## Reinforcement Learning

COMP9414: Artificial Intelligence

#### Lecture Overview

- Introduction
- Elements of Reinforcement Learning
- Exploration vs Exploitation
- The agent-environment interface
- Value functions
- Temporal difference prediction

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### Initial ideas

Instrumental or operational conditioning. Stimulus-behavior learning.

Thorndike, 1911

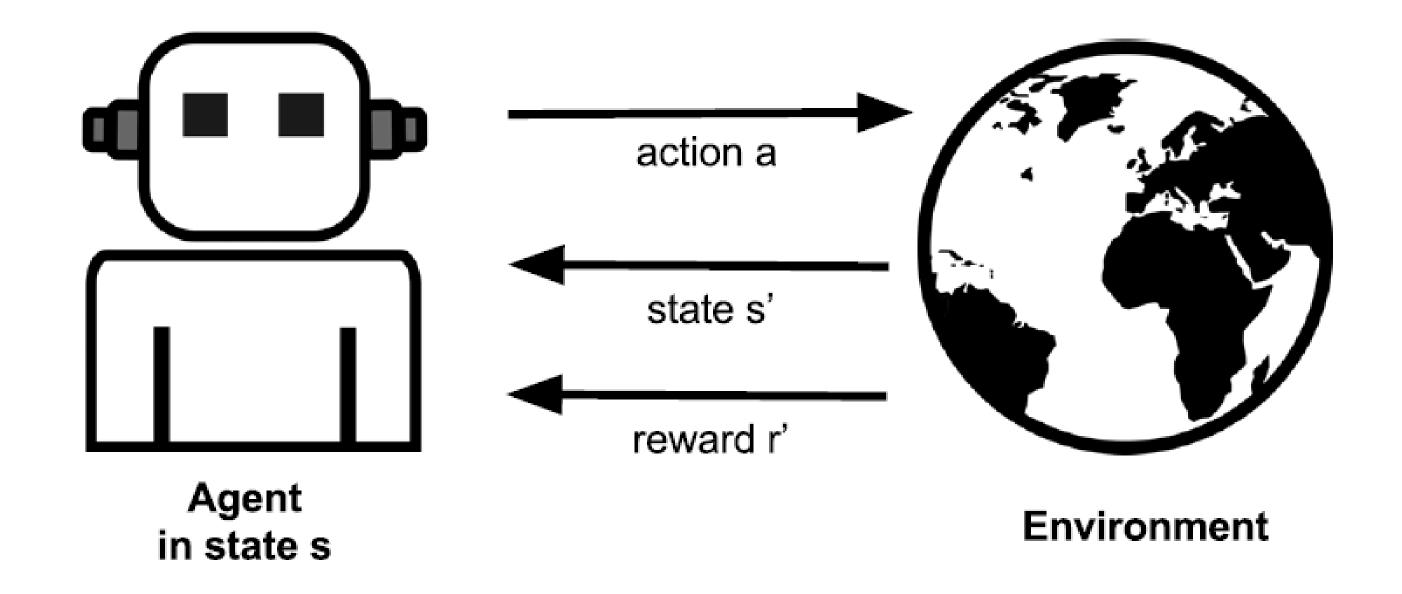


## Reinforcement learning

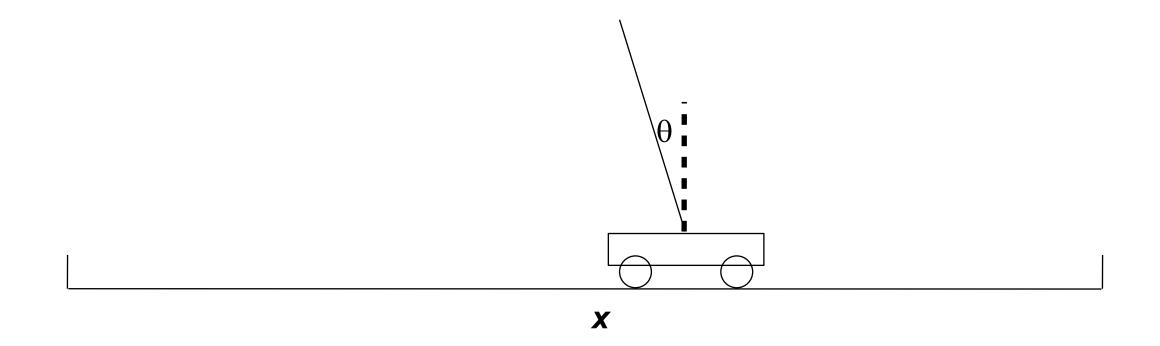
- Idea of learning by interaction with the environment.
- With no explicit instructor but with a direct sensorimotor connection.
- Awareness of how our environment answers to what we do.



## Reinforcement Learning



## Pole balancing



- Pole balancing can be learned the same way
- Reward might be only received at the end
  - after falling or hitting the end of the track

## Pole balancing



## And you think pole balancing is trivial?



## Types of learning

#### Supervised Learning

- Agent is given examples of input/output pairs
- Learns a function from inputs to outputs that agrees with the training examples and generalises to new examples

#### Unsupervised Learning

- Agent is only given inputs
- Tries to find structure in these inputs

#### Reinforcement Learning

- Training examples presented one at a time
- Must guess best output based on a reward, tries to maximise (expected) rewards over time

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## Reinforcement Learning

- RL is to learn what to do, mapping from situations to actions.
- An agent should be able to sense the environment states and perform actions to affect such states.
- Actions might affect not only immediate reward.
- An important challenge is the exploration/ exploitation trade-off problem.

state s'

reward r'

Environment

Agent

in state s

There are four essential elements:

#### Policy

- Informs how to act in a particular situation.
- Set of stimulus-response rules or associations.
- Can be stochastic.

There are four essential elements:

#### Reward function

- Defines the aim of an RL problem.
- Maps each perceived state (or state-action pair) into a number, the reward.
- The goal is to maximize the long-term reward.
- In biological systems may correspond to pain and pleasure feelings.
- Can be stochastic.

There are four essential elements:

#### Value function

- Shows what it's good in the long run (the reward in an immediate sense).
- In biological systems corresponds to more refined judgments of foresight about the future from one state.
- Actions are decided based on the value.
- It's much harder to determine values than rewards.

There are four essential elements:

- Optionally, a model of the environment
  - Imitates the environment behaviour.
  - Can predict states and reward obtained.
  - The use of models of the environment is still relatively new.

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## Exploration / Exploitation Trade-off

- Most of the time, the agent chooses what it thinks the "best" action is.
- But to learn, it must occasionally choose something different from the preferred action.

## Exploration / Exploitation Trade-off

Should I stay or should I go now?
Should I stay or should I go now?
If I go, there will be trouble
And if I stay it will be double

-- The Clash



## Exploration / Exploitation Trade-off

- The greedy action exploits the current knowledge.
- The non-greedy action explores.
- Exploitation maximises the immediate reward and exploration in the long run.
- There is a conflict between exploration and exploitation.



- We denote the real action value as  $q_*(a)$ .
- We denote the estimated value at time-step t as  $Q_t(a)$ .
- Simple Estimation: to average received rewards when action **a** has been selected K<sub>a</sub> times.

$$Q_t(a) = \frac{R_1 + R_2 + \dots + R_{K_a}}{K_a}.$$

- If  $K_a = 0$ ,  $Q_t(a)$  is defined with an arbitrary value, e.g.,  $Q_t(a) = 0$  (not necessarily the best).
- As  $K_a \rightarrow \infty$ ,  $Q_t(a)$  converges to  $q_*(a)$ .

$$Q_t(a) = \frac{R_1 + R_2 + \dots + R_{K_a}}{K_a}.$$

#### Greedy method

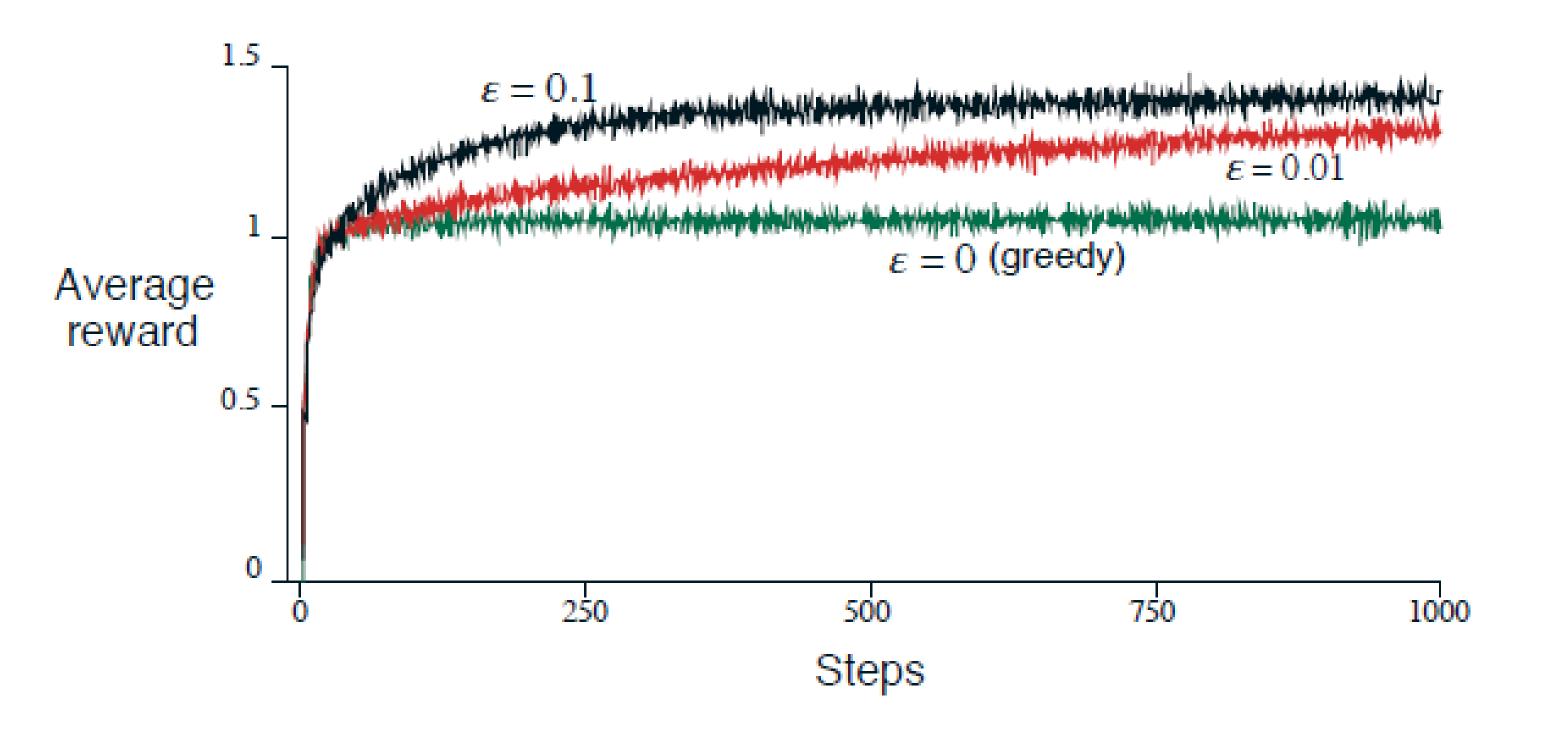
- The simplest way to choose an action: the action with the highest estimated value.
- $A_t^*$  where  $Q_t(A_t^*) = \max_a Q_t(a)$ .

