epsilon^2 + \sigma_{z_2}^2} \tag{65}$$

$$\beta_1 = 0 \tag{66}$$

Thus, the profit of each trader is given by

$$E[\pi_A^{0S}] = 0 \tag{67}$$

$$E[\pi_S^{0S}] = E[z_1(p_1 - p_2)] = -\frac{\lambda_1 + \lambda_{22} - \lambda_{12}}{4}(\tilde{\beta_1}^{0S})^2\sigma_{z_2}^2 \tag{68}$$

$$E[\pi_I^{0S}] = E[x(v - p_2)] = \beta^{0S}\sigma^2 \tag{69}$$

Table 2: Payoff matrix for anticipatory trader and spoofer.

		Spoofer	
		Trade	No trade
Anticipatory trader	Trade	$(E[\pi_A^{AS}], E[\pi_S^{AS}])$	$(E[\pi_A^{A0}], 0)$
	No trade	$(0, E[\pi_S^{0S}])$	$(0, 0)$

When there is only the spoofer, all orders in the first period are uninformed. Therefore, the market maker sets $\lambda_1 = 0$. However, in the second period, he adjusts the price impact λ_{12} of the first-period order flow as he learns that order flows of two periods are correlated. As there is no anticipatory trader to be preyed upon, the spoofer suffers from a loss to the informed trader.

Proposition 5.4 *There exists a unique linear pure strategy equilibrium in which both the spoofer and the anticipatory trader use pure strategies and make positive profits.*

The payoff matrix for both players is presented in table 2. Obviously, "Trade" is the dominant strategy for the anticipatory trader as he makes zero profit if he chooses to not trade. When the anticipatory trader plays "trade", the optimal strategy for the spoofer is to trade. Therefore, in equilibrium, both traders opt to trade and make positive profits. Order anticipation strategies are profitable against traditional orders entered by big players. But with spoofers in the mix, the game looks quite different. When the order anticipation HFT wants to jump ahead of the spoofer, the HFT falls prey to the spoofer and loses money. In short, spoofing poses the risk of making order anticipation strategies unprofitable. However, spoofing is only profitable if order anticipation algorithms are active. When the anticipatory traders choose to not trade, the spoofer gets fooled by his own strategy and loses money. Zaloom [2003] documented the incidence in which the spoofer falls prey to his own strategy and gets fooled by other traders. " Traders learned to identify a spoofer by watching changes in the aggregate number of bids or offers on the screen creating a novel strategy for profit. By riding the tail of a spoofer, a small trader could make money on market direction. Traders who dealt in large contract sizes aspired to "take out" the Spoofer by calling his bluff, selling into his bid, and waiting for him to balk. There was great symbolic capital attached to "taking out" a spoofer by matching wits with this high-risk player. Taking out the Spoofer showed the prowess of a trader in one-to-one combat". The spoofer and the anticipatory trader are two sides of the same coin, the existence of one keeps the other in check.

Proposition 5.5 *In equilibrium, the optimal intensities of the anticipatory trader* *and the informed trader decrease with θ_{z_2} and θ_ϵ. Mathematically,*

$$\frac{\partial\beta}{\partial\theta_{z_2}} < 0, \frac{\partial\beta}{\partial\theta_\epsilon} < 0 \tag{70}$$

$$\frac{\partial\beta_1}{\partial\theta_{z_2}} < 0, \frac{\partial\beta_1}{\partial\theta_\epsilon} < 0 \tag{71}$$

The proposition 5.5 shows that the anticipatory trader strategically responds to the spoofer by reducing his participation when the spoofer increases spoofing intensity. When θ_{z_2} is higher, the signal of the anticipation HFT becomes noisier. Therefore, he becomes less active in the market. This result is consistent with the testimony of Mr.May in the second section. Surprisingly, spoofing affects the informed trader unfavorably. However, the informed trader is less responsive to spoofing than the anticipatory trader. In other words, spoofing only has indirect effects on the informed trader's strategy as a result of changes in other traders' strategies. This argument is clearly illustrated in Section 8.

6 The spoofer's signal optimization problem

In the previous section, z_2 is treated as a given random variable. In this section, the spoofer is allowed to optimally choose z_2 to maximize his ex-ante expected profit. z_2 is characterized by the variance $\sigma^2_{z_2}$, which can be interpreted as the spoofing intensity of the spoofer. From the previous section, $\sigma^2_{z_2}$ has a mixed effect on the spoofer's profit. The higher the spoofing intensity, the lower the trading intensity of the anticipatory trader, thus reducing the real trading volume of the spoofer. However, the higher the spoofing intensity, the higher the profit per share. From the equation (49), the spoofer's expected profit is given by.

$$E[\pi_S] = E[z_1(p_1 - p_2)] = \frac{\lambda_1 + \lambda_{22} - \lambda_{12}}{4}(\beta_1)^2\sigma^2_{z_2} \tag{72}$$

The spoofer's signal optimization problem is to choose $\sigma^2_{z_2}$ to maximize his expected profit $E[\pi_S]$ subject to constraints (42), (43), (44), (45), (46), (47), and $\lambda_1 + \lambda_{22} - \lambda_{12} > 0$.

Proposition 6.1 *Given $\sigma^2, \sigma^2_\epsilon, \sigma^2_u$, the spoofer's signal optimization problem has a global maximum.*

From the proof in the Appendix, we can see that not spoofing or $\sigma_z = 0$ is not the optimal solution to the spoofer's signal optimization problem as he makes a zero profit. Too high spoofing variance σ_z is also not optimal as the higher the spoofing intensity is, the lower the anticipatory trader order flow is. When σ_{z_2} approaches infinity, β_1 goes to zero as the order anticipation HFT protects himself by reducing his market participation. As a result, the profit of the spoofer approaches zero.

7 Market quality

7.1 Market efficiency

Two important aspects of market efficiency are price accuracy (or price discovery) and the liquidity of the market. First, liquidity is a multi-dimensional concept with most measures only capturing one of its many aspects. Under the Kyle framework, market liquidity is defined as the inverse of the Kyle lambdas λ_1, λ_{12}, λ_{22} which are price impacts of trading. Those λs measure how much the price moves with one unit of share. The lower the price impact, the deeper and more liquid the market is. Second, price discovery is measured by how much information is incorporated into the price of an asset. More accurate pricing stocks can generate more efficient capital allocations and foster investor's sense of fairness. For Kyle's setting, price discovery is measured by the forecast error variance of the market maker.

$$\Sigma_1 = E[(v - p_1)^2] \tag{73}$$

$$\Sigma_2 = E[(v - p_2)^2] \tag{74}$$

Proposition 7.1 *In equilibrium, the price impacts of the first period λ_1 and λ_{12} are decreasing in σ_{z_2} while the price impact of the second period of the second period is increasing in σ_{z_2}. Mathematically*

$$\frac{\partial \lambda_1}{\partial \theta_{z_2}} \leq 0, \frac{\partial \lambda_{12}}{\partial \theta_{z_2}} \leq 0 \tag{75}$$

$$\frac{\partial \lambda_{22}}{\partial \theta_{z_2}} \geq 0, \tag{76}$$

In the first period, price impacts are decreasing in the spoofing intensity. The more the spoofing variance, the lower the price impact. As the spoofer adds more noise to the order anticipation HFT's signal, the anticipatory trader reduces his trading activities in the first period. This makes aggregate order in the first period less informed. Therefore, spoofing leads to a lower price impact in the first period. However, in the second period, spoofing increases the price impact. If we consider each period separately, spoofing has a mixed effect on liquidity. But if we combine them, the best measure is $\lambda_1 + \lambda_{22}$ which is the proxy for the welfare of uninformed traders. This one we will consider in the next section.

Proposition 7.2 *In equilibrium, the price discovery measure of the first period Σ_1 is increasing in σ_z while the price discovery measure of the second period Σ_2 is the same for all different models (AS, A0, 0S, Kyle). Mathematically*

$$\frac{\partial \Sigma_1}{\partial \theta_{z_2}} \leq 0 \tag{77}$$

$$\Sigma_2 = \frac{\sigma^2}{2} \tag{78}$$

In the first period, the market maker's forecast error variance is decreasing in spoofing intensity. As the result of a proposition 7.2, we can have $\Sigma_1^{Kyle} \geq \Sigma_1^{AS} \geq \Sigma_1^{A0}$. Compared to the standard Kyle model, both spoofing and order anticipation speed up the price discovery. However, the improvement of price discovery is at the expense of the informed trader in the form of information leakage. In a short duration, spoofing delays price discovery by adding more noise to the order anticipation HFTs' signal, thus reducing information leakage. In the second period, the market maker's forecast error variances are the same across models. The reason is that both the spoofer and anticipatory trader are short-term traders, they tend to close their positions within a short timeframe and their net positions are zero within two periods.

7.2 Wealth transfer and market welfare

Trading is a zero-sum game, so if someone has expected profits from the trade, the other has to suffer the loss. To understand how spoofing affects market welfare, we need to study how this practice affects the wealth positions of all market participants and the implications of these effects.

Proposition 7.3 *In equilibrium, the expected profit of the anticipatory trader and the loss to uninformed traders are decreasing in σ_z. Mathematically*

$$\frac{\partial E[\pi_I]}{\partial \theta_{z_2}} \leq 0 \tag{79}$$

$$\frac{\partial E[\pi_U]}{\partial \theta_{z_2}} \geq 0 \tag{80}$$

For the uninformed trader, the higher the spoofing intensity, the lower the loss to the uninformed trader. From proposition 5.5, when the spoofer increases the spoofing variance, the trading intensities of the anticipatory trader and the informed trader decrease. As a result, uninformed traders are less likely to be exploited by other traders. In other words, uninformed traders indirectly benefit from spoofing.

