The Convex Hulls

Implementation of Two-dimensional Convex Hull Algorithms and Comparing Their Performances

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Outline

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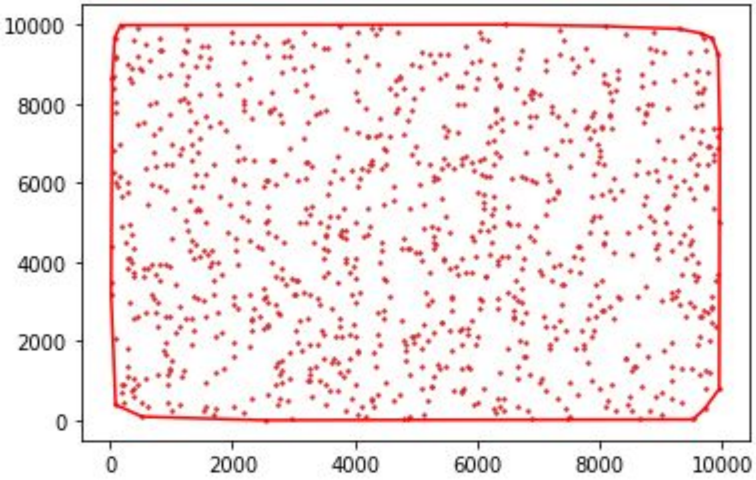
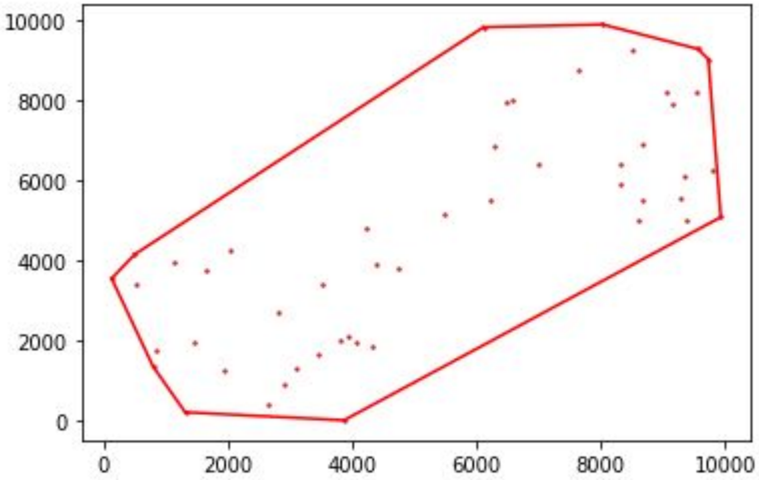
1. Introduction

What is Convex Hull?

Convex hull is the smallest convex set which contains all the points in it.





Why Convex Hull?

It has wide application like in

- Math-> analyze asymptotic beh. of poynomials
- Statistic -> visualize the spread of 2D sample points
- Economic -> When actual data non-complex, made convex
- Geometric modeling -> In Bezier curve, quickly detecting intersections of curves

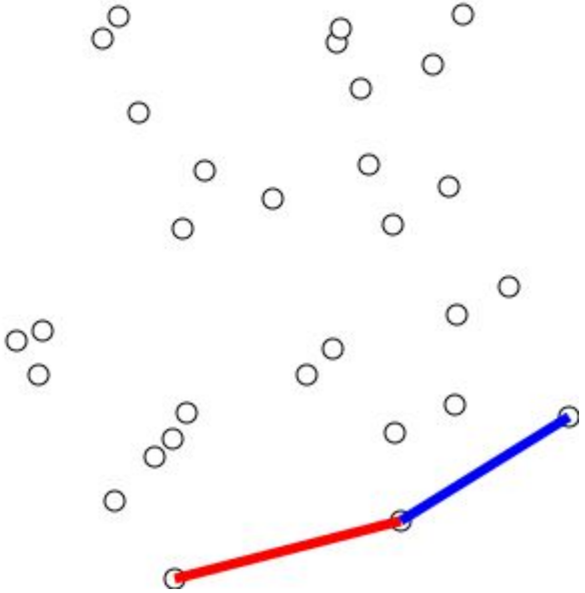
2. Background

Algorithms

- In general, most of the algorithms can be solved in time O(nlogn) for 2D or 3D
    - Graham Scan -> O(nlogn)
    - Gift wrapping (Jarvis March) -> O(nlogn)
    - Quickhull -> ave. O(nlogn) -> wor. O(n^2)
    - Divide & Conquer -> O(nlogn)