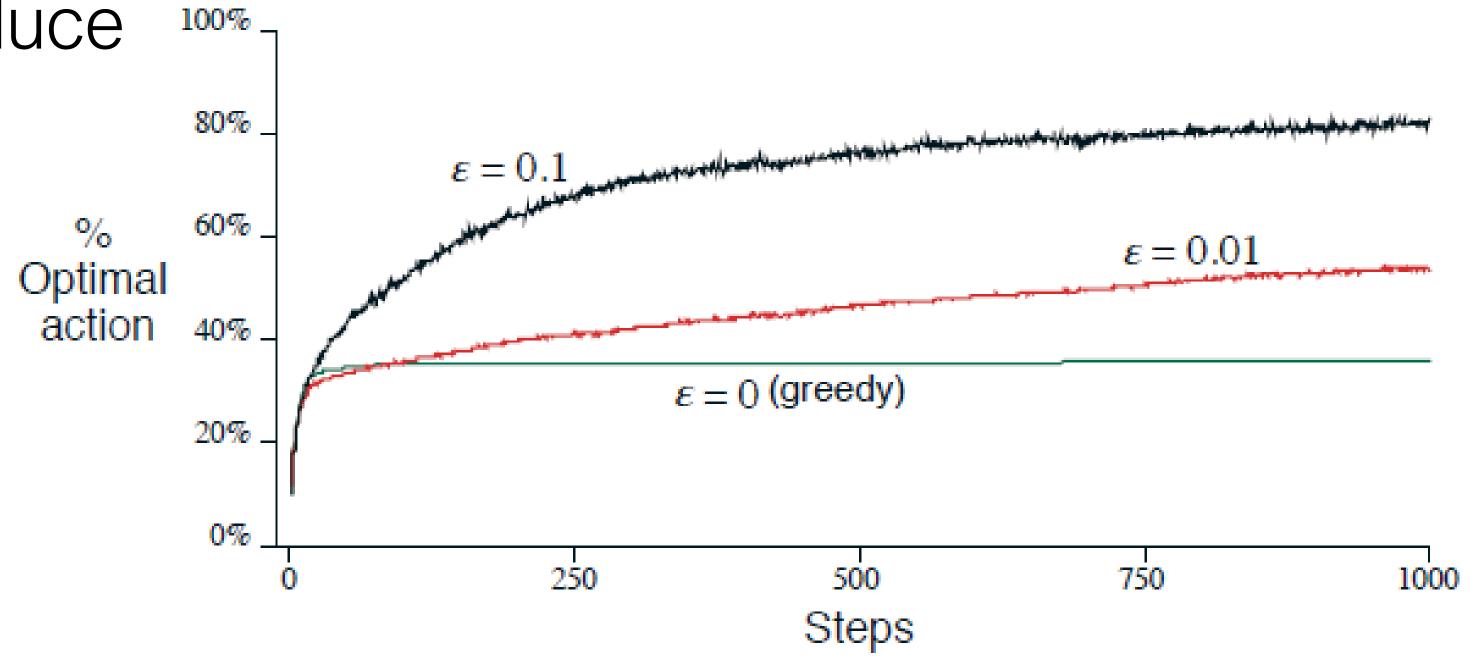
#### ε-greedy method

- A simple alternative: to choose the best action most of the time, and sometimes (with a small probability ε) a random one.
- $Q_t(a)$  converges to  $q_*(a)$  with probability 1-  $\epsilon$ .

• 2000 agents averaged.

It's possible to reduce
 ε over time





#### Softmax method

- ε-greedy effectively trades off exploration and exploitation, but the selection is equitable (or fair) among actions.
- Sometimes, the worst action is very bad.
- High temperatures give almost equal probability for all actions.
- Low temperatures make a bigger difference in the probability.

$$\frac{e^{Q_t(a)/\tau}}{\sum_{i=1}^n e^{Q_t(i)/\tau}}$$

## Incremental implementation

A simple implementation: record all the rewards.

$$Q_t(a) = \frac{R_1 + R_2 + \dots + R_{K_a}}{K_a}$$

 Problem: growing use of memory and computational cost over time.

## Incremental implementation

 Denote Q<sub>k</sub> the estimated reward at time-step k, i.e., the average of the k-1 first rewards, then:

$$Q_{k+1} = \frac{1}{k} \sum_{i=1}^{k} R_i$$

$$= \frac{1}{k} \left( R_k + \sum_{i=1}^{k-1} R_i \right)$$

$$= \frac{1}{k} \left( R_k + (k-1)Q_k \right)$$

$$= \frac{1}{k} \left( R_k + kQ_k - Q_k \right)$$

$$= Q_k + \frac{1}{k} \left[ R_k - Q_k \right],$$

## Non-stationary problems

- Methods of average are appropriate for stationary problems.
- In non-stationary problems, make sense to give more weight to more recent rewards than the past ones.
- Using a constant parameter step-size,  $0 < \alpha \le 1$ :

$$Q_{k+1} = Q_k + \alpha \left[ R_k - Q_k \right]$$

#### Contextual or associative search

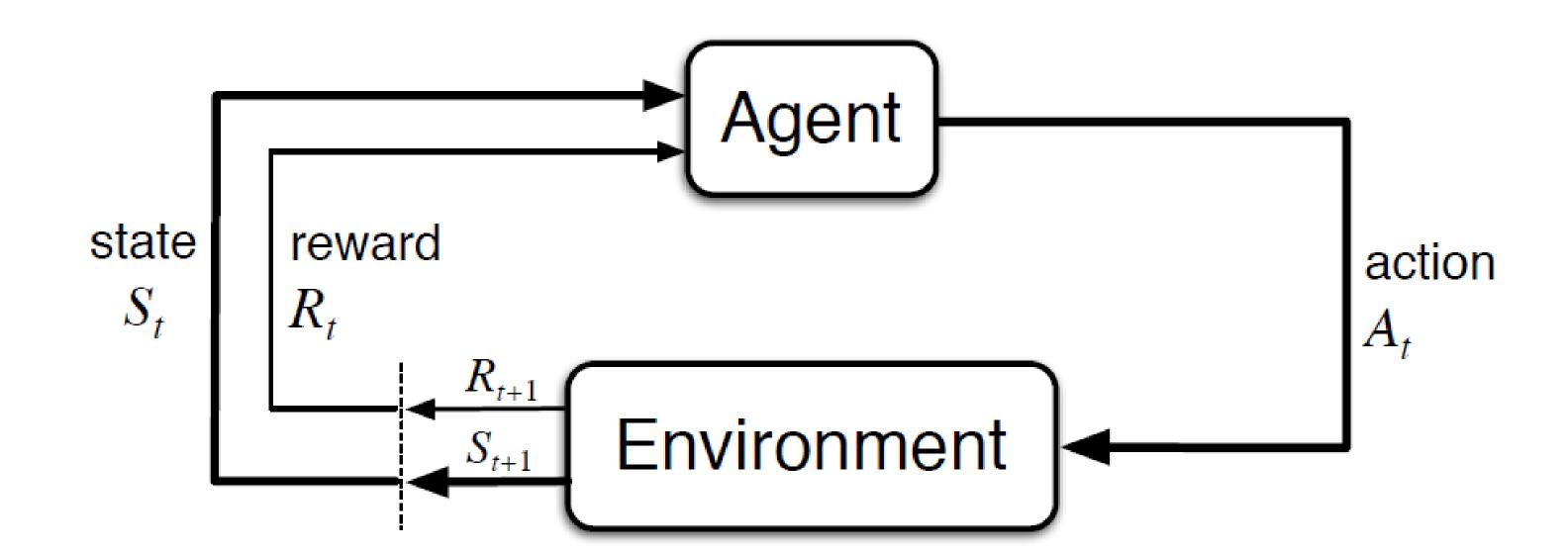
- So far, only non-associative tasks, i.e., no association between actions and situations.
- In an RL problem, there is more than one situation.
- The aim is to learn a policy: to map situations into actions.
- For instance, a set of n-armed bandits with changing colours.
- If actions also affect the following situation as well as the reward, then it corresponds to a full RL problem.

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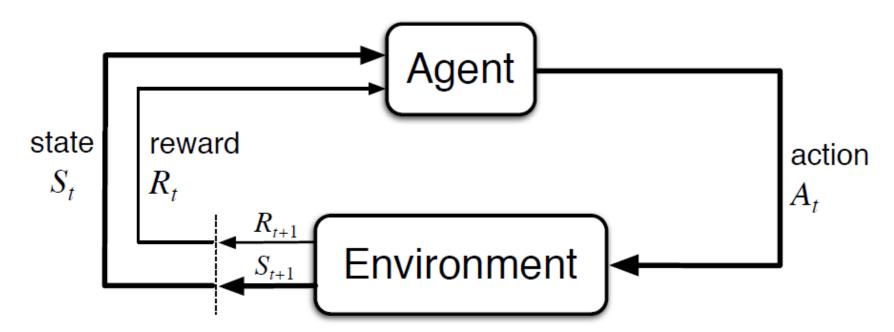
## The agent-environment interface

- Any method able to solve the problem is considered an RL method.
- Agent: comprises the learner and the one making the decisions.
   (although they can be separated!).
- Environment: everything external to the agent that it interacts with.



