Control Systems - II

PID Control Implementation for a Mass-Spring-Damper System Using MATLAB & Simulink

Submitted To:

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Introduction:

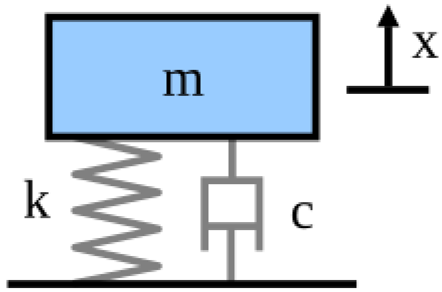


Fig1.1: Mass Spring Damper Block Diagram **[1]**

The mass-spring-damper model consists of discrete mass nodes distributed throughout an object and interconnected via a network of springs and dampers.

This form of model is also well-suited for modelling objects with complex material behavior such as those with nonlinearity or viscoelasticity. **[1]**

Here is the derivation of the transfer function of the mass-spring system:

$$\ddot{m}\{x\}(t) + \dot{b}\{x\}(t) + kx(t) = F(t)$$

Taking Laplace Transform,

$$m\, s^2 X(s) + b\, s\, X(s) + k\, X(s) = F(s)$$

$$X(s)(\, ms^2 + b\, s + k) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{(m\, s^2 + b\, s + k)}$$

Non-Linear Simulink Model:

Keeping in mind the transfer function of the mass-spring system, non-linear model is designed in MATLAB Simulink.

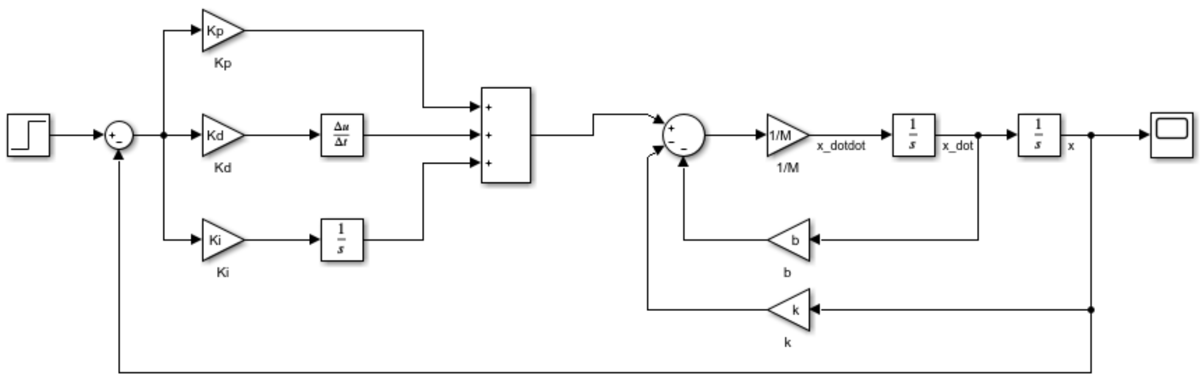


Fig1.2: Non-Linear Mass Spring Damper System Model

With the following reference values, the model is simulated.

```
clear; clc;
% Define system parameters
M = 1;
b = 10;
k = 20;

% Define PID controller gains
Kp = 350;
Ki = 300;
Kd = 50;

% Set step input value and simulation time
Step_Value = 1;

% THEN run the simulation
sim('MassSpringDamper')

% Define Laplace variable
s = tf('s');

% Mass-Spring-Damper transfer function
% Output is displacement X(s) / Force input F(s)
G = 1 / (M*s^2 + b*s + k);    % Open-loop system
% Define PID controller
Gc = pid(Kp, Ki, Kd);

% Closed-loop system with unity feedback
sys_cl = feedback(Gc * G, 1);

% Plot step response
figure;
step(sys_cl);
title('Step Response of Closed-Loop MSD System with PID');
xlabel('Time (s)');
```

```
ylabel('Displacement');

% Performance characteristics
disp('Step Response Info:');
stepinfo(sys_cl)

disp('Damping Characteristics:');
damp(sys_cl)
```

Where, accordingly to this code, the step response of closed-loop model is shown in Fig1.3:

