COMP0003 Theory of Computation
Exercises IV: CFL pumping lemma and Turing machines

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1 Pumping lemma for CFLs

Exercise 1. Prove that the language $L = \{a^i b^j c^k \mid k = i * j;\ i, j, k \geq 1\}$ is not a CFL. ($\Sigma = \{a, b, c, \}$)

2 Turing machines

Exercise 2. Give a high-level overview of a Turing machine that is able to recognize $L = \{w \mid w \text{ contains an equal number of 0's and 1's}\}$ ($\Sigma = \{0, 1\}$).

Exercise 3. Give a high-level overview of a Turing machine that is able to recognize $L = \{0^{n^2} \mid n \geq 0\}$ ($\Sigma = \{0\}$). That is, L is the set of strings consisting of a square number of zeros.

3 Decidability

Exercise 4. Let $L = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) = \Sigma^*\}$. That is, L is the language containing strings corresponding to DFAs that recognize all strings in an alphabet Σ. Is L (a) decidable, (b) Turing-recognizable, (c) co-Turing-recognizable, or (d) none of the above? Explain your reasoning.

Exercise 5. Let $L = \{\langle M \rangle \mid M \text{ is a DFA that doesn't accept any string containing an odd number of 1's}\}$. Show that L is decidable.