8 Model calibration

The main purpose of this section is to simulate the model numerically to ensure that the calibrated model is consistent with our findings. From the previous section, we have proved that the model can be solved numerically if a set of $\sigma^2, \sigma_u^2, \sigma_\epsilon^2, \sigma_{z_2}^2$ is given. We interpret the traded asset as a typical stock in the US stock market. Specifically, we choose SPY (SPDR S&P 500 ETF Trust), as it tracks the S&P 500 index. Duong and Taub [2023] used trading data in Feburary, 2018 and estimated $\sigma^2 = 0.00039$ and $\sigma_u^2 = 731,957$. In order to reduce the computation, we convert the unit of σ_u to thousand shares, $\sigma_u = 0.8555$. The only remaining parameter σ_ϵ^2 which is hard to observe but an important one that determines the nature of the equilibrium. We start our analysis with $\theta_\epsilon = \frac{\sigma_\epsilon^2}{\sigma_u^2} = 0.4$, then we explore the variation in θ_ϵ in subsequent analysis. The optimal signal $\sigma_{z_2}^2$ can be recovered from the spoofer's signal optimization problem.

Parameter	σ^2	σ_u^2	σ_ϵ^2
Value	0.00039	0.732	0.2927
Unit	Dollar squared	Thousand shares squared	Thousand shares squared

Table 3: Parameter values

Using these parameters, we solve 2 separate models numerically. The baseline model is the economy with only the anticipatory trader $\sigma_{z_2} = 0$. The second model is the economy with both the spoofer and the order anticipation HFT. In this model, we allow the spoofer to choose its optimal spoofing strategy. The solutions of two models are given as follows

	Baseline (A0)	Spoofer's optimization
σ_{z_2}	0	1.072
β_1	0.3181	0.1797
β	57.83	51.96
λ_1	0.0083	0.00464
λ_{12}	0.0051	0.0027
λ_{22}	0.01029	0.0111

Table 4: Solutions to AS and A0 models

The solutions to AS and A0 models are consistent with the findings of Duong and Taub [2023]. In that paper, we measure the slope of the book by running a simple regression of price against quantity in each snapshot of the limit order book and recover an estimate of λ. The estimated $\lambda = 0.0000394$/share or 0.0394/thousands shares for SPY. It is in the same magnitude of our calibrated λ_{22}. Furthermore, Duong and Taub [2023] also presents that the average and median volumes of SPY per message are 169.5 shares and 100 shares. The optimal spoofing deviation for the spoofer is 1072 shares, which is over 10 times the median volume per message and 6.3 times the average volume per message. This scale is in line with the spoofing orders recorded in many spoofing cases. For example, in the complaint against Igor B.Oystacher [14], the CFTC presented evidences that Igor B.Oystacher used big orders to give the false sense of market depth. At 8:02:34.360 a.m. on November 30, 2012, he was alleged to have opened a short position of 10 futures contracts in natural gas while placing seven visible orders of 103 contracts. His strategy led to an 11 times increase in the visible market depth. Unsurprisingly, even though the spoofer needs to send a big order to mislead the order anticipation HFT, it is not optimal to send too big

[14] Complaint Case: 1:15-cv-09196 https://www.cftc.gov/sites/default/files/idc/groups/public/@lrenforcementactions/documents/legalpleading/enfigorcomplnt101915.pdf

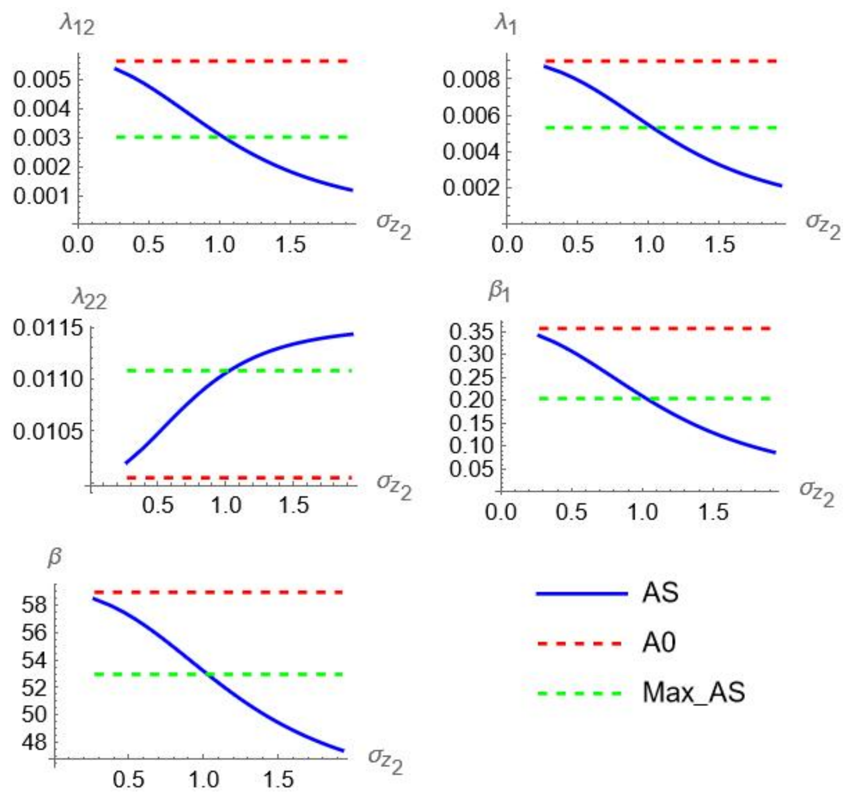


Figure 1: Numerical solutions to the models when $\theta\epsilon = 0.4$

an order. The optimal variance of the spoofing order in this case is about 1.23 times more than the variance of noise trade. The big order exposes the spoofer to the risk of execution and detection by other traders.

In order to help intuition, we study the variation of $\sigma^2_{z_2}$ and its effects on other traders' strategies. We also compare the results with the baseline model above.

Figure 1 presents the numerical solutions to different models for the various values of σ_{z_2}. The green dashed line is the outcome for the economy (AS) with both traders and the spoofer maximizing his signal. The red dashed line represents the solutions to the baseline model with only the anticipatory trader. The solid blue line is the outcome of the AS models with various values of σ_{z_2}. Looking across panels of Figure 1, it is obvious that $\lambda_1, \lambda_{12}, \beta_1, \beta$ are decreasing in σ_{z_2} and lie below the red line of the baseline model which indicates those values of AS models is less than those of the baseline model. It can be explained that when the spoofer adds more noise to the market, there is more buffer liquidity in the market, thus leading to a decrease in the price impact of the first period. At the same time, the spoofer makes the anticipatory trader's signal less accurate. The order anticipation HFT protects itself by reducing its trading intensity. Contrarily, λ_{22} is an increasing function of σ_{z_2} and lies above the baseline line. It is notable that λ_{22}, β are also relatively insensitive to changes in σ_{z_2} while σ_{z_2} variations affects $\beta_1, \lambda_1, \lambda_{12}$ significantly. This is due to the fact that the spoofer directly influences the strategies of the anticipatory HFT but has indirect effects on the informed trader. In all panel of Figure 1, the green dashed lines cross the blue line at the optimal value of $\sigma_{z_2} = 1.072$.

Figure 2 shows the ex-ante profits(loss) of all traders in the model. In the case of the spoofer, his profit is a concave function in σ_z. The green dashed line is tangent to his profit curve at the optimal value of $\sigma_{z_2} = 1.072$. The profits of the informed trader and the anticipatory trader are decreasing in spoofing intensity and lie below the red dashed line. However, the profit of the anticipatory HFT is more sensitive to σ_z than that of the informed trader. It is understandable as the order anticipation trader has a noisier signal as the spoofer increases his spoofing intensity. The anticipatory trader reduces his trading intensity rapidly to avoid the loss, thus leading to the sensitivity of his trading strategy to the spoofer's strategy. In the case of the informed trader, he has private information therefore, he is less sensitive to the spoofer's strategy.

Figure 3 and figure 4 present how the model solution is sensitive to variations in θ_ϵ. Looking across panels of Figure 3, it is obvious that the dependencies of $\lambda_1, \lambda_{12}.\beta_1, \beta, \lambda_{22}$ on θ_ϵ is similar to those dependencies on θ_{z_2}. It is understandable as both θ_ϵ and θ_{z_2} are noises to the anticipatory's trader. The only difference between them is the sources of noise. The optimal spoofing deviations for the spoofer range from above 1000-1700 shares, which are over 10 times the median volume per message. This scale is in line with the spoofing orders recorded in many spoofing cases we presented above. If we examine Figure 3, the profit of the spoofer is decreasing in θ_ϵ. As θ_ϵ increases, the signal of the anticipatory traders becomes noisier. As a result, he scales back his trading participation, thus reducing the profit of the spoofer. In other words, the more accurate the signal of the anticipatory trader, the more easily he can be exploited.