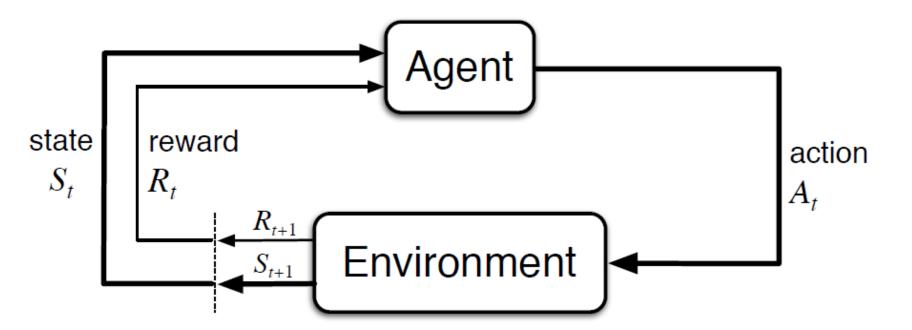
## The agent-environment interface

- Reward: numeric value the agent tries to maximise.  $R_{t+1} \in \mathbb{R}$ .
- $S_t \in S$ . S set of possible states.
- $A_t \in A(S_t)$ .  $A(S_t)$  actions available at  $S_t$ .
- The agent implements a map from the states toward the action selection probability.
- This is called agent policy  $\pi_t$  where  $\pi_t(a|s)$  is the probability of  $A_t = a$  and  $S_t = s$ .
- RL methods detail how an agent updates its policy as a result of its experience.



## The agent-environment interface

- Example: recycling robot.
- The agent decides if (i) actively search for a can, (ii) remains stationary and waits for a can, or (iii) gets back to the home base to recharge (three possible actions).
- The state is determined by the battery state.
- Reward: Mostly zero, positive when collects a can and negative (much higher) when the battery runs empty.



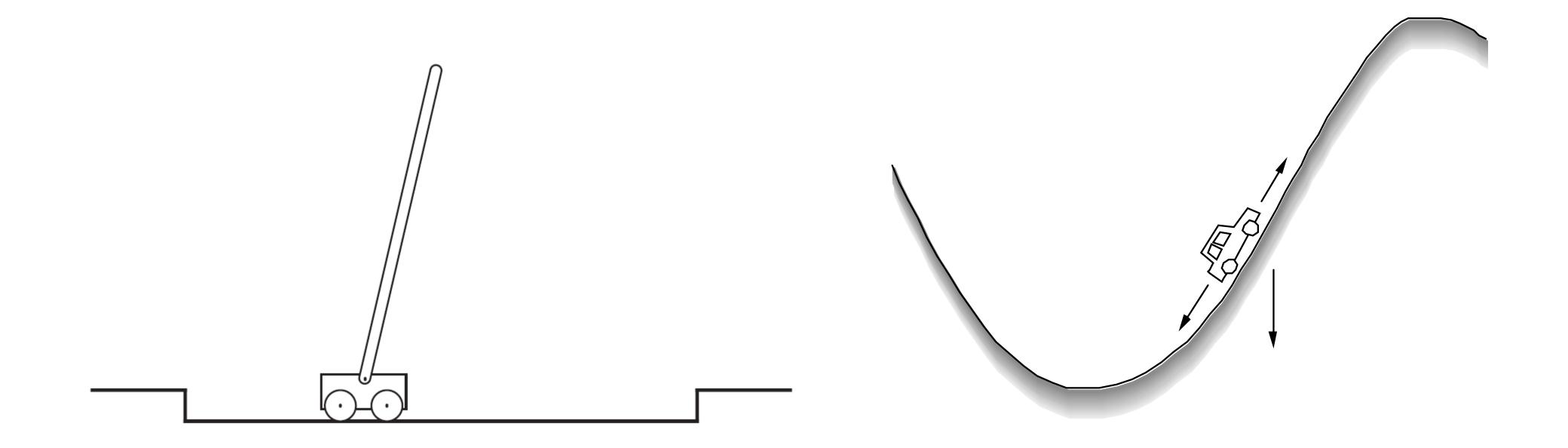
#### Goals and rewards

- The agent's goal is to maximise the amount of total reward, not the immediate reward.
- A robot learning to walk receives a reward proportional to the forward movement.
- A robot learning to escape from a maze receives a reward equal to zero until escapes and then receives +1.
- Another strategy is giving a reward of -1 after each movement till escaping.
- An agent learning to play chess receives +1 for winning and -1 for losing.

#### Goals and rewards

- The chess player should be rewarded only for winning and not for taking opponent's pieces.
- Otherwise, the agent will learn to maximise subgoals.
- In summary, the reward signal is the way to communicate to the agent **what** you want it to achieve, **not how** you want it achieved.

## Episodic and non-episodic tasks



The pole-balancing task.

The mountain car task.

#### Returns

• If the reward sequence received is  $R_{t+1}$ ,  $R_{t+2}$ ,  $R_{t+3}$ , .... We want to maximise the expected return  $G_t$ .

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \dots + R_T$$

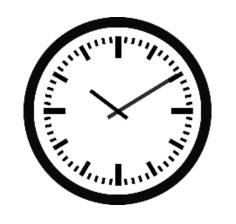
- In tasks with final state and that can be divided into subsequences (episodes)
- Each episode finishes in the final state and the task starts over from an initial state.
- These tasks are known as episodic tasks.

#### Returns

- Tasks intended to be performed continuously without limit are referred to as continuous tasks (or non-episodic).
- The return could be infinite, given that  $T = \infty$ .
- In this case, the agent maximises the discounted rewards, choosing actions to maximise the discounted return:

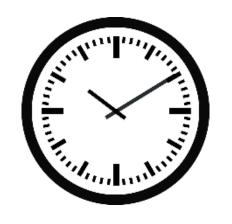
$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

• Discount rate  $0 \le \gamma < 1$ . Determine the present value of future rewards. If  $\gamma = 0$ , the agent is myopic. If  $\gamma \to 1$  the agent is foresighted.



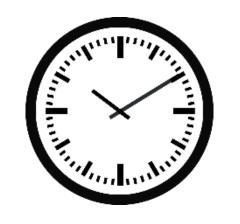
Y	Reward sequence	Return
0.5	1000	
0.5	00200	
0.9	00200	
0.5	-12632000	

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



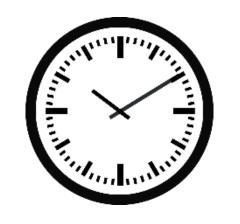
Y	Reward sequence	Return
0.5	1000	1
0.5	00200	
0.9	00200	
0.5	-12632000	

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



Υ	Reward sequence	Return
0.5	1000	1
0.5	00200	0.5
0.9	00200	
0.5	-12632000	

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



Y	Reward sequence	Return
0.5	1000	1
0.5	00200	0.5
0.9	00200	1.62
0.5	-12632000	

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

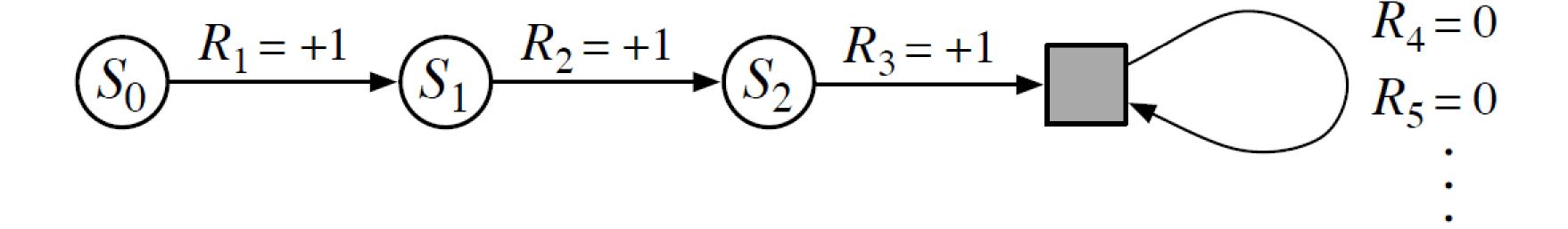


Υ	Reward sequence	Return
0.5	1000	1
0.5	00200	0.5
0.9	00200	1.62
0.5	-12632000	2

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

#### Unified Notation

A final absorbing state with reward equal to zero.



• It is possible that  $T = \infty$  or  $\gamma = 1$ , but not both.

$$G_t = \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1}$$

### The Markov property

- In RL, state means any information available for the agent (either processed or not).
- The state should not inform everything about the environment to the agent. For instance, an agent playing blackjack should not know the next card in the deck.
- We do not blame the agent for not knowing something important, but we do for knowing something and then forgetting it.
- Ideally, a state should contain compact information about the past, retaining relevant information. This is called the Markov property. For instance, the chess board.

### The Markov property

- Sometimes, the property cannot be fully satisfied.
- In pole-balancing, the state satisfies the property if the exact position and velocity of the cart are specified, and the pole angle and its change rate.
- However, there may exist distortions, such as delays and other effects as the temperature of the wheels.
- Some studies have even used a simple region division: left, right, and centre.

#### Markov Decision Processes

- An RL task with the Markov property is called Markov decision process (MDP).
- Markov decision process (MDP):  $< S, A, \delta, r >$ .
  - S is a finite set of states,
  - A is a set of actions,
  - $\delta$  is the transition function  $\delta: S \times A \rightarrow S$ , and,
  - r is the reward function  $r: S \times A \rightarrow \mathbb{R}$ .

### Recycling robot MDP

- At each moment, the robot decides if (i) actively search for a can, (ii) waits for someone to bring a can, or (iii) gets back to the home base to recharge.
- The best strategy is to actively search for cans.
- In case the battery runs out, the robot needs to be rescued leading to a negative reward.
- The agent solely decides as a function of the energy level of the battery. Two levels: high, low.
- S = {high, low}.
- Possible decisions (agent's actions): wait, search, recharge.
- A(high) = {search, wait}.
- A(low) = {search, wait, recharge}.

