COMP 4418 – Exercise Sheet: Game Theory I

Exercise I: Iterated Dominance

Consider the following two games. Find all Pareto-optimal outcomes and decide whether the games can be solved by iterated strict dominance.

a)

	w	x	y	z
a	0, 0	3, 2	2, 1	1, 0
b	3, 1	0, 1	5, 1	3, 2
c	3, 2	2, 1	0, 5	0, 1
d	5, 1	1, 4	4, 0	0, 0

b)

	b_1	b_2
a_1	(2, 3, 2)	(0, 5, 2)
a_2	(1, 4, 1)	(2, 1, 1)
a_3	(1, 1, 1)	(5, 4, 2)

c_1

	b_1	b_2
a_1	(4, 5, 1)	(1, 0, 1)
a_2	(2, 0, 3)	(1, 5, 3)
a_3	(1, 2, 0)	(2, 2, 1)

c_2

Exercise II: Maximin Strategies and Security Levels

a) Consider the following formulation of rock-paper-scissors. What are the maximin strategies and the security levels of both players?

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

b) Model the situation with well as a fourth option that beats rock and scissor but loses again paper. What are the maximin strategies and the security levels of both players?

c) Assume there is lava as a fourth option. Lava beats all other option, but if both players play lava, they both experience a super lose with a utility of -100. What are the maximin strategies and the security levels of both players?

Exercise III: Independence

Assume that $A = \{a, b, c\}$ and let $\succsim$ denote a rational and independent preference relation on $\mathscr{L}(A)$ such that $[1 : a] \succ [1 : b]$ and $[\frac{1}{2} : b, \frac{1}{2} : c] \sim [\frac{2}{3} : a, \frac{1}{3} : c]$. Show the following statements.

a) $[1 : c] \succ [1 : a]$.

b) If $\succsim$ is additionally continuous, then it can be represented by the vNM utility function u given by $u(c) = 1$, $u(a) = \frac{1}{4}$, $u(b) = 0$.

Exercise Sheet IV: Preferences over Lotteries

Let $\succsim$ denote the rational preference relation over a set $A = \{x_1, \dots, x_m\}$ given by $x_1 \succ x_2 \succ \cdots \succ x_m$. Decide for each of the following relations on $\mathscr{L}(A)$ whether they are rational, continuous, and transitive. Prove your answers!

a) The relation $\succsim_1$ is defined by $L_1 \succsim_1 L_2$ if and only if $x \succsim y$ for all $x, y \in A$ with $L_1(x) > 0$ and $L_2(y) > 0$.

b) We define $\max(\succsim, X)$ as the most preferred alternative in X with respect to $\succsim$ and $\Delta(L_1, L_2) = \max(\succsim, \{x \in A \colon L_1(x) \neq L_2(x)\})$.
The relation $\succsim_2$ is defined by $L_1 \succsim_2 L_2$ if and only if $L_1 = L_2$ or $L_1(\Delta(L_1, L_2)) \geq L_2(\Delta(L_1, L2))$.

c) The relation $\succsim_3$ is defined by $L_1 \succsim_3 L_2$ if and only if $L_1(x_1) \geq L_2(x_1)$.