9 Policy discussion

There are many reasons that were cited by regulators to justify the prohibition of spoofing. But the most common ones are to protect market integrity and fairness. The DOJ[15] has stated that spoofing "poses a significant risk of eroding confidence in U.S. markets" and " protecting the integrity of our markets remains a significant priority in our fight against economic crime". James McDonald, the CFTC's Director of Enforcement, also has commented "Spoofing undermines the integrity of our markets and gives those engaging in the

[15] Press release. U.S. Dep't of Justice, Eight Individuals Charged with Deceptive Trading Practices Executed on U.S. Commodities Markets (Jan. 29, 2018) https://www.justice.gov/opa/pr/eight-individuals-charged-deceptive-trading-practices-executed-us-commodities-markets

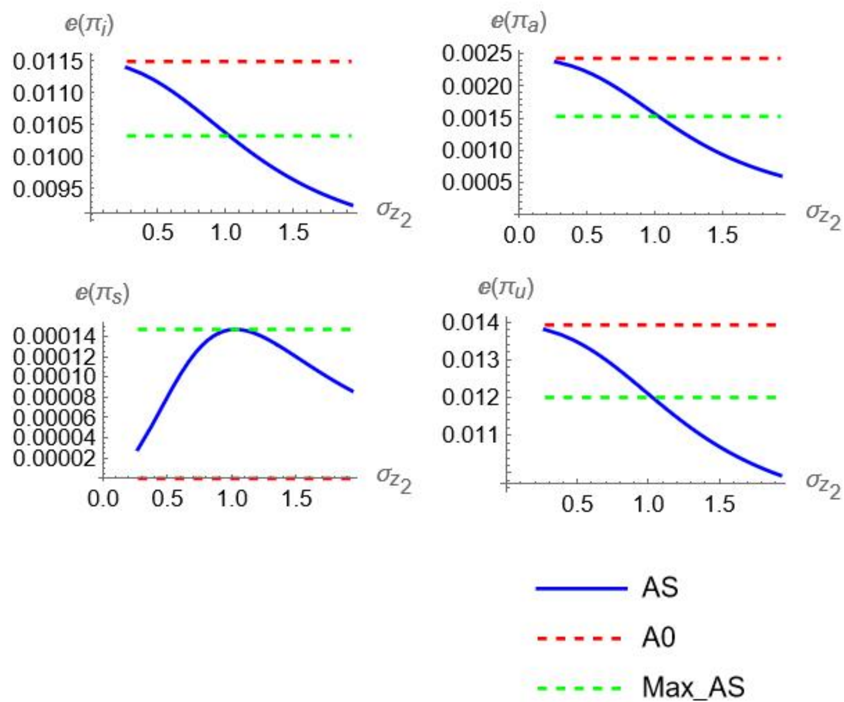


Figure 2: Ex-ante profits of traders when $\theta\epsilon = 0.4$

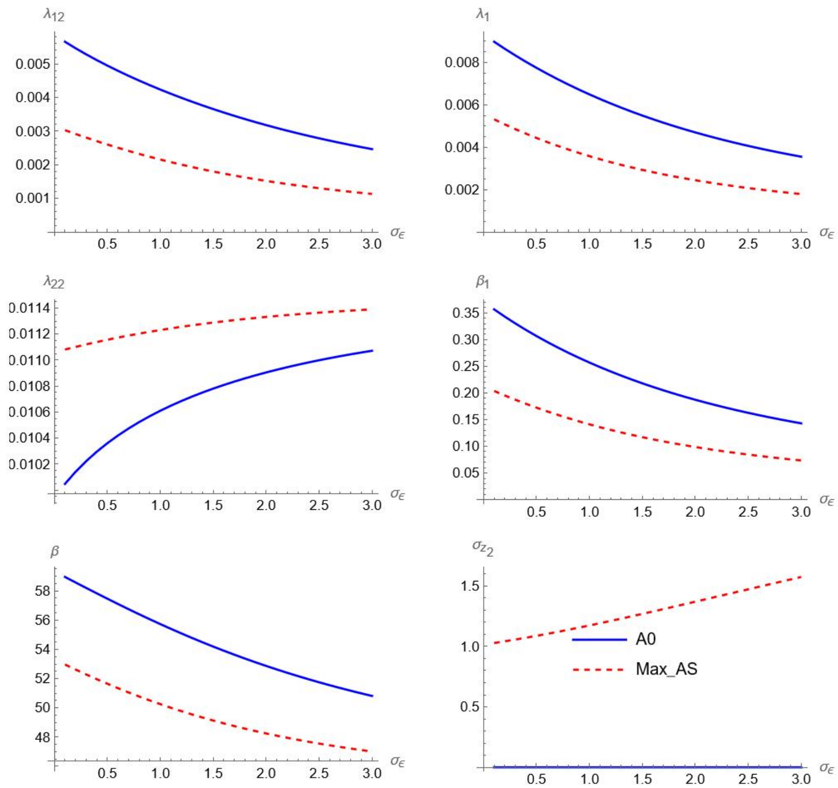


Figure 3: Numerical solution for different values of $\theta\epsilon$

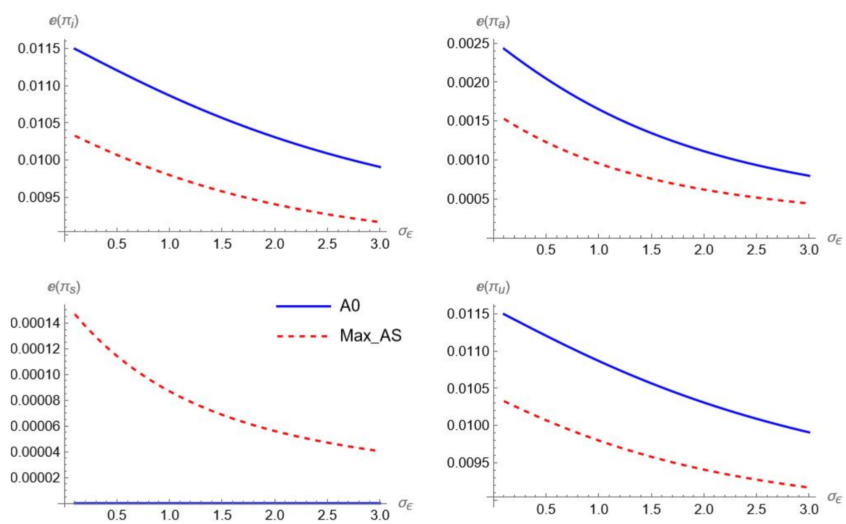


Figure 4: Ex-ante profits of traders for different values of $\theta\epsilon$

unlawful conduct an unfair advantage over law-abiding market participants"[16]. Even though market integrity is one of the main missions of securities and commodities regulators, it is poorly defined by regulators. Although this approach can give regulators more flexibility to interpret what they perceive as challenges when they arise, the lack of clarifications makes it impossible to assess the progress of securities regulators toward achieving these goals. This also leads to different interpretations of market integrity by market participants. Austin [2017] presents different definitions of what market integrity should encompass. In a narrow sense, the market integrity is often defined as "the ability of investors to transact in a fair and informed market where prices reflect information." This definition is close to market efficiency. Austin [2017] also suggests that market integrity and market fairness may be equivalent. Shefrin and Statman [1993] defined market fairness as a claim to 7 entitlements: freedom from coercion, freedom from misrepresentation, equal information, equal processing power, freedom from impulse, efficient prices, and equal bargaining power. Apart from market efficiency, this definition extends to equal access to information and equal information processing. In this paper, we adopt the framework from Fox, Glosten, and Guan [2022] to evaluate spoofing through 3 main aspects: efficiency considerations, wealth transfer from an ex-ante perspective, and fairness considerations.

9.1 Efficiency

For price accuracy, we find that in a very short time frame, spoofing delays the price discovery. The main driver of this result is that spoofing makes the anticipatory trader less active in the market, thus delaying the information dissipation in the market. The improvement in price discovery caused by the order anticipation strategies is at the expense of informed traders in the form of information leakage. while order anticipation strategies can foster price discovery in the short term, the widespread order anticipation HFTs will harm price discovery in the long run as they discourage informed traders from researching and finding mispricing assets. Informed traders will have less incentive to create information. With spoofing, the participation of anticipatory traders can be kept in check. In the longer timeframe, both spoofing and order anticipation strategies have a limited effect on price discovery as both spoofers and order anticipators are short-term traders. Their daily net positions are usually zero.

Contrary to Fox et al. [2022]'s arguments, our above model shows that spoofing has positive impacts on liquidity and market welfare. From our above analysis, with spoofing both anticipatory traders and informed traders reduce their trading intensities. As an indirect effect, uninformed traders suffer less loss and the market liquidity improves.

Another aspect is to examine how spoofing affects market confidence. Our results are consistent with Fox et al. [2022]'s arguments that spoofing does not decrease the wealth position of ordinary investors, and any additional risk-

[16] Release Number 7686-18, https://www.cftc.gov/PressRoom/PressReleases/7686-18

related spoofing can be diversified away. However, we disagree with their arguments that misperceptions that spoofing occurs may harm ordinary investors can reduce their participation. Misperceptions come mainly from the enforcement agenda of regulators and should be corrected through education rather than serving as a ground for banning spoofing.

9.2 Wealth transfer

To evaluate spoofing, we need to consider how spoofing affects different members of our society. From the above analysis, spoofing has no or little impact on HFTs that use arbitrage and market-making strategies. For informed traders, they benefit from spoofing as spoofing reduces the participation of anticipatory traders. Uninformed traders also indirectly benefit from spoofing as they suffer less loss. The only victim of spoofing, in this case, is those order anticipation HFTs. The higher the spoofing intensity is, the lower the profit order anticipators can make.

9.3 Fairness considerations

From our above analysis, legalizing spoofing doesn't harm informed traders and they actually benefit from spoofing. For informed traders, even though they reduce their trading intensity, they also benefit from less active anticipatory traders.

To study the fairness of spoofing, we examine the arguments put forward by the Department of Justice in the cases against Andre Flotron [17] and B. Oystacher. First, regulators tend to paint HFTs as the innocent targets of spoofing, at least in part, to give the jury a reason to care about the crime it was trying to prove. However, if we examine HFTs' testimony, we find that the only harmful HFTs that use order anticipation strategies are vulnerable to spoofing. Other good HFTs using market-making, arbitrage, and news feed strategies are not harmed by spoofing. Order anticipation HFTs are sophisticated traders with pattern recognition algorithms. They only fall victim to spoofing as their algorithms get detected. Therefore, the argument that HFTs are the innocent targets of spoofing seems far from the truth. Second, a spoofer is claimed to have conducted fraudulent misrepresentation of the price. According to regulators, a spoofer fraudulently induces other traders into filling its "real orders" using the spoof order. For example, if an HFT trades against a spoofer, the terms of that transaction are fully and accurately disclosed in the market. No one forces the HFT to trade and the transactions are executed with exactly disclosed terms. Furthermore, the limit order book is the second order information. There is no law to require traders to fully disclose their trade intentions. One of these examples is iceberg orders are legal in most jurisdictions. The responsibility for that error should come with anticipatory algorithms, not spoofers.

