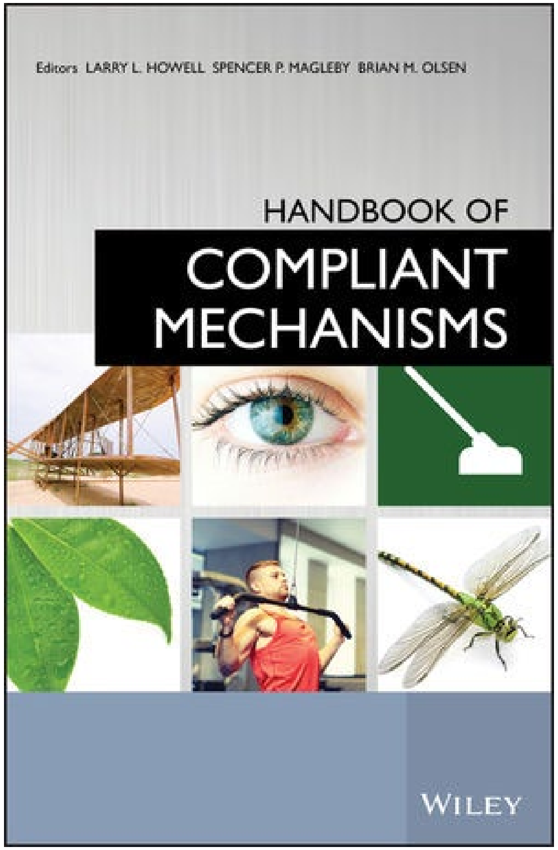
MECHENG 5751 – Design and Manufacturing of Compliant Mechanisms and Robots

CM L2: Pseudo Rigid Body Models

Reading: Appendix of Ch5, Handbook of Compliant Mechanisms
(digital book available in OSU library)

https://onlinelibrary.wiley.com/doi/epub/10.1002/9781118516485



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Compliant Mechanism Design Process

- Max deflection occurs when max stress equals strength (S), or

$$\delta_{\max} = \frac{2SL^2}{3Eh} = \frac{2}{3}\frac{S}{E}\frac{L^2}{h}$$

- Deflection depends on
  - Material properties (S/E)
  - Geometry (L^2/h)
  - Boundary conditions (2/3)

- Compliant mechanism design process
  - Material selection (consider fabrication process)
  - Design of geometric dimensions
  - Evaluation: design analysis, FEA simulation

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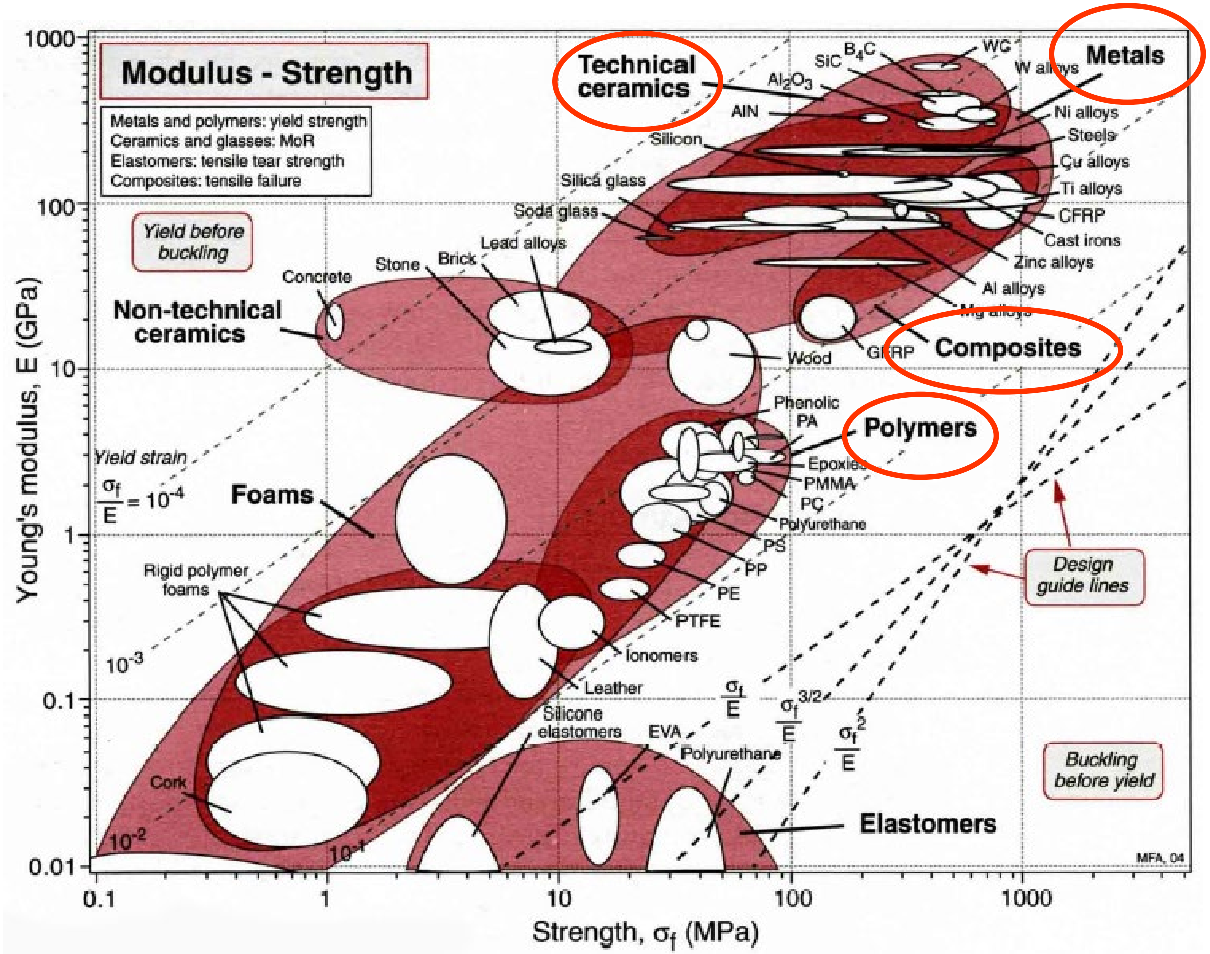
Material Selection Chart (E vs. S)