### Recycling robot MDP

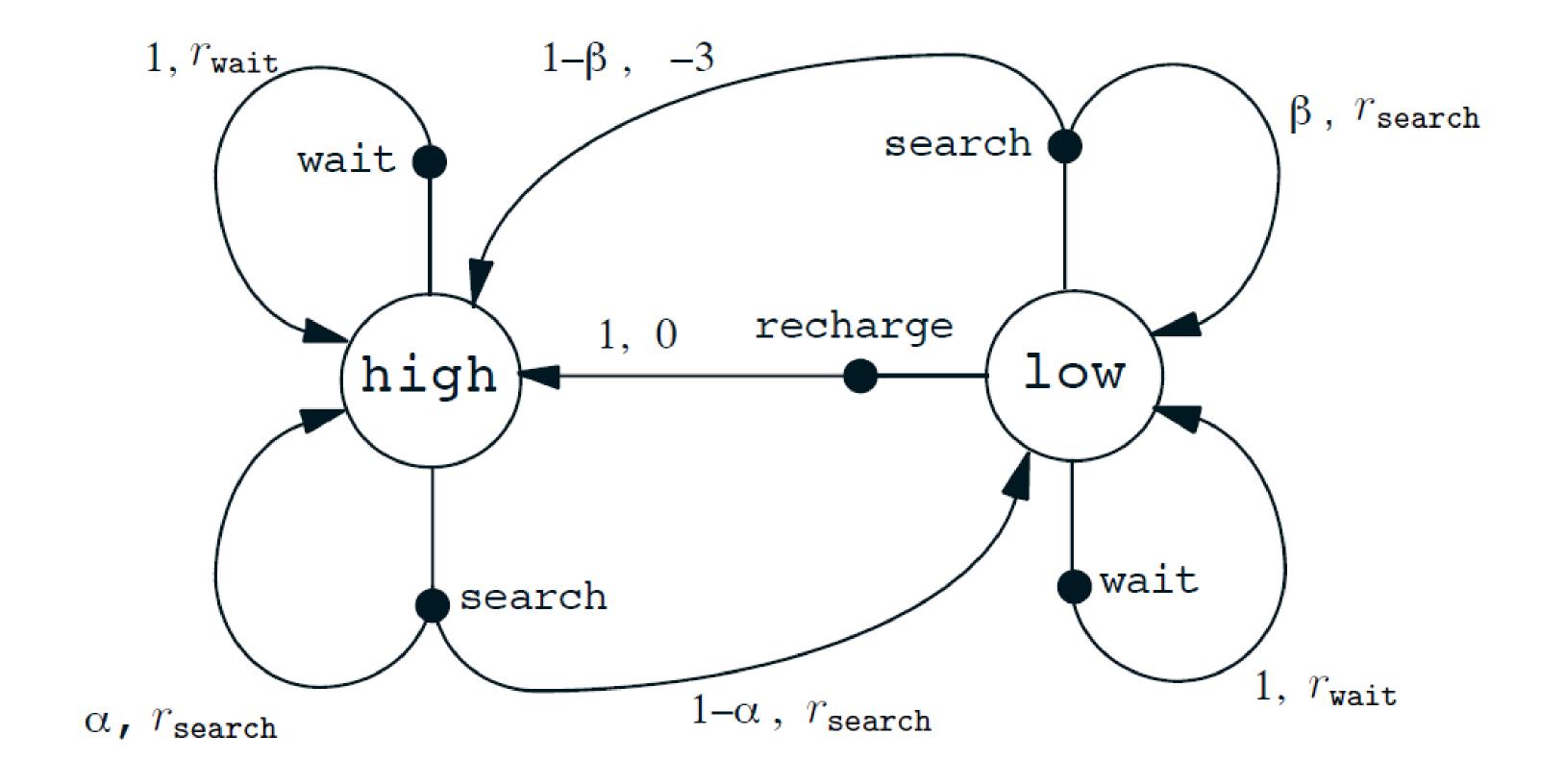
Transition probabilities and expected reward:

s	s'	a	p(s' s,a)	r(s, a, s')	
high	high	search	$\alpha$	$r_{\mathtt{search}}$	
high	low	search	$1-\alpha$	$r_{\mathtt{search}}$	
low	high	search	$1-\beta$	-3	
low	low	search	$\beta$	$r_{\mathtt{search}}$	r . > r
high	high	wait	1	$r_{\mathtt{wait}}$	r <sub>search</sub> > r <sub>wait</sub>
high	low	wait	0	$r_{\mathtt{wait}}$	
low	high	wait	0	$r_{\mathtt{wait}}$	
low	low	wait	1	$r_{\mathtt{wait}}$	
low	high	recharge	1	0	
low	low	recharge	0	0.	

 Assumption: cans cannot be collected when going back to the home base or when the battery is depleted.

## Recycling robot MDP

Transition graph:



Transition probabilities from one action always sum to 1.

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- Value function estimations.
  - State-value function, or
  - Action-value function (for state-action pairs)
- The function estimates how good it is for the agent to be in a given state, in terms of future reward (or expected return).
- The value of a state s under a policy  $\pi$ , denoted  $v_{\pi}(s)$  or  $V^{\pi}(S)$ , is the expected return when starting in s and following  $\pi$  thereafter:

$$v_{\pi}(s) = \mathbb{E}_{\pi}[G_t \mid S_t = s] = \mathbb{E}_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s\right]$$

- The value of a terminal state, if any, is zero.
- The value of taking action a in state s under policy  $\pi$ , is denoted  $q_{\pi}(s,a)$  or  $Q^{\pi}(S,A)$ :

$$q_{\pi}(s, a) = \mathbb{E}_{\pi}[G_t \mid S_t = s, A_t = a] = \mathbb{E}_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s, A_t = a\right]$$

- Value functions  $v_{\pi}(s)$  and  $q_{\pi}(s,a)$  can be estimated from experience.
- If there are many states, it's impractical to keep values for each state.
- In this case, parameterized function approximators are used to keep  $v_{\pi}(s)$  and  $q_{\pi}(s,a)$ .

Try to maximise expected future reward:

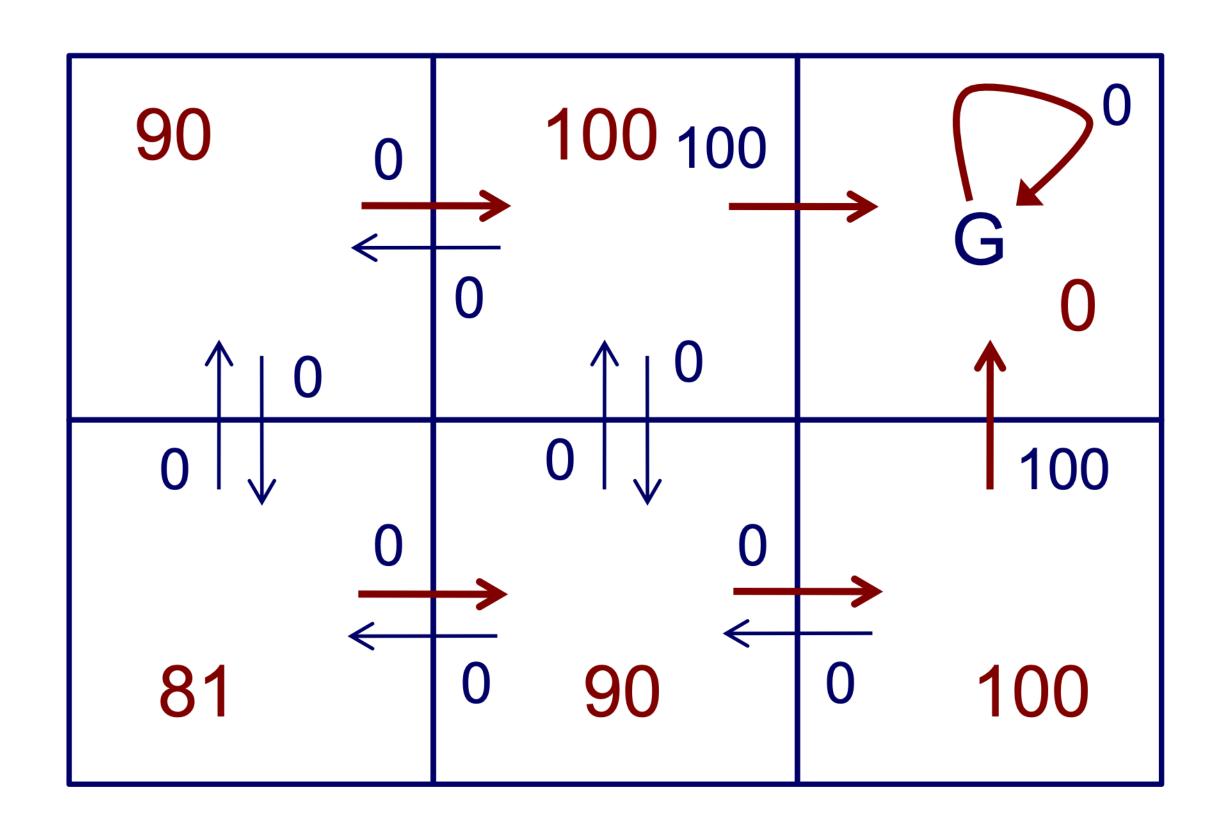
$$V^{\pi}(s_{t}) = r_{t} + \gamma r_{t+1} + \gamma^{2} r_{t+2} + \dots$$
$$= \sum_{i=0}^{\infty} \gamma^{i} r_{t+i}$$

- $V^{\pi}(s_t)$  is the value of state  $s_t$  under policy  $\pi$
- $\gamma$  is a discount factor [0..1]

- $V^{\pi}(s)$  is the expected value of following policy  $\pi$  in state s
- $V^*(s)$  be the maximum discounted reward obtainable from s.
  - i.e. the value of following the optimal policy
- We make the simplification that actions are deterministic, but we don't know which action to take.
  - Other RL algorithms relax this assumption

- The red arrows show  $\pi^*$ , the optimal policy, with  $\gamma=0.9$
- $V^*(s)$  values shown in red

$$V^{\pi}(s_{t}) = r_{t} + \gamma r_{t+1} + \gamma^{2} r_{t+2} + \dots$$
$$= \sum_{i=0}^{\infty} \gamma^{i} r_{t+i}$$



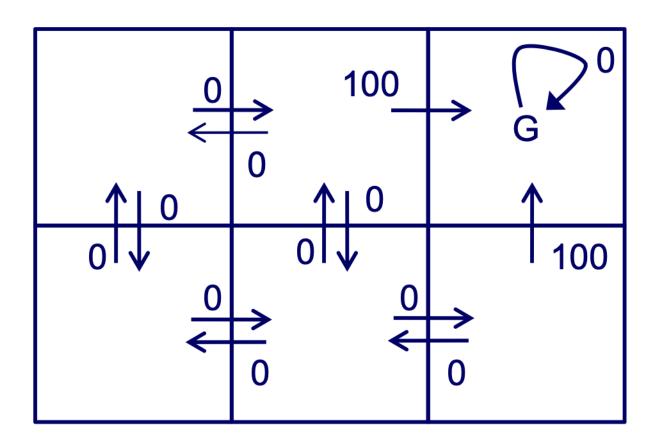
### Q-values

How to choose an action in a state?