[17] Court Docket No.:3:17-cr-00220-JAM, https://www.justice.gov/criminal/criminal-vns/united-states-v-andre-flotron

10 Conclusion

In recent years, spoofing has become a main target of manipulative crackdowns by regulators. This paper provides a two-period model of strategic interactions between a spoofer and an anticipatory trader who employs pattern recognition algorithms to predict the incoming order. Detecting this strategy, the spoofer submits a spoofing order to mislead the anticipatory trader about the incoming order. The order anticipation HFT protects itself by reducing its market participation. A pure strategy spoofing equilibrium exists and both traders make positive profits.

We show that while spoofing delays price discovery in a short horizon, price dislocation will be so brief as to have little economic efficiency implications. Moreover, spoofing improves market liquidity and market welfare. By studying different recounts of traders, we find that spoofing resemblance practice has existed for centuries. The introduction of electronic trading systems has altered regulators' ethical judgments of spoofing. Furthermore, we study different legal cases on spoofing and find that the main victim of spoofing is order anticipation HFTs. They get exploited because their algorithms are detected and easy to get tricked by spoofers.

A Proofs and derivations

A.1 Proof of Proposition 5.1

Combining the equations (44) and (46), we can arrive at the following expression:

$$\lambda_1 = \frac{a_1\sigma^2\beta(a_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (a_1^2 + a_2^2))}{a_1^2\sigma_u^2(2a_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1) + \sigma_u^2(a_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1)(a_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (a_1^2 + a_2^2))} \tag{81}$$

Inserting this expression and equations (42) and (43) and (46) into (47), we can obtain the following equation.

$$F(\beta_1, \theta_\epsilon, \theta_{z_2}, k) = 0 \tag{82}$$

Given $(\theta_\epsilon, \theta_{z_2}, k)$, the function $F(\beta_1, \theta_\epsilon, \theta_{z_2}, k)$ is a 6^{th} polynomial defined as follows:

$$\begin{aligned} F(\beta_1, \theta_\epsilon, \theta_{z_2}, k) = {} & 32 + \beta_1(-64 - 64k - 64\theta_\epsilon - 64\theta_{z_2}) + \beta_1^2(64k + 96k^2 + 192k\theta_\epsilon + 128k^2\theta_\epsilon + 32\theta_{z_2} + 192k\theta_{z_2}) + \beta_1^3(-128k^2 - 576k^2\theta_\epsilon - 192k^3\theta_\epsilon - 128k^2\theta_\epsilon^2 - 48\theta_{z_2} - 48k\theta_{z_2} - 384k^2\theta_{z_2} - 32\theta_\epsilon\theta_{z_2} - 128k^2\theta_\epsilon\theta_{z_2} - 32\theta_{z_2}^2) + \beta_1^4(384k^3\theta_\epsilon + 320k^4\theta_\epsilon + 64k^3\theta_\epsilon^2 + 160k^4\theta_\epsilon^2 + 80k^2\theta_{z_2} + 384k^3\theta_{z_2} + 16k\theta_\epsilon\theta_{z_2} + 80k^2\theta_\epsilon\theta_{z_2} + 64k^3\theta_\epsilon\theta_{z_2} + 10\theta_{z_2}^2 + 16k\theta_{z_2}^2) + \beta_1^5(-512k^4\theta_\epsilon - 128k^4\theta_\epsilon^2 - 128k^5\theta_\epsilon^2 - 64k^4\theta_\epsilon^3 - 64k^2\theta_{z_2} - 256k^4\theta_{z_2} - 64k^2\theta_\epsilon\theta_{z_2} - 64k^3\theta_\epsilon\theta_{z_2} - 32k^2\theta_\epsilon^2\theta_{z_2} - 64k^4\theta_\epsilon^2\theta_{z_2} - 8\theta_{z_2}^2 - 8k\theta_{z_2}^2 - 4\theta_\epsilon\theta_{z_2}^2 - 32k^2\theta_\epsilon\theta_{z_2}^2 - 4\theta_{z_2}^3) + \beta_1^6(-512k^5\theta_\epsilon^2 + 256k^6\theta_\epsilon^2 - 128k^5\theta_\epsilon^3 + 64k^6\theta_\epsilon^3 - 192k^3\theta_\epsilon\theta_{z_2} + 128k^4\theta_\epsilon\theta_{z_2} - 256k^5\theta_\epsilon\theta_{z_2} - 64k^3\theta_\epsilon^2\theta_{z_2} + 48k^4\theta_\epsilon^2\theta_{z_2} - 128k^5\theta_\epsilon^2\theta_{z_2} - 16k\theta_{z_2}^2 + 16k^2\theta_{z_2}^2 - 64k^3\theta_{z_2}^2 - 8k\theta_\epsilon\theta_{z_2}^2 + 12k^2\theta_\epsilon\theta_{z_2}^2 - 64k^3\theta_\epsilon\theta_{z_2}^2 + \theta_{z_2}^3 - 8k\theta_{z_2}^3) \end{aligned} \tag{83}$$

Assume that $\beta_1 \leq 0$. From the second order condition of the spoofer problem, we have $\lambda_1 + \lambda_{22} - \lambda_{12} > 0$. Combining this condition and equation (47), λ_{22} is not positive. From the equation (43), $a_2 \leq 0$ or $\beta_1 > 1$, contradicting with our assumption. So $\beta_1 > 0$. Now, we will prove that $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1)$ have a unique equation β_1 in $(0, \frac{1}{2})$. We have

$$F(0, \theta_\epsilon, \theta_{z_2}, 1) = 32 > 0 \tag{84}$$

$$\begin{aligned} F(\frac{1}{2}, \theta_\epsilon, \theta_{z_2}, 1) = {} & -8 - 20\theta_\epsilon - 14\theta_\epsilon^2 - 3\theta_\epsilon^3 - 17\theta_{z_2} - 19\theta_\epsilon\theta_{z_2} - \frac{21\theta_\epsilon^2\theta_{z_2}}{4} - \frac{31\theta_{z_2}^2}{8} - \frac{33\theta_\epsilon\theta_{z_2}^2}{16} - \frac{15\theta_{z_2}^3}{64} < 0 \end{aligned} \tag{85}$$

As $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1)$ is a continuous function in β_1, there must exist at least one solution of (82) in $(0, \frac{1}{2})$. Now, we take first derivative of $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1)$ and obtain:

$$F'(\beta_1, \theta_\epsilon, \theta_{z_2}, 1) = -28 - (10 - 16\beta_1)^2 - 128\beta_1^2 - 64\theta_\epsilon(1 - 10\beta_1 + 36\beta_1^2 - 44\beta_1^3 + 40\beta_1^4) - 128\theta_\epsilon^2\beta_1^2(3 - 7\beta_1 + 10\beta_1^2) - 1536\beta_1^5\theta_\epsilon^2 - 320\beta_1^4\theta_\epsilon^3 - 384\beta_1^5\theta_\epsilon^3 - 32\theta_{z_2}(2 - 14\beta_1 + 45\beta_1^2 - 56\beta_1^3 + 50\beta_1^4) - 160\theta_{z_2}\beta_1^2\theta_\epsilon(2 + (1 - 2\beta_1)^2) - (1920\beta_1^5\theta_\epsilon + 480\beta_1^4\theta_\epsilon^2 + 864\beta_1^5\theta_\epsilon^2)\theta_{z_2} - 8\theta_{z_2}^2\beta_1^2(12 - 13\beta_1 + 10\beta_1^2) - (384\beta_1^5 + 180\beta_1^4\theta_\epsilon + 360\beta_1^5\theta_\epsilon)\theta_{z_2}^2 - (20\beta_1^4 + 42\beta_1^5)\theta_{z_2}^3 \tag{86}$$

For any β_1, $F'(\beta_1, \theta_\epsilon, \theta_{z_2}, 1) < 0$. Therefore, $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1)$ is an decreasing function in β_1 on $(0, +\infty)$. So $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1) = 0$ has unique solution β_1 and $0 < \beta_1 < \frac{1}{2}$.

A.2 Proof of Proposition 5.2

When the spoofer does not trade, his real trading volume and spoofing volume are both zero. Therefore, $\beta_2 = 0$ and $\sigma_z^2 = 0$. This problem is a special case of the proposition 5.1 with $\sigma_z^2 = 0$.

A.3 Proof of Proposition 5.3

Similar to the proof of the proposition 5.1, $\tilde{\beta_1}$ is the solution to the following equation

$$F(\beta_1, \theta_\epsilon, \theta_{z_2}, 0) = 0 \tag{87}$$

Inserting $k = 0$ into equation (83) to arrive at the following 5^{th} polynomial

$$F(\beta_1, \theta_\epsilon, \theta_{z_2}, 0) = 32 + \beta_1(-64 - 64\theta_\epsilon - 64\theta_{z_2}) + 32\beta_1^2\theta_{z_2} + 10\beta_1^4\theta_{z_2}^2 + \beta_1^6\theta_{z_2}^3 + \beta_1^3(-48\theta_{z_2} - 32\theta_\epsilon\theta_{z_2} - 32\theta_{z_2}^2) + \beta_1^5(-8\theta_{z_2}^2 - 4\theta_\epsilon\theta_{z_2}^2 - 4\theta_{z_2}^3) \tag{88}$$

Taking derivatives of equation (88) and obtaining

$$F'(\beta_1, \theta_\epsilon, \theta_{z_2}, 0) = -64 - 64\theta_\epsilon + (-64 + 64\beta_1 - 144\beta_1^2 - \beta_1^2\theta_\epsilon)\theta_{z_2} + (-96\beta_1^2 + 40\beta_1^3 - 40\beta_1^4 - 20\beta_1^4\theta_\epsilon)\theta_{z_2}^2 + (-20\beta_1^4 + 6\beta_1^5)\theta_{z_2}^3 \tag{89}$$

We have $F(0, \theta_\epsilon, \theta_{z_2}, 0) = 32$, and $F(\frac{1}{2}, \theta_\epsilon, \theta_{z_2}, 0) = -32\theta_\epsilon - 30\theta_{z_2} - 4\theta_\epsilon\theta_{z_2} - \frac{29\theta_{z_2}^2}{8} - \frac{\theta_\epsilon\theta_{z_2}^2)}{8} - \frac{7\theta_{z_2}^3}{64} \leq 0$. It is easy to see that $F'(\beta_1, \theta_\epsilon, \theta_{z_2}, 0) \leq 0$ for all β_1 in (0,1). Therefore, $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 0)$ has unique solution in $(0, \frac{1}{2})$.