Source: *Materials Selection in Mechanical Design (2004) by Ashby*
Software: *CES Material & Process Selectors*

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Ratios for Several Materials

Material	E[Mpsi (GPa)]		S_y[kpsi (MPa)]		(S_y/E) x 1000
Steel (1010 hot rolled)	30	(207)	26	(179)	0.87
Steel (4140 Q&T@400)	30	(207)	238	(1641)	7.9
Aluminum (1100 annealed)	10.4	(71.7)	5	(34)	0.48
Aluminum (7075 heat treated)	10.4	(71.7)	73	(503)	7.0
Titanium (Ti-35A annealed)	16.5	(114)	30	(207)	1.8
Titanium (Ti-13 heat treated)	16.5	(114)	170	(1170)	10
Beryllium copper (CA170)	18.5	(128)	170	(1170)	9.2
Polycrystalline silicon	24.5	(169)	135	(930)	5.5
Polyethylene (HDPE)	0.2	(1.4)	4	(28)	20
Nylon (type 66)	0.4	(2.8)	8	(55)	20
Polypropylene	0.2	(1.4)	5	(34)	25
Kevlar (82 vol %) in epoxy	12	(86)	220	(1517)	18
E-glass (73.3 vol %) in epoxy	8.1	(56)	238	(1640)	29

Source: *Compliant Mechanisms*

Linear vs. Nonlinear Deflections

- Deflection analysis assumptions
  - Deflection is small compared to the dimensions of the structure
  - Material is elastic: Hooke's law $\sigma = E\varepsilon$ (E: Young's modulus)
- Linear Deflection
  - Deflection is small enough so that linear beam equations are sufficient
- Structural nonlinearities
  - Material nonlinearity: Hooke's law does not apply
    - Plasticity, nonlinear elasticity, hyperelasticity and creep
  - Geometric nonlinearity (this class)
    - Large deflections with a small strain
    - stress stiffening, large strains.