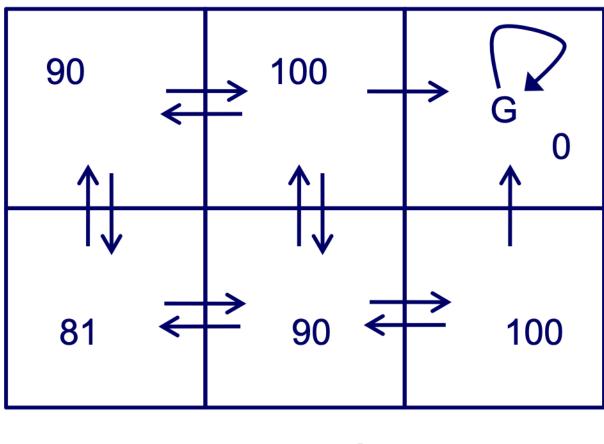
$$Q(s,a) = r(s,a) + \gamma V^*(s')$$

- The Q-value for an action, a, in a state, s, is the immediate reward for the action plus the discounted value of following the optimal policy after that action
- V\* is value obtained by following the optimal policy
- $s' = \delta(s, a)$  is the succeeding state, assuming the optimal policy

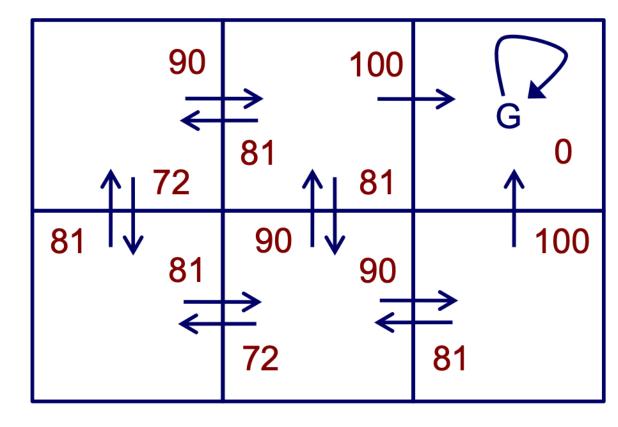
## Q-values



r(s, a) (immediate reward) values



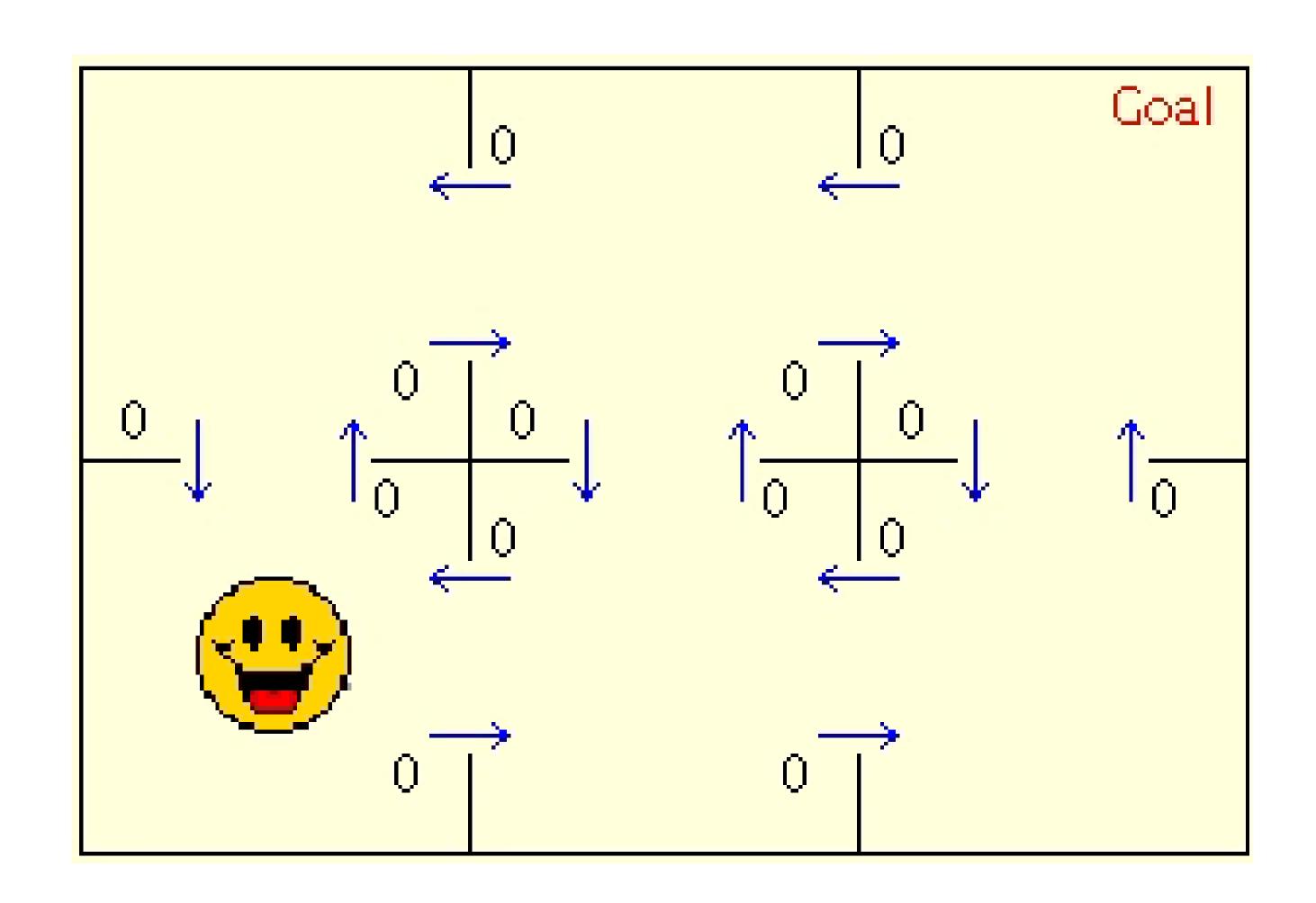
V\*(s) values

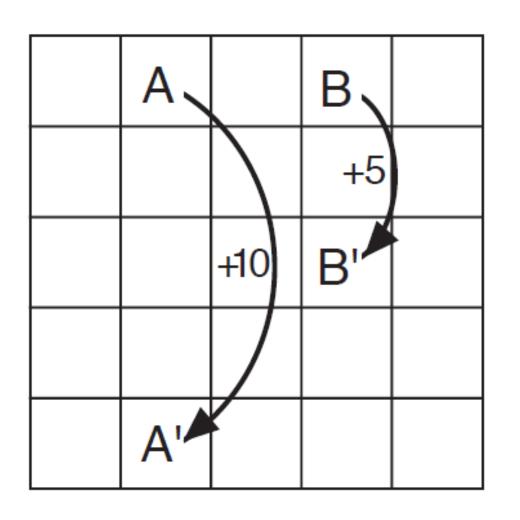


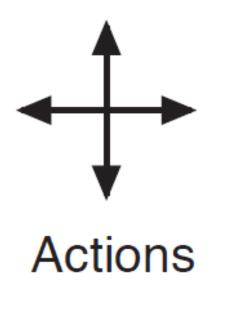
Q(s, a) values

$$\gamma = 0.9$$

## Grid world example

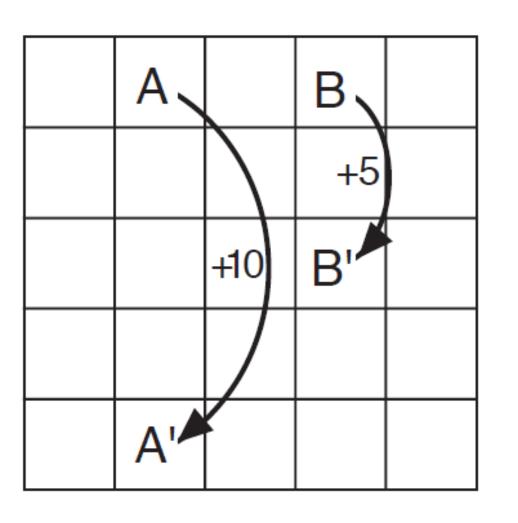


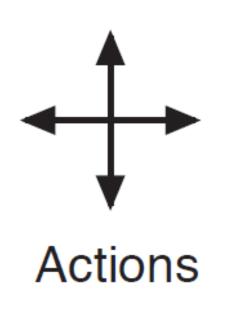




3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

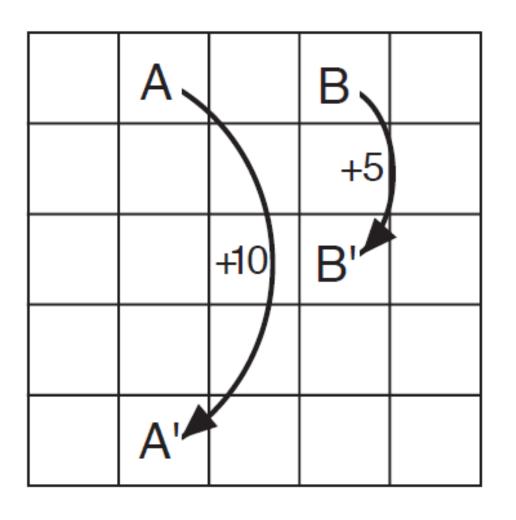
- Cells correspond to the states.
- 4 possible actions.
- Actions leading the agent out of the environment do not change the position but give reward = -1.
- All other actions give reward = 0, except movements from A and B.