A.4 Proof of Proposition 5.5

Using equation (82) with the case in which both traders choose to trade or $k = 1$

$$F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1) = 0 \tag{90}$$

From the proposition 5.1, , the optimal β_1 is an implicit function in θ_ϵ and θ_{z_2}. Taking derivative of (90) with respect to θ_{z_2}.

$$\frac{\partial F}{\partial \beta_1}\frac{\partial \beta_1}{\partial \theta_{z_2}} + \frac{\partial F}{\partial \theta_{z_2}} = 0 \tag{91}$$

The partial derivative of $F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1)$ is given by the following equation

$$\begin{aligned}\frac{\partial F}{\partial \theta_{z_2}} = {} & -16\beta_1(4 - 14\beta_1 + 30\beta_1^2 - 29\beta_1^3 + 20\beta_1^4) - 32\beta_1^3\theta_\epsilon(5 - 5\beta_1 + \\ & 4\beta_1^2) - (320\beta_1^6\theta_\epsilon + 96\beta_1^5\theta_\epsilon^2 + 144\beta_1^6\theta_\epsilon^2) - 4\theta_{z_2}((4-2\beta_1)^2 + 3\beta_1 + \\ & 2\beta_1^2) - (128\beta_1^6 + 72\beta_1^5\theta_\epsilon + 120\beta_1^6\theta_\epsilon)\theta_{z_2} - (12\beta_1^5 + 21\beta_1^6)\theta_{z_2}^2\end{aligned} \tag{92}$$

As $4 - 14\beta_1 + 30\beta_1^2 - 29\beta_1^3 + 20\beta_1^4 > 0$ for any β_1 and $\beta_1 > 0$, $\frac{\partial F}{\partial \theta_{z_2}} < 0$. Combining this condition with (71) and $\frac{\partial F}{\partial \beta_1} < 0$, we can conclude that $\frac{\partial \beta_1}{\partial \theta_{z_2}} < 0$.

Inserting $a_1 = \beta_1, a_2 = 1 - \beta_1$ into the equation (47) and obtaining

$$\beta^2 = \frac{\sigma_u^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)}{\sigma^2(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (\beta_1^2 + (1-\beta_1)^2))} \tag{93}$$

We denote the right side of equation (93) as $G(\beta_1, \theta_{z2})$. Taking derivative both sides of (93)

$$\frac{\partial \beta^2}{\partial \theta_{z_2}} = \frac{\partial G}{\partial \beta_1}\frac{\partial \beta_1}{\partial \theta_{z_2}} + \frac{\partial G}{\partial \theta_{z_2}} \tag{94}$$

Combining equations (94) and (91) to arrive at the following expression

$$\frac{\partial F}{\partial \beta_1}\frac{\partial \beta^2}{\partial \theta_{z_2}} = -\frac{\partial G}{\partial \beta_1}\frac{\partial F}{\partial \theta_{z_2}} + \frac{\partial G}{\partial \theta_{z_2}}\frac{\partial F}{\partial \beta_1} \tag{95}$$

We need to prove that $\frac{\partial \beta^2}{\partial \theta_{z_2}} \leq 0$. This condition is equivalent to:

$$4(1-2\beta_1)^2(\beta_1^2)\frac{\partial F}{\partial \beta_1} + 8(-1+2\beta_1)(4+\beta_1(4\theta_\epsilon + \theta_{z_2}))\frac{\partial F}{\partial \theta_{z_2}} \geq 0 \tag{96}$$

Inserting equations (92) and (86) into the above the inequality and simplify

$$-16\beta_1(-1+2\beta_1)(N_0 + N_1\theta_{z_2} + N_2\theta_{z_2}^2 + N_3\theta_{z_2}^3) \geq 0 \tag{97}$$

Where N_0, N_1, N_2, N_3 are defined as follows:

$$N_3 = \beta_1^5 + 10\beta_1^6 + 21\beta_1^7 \tag{98}$$

$$N_2 = 8\beta_1^3 + 72\beta_1^4 - 14\beta_1^5 + 8\beta_1^6 + 192\beta_1^7 + 15\beta_1^5\theta_\epsilon + 102\beta_1^6\theta_\epsilon + 180\beta_1^7\theta_\epsilon \tag{99}$$

$$\begin{aligned}N_1 = {} & 16\beta_1 + 160\beta_1^2 - 448\beta_1^3 + 1016\beta_1^4 - 912\beta_1^5 + 800\beta_1^6 + (88\beta_1^3 + \\ & 360\beta_1^4 - 112\beta_1^5 + 256\beta_1^6 + 960\beta_1^7)\theta_\epsilon + (72\beta_1^5 + 336\beta_1^6 + 432\beta_1^7)\theta_\epsilon^2\end{aligned} \tag{100}$$

$$\begin{aligned}N_0 = {} & 128 - 480\beta_1 + 1104\beta_1^2 - 1184\beta_1^3 + 832\beta_1^4 + (112\beta_1 + 64\beta_1^2 - \\ & 256\beta_1^3 + 1184\beta_1^4 - 768\beta_1^5 + 1280\beta_1^6)\theta_\epsilon + (224\beta_1^3 + 288\beta_1^4 - \\ & 224\beta_1^5 + 896\beta_1^6 + 768\beta_1^7)\theta_\epsilon^2 + (112\beta_1^5 + 352\beta_1^6 + 192\beta_1^7)\theta_\epsilon^3\end{aligned} \tag{101}$$

As β_1 lies in $[0, \frac{1}{2}]$, $-1 + 2\beta_1 \leq 0$ and it easy to see that N_0, N_1, N_2, N_3 are non-negative. Therefore, β is decreasing in σ_{z_2}.

A.5 Proof of Proposition 6.1

We can rewrite the optimization into another form by inserting equation (47) into equation (49) to obtain

$$E[\pi_S] = \frac{\lambda_1 + \lambda_{22} - \lambda_{12}}{4}(\beta_1)^2\sigma_{z_2}^2 = \sigma_{z_2}^2\frac{\lambda_{22}}{8}\frac{\beta_1\beta^2\sigma^2}{\beta^2\sigma^2 + \sigma_\epsilon^2 + \sigma_{z_2}^2} \tag{102}$$

Combining equations (43) and (102) to arrive at the following expression:

$$E[\pi_S] = \theta_{z_2}\frac{\sigma^2\beta(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1 - \beta_1)}{16(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)}\frac{\beta_1\beta^2\sigma^2}{\beta^2\sigma^2 + (\theta\epsilon + \theta_{z_2})\sigma_u^2} \tag{103}$$

We denote $H(\beta_1, \theta_\epsilon, \theta_{z_2}) = \frac{\beta^2\sigma^2}{\beta^2\sigma^2 + (\theta\epsilon + \theta_{z_2})\sigma_u^2}$. Substituting equation (46) into this expression to get

$$H(\beta_1, \theta_\epsilon, \theta_{z_2}) = \frac{(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)}{(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1) + (\theta\epsilon + \theta_{z_2})(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (\beta_1^2 + (1 - \beta_1)^2))} \tag{104}$$

With $a_1 = \beta_1, a_2 = 1 - a_1$, we have the inequality $a_1^2 + a_2^2 = a_1 + (1 - a_1)^2 \geq \frac{1}{2}$. By using this inequality and (46), we arrive at the following upper bound of β

$$\beta^2 \leq \frac{2\sigma_u^2}{\sigma^2} \tag{105}$$

Combining this inequality and β_1 in $(0, \frac{1}{2})$, we have

$$\frac{\sigma^2\beta(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1 - \beta_1)}{16(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \leq \frac{\sigma_u\sigma}{8\sqrt{2}} \tag{106}$$

Using the same inequality $a_1^2 + a_2^2 = a_1 + (1 - a_1)^2 \geq \frac{1}{2}$, we can have the following inequality

$$H(\beta_1, \theta_\epsilon, \theta_{z_2}) \leq \frac{2}{2 + \theta_\epsilon + \theta_{z_2}} \tag{107}$$

Combining the inequalities (106), (107) and equation (103) to obtain

$$E[\pi_S] \leq \frac{\sigma_u\sigma}{4\sqrt{2}}\frac{\theta_{z_2}}{2 + \theta_\epsilon + \theta_{z_2}}\beta_1 \leq \frac{\sigma_u\sigma}{4\sqrt{2}}\beta_1 \tag{108}$$

From the proposition 5.5, β_1 is a decreasing continuous function of σ_{z_2} and β_1 in $(1, \frac{1}{2})$, there exists a constant β_0 in $[0, 1]$ such that $\lim_{\sigma_{z_2} \to +\infty} \beta_1 = \beta_0$ and β_0 is the lower bound of β_1. We consider the case $\beta_0 > 0$. From the proposition 5.1, we have

$$\begin{aligned} F(\beta_1, \theta_\epsilon, \theta_{z_2}, 1) = {} & 32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 - 64\beta_1\theta_\epsilon + 320\beta_1^2\theta_\epsilon - 768\beta_1^3\theta_\epsilon + \\ & 704\beta_1^4\theta_\epsilon - 512\beta_1^5\theta_\epsilon - 128\beta_1^3\theta_\epsilon^2 + 224\beta_1^4\theta_\epsilon^2 - 256\beta_1^5\theta_\epsilon^2 - 256\beta_1^6\theta_\epsilon^2 - \\ & 64\beta_1^5\theta_\epsilon^3 - 64\beta_1^6\theta_\epsilon^3 + (-64\beta_1 + 224\beta_1^2 - 480\beta_1^3 + 464\beta_1^4 - \\ & 320\beta_1^5 - 160\beta_1^3\theta_\epsilon + 160\beta_1^4\theta_\epsilon - 128\beta_1^5\theta_\epsilon - 320\beta_1^6\theta_\epsilon - 96\beta_1^5\theta_\epsilon^2 - \\ & 144\beta_1^6\theta_\epsilon^2)\theta_{z_2} + (-32\beta_1^3 + 26\beta_1^4 - 16\beta_1^5 - 64\beta_1^6 - 36\beta_1^5\theta_\epsilon - \\ & 60\beta_1^6\theta_\epsilon)\theta_{z_2}^2 + (-4\beta_1^5 - 7\beta_1^6)\theta_{z_2}^3 = 0 \end{aligned} \tag{109}$$