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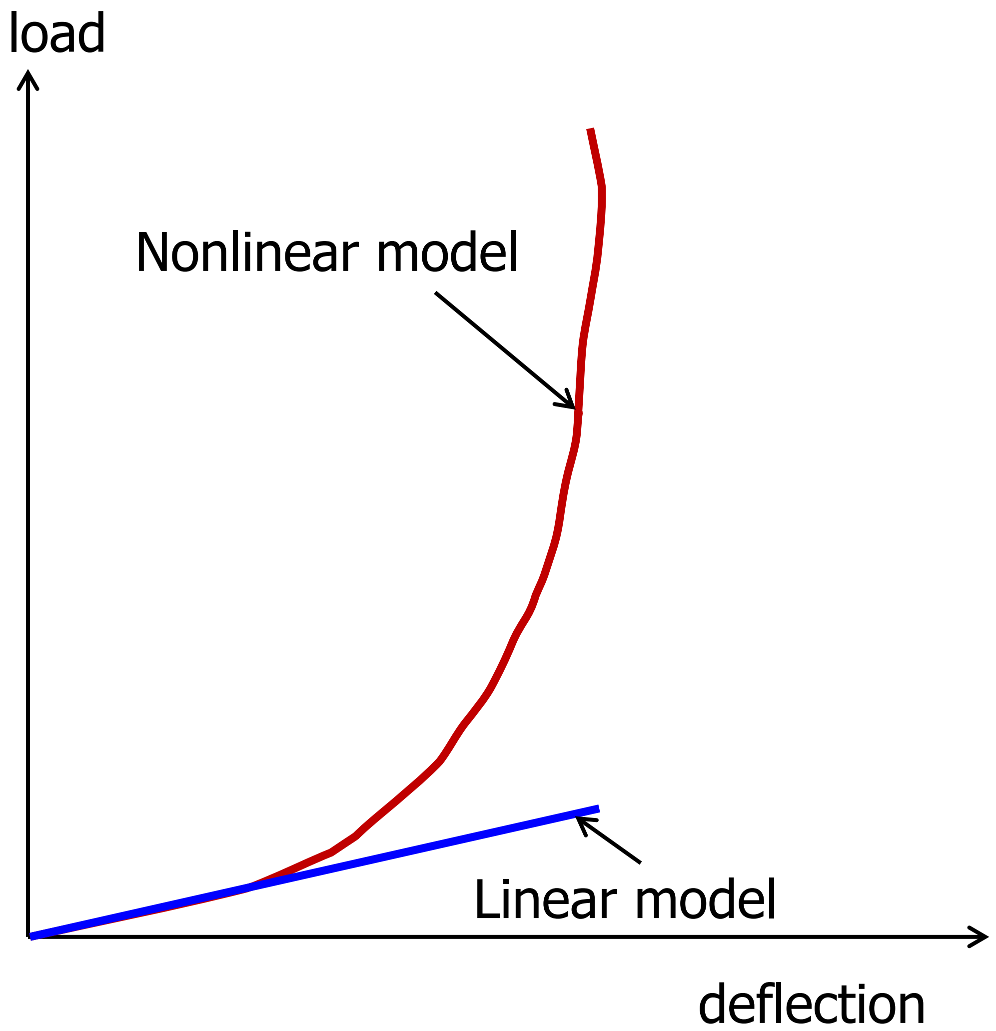
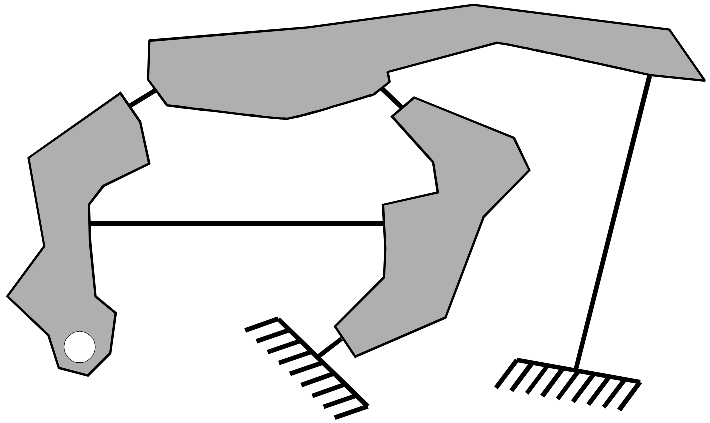
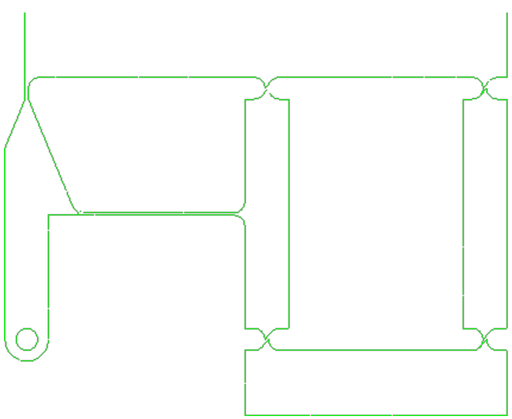
Small vs. Large Deflection Formulation

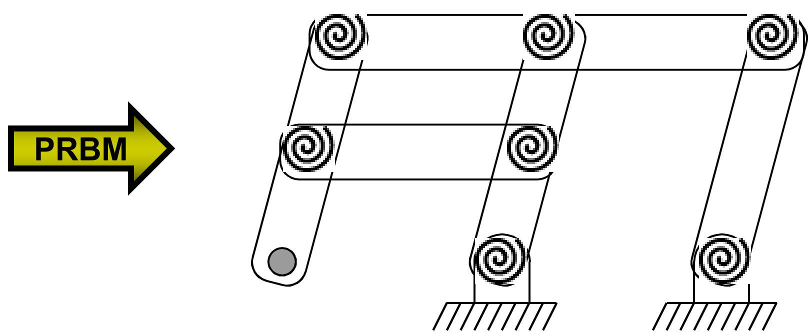


What is Pseudo-Rigid-Body Model?