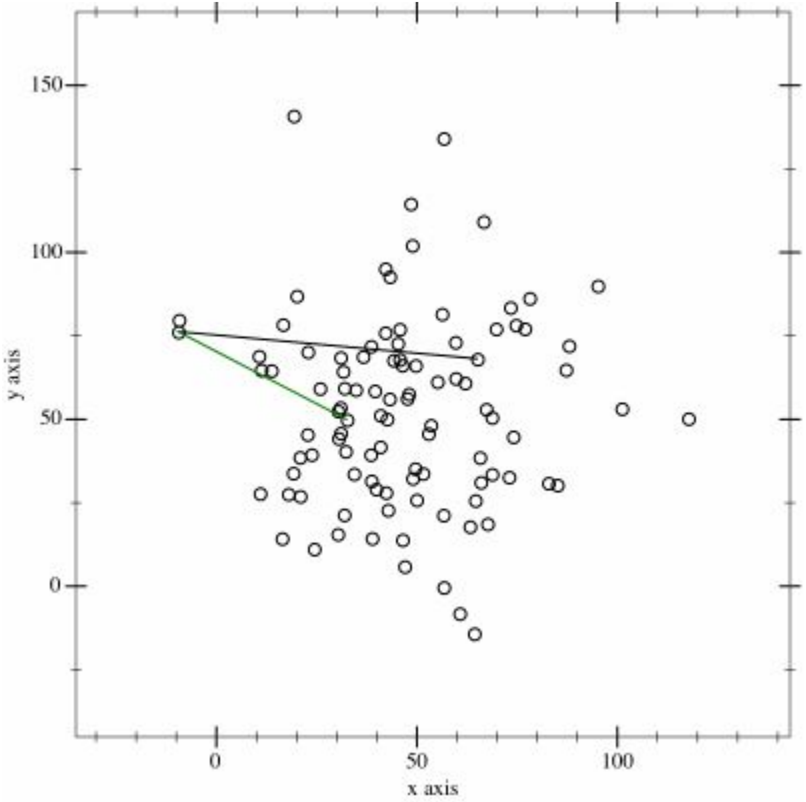
Graham Scan

- Ronald Graham 1972
- O(nlogn)
- First order vertices based on polar angle
- By left test, move around the points
- If left test is true add to stack or delete from until it is true



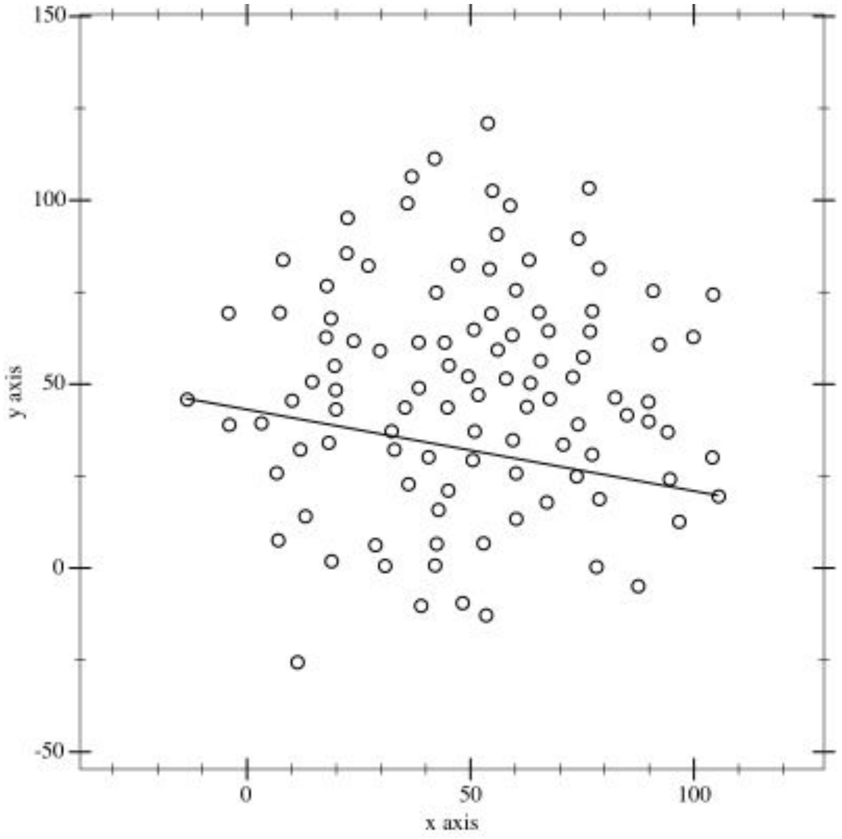
Jarvis March

- R. A. Jarvis-1973
- O(nh)
- It takes smallest point based on x or y.
- Sort all points based on polar angle
- Beside Graham, only calculates the points on the convex hull.



