# The Quasi-Balanced Aximetric Evolution model

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## 1 Model Description

### 1.1 The Equations

The tendency equation for tangential wind speed in $r-z$ coordinates is:

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial r} - w\frac{\partial v}{\partial z} - \frac{uv}{\partial r} - fu + \dot{V} \tag{1}$$

Where $\dot{V}$ is the momentum source due to friction force.

The Sawyer-Elliassen (SE) equation in radialheight coordinates has the form:

$$\begin{aligned}
&\frac{\partial}{\partial r}\left[-g\frac{\partial \chi}{\partial z}\frac{1}{\rho r}\frac{\partial \psi}{\partial r} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial \psi}{\partial z}\right] + \\
&\frac{\partial}{\partial z}\left[\left(\xi\chi(\zeta + f) + C\frac{\partial \chi}{\partial r}\right)\frac{1}{\rho r}\frac{\partial \psi}{\partial z} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial \psi}{\partial r}\right] = \\
&g\frac{\partial}{\partial r}\left(\chi^2\dot{\theta}\right) + \frac{\partial}{\partial z}\left(C\chi^2\dot{\theta}\right) + g\frac{\partial F_\lambda}{\partial r} + \frac{\partial}{\partial z}\left(CF_\lambda\right)
\end{aligned} \tag{2}$$

where $\chi = 1/\theta$, $C = v^2/r + fv$, $\xi = 2v/r + f$, $\zeta = (1/r)(\partial(rv)/\partial r)$ is the vertical component of relative vorticity; $\eta = \zeta + f$ is the absolute vorticity; $\dot{\theta} = d\theta/dt$ is diabatic heating rate. $\psi$ is a stream function that satisfies

$$u = -\frac{1}{r\rho}\frac{\partial \psi}{\partial z}, w = \frac{1}{r\rho}\frac{\partial \psi}{\partial r}$$

The heating source can be defined as a function of r and z as:

$$\dot{\theta} = M\cos\left(\pi\delta_r/W\right)\cos\left(\pi\delta_z/H\right) \tag{3}$$

Where $M$, $W$, $H$ is the magnitude, width and height of the source accordingly. $\delta_r$, $\delta_z$ is the relative distance is the heating center. Note that in this formula, a potential radius $R$ is used such as:

$$1/2fR^2 = rv + 1/2fr^2$$

For a prescribed tangential wind distribution $v(r,z)$ and environment field of pressure and temperature, a complete balanced fields of a TC can be calculated using a unapproximated method of Smith (2006). After a diabatic heating source is defined by (3)in potential radius coordinates and then transformed into physical coordinates, a solution for the toroidals stream function $\psi$ and hence radial and vertical wind speed can be obtained by solving the SE equation (2) using an over-successive relaxation similar to Bui *et al.* (2009) . Then, equation (1) can be integrated to obtain tangential wind speed in the next timestep. After each timestep, potential temperature and density can be *re-balanced* using the method of Smith (2006).

To prevent the collapse of the heating source during the integration, a vertical and a radial diffusion terms are introduced into the right hand side of 1 as:

$$D_z = K_z \frac{\partial^2 v}{\partial z^2}$$

$$D_r = \frac{1}{r}\frac{\partial}{\partial z}(rK_r\frac{\partial v}{\partial r})$$

Where the vertical and raidal diffusion coefiction $K_z$ and $K_r$ take the value of 10 and $10^4$ $m^2s^{-1}$ accordingly **(why we take three-order different values for $K_z$ and $K_r$?)**.

If friction force is introcuded, it is parameterized as:

$$\dot{V} = -\frac{C_d V_s^2 \exp{(-\frac{z}{z_0})^2}}{h}$$

where $h$ is some assumed boundary layer depth, say 600 m, $z_0$ is a vertical scale (also take the value of 600 m) $C_d$ is the drag coefficient, typically $2 \times 10^{-3}$ and $Vs = 0.9v(r, 0)$ is the prescribed tangential wind at the surface

## 1.2 Numerical Methods

The over-successive relaxation method is the same as the one used in Bui *et al.* (2009). The diffusion terms are diffentiated as:

$$D_z(k) = \frac{K_z}{\Delta z^2}(v_{k+1} + v_{k-1} - 2v_k)$$

$$D_r(i) = \frac{K_r}{\Delta r^2}(v_{i+1} + v_{i-1} - 2v_i) + \frac{K_r}{2r_i\Delta r}(v_{i+1} - v_{i-1})$$

where $i$ and $k$ are the radial and vertical gridpoint number accordingly.

**References**

H. H. Bui, R. K. Smith, M. T. Montgomery, and J. Peng, 2009: Balanced and unbalanced aspects of tropical-cyclone intensification. *Q. J. R. Meteor. Soc.*, **135**, 1715-1731.

Smith RK. 2006: Accurate determination of a balanced axisymmetric vortex. *Tellus*, **58A**, 98-103.