Compliant Mechanism



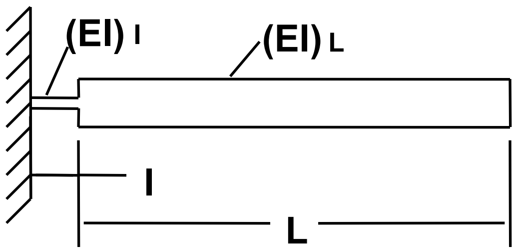


The pseudo-rigid-body model

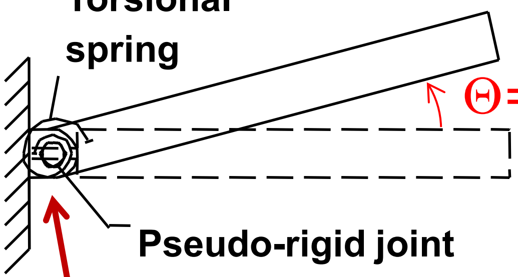
Why Pseudo-Rigid-Body Model?

- Solving differential equations or using elliptical integral or using FEA are not appropriate for design
  - Too complicated, depending on boundary/loading conditions
  - Solution process too slow.
  - Not intuitive to designers.
- Models compliant mechanisms as rigid-body mechanisms
  - Allows use of decades of research in mechanical systems
  - Unifies compliant mechanism and rigid-body mechanism theories

PRBM 1: Small-Length Flexural Pivot



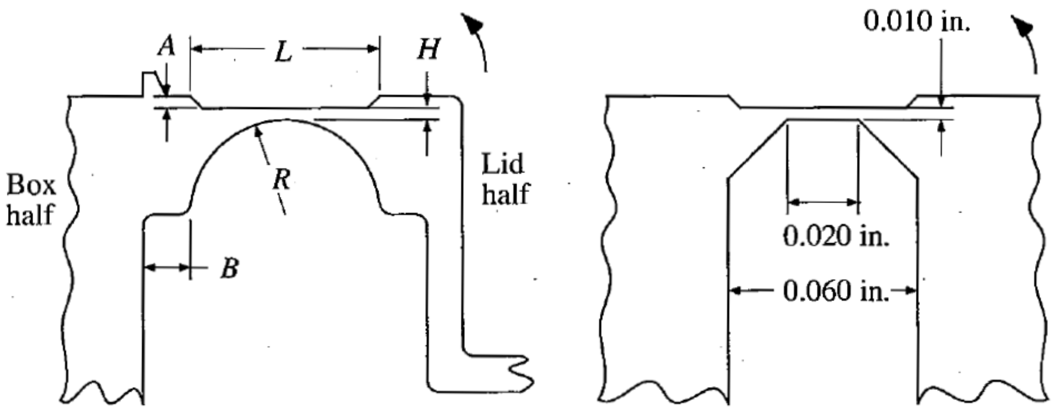
l<<L



$$K = \frac{(EI)_l}{l}$$

Design Practice

- Living hinge: extremely short and thin small-length flexural pivots
- PRBM is a pin joint at the center of the flexible segment.
- If other compliant elements are present, then can ignore spring for living hinge



$0.060 \text{ in.} \leq L \leq 0.090 \text{ in.}$
$0.008 \text{ in.} \leq H \leq 0.020 \text{ in.}$
$A \approx 0.010 \text{ in.}$
$0.030 \text{ in.} \leq B$

Figure 5.37. Two common types of living hinge geometries used in container lids.