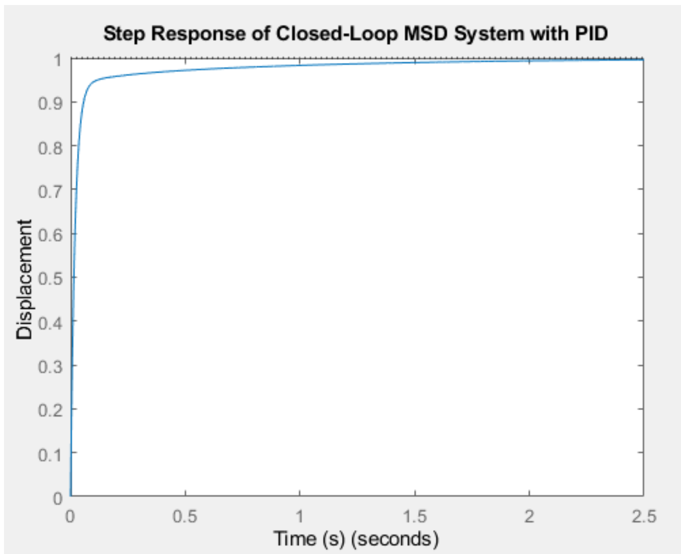


Fig1.3: Step Response Closed-Loop MSD System w/ PID Controller

The Step Response Info using stepinfo() in shown in Table1.1:

Rise Time	0.0548
Settling Time	0.8308
Settling Min	0.9006
Settling Max	0.9959
Overshoot	0
Undershoot	0
Peak	0.9959
Peak Time	2.4775

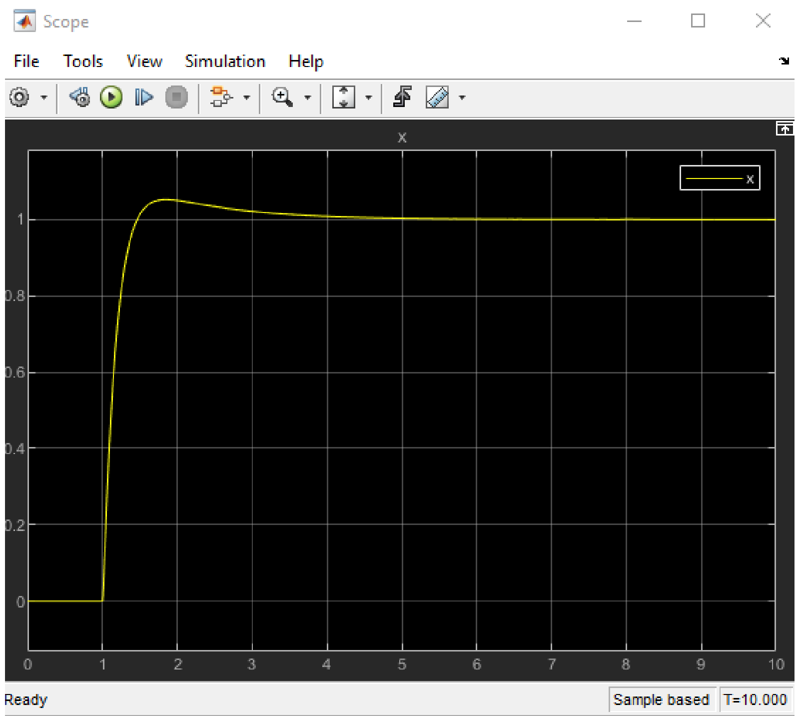
Table1.1: MSD System Step Response Info

Whereas, the damping characteristics are shown in Table1.2:

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-9.57e-01	1.00e+00	9.57e-01	1.04e+00
-5.90e+00	1.00e+00	5.90e+00	1.70e-01
-5.31e+01	1.00e+00	5.31e+01	1.88e-02

Table1.2: MSD System's Damping Characteristics

Finally, the response of the displacement of non-linear model is shown in Fig1.4.



Where **x** — represents the **displacement** of the mass from its equilibrium position as a function of time.

REFERENCES

[1] https://en.wikipedia.org/wiki/Mass-spring-damper_model