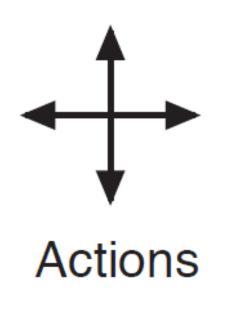




3.3	8.8	4.4	5.3	1.5
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-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

- All actions equal probability.
- Discount factor  $\gamma = 0.9$ .
- Negative values near the lower edge.
- The best state is A, but expected return is lower than 10, the immediate reward.
- B is valued more than 5, the immediate rewards.



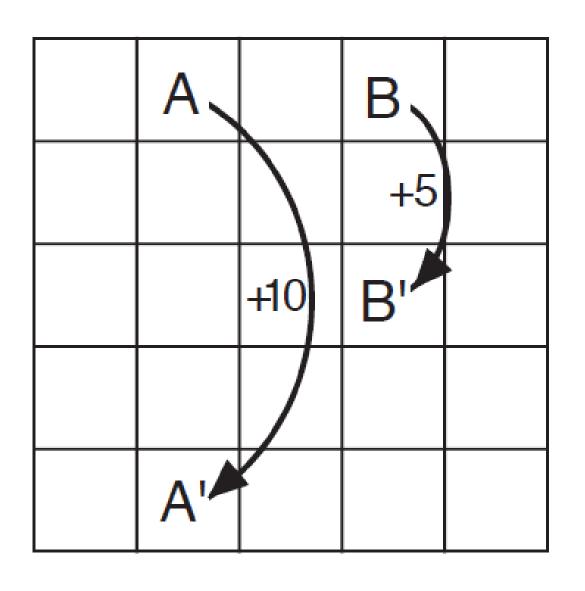


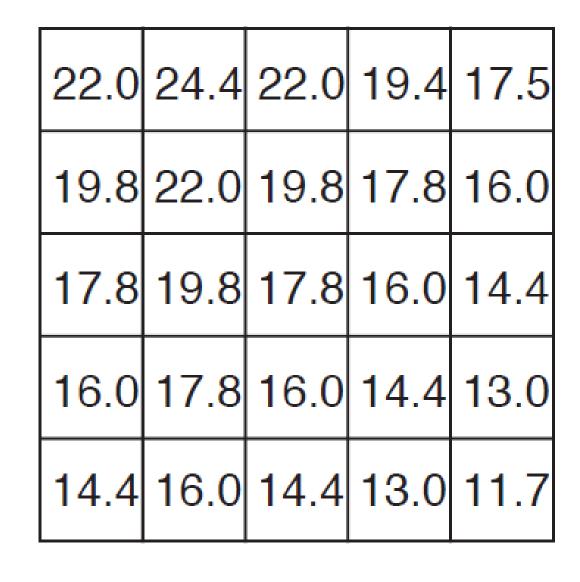
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-1.9	-1.3	-1.2	-1.4	-2.0

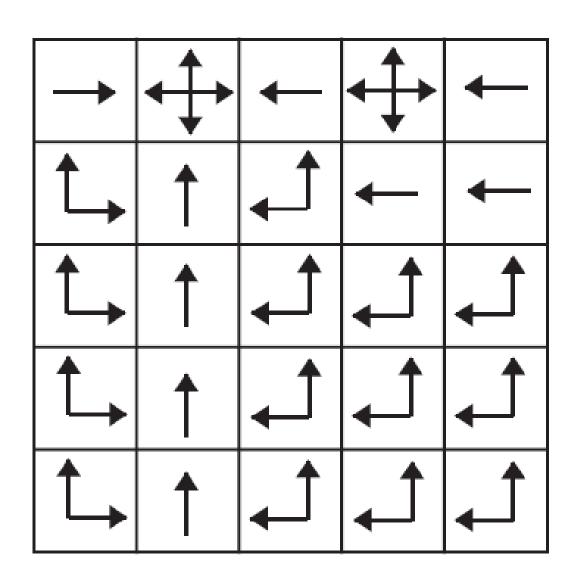
- The best state is A, but expected return is lower than 10, the immediate reward.
- B is valued more than 5, the immediate rewards.
- Why?



2 minutes







a) gridworld

b) 
$$v_*$$

c) 
$$\pi_*$$

Optimal value function and optimal policy for the grid world.

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## Temporal-difference (TD) prediction

- TD is one central and novel idea in RL.
- Monte Carlo-like methods must wait until the end of the episode to update  $V(S_t)$  (only at that point  $G_t$  is known):

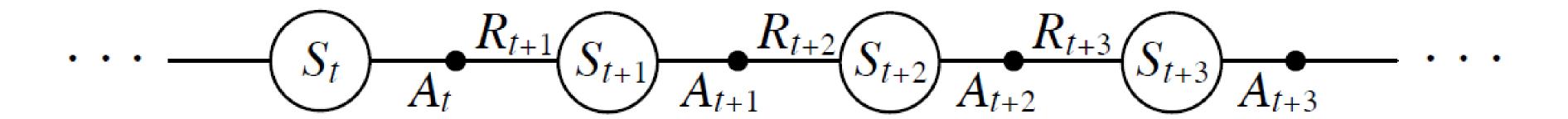
$$V(S_t) \leftarrow V(S_t) + \alpha \left[ G_t - V(S_t) \right]$$

• The simplest TD method is called TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha \left[ R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right]$$

## Temporal-difference (TD) prediction

- Approximations can be on-policy or off-policy.
- TD control learns an action-value function instead of a state-value function.
- We estimate  $q_{\pi}(s,a)$  for the current policy  $\pi$ .
- Therefore, we consider transitions from state-action pair to state-action pair.



### Sarsa: On-Policy TD Control

- Updates after each transition from a non-terminal S<sub>t</sub>.
- If  $S_{t+1}$  is terminal,  $Q(S_{t+1}, A_{t+1})$  is defined as zero.

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[ R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$$

- Each element of the 5-tuple ( $S_t$ ,  $A_t$ ,  $R_{t+1}$ ,  $S_{t+1}$ ,  $A_{t+1}$ ) is used, this gives the name to the algorithm.
- On-policy methods continuously estimate  $q_{\pi}$  for policy  $\pi$ , and at the same time change  $\pi$  greedily towards  $q_{\pi}$ .

### Sarsa: On-Policy TD Control

On-policy TD algorithm:

```
Initialize Q(s,a), \forall s \in S, a \in A(s), arbitrarily, and Q(terminal-state, \cdot) = 0
Repeat (for each episode):
   Initialize S
   Choose A from S using policy derived from Q (e.g., \varepsilon-greedy)
   Repeat (for each step of episode):
       Take action A, observe R, S'
       Choose A' from S' using policy derived from Q (e.g., \varepsilon-greedy)
      Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]
       S \leftarrow S' : A \leftarrow A' :
   until S is terminal
```

## Q-Learning: Off-Policy TD Control

- A simple but important breakthrough is an off-policy TD algorithm.
- The simplest way is one-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[ R_{t+1} + \gamma \max_{a} Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

- The learned action-value function Q directly approximates  $q^*$ , the optimal action-value function, regardless the followed policy  $\pi$ .
- The policy still has an effect in which state-action pairs are visited and updated.

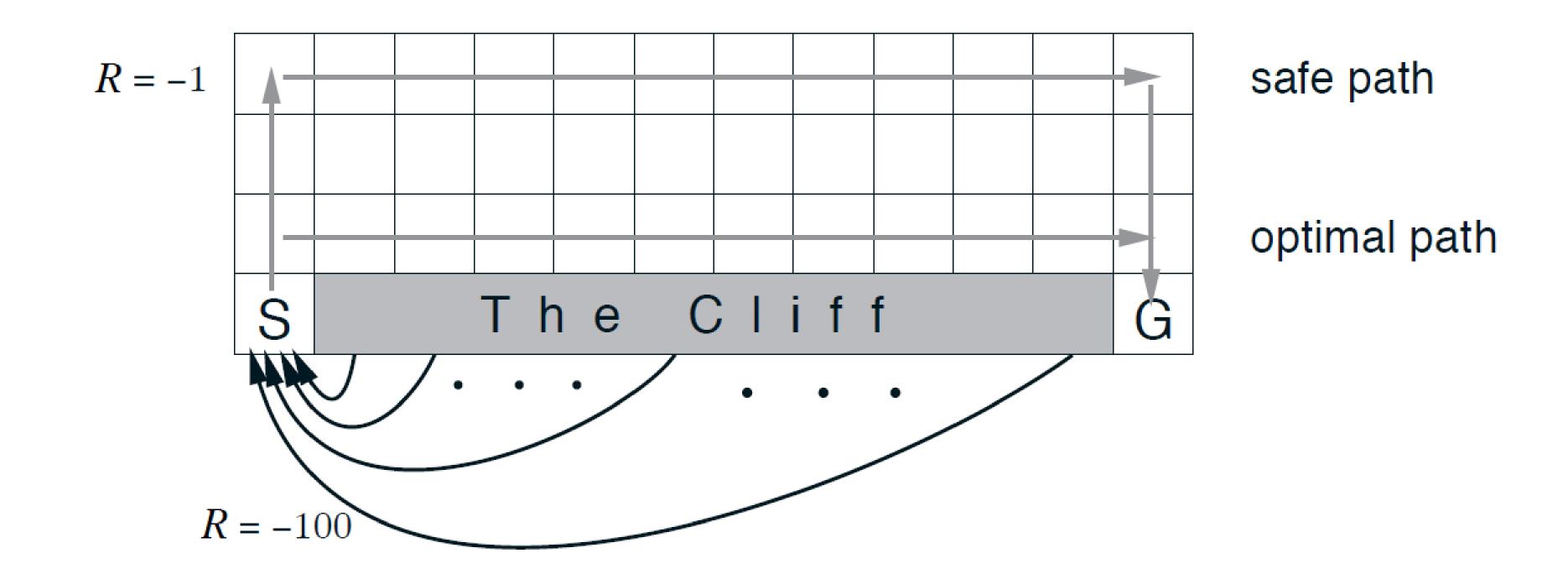
## Q-Learning: Off-Policy TD Control

Off-policy TD algorithm:

```
Initialize Q(s,a), \forall s \in S, a \in \mathcal{A}(s), arbitrarily, and Q(terminal\text{-}state, \cdot) = 0
Repeat (for each episode):
Initialize S
Repeat (for each step of episode):
Choose A from S using policy derived from Q (e.g., \varepsilon-greedy)
Take action A, observe R, S'
Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]
S \leftarrow S';
until S is terminal
```