Quickhull

- Bradford Barber-1996
- Worst case O(n^2) Average case O(nlogn)
- Divides points into 2
- Find farthest point by calculating triangle area as recursive until no points left on recursive function



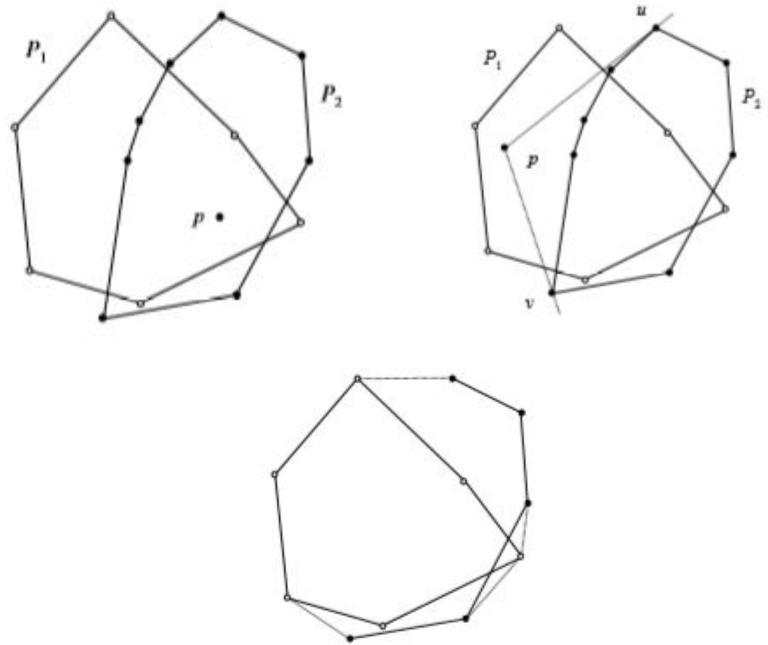
Divide and Conquer

- Quickhull recursively
- Then Merge them

Merge Hull (Merging Process of D&C)

1. P1 & P2 are convex hulls, find p internal P1
2. If p internal P2 -> step 3
   Else -> step 4
3. Sort based on polar for p
4. Find u and v and eliminate middle chain in P2
5. Graham Scan

- Merge: p1 -> m p2-> n So, O(m+n)=O(N)
- In Total O(NlogN)



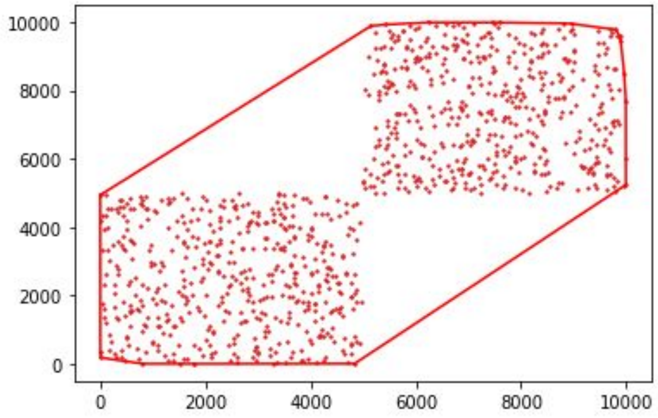
3. Progress

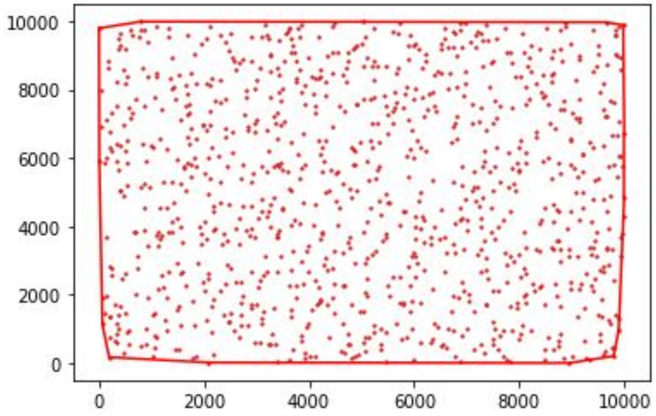
Language & Libraries

- Python 3 for coding
- The libraries:
  - random for point generation
  - math for tan calculations
  - numpy as array
  - time for timing calculations
  - matplotlib to show visual results

Features

- Clustered and non-clustered points
- from 1000 to 10.000.000 random point size
- Time calculation





