

# Properties of breadth-first search

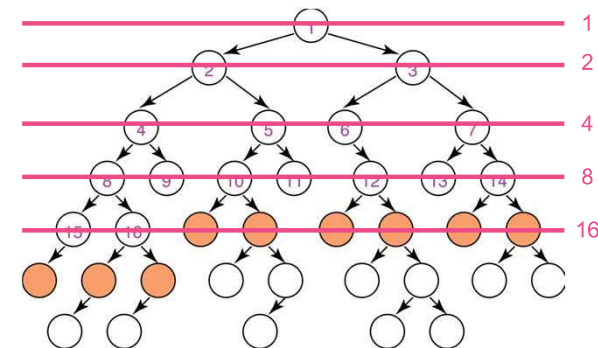
**Complete?** Yes (if breadth,  $b$ , is finite, the shallowest goal is at a fixed depth,  $d$ , and will be found before any deeper nodes are generated)

**Time?**  $1 + b^1 + b^2 + b^3 + \dots + b^d = \frac{b^{d+1} - 1}{b - 1} = O(b^d)$

**Space?**  $O(b^d)$  keeps every node in memory; generate all nodes up to level  $d$  )

**Optimal?** Yes, but only if all actions have the same cost

**Space** is the big problem for BFS. It grows **exponentially** with depth



# Tree-Search DFS vs. Graph-Search DFS

## Tree-Search DFS

- We covered Tree-Search DFS in this course
- Can **be used** for iterative deepening dfs
- Checks new states against those on the **path** from the **root** to the **current node** to avoid infinite loops in finite state space.
- Does not avoid the proliferation of redundant paths.
- Both are non-optimal
- The space complexity can be  $O(bm)$

## Graph-Search DFS

- You probably covered Graph-Search in COMP2521
- Can **not** be used for iterative deepening dfs!
- Keeps a record of every visited, expanded node
- Both are non-optimal
- The space complexity is as bad as BFS:  $O(b^m)$

# Properties of Iterative Deepening Search

Complete? Yes.

Time:  $O(b^d)$

Space?  $O(bd)$

Optimal? Yes, if step costs are identical.

In general, iterative deepening is the preferred search strategy for a large search space where depth of solution is not known

# Properties of Uniform-Cost Search

**Complete?** Yes, if  $b$  is finite and if transition  $cost \geq \epsilon$  with  $\epsilon > 0$

**Time?** Worst case,  $O(b^{\lceil C^*/\epsilon \rceil})$  where  $C^*$  = cost of the optimal solution  
every transition costs at least  $\epsilon$   
 $\therefore$  cost per step is  $\frac{C^*}{\epsilon}$

**Space?**  $O(b^{\lceil C^*/\epsilon \rceil})$ ,  $b^{\lceil C^*/\epsilon \rceil} = b^d$  if all step costs are equal

**Optimal?** Yes – nodes expanded in increasing order of lower path cost,  $g(n)$

# Complexity Results for Uninformed Search

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Time	$O(b^d)$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^k)$	$O(b^d)$
Space	$O(b^d)$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bk)$	$O(bd)$
Complete?	Yes <sup>1</sup>	Yes <sup>2</sup>	No	No <sup>4</sup>	Yes <sup>1</sup>
Optimal ?	Yes <sup>3</sup>	Yes	No	No	Yes <sup>3</sup>

$b$  = branching factor,  $d$  = depth of the shallowest solution,  
 $m$  = maximum depth of the search tree,  $k$  = depth limit.

1 = complete if  $b$  is finite.

2 = complete if  $b$  is finite and step costs  $\geq \epsilon$  with  $\epsilon > 0$ .

3 = optimal if actions all have the same cost.

4 = incomplete if the goal is not within the depth bound

# Properties of Greedy Best-First Search

**Complete:** No. Can get stuck in loops.  
(Complete in finite space with repeated-state checking)

**Time:**  $O(b^m)$ , where  $m$  is the maximum depth in search space.

**Space:**  $O(b^m)$  (retains all nodes in memory)

**Optimal:** No.

Greedy Search is not efficient

However, a good heuristic can reduce time and memory costs substantially.

# Optimality of A\*

- Heuristic  $h$  is said to be **admissible** if

$\forall n, h(n) \leq h^*(n)$  where  $h^*(n)$  is the true cost from  $n$  to goal

- If  $h$  is **admissible** then  $f(n)$  never overestimates the actual cost of the best solution through  $n$ .

$f(n) = g(n) + h(n)$ , where  $g(n)$  is the actual cost to  $n$  and  $h(n)$  is an underestimate

- Example:  $h = \text{straight line distance}$  is admissible because the shortest path between any two points is a line.
- A\* is optimal if  $h$  is **admissible**.
- Admissible heuristics are by nature optimistic because they think the cost of solving the problem is less than it actually is.

# Optimality of A\* Search

**Complete:** Yes, unless infinitely many nodes with  $f \leq \text{cost of solution}$

**Time:** Exponential in *relative error in  $h \times \text{length of solution}$*

**Space:** Keeps all expanded nodes in memory

**Optimal:** Yes (assuming  $h$  is admissible).