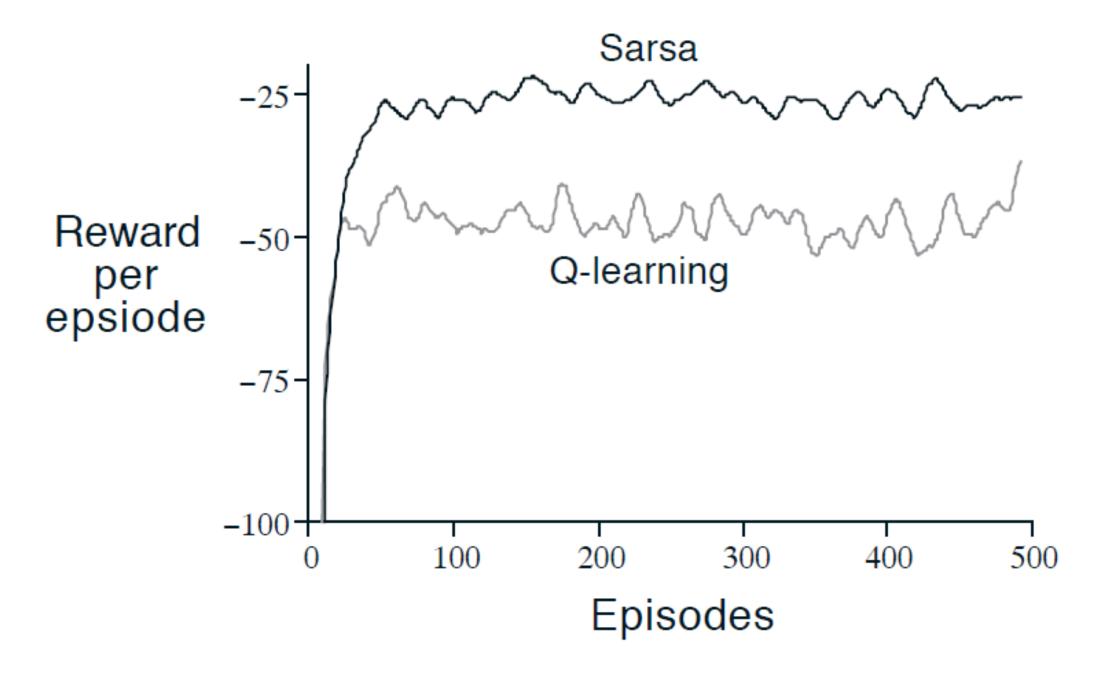
## The Cliff Walking

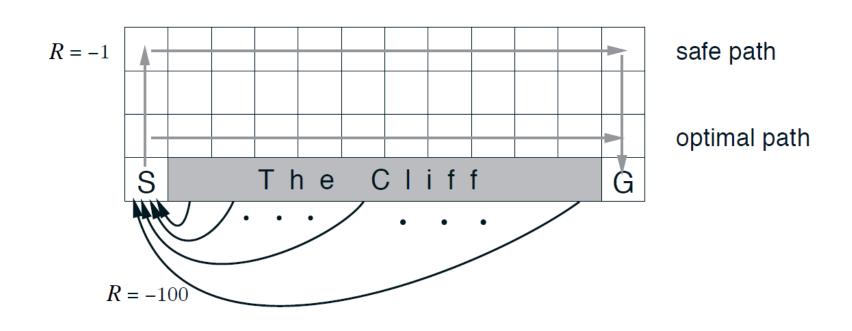
- Reward of -1 for all transitions, except in the cliff.
- The cliff gives a negative reward of -100 and sends the agent back to the start position.



### The Cliff Walking

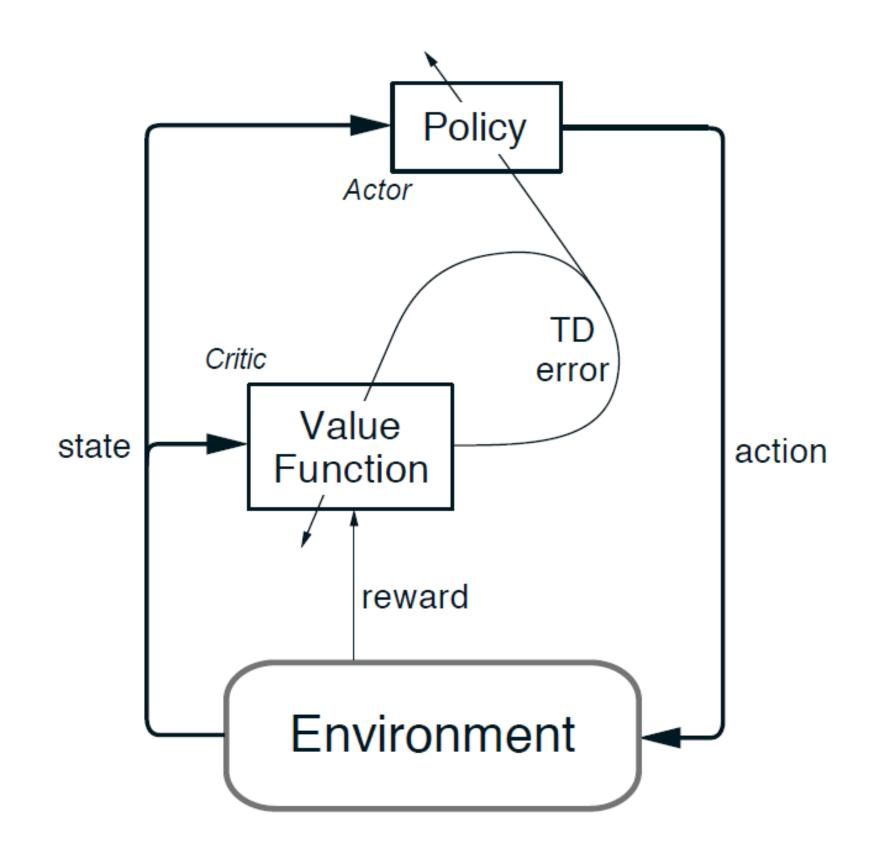
- $\epsilon$ -greedy, with  $\epsilon$ =0.1.
- Q-learning learns the optimal path. Sarsa learns the longest, safest path.
- However, overall Q-learning behaviour is worse.
- If ε is gradually reduced, both methods converge asymptotically to the optimal policy.





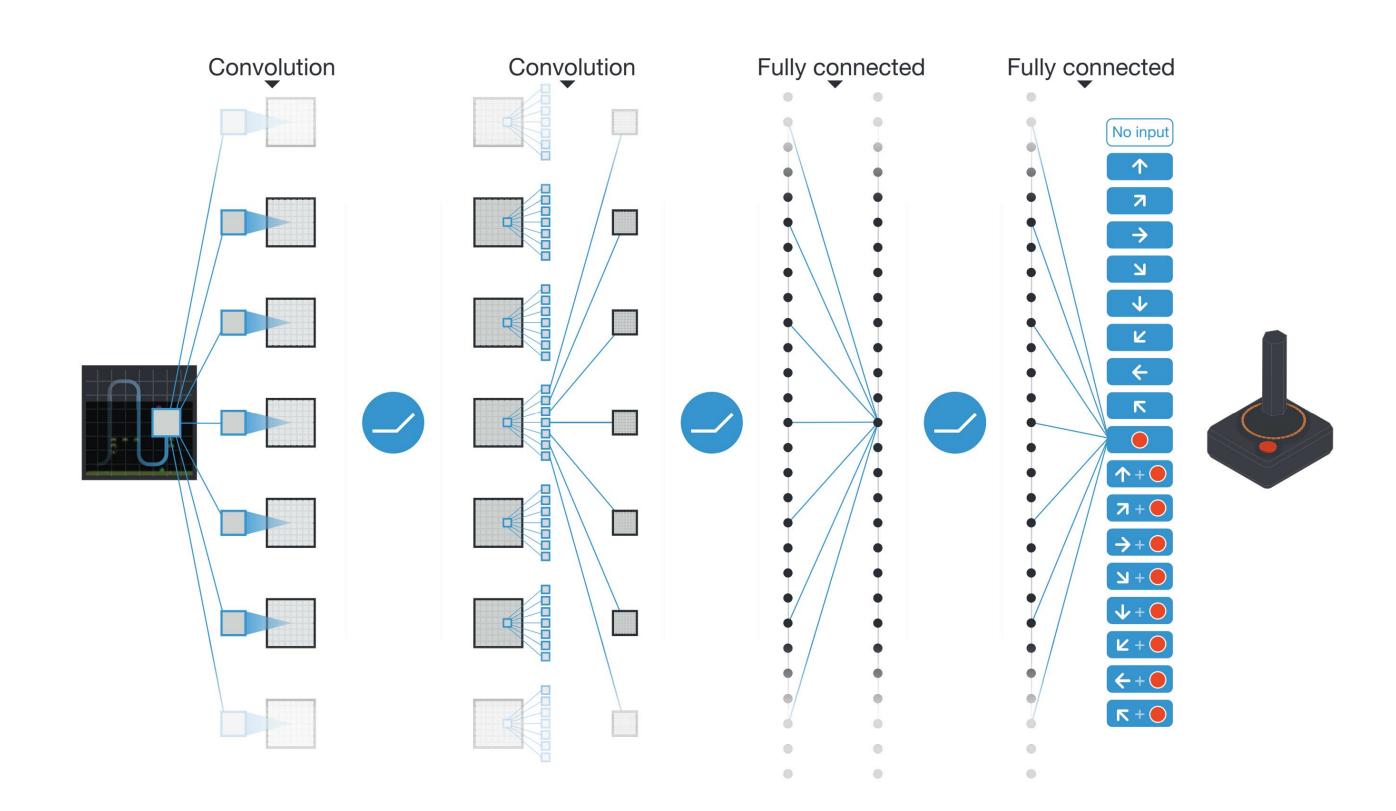
#### Actor-critic methods

- Policy approximation.
- Learning is always on-policy.
- The actor structures the policy.
- The critic must learn and critique the followed policy.
- Minimal computation to select actions, even in continuous-valued actions or for stochastic policies.
- The separate actor in actor-critic is more appealing as psychological and biological models.