If $\beta_0 > 0$, the left side of (105) goes to infinity when σ_{z_2} goes to infinity. This contradicts with the equation. So, $\lim_{\sigma_{z_2}\to+\infty}\beta_1 = 0$. If we take limit the both sides of (107), we have

$$0 \leq \lim_{\sigma_{z_2}\to+\infty} E[\pi_S] \leq \lim_{\sigma_{z_2}\to+\infty} \frac{\sigma_u\sigma}{4\sqrt{2}}\beta_1 = 0 \tag{110}$$

So, we have $\lim_{\sigma_{z_2}\to+\infty} E[\pi_S] = 0$. When $\sigma_{z_2} = 0$, then $E[\pi_S] = 0$. From the proposition 5.1, we can deduce $E[\pi_S]$ is a continuous function in σ_{z_2}, $0 \leq E[\pi_S] \leq \frac{\sigma_u\sigma}{8\sqrt{2}}$ and $\lim_{\sigma_{z_2}\to+\infty} E[\pi_S] = \lim_{\sigma_{z_2}\to 0} E[\pi_S] = 0$. Therefore, $E[\pi_S]$ has the global maximum.

A.6 Proof of Proposition 7.1

From the equation (42), we have the equation for λ_{12} as follows

$$\lambda_{12} = \frac{\sigma^2\beta(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + \beta_1)}{2\sigma_u^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \tag{111}$$

We denote the right side of equation (111) as $R_{12}(\beta, \beta_1, \theta_{z2})$. Taking derivative both sides of (111)

$$\frac{\partial\lambda_{12}}{\partial\theta_{z_2}} = \frac{\partial R_{12}}{\partial\beta_1}\frac{\partial\beta_1}{\partial\theta_{z_2}} + \frac{\partial R_{12}}{\partial\beta}\frac{\partial\beta}{\partial\theta_{z_2}} + \frac{\partial R_{12}}{\partial\theta_{z_2}} \tag{112}$$

The partial derivatives of $R_{12}(\beta, \beta_1, \theta_{z2})$ are given by

$$\frac{\partial R_{12}}{\partial\beta_1} = \frac{\beta\sigma^2(2 + \beta_1(1-\beta_1)(4\theta_\epsilon + \theta_{z_2}))}{\sigma_u^2(2 + \beta^2(4\theta_\epsilon + \theta_{z_2}))^2} \tag{113}$$

$$\frac{\partial R_{12}}{\partial\beta} = \frac{\sigma^2(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + \beta_1)}{2\sigma_u^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \tag{114}$$

$$\frac{\partial R_{12}}{\partial\theta_{z_2}} = -\frac{\beta\beta_1^2(-1 + 2\beta_1)\sigma^2}{2\sigma_u^2(2 + \beta_1^2(4\theta_\epsilon + \theta_{z_2}))} \tag{115}$$

Based on the results of the proposition 5.1, $0 \leq \beta_1 \leq \frac{1}{2}$. Therefore, $\frac{\partial R_{12}}{\partial\beta_1} \geq 0$, $\frac{\partial R_{12}}{\partial\beta} \geq 0$ and $\frac{\partial R_{12}}{\partial\theta_{z_2}} \leq 0$. Combining with the proposition 5.5, $\frac{\partial\lambda_{12}}{\partial\theta_{z_2}} \leq 0$.

From the equation (43), we have the equation for λ_{22} as follows

$$\lambda_{22} = \frac{\sigma^2\beta(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1 - \beta_1)}{2\sigma_u^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \tag{116}$$

Squaring both sides of equation (116), then combining with equation (47)

$$\lambda_{22}^2 = \frac{\sigma^2(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1 - \beta_1)^2}{4\sigma_u^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + \beta_1^2 + (1-\beta_1)^2)} \tag{117}$$

We denote the right side of equation (117) as $R_{22}(\beta_1, \theta_{z2})$. Taking derivative both sides of (116)

$$\frac{\partial \lambda_{22}^2}{\partial \theta_{z_2}} = \frac{\partial R_{22}}{\partial \beta_1}\frac{\partial \beta_1}{\partial \theta_{z_2}} + \frac{\partial R_{22}}{\partial \theta_{z_2}} \tag{118}$$

We need to prove that $\frac{\partial \lambda_{22}^2}{\partial \theta_{z_2}} \geq 0$. Substituting the partial derivatives of R_{22}, then simplify to obtain the following condition

$$-2\frac{\partial F}{\partial \theta_{z_2}}(-16 - 16\beta_1^2\theta_\epsilon^2 + 4(-1 + 3\beta_1 - 8\beta_1^2 + 4\beta_1^3)\theta_{z_2} - \beta_1^2\theta_{z_2}^2 + 8\theta_\epsilon(-2 + 6\beta_1 + 8\beta_1^3 - \beta_1^2(16 + \theta_{z_2}))) + \frac{\partial F}{\partial \beta_1}\beta_1(-1 + 2\beta_1)\left(4 - 4\beta_1 + 2\beta_1^3(4\theta_\epsilon + \theta_{z_2}) + \beta_1^2(8 + 4\theta_\epsilon + \theta_{z_2})\right) \leq 0 \tag{119}$$

Inserting equations (92) and (86) into the above the inequality and simplify

$$M_0 + M_1\theta_{z_2} + M_2\theta_{z_2}^2 + +M_3\theta_{z_2}^3 + M_4\theta_{z_2}^4 \leq 0 \tag{120}$$

Where M_0, M_1, M_2, M_3, M_4 are defined as follows

$$M_0 = -1536\beta_1 + 4352\beta_1^2 - 7936\beta_1^3 + 3072\beta_1^4 + 1024\beta_1^5 - 6144\beta_1^6 + (-1792\beta_1 + 9984\beta_1^2 - 39936\beta_1^3 + 80128\beta_1^4 - 119808\beta_1^5 + 93184\beta_1^6 - 61440\beta_1^7)\theta_\epsilon + (-5376\beta_1^3 + 16896\beta_1^4 - 48640\beta_1^5 + 43008\beta_1^6 - 43008\beta_1^7 - 16384\beta_1^8 - 24576\beta_1^9)\theta_\epsilon^2 + (-5376\beta_1^5 + 3840\beta_1^6 - 15360\beta_1^7 - 13312\beta_1^8 - 8192\beta_1^9 - 24576\beta_1^{10})\theta_\epsilon^3 + (-1792\beta_1^7 - 3072\beta_1^8 - 5120\beta_1^9 - 6144\beta_1^{10})\theta_\epsilon^4 \tag{121}$$

$$M_1 = -256\beta_1 + 768\beta_1^2 - 3072\beta_1^3 + 64\beta_1^4 + 6144\beta_1^5 - 22016\beta_1^6 + 18432\beta_1^7 - 15360\beta_1^8 + (-2112\beta_1^3 + 5760\beta_1^4 - 22784\beta_1^5 + 21504\beta_1^6 - 39168\beta_1^7 + 16384\beta_1^8 - 39936\beta_1^9)\theta_\epsilon + (-3456\beta_1^5 + 2688\beta_1^6 - 20736\beta_1^7 - 7680\beta_1^8 - 9216\beta_1^9 - 36864\beta_1^{10})\theta_\epsilon^2 + (-1600\beta_1^7 - 2304\beta_1^8 - 8960\beta_1^9 - 15360\beta_1^{10})\theta_\epsilon^3 \tag{122}$$

$$M_2 = -192\beta_1^3 + 384\beta_1^4 - 2656\beta_1^5 + 2688\beta_1^6 - 7104\beta_1^7 + 5120\beta_1^8 - 8448\beta_1^9 + (-720\beta_1^5 + 624\beta_1^6 - 7488\beta_1^7 - 1344\beta_1^8 - 3072\beta_1^9 - 13824\beta_1^{10})\theta_\epsilon + (-528\beta_1^7 - 576\beta_1^8 - 4800\beta_1^9 - 9216\beta_1^{10})\theta_\epsilon^2 \tag{123}$$

$$M_3 = -48\beta_1^5 + 48\beta_1^6 - 816\beta_1^7 - 64\beta_1^8 - 320\beta_1^9 - 1536\beta_1^{10} + (-76\beta_1^7 - 48\beta_1^8 - 1040\beta_1^9 - 2112\beta_1^{10})\theta_\epsilon \tag{124}$$

$$M_4 = -4\beta_1^7 - 80\beta_1^9 - 168\beta_1^{10} \tag{125}$$

For $0 \leq \beta_1 \leq \frac{1}{2}$ and $\theta_\epsilon \geq 0$, it is easy to prove that M_0, M_1, M_2, M_3, M_4 are non-positive. Therefore the inequality (120) holds or $\frac{\partial \lambda_{22}^2}{\partial \theta_{z_2}} \geq 0$.

From the proposition 7.3, $\lambda_1 + \lambda_{22}$ is decreasing in σ_ϵ while λ_{22} is increasing in σ_ϵ . Therefore λ_1 is decreasing in σ_ϵ.