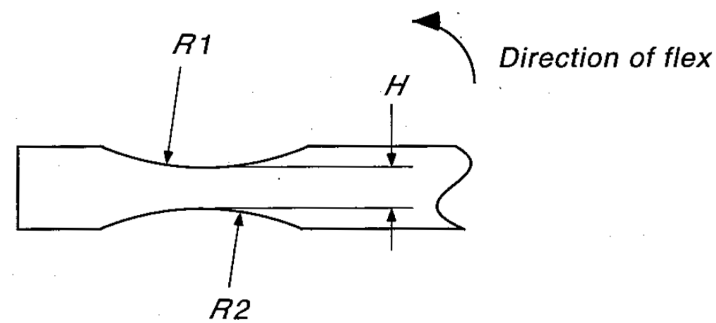
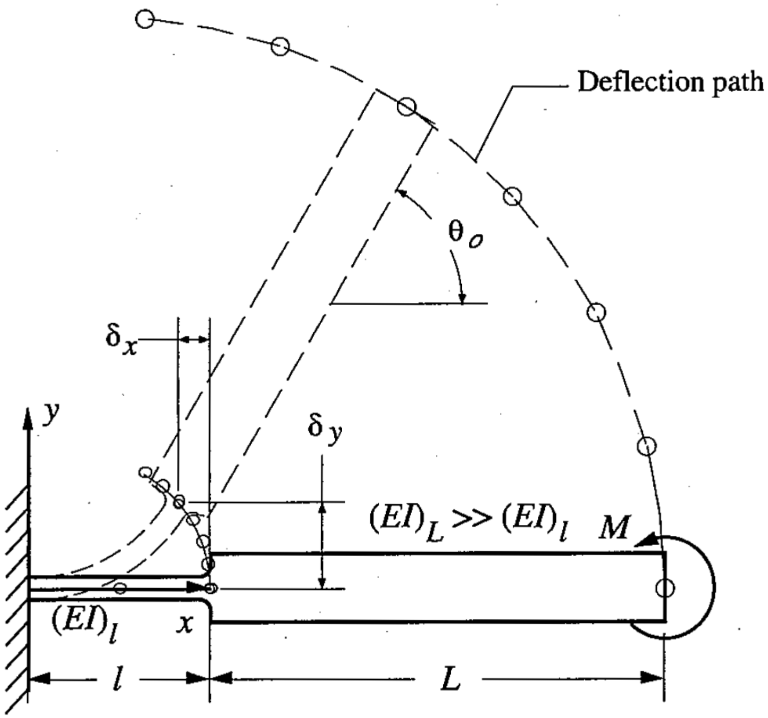
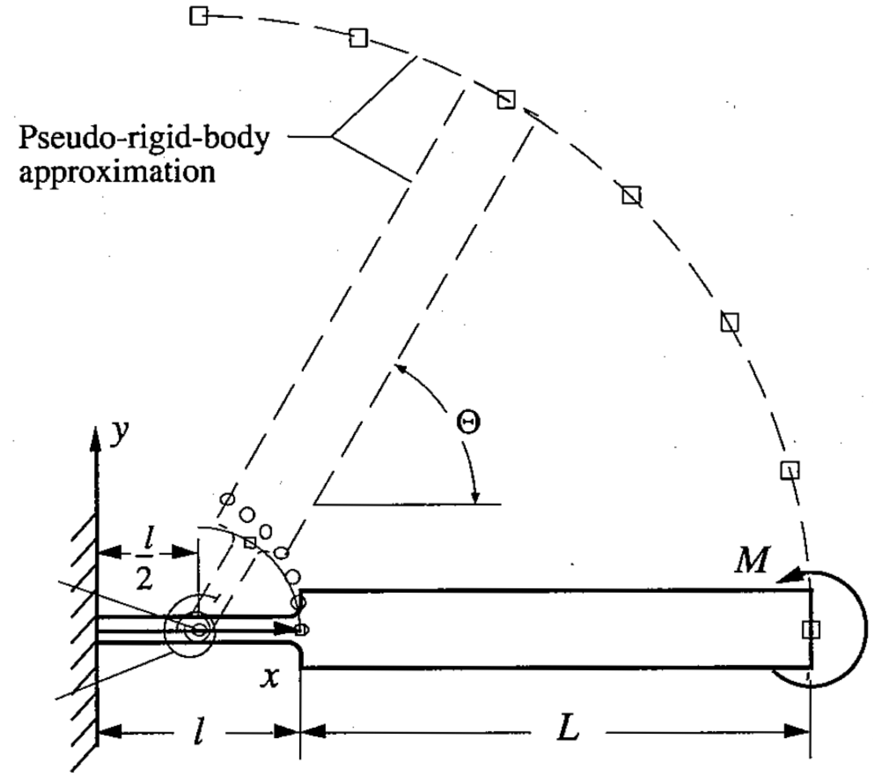


Figure 5.38. Hinge that requires less than 180° of flex.