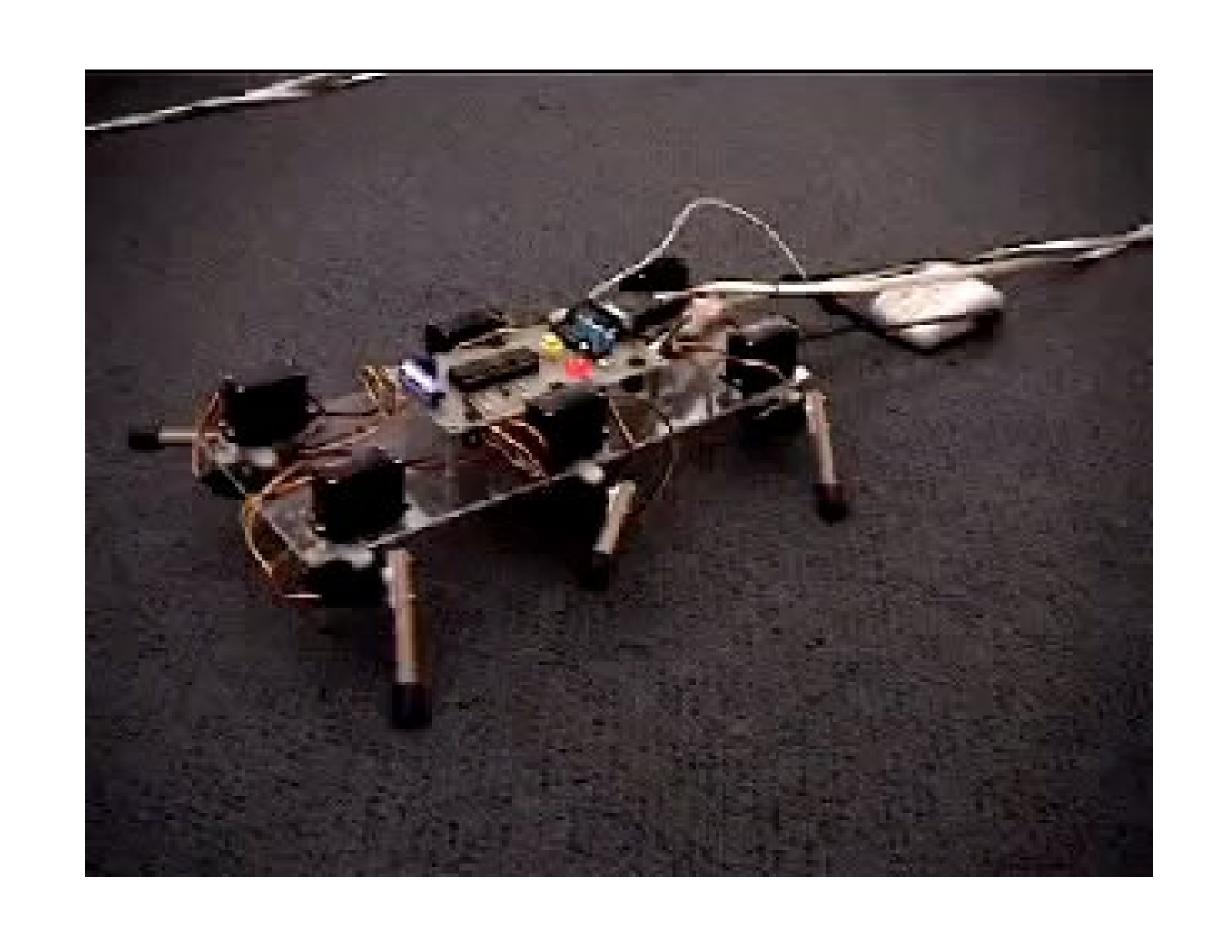
### Deep Q-Network

- Proposed by Mnih et al. in 2015.
- Human-level control through deep reinforcement learning.
- Tested in 49 Atari games.



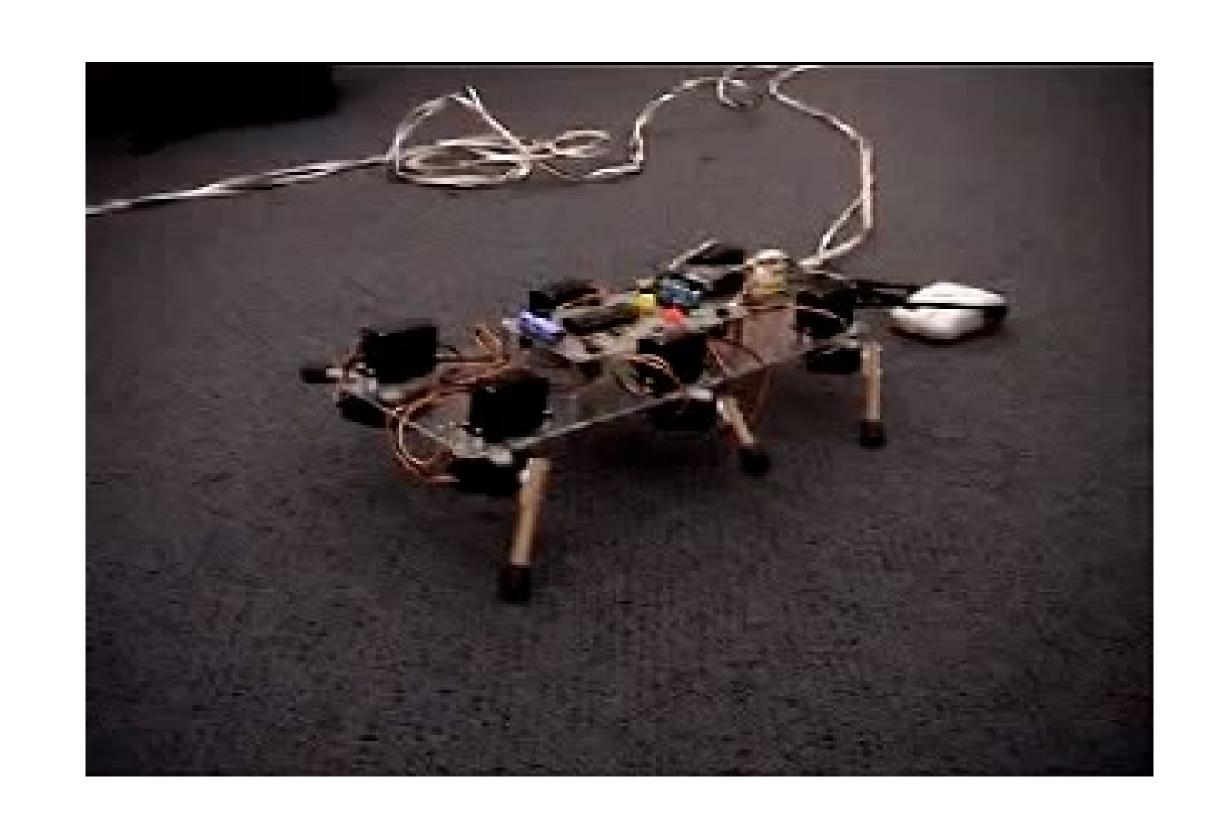
### Examples

- Stumpy A simple learning robot.
- Stumpy receives a *reward* after each action. Did it move forward or not?
- After each move, updates its policy.



### Examples

- Stumpy A simple learning robot.
- Continues trying to maximise its reward.
- Stumpy after 30 minutes.



### Examples

Another example



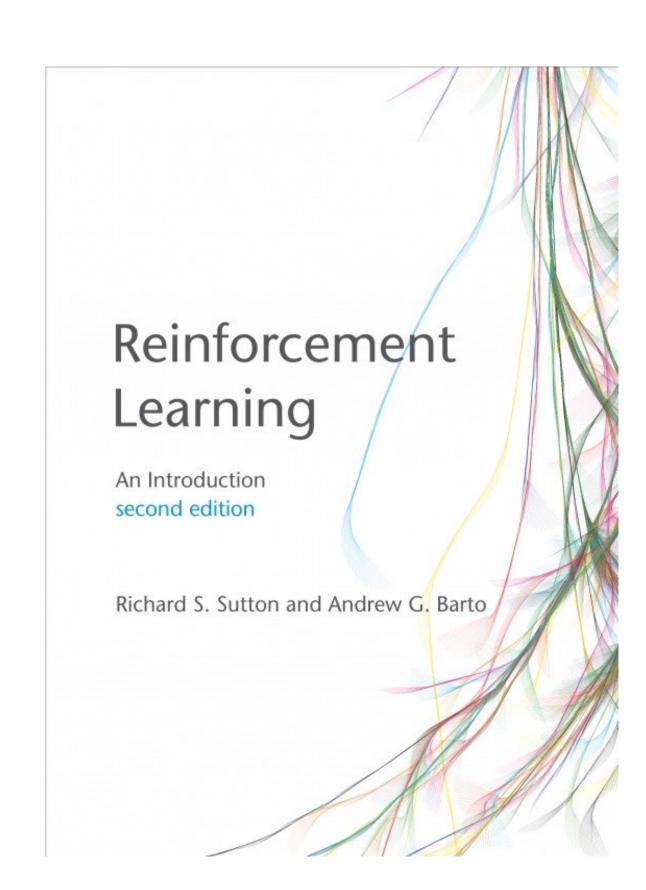
# Pepper Learning Bilboquet from Human Demonstration

SoftBank Robotics Europe Al Lab

Asya Grechka, Nikolas Hemion August 2016

#### Reference

- For a more comprehensive introduction, you should definitely have a look at:
  - Sutton, R. S., & Barto, A. G. (2018). *Reinforcement learning: An introduction*. MIT press.
  - <a href="http://www.incompleteideas.net/book/">http://www.incompleteideas.net/book/</a> the-book-2nd.html



#### Feedback

 In case you want to provide anonymous feedback on these lectures, please visit:

• https://forms.gle/KBkN744QuffuAZLF8

Muchas gracias!