A.7 Proof of Proposition 7.2

By the definition, the forecast error variance of the market maker is defined by

$$\Sigma_1 = E[(v - p_1)^2] = (1 - \lambda_1 \beta_1 \beta)\sigma^2 \tag{126}$$

From the propositions (5.5), (7.1), $\lambda_1, \beta_1, \beta$ are decreasing in σ_u. Therefore, Σ_1 is increasing in σ_u. Similarly, the forecast error variance of the market maker in the second period is defined by

$$\Sigma_2 = E[(v - p_2)^2] = \frac{1}{2}\sigma^2 \tag{127}$$

A.8 Proof of proposition 7.3

The expected profit to uninformed traders is given by $E[\pi_U] = -(\lambda_1 + \lambda_{22})\sigma_u^2$. Now we will prove that $\lambda_1 + \lambda_{22}$ is decreasing in σ_{z_2}. Similar to the proof of 7.1, we consider $(\lambda_1 + \lambda_{22})^2$

$$(\lambda_1 + \lambda_{22})^2 = B_3(B_1 + B_2)^2 \frac{\sigma^2}{\sigma_u^2} \tag{128}$$

Where B_1, B_2, B_3 are defined as follows

$$B_1 = \frac{\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1 - \beta_1}{2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \tag{129}$$

$$B_2 = \frac{\beta_1(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (\beta_1^2 + (1-\beta_1)^2))}{\beta_1^2(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1) + (\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + 1)(\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (\beta_1^2 + (1-\beta_1)^2))} \tag{130}$$

$$B_3 = \frac{2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1}{\beta_1^2\theta_\epsilon + \frac{1}{4}\beta_1^2\theta_{z_2} + (\beta_1^2 + (1-\beta_1)^2)} \tag{131}$$

We denote $K(\beta_1, \theta_\epsilon, \theta_{z_2}) = B_3(B_1+B_2)^2$. We can factorize K into $K(\beta_1, \theta_\epsilon, \theta_{z_2}) = (B_1 + B_2)(B_3B_1 + B_3B_2)$. Taking the first derivative of K with respect to θ_{z_2}, we have the following expression.

$$\frac{\partial(\lambda_1 + \lambda_{22})^2}{\partial\theta_{z_2}} = (\frac{\partial K}{\partial\beta_1}\frac{\partial\beta_1}{\partial\theta_{z_2}} + \frac{\partial K}{\partial\theta_{z_2}})\frac{\sigma^2}{\sigma_u^2} \tag{132}$$

If we rearrange the terms of B_1, we have

$$B_1 = \frac{1}{4} + \frac{1 - 2\beta_1}{4(2\beta_1^2\theta_\epsilon + \frac{1}{2}\beta_1^2\theta_{z_2} + 1)} \tag{133}$$

As $0 \le \beta_1 \le \frac{1}{2}$, $1 - 2\beta_1 \ge 0$. Therefore, given β_1, B_1 is decreasing in θ_{z_2} or the partial derivative of B_1 with respect to θ_{z_2} is nonpositive. Now we consider the partial derivatives of B_1B_3, B_1B_2, B_2 with respect to θ_{z_2}

$$\frac{\partial(B_1B_3)}{\partial\theta_{z_2}} = -\frac{2\beta_1^3(1-2\beta_1)}{(4 - 8\beta_1 + \beta_1^2(8 + 4\theta_\epsilon + \theta_{z_2}))^2} \tag{134}$$

$$\frac{\partial(B_2B_3)}{\partial\theta_{z_2}} = -\frac{8\beta_1^4(16-16\beta_1+4x\beta_1+\beta_1^3x^2)}{(16-32\beta_1-8\beta_1^3+8\beta_1^2(6+x)+\beta_1^4(x+16)x)^2} \tag{135}$$

$$\frac{\partial(B_2)}{\partial\theta_{z_2}} = -\frac{4\beta_1^3(16(1-2\beta_1)^2+8\beta_1^2x(1-2\beta_1)+8\beta_1^2(1+(3-4\beta_1)^2)+\beta_1^4x(16+x))}{(16-32\beta_1-8\beta_1^3+8\beta_1^2(6+x)+\beta_1^4(x+16)x)^2} \tag{136}$$

Where $x = 4\theta_\epsilon + \theta_{z_2}$ With $0 \leq \beta_1 \leq \frac{1}{2}$, it is easy to see that $\frac{\partial(B_2)}{\partial\theta_{z_2}} \leq 0$, $\frac{\partial(B_2B_3)}{\partial\theta_{z_2}} \leq 0$ and $\frac{\partial(B_1B_3)}{\partial\theta_{z_2}} \leq 0$. Combining with B_1, B_2, B_3 are non negative, we arrive at the conclusion $\frac{\partial K}{\partial\theta_{z_2}}) \leq 0$.

Similarly, now we will prove that $\frac{\partial K}{\partial\beta_1} \geq 0$. Using $x = 4\theta_\epsilon + \theta_{z_2}$. The partial derivative of K with respect to θ_{z_2} can be written as

$$\frac{\partial K}{\partial\beta_1} = \frac{T_0+T_1x+T_2x^2+T_3x^3+T_4x^4+T_5x^5+T_6x^6+T_7x^7+T_8x^8+T_9x^9}{(2(2+\beta_1^2x)^2(4-8\beta_1+\beta_1^2(8+x))^2(16-32\beta_1-8\beta_1^3x+8\beta_1^2(6+x)+\beta_1^4x(16+x))^3} \tag{137}$$

Where $T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9$ are defined as follows

$$T_0 = 524288 - 3407872\beta_1 + 10223616\beta_1^2 - 18612224\beta_1^3 + 20709376\beta_1^4 - 13369344\beta_1^5 + 1310720\beta_1^6 + 3932160\beta_1^7 - 3407872\beta_1^8 \tag{138}$$

$$T_1 = -65536\beta_1 + 1835008\beta_1^2 - 9895936\beta_1^3 + 26476544\beta_1^4 - 44367872\beta_1^5 + 45350912\beta_1^6 - 26542080\beta_1^7 - 1310720\beta_1^8 + 11927552\beta_1^9 - 10485760\beta_1^{10} + 1310720\beta_1^{11} \tag{139}$$

$$T_2 = -131072\beta_1^3 + 2228224\beta_1^4 - 10108928\beta_1^5 + 22790144\beta_1^6 - 32145408\beta_1^7 + 24215552\beta_1^8 - 5980160\beta_1^9 - 13385728\beta_1^{10} + 14155776\beta_1^{11} - 9895936\beta_1^{12} + 1310720\beta_1^{13} \tag{140}$$

$$T_3 = -114688\beta_1^5 + 1359872\beta_1^6 - 5124096\beta_1^7 + 9158656\beta_1^8 - 10444800\beta_1^9 + 4734976\beta_1^{10} - 45056\beta_1^{11} - 4046848\beta_1^{12} + 1228800\beta_1^{13} - 1245184\beta_1^{14} - 262144\beta_1^{15} \tag{141}$$

$$T_4 = -57344\beta_1^7 + 462848\beta_1^8 - 1420288\beta_1^9 + 1786880\beta_1^{10} - 1719296\beta_1^{11} + 367616\beta_1^{12} - 839680\beta_1^{13} + 487424\beta_1^{14} - 1343488\beta_1^{15} + 458752\beta_1^{16} - 262144\beta_1^{17} \tag{142}$$

$$T_5 = -17920\beta_1^9 + 84992\beta_1^{10} - 210176\beta_1^{11} + 101376\beta_1^{12} - 114944\beta_1^{13} - 126976\beta_1^{14} - 87040\beta_1^{15} - 122880\beta_1^{16} - 16384\beta_1^{17} - 65536\beta_1^{18} \tag{143}$$

$$T_6 = -3584\beta_1^{11} + 5632\beta_1^{12} - 12992\beta_1^{13} - 25280\beta_1^{14} + 11776\beta_1^{15} - 59648\beta_1^{16} + 21504\beta_1^{17} - 36864\beta_1^{18} \tag{144}$$

$$T_7 = -448\beta_1^{13} - 704\beta_1^{14} + 240\beta_1^{15} - 5568\beta_1^{16} + 2368\beta_1^{17} - 5888\beta_1^{18} \tag{145}$$

$$T_8 = -32\beta_1^{15} - 152\beta_1^{16} + 48\beta_1^{17} - 368\beta_1^{18} \tag{146}$$

$$T_9 = -\beta_1^{17} - 8\beta_1^{18} \tag{147}$$

With $0 \leq \beta_1 \leq \frac{1}{2}$, it is easy to see that the denominator of (137) is positive. We only need to prove that the numerator of(137) which we denote as $D(\beta_1, x)$ is positive. From the equation (83), we have $F(\beta_1, \theta_{z_2}, \theta_\epsilon, 1) = 0$. With $\theta_{z_2} = x - 4\theta_\epsilon$, we can write the equation in the following way.

$$32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 - 64\beta_1 x + 224\beta_1^2 x - 480\beta_1^3 x + 464\beta_1^4 x - 320\beta_1^5 x - 32\beta_1^3 x^2 + 26\beta_1^4 x^2 - 16\beta_1^5 x^2 - 64\beta_1^6 x^2 - 4\beta_1^5 x^3 - 7\beta_1^6 x^3 + \theta_\epsilon(48\beta_1(2-3\beta_1)^2 + 144\beta_1^3(5-8\beta_1) + 768\beta_1^5 + 96\beta_1^3 x - 48\beta_1^4 x + 192\beta_1^6 x + 12\beta_1^5 x^2 + 24\beta_1^6 x^2) = 0 \tag{148}$$

With $0 \leq \beta_1 \leq \frac{1}{2}$, it is obvious that $48\beta_1(2-3\beta_1)^2 + 144\beta_1^3(5-8\beta_1) + 768\beta_1^5 + 96\beta_1^3 x - 48\beta_1^4 x + 192\beta_1^6 x + 12\beta_1^5 x^2 + 24\beta_1^6 x^2 \geq 0$. As the coefficient of θ_ϵ is nonnegative , We substitute $\frac{x}{4} \geq \theta_\epsilon$ into the equation (147) and simplify to obtain the following inequality.

$$32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 + (-16\beta_1 + 80\beta_1^2 - 192\beta_1^3 + 176\beta_1^4 - 128\beta_1^5)x + (-8\beta_1^3 + 14\beta_1^4 - 16\beta_1^5 - 16\beta_1^6)x^2 + (-\beta_1^5 - \beta_1^6)x^3 \geq 0 \tag{149}$$