Kinematics and Statics of Small-Length Flexural Pivots





$$\theta_o = \frac{Ml}{EI}$$

$$\delta_y = \frac{(1-\cos\theta_o)l}{\theta_o}$$

$$\delta_x = \left(1 - \frac{\sin\theta_o}{\theta_o}\right)l$$

$$\Theta = \theta_o = \frac{Ml}{EI},$$

$$a = \frac{l}{2} + \left(L + \frac{l}{2}\right)\cos\Theta$$

$$b = \left(L + \frac{l}{2}\right)\sin\Theta$$

$$M = K\Theta = \frac{EI}{l}\Theta$$

Approximation Error of Small Length Pivot PRBM

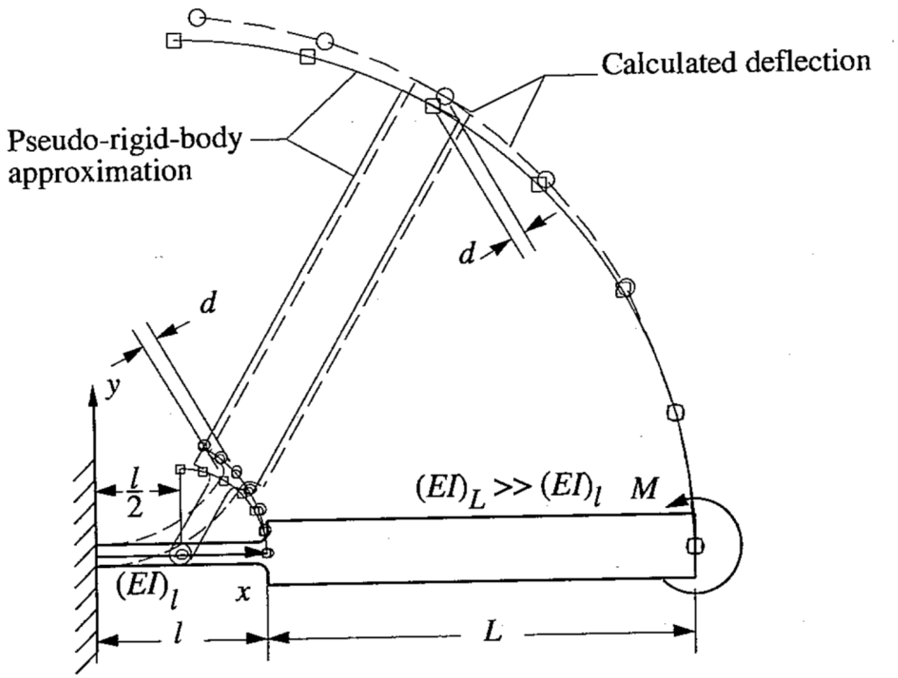


Figure 5.2. Error associated with the small-length flexural pivot approximation.

Actual

$$\theta_o = \frac{Ml}{EI}$$

$$a = l - \delta_x + L\cos\theta_o = \frac{(1-\cos\theta_o)l}{\theta_o} + L\cos\theta_o$$

$$b = \delta_y + L\sin\theta_o = \left(1 - \frac{\sin\theta_o}{\theta_o}\right)l + L\sin\theta_o$$

