COMP0003 Theory of Computation
Exercises II: Regexes, pumping lemma, and PDAs

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1 Regexes

Exercise 1. Use the recursive definition of regular expressions (not DFAs or NFAs) to prove that regular languages are closed under reversal, i.e. if L is a regular language, then $L^R = \{w \mid w^R \text{ (w written backwards)} \in L\}$ is also a regular language.

Exercise 2. Use DFAs/NFAs and the GNFA reduction procedure from class to write a regex that recognizes the set of strings that do NOT contain the substring "`bb`" ($\Sigma = \{\texttt{a, b}\}$).

2 Pumping lemma

Exercise 3. Prove that the language $L = \{ww^R \mid \text{ for some } w \in \Sigma^*\}$ is not regular. (You can assume $\Sigma = \{\texttt{a, b}\}$.)

Exercise 4. Prove that the language $L = \{a^i b^j \mid i > j\}$ is not regular.

3 Pushdown Automata

Exercise 5. Design a PDA that recognizes the language in Exercise 3.

Exercise 6. Design a PDA that recognizes the language in Exercise 4.