As $0 \leq \beta_1 \leq \frac{1}{2}$, we can verify that all coefficients of x, x^2, x^3 are not positive. Therefore we can have 2 below inequalities.

$$32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 + (-16\beta_1 + 80\beta_1^2 - 192\beta_1^3 + 176\beta_1^4 - 128\beta_1^5)x \geq 0 \tag{150}$$

$$32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 \geq 0 \tag{151}$$

Solving the inequality (151), we have a stricter condition for β_1

$$\beta_1 \leq \frac{1}{12}\left(5 - \frac{23}{\sqrt[3]{\left(12\sqrt{87} - 19\right)}} + \sqrt[3]{\left(12\sqrt{87} - 19\right)}\right) = \beta_1^* \tag{152}$$

Using the inequality (150), we can rewrite the coefficient of x in the right side of (150) as follows

$$-16\beta_1 + 80\beta_1^2 - 192\beta_1^3 + 176\beta_1^4 - 128\beta_1^5 = -5\beta_1 - 11\beta_1 + 80\beta_1^2 - 192\beta_1^3 + 176\beta_1^4 - 128\beta_1^5 \leq -5\beta_1 \tag{153}$$

As $0 \leq \beta_1 \leq \beta_1^*$, we can verify that $-11\beta_1 + 80\beta_1^2 - 192\beta_1^3 + 176\beta_1^4 - 128\beta_1^5 \leq 0$. Therefore, the inequality (153) holds. Combining inequalities (150), (151),(153), we have the constraint for x as follows

$$32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3 - 5\beta_1 x \geq 0 \tag{154}$$

We can computationally verify that $T_4, T_5, T_6, T_7, T_8, T_9$ are not positive for all $0 < \beta_1 \leq \beta_1^*$. Now, we consider T_2, T_3. Using the equations (140), (141), we can obtain the following expressions.

$$T_2 - 50000\beta_1^3 = 16\beta_1^3(-11317 + 139264\beta_1 - 631808\beta_1^2 + 1424384\beta_1^3 - 2009088\beta_1^4 + 1513472\beta_1^5 - 373760\beta_1^6 - 836608\beta_1^7 + 884736\beta_1^8 - 618496\beta_1^9 + 81920\beta_1^{10}) \leq 0 \tag{155}$$

$$T_3 - 20000\beta_1^5 = -32\beta_1^5(4209 - 42496\beta_1 + 160128\beta_1^2 - 286208\beta_1^3 + 326400\beta_1^4 - 147968\beta_1^5 + 1408\beta_1^6 + 126464\beta_1^7 - 38400\beta_1^8 + 38912\beta_1^9 + 8192\beta_1^{10}) \leq 0 \tag{156}$$

Similarly, we can verify that both inequalities (153) and (154) hold for all $0 < \beta_1 \leq \beta_1^*$. Therefore, we have the following inequality

$$D(\beta_1, x) \geq T_0 + T_1 x + (T_2 - 50000\beta_1^3)x^2 + (T_3 - 20000\beta_1^5)x^3 + T_4 x^4 + T_5 x^5 + T_6 x^6 + T_7 x^7 + T_8 x^8 + T_9 x^9 \tag{157}$$

If $\beta_1 = 0$, $D = 524288 > 0$, now we only need to consider the case when $\beta_1 > 0$. We denote the right side of (157) as $D_1(\beta_1, x)$ We consider 2 following cases:

1. $T_1 < 0$

   When $T_1 < 0$, it is obvious that $D_1(\beta_1, x)$ is a polynomial of x with almost all of coefficients $T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9$ nonpositive. Combining with the inequality (154), we have that

   $$D_1(\beta_1, x) \geq D_1(\beta_1, \frac{32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3}{5\beta_1}) \tag{158}$$

   We can verify that $D_1(\beta_1, \frac{32-128\beta_1+160\beta_1^2-128\beta_1^3}{5\beta_1}) > 0$ for all $0 < \beta_1 \leq \beta_1^*$.

2. $T_1 \geq 0$, we have the following inequality

   $$D(\beta_1, x) \geq T_0 + (T_2 - 50000\beta_1^3)x^2 + (T_3 - 20000\beta_1^5)x^3 + T_4 x^4 + T_5 x^5 + T_6 x^6 + T_7 x^7 + T_8 x^8 + T_9 x^9 \tag{159}$$

   We denote the right side of (159) as $D_2(\beta_1, x)$. Similarly, we can see that $D_2(\beta_1, x)$ is a polynomial of x with almost all of coefficients except T_0 nonpositive. Combining with the inequality (154), we have that

   $$D_2(\beta_1, x) \geq D_2(\beta_1, \frac{32 - 128\beta_1 + 160\beta_1^2 - 128\beta_1^3}{5\beta_1}) \tag{160}$$

   We can verify that $D_2(\beta_1, \frac{32-128\beta_1+160\beta_1^2-128\beta_1^3}{5\beta_1}) > 0$ for all $0 < \beta_1 \leq \beta_1^*$.

So we can conclude that the loss to uninformed traders is decreasing in σ_{z_2}.

B List of legal cases on spoofing

Defendant	Year	Market	Spoofing period	Victim	Penalty
Joseph R. Blackwell	2001	Stocks	2000		Fined $3,212.67
Robert Monski	2001	Stocks	1997		Fined $15,000,
Igor Oystacher	2016	E-mini S&P 500	2011-2014	Citadel, HTG Capital	Fined $2.5 million
Michael Coscia	2015	CME. NYMEX	2011	D.E. Shaw, Citadel	36 months
Navinder Sarao	2017	E-mini S&P 500	2010-2014		Home confinement
David Liew	2017	Metal contracts	2009-2012		Pending
Jiongsheng Zhao	2018	E-mini S&P 500	2012 -2016		Time served
Andre Flotron	2018	Metal contracts			Acquitted
John Edmonds	2018	Metal contracts	2009-2015		Pending
Kamaldeep Gandhi	2018	E-Mini S&P 500	2012-2014		Pending
Krishna Mohan	2018	E-Mini S&P 500	2012-2014		Pending
Edward Bases	2018	COMEX. NYMEX	2008-2014		1 year in prison
John Pacilio	2018	COMEX. NYMEX	2008-2014		1 year in prison
Jitesh Thakkar	2019	E-mini S&P	2011-2015		Dismissed
Corey Flaum	2019	Metal contracts	2007-2016		Pending
Christian Trunz	2019	Metal contracts	2006-2010		Pending
Xiasong Wang	2019	Stocks			Pending
Jiali Wang	2019	Stocks			Pending
Gregg Smith	2019	Metal contracts	2008 -2016	Citadel , Quantlab	2 years in prison
Michael Nowak	2019	Metal contracts	2008 -2016	Citadel , Quantlab	1 year in prison
Christopher Jordan	2019	Metal contracts	2006-2010	Citadel , Quantlab	Pending
Jeffrey Ruffo	2019	Metal contracts	2008 -2016	Citadel , Quantlab	Acquitted
James Vorley	2020	Metal contracts	2008-2013	Quantlab Financial	Pending
Cedric Chanu	2020	Metal contracts	2008-2013	Quantlab Financial	Pending
Merrill Lynch Commodities	2019	Metal contracts	2008-2014		$25 million
Tower Research Capital LLC	2019	E-Mini S&P 500	2012-2013		$67.4 million
Propex Derivatives Pty Ltd.	2020	E-Mini S&P 500	2012-2016		$1 million
Bank of Nova Scotia	2020	Metal contracts	2008-2016		$60.4 million
JP Morgan Chase & Co.	2020	Metal contracts	2008 -2016	Citadel , Quantlab	$920 million
Nielsen	2020	Arrayit	2020		3 years supervision
Nicholas Mejia Scrivener	2020	Stocks	2015-2016		$205,270
Xuepeng Xie	2021	Stocks			$2,708,778

References

Janet Austin. What exactly is market integrity? an analysis of one of the core objectives of securities regulation. William Mary Business Law Review, 8 (2), 01 2017.

Dan Bernhardt and Bart Taub. Front-running dynamics. Journal of Economic Theory, 138(1):288–296, 2008.

Hai Duong and Bart Taub. The value of information in the stock market. University of Glasgow(2023), 2023.

Merritt B. Fox, Lawrence R. Glosten, and Sue S. Guan. Spoofing and its regulation. Columbia Business Law Review, 2021(3), 2022.

Larry Harris. What to do about high-frequency trading. Financial Analysts Journal, 69(2):6–9, 2013.

Andrei Kirilenko, Albert S. Kyle, Mehrdad Samadi, and Tugkan Tuzun. The flash crash: High-frequency trading in an electronic market. The Journal of Finance, 72(3):967–998, 2017.

A. Kyle. Continuous auctions and insider trading. Econometrica, 53:1315–1355, 1985.

Kyong Shik Eom Lee, Eun Jung and Kyung Suh Park. Microstructure-based manipulation: Strategic behavior and performance of spoofing traders. Journal of Financial Markets, 16(2):227–252, 2013.

Donald MacKenzie. Spoofing: Law, materiality and boundary work in futures trading. Economy and Society, 2022.

Hersh Shefrin and Meir Statman. Ethics, fairness and efficiency in financial markets. Financial Analysts Journal, 49(6):21–29, 1993.

Basil Williams and Andrzej Skrzypacz. Spoofing in equilibrium. Stanford University Graduate School of Business Research Paper (2020), 2020. URL `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3742327`.

Ziyi Xu and Xue Cheng. Are large traders harmed by front-running hfts? Procedia Computer Science, 221:501–508, 2023. ISSN 1877-0509. Tenth International Conference on Information Technology and Quantitative Management (ITQM 2023).

Caitlin Zaloom. Ambiguous numbers: Trading technologies and interpretation in financial markets. American Ethnologist, 30(2):258–272, 2003. ISSN 00940496, 15481425.