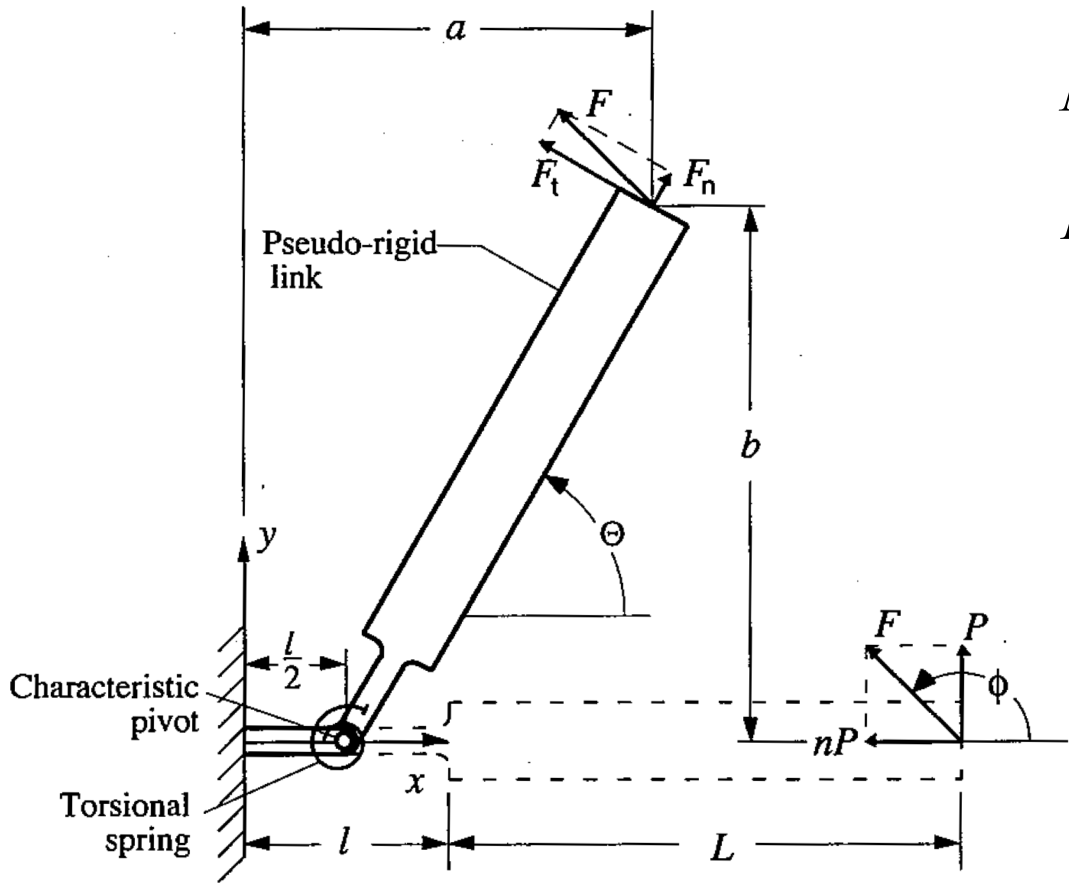
PRBM

$$\Theta = \theta_o = \frac{Ml}{EI},$$

$$a = \frac{l}{2} + \left(L + \frac{l}{2}\right)\cos\Theta$$

$$b = \left(L + \frac{l}{2}\right)\sin\Theta$$

Nonfollower End Force

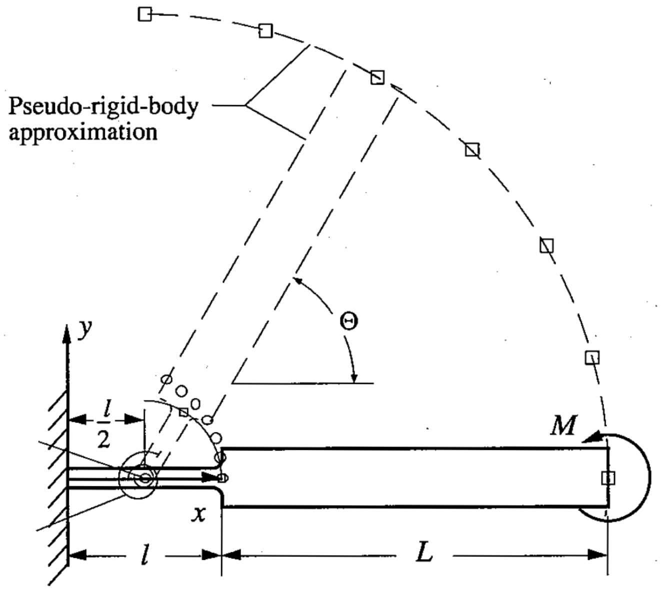


$$M = F_t\left(L + \frac{l}{2}\right)$$

$$F_t = F\sin(\phi - \Theta)$$

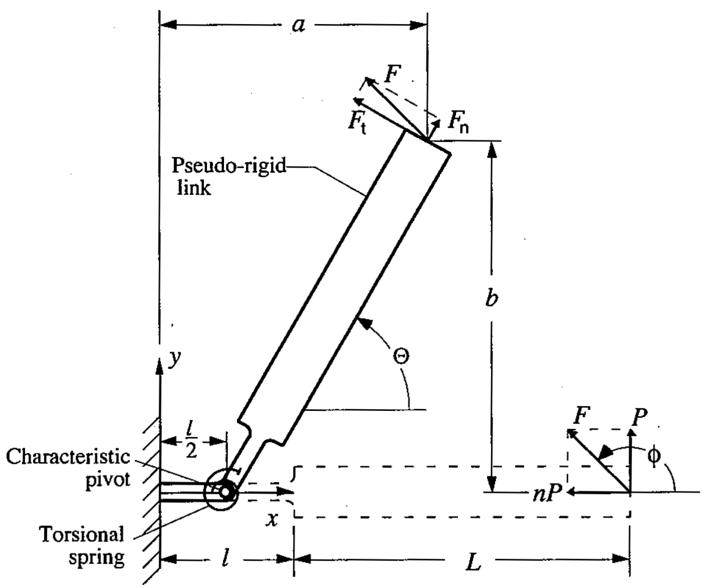
Figure 5.4. Pseudo-rigid-body model of a beam with a small-length flexural pivot and a force at the free end.