4. Results

Graham Scan	Uniform(s)	Clustered(s)	Expected
1,000	0.00348	0.00343	O(nlogn)=3,000
10,000	0.04643	0.04083	O(nlogn)=40,000
100,000	1.26024	0.48540	O(nlogn)=500,000
1,000,000	7.10247	7.25312	O(nlogn)=6,000,000
2,000,000	14.91750	14.30834	O(nlogn)=12,602,060
3,000,000	23.81629	23.92488	O(nlogn)=19,431,363
4,000,000	31.19035	30.92443	O(nlogn)=26,408,240
5,000,000	40.74763	38.05435	O(nlogn)=33,494,850
10,000,000	87.29779	80.77861	O(nlogn)=70,000,000

Jarvis' March	Uniform(s)	Clustered(s)	Expected
1,000	0.01776 h = 23	0.01535 h = 18	u->O(nh)=23,000 c->O(nh)=18,000
10,000	0.23531 h = 26	0.25829 h = 26	u->O(nh)=260,000 c->O(nh)=260,000
100,000	5.52423 h = 37	4.01435 h = 35	u->O(nh)=3,700,000 c->O(nh)=3,500,000
1,000,000	66.89897 h = 33	69.90606 h = 41	u->O(nh)=33,000,000 c->O(nh)=41,000,000
2,000,000	174.11069 h = 47	115.76362 h = 44	u->O(nh)=94,000,000 c->O(nh)=88,000,000
3,000,000	244.66016 h = 45	295.58773 h = 48	u->O(nh)=135,000,000 c->O(nh)=144,000,000
4,000,000	340.58623 h = 40	414.12098 h = 48	u->O(nh)=160,000,000 c->O(nh)=192,000,000
5,000,000	353.88278 h = 47	469.60794 h = 44	u->O(nh)=235,000,000 c->O(nh)=220,000,000
10,000,000	356.32224 h=50	1145.9719 h=66	u->O(nh)=500,000,000 c->O(nh)=660,000,000

Quickhull	Uniform(s)	Clustered(s)	Expected
1,000	0.00512	0.00384	O(nlogn)=3,000 O(n*n)=10^6
10,000	0.05689	0.03930	O(nlogn)=40,000 O(n*n)= 10^8
100,000	0.71644	0.55157	O(nlogn)=500,000 O(n*n)=10^10
1,000,000	5.52458	5.60349	O(nlogn)=6,000,000 O(n*n)=10^12
2,000,000	11.87036	11.54690	O(nlogn)=12,602,060 O(n*n)=4*10^12
3,000,000	X	17.12558	O(nlogn)=19,431,363 O(n*n)=9*10^12
4,000,000	X	X	O(nlogn)=26,408,240 O(n*n)=16*10^12
5,000,000	X	X	O(nlogn)=33,494,850 O(n*n)=25*10^12
10,000,000	X	X	O(nlogn)=70,000,000 O(n*n)=10^14

Merge Hull	Uniform(s)	Clustered(s)	Expected
1,000	0.00414	0.00391	O(nlogn)=3,000
10,000	0.04171	0.03664	O(nlogn)=40,000
100,000	0.52161	0.44887	O(nlogn)=500,000
1,000,000	6.18138	6.00312	O(nlogn)=6,000,000
2,000,000	12.24851	13.15488	O(nlogn)=12,602,060
3,000,000	17.88851	26.40263	O(nlogn)=19,431,363
4,000,000	25.77283	31.05477	O(nlogn)=26,408,240
5,000,000	32.26809	42.65405	O(nlogn)=33,494,850
10,000,000	X	X	O(nlogn)=70,000,000

Clustered	Graham	Jarvis	Quickhull	Merge
1,000,000	7.25312	69.90606 h = 41	5.60349	6.00312
2,000,000	14.30834	115.76362 h = 44	11.54690	13.15488
3,000,000	23.92488	295.58773 h = 48	17.12558	26.40263
4,000,000	30.92443	414.12098 h = 48	X	31.05477
5,000,000	38.05435	469.60794 h = 44	X	42.65405
10,000,000	80.77861	1145.9719 h=66	X	X

Uniform	Graham	Jarvis	Quickhull	Merge
1,000,000	7.10247	66.89897 h = 33	5.52458	6.18138
2,000,000	14.91750	174.11069 h = 47	11.87036	12.24851
3,000,000	23.81629	244.66016 h = 45	X	17.88851
4,000,000	31.19035	340.58623 h = 40	X	25.77283
5,000,000	40.74763	353.88278 h = 47	X	32.26809
10,000,000	87.29779	356.32224 h=50	X	X

Demo