Stress



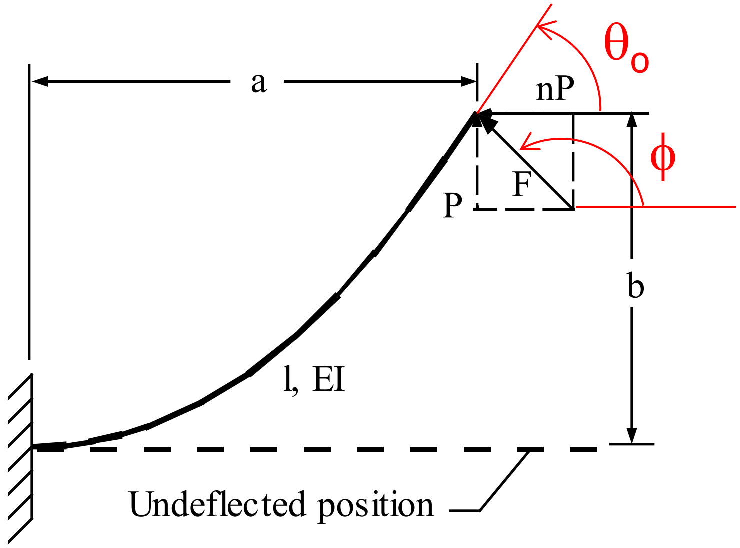
$$\delta_{\max} = \frac{M_{\max}(h/2)}{I}$$

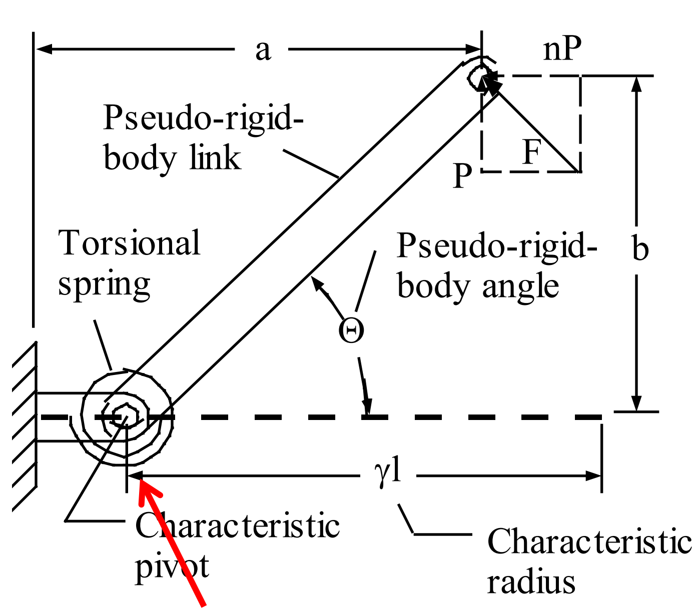
$$M_{\max} = M$$



$$M_{\max} = Pa + nPb \approx F_t(L + l/2)$$

PRBM 2: Fixed-Pinned Beams





$$K = \gamma K_\Theta \frac{EI}{l}$$

Kinematics and Statics

$\theta_o = c_\theta \Theta$ Beam tip angle

$a = l\left[1-\gamma(1-\cos\Theta)\right]$ Horizontal tip coordinate

$b = \gamma l \sin\Theta$ vertical tip coordinate

$\sigma_{\max} = \dfrac{M_{\max}(h/2)}{I}$ Maximum stress

γ, c_Θ K_Θ depends on force angle ϕ
See Table E.1 for values of γ, c_Θ K_Θ

PRBM Parameters for Fixed-Pinned Beams

Most common case

n=cot(ϕ) is the ratio the buckling force (nP) over the lateral force (P).

TABLE E.1. Numerical data for γ, c_θ, and K_Θ for various angles of force

n	ϕ	γ	$\Theta_{max}(\gamma)$	c_θ	K_Θ	$\Theta_{max}(K_\Theta)$
0.0	90.0	0.8517	64.3	1.2385	2.67617	58.5
0.5	116.6	0.8430	81.8	1.2430	2.63744	64.1
1.0	135.0	0.8360	94.8	1.2467	2.61259	67.5
1.5	146.3	0.8311	103.8	1.2492	2.59289	65.8
2.0	153.4	0.8276	108.9	1.2511	2.59707	69.0
3.0	161.6	0.8232	115.4	1.2534	2.56737	64.6
4.0	166.0	0.8207	119.1	1.2548	2.56506	66.4
5.0	168.7	0.8192	121.4	1.2557	2.56251	67.5
7.5	172.4	0.8168	124.5	1.2570	2.55984	69.0
10.0	174.3	0.8156	126.1	1.2578	2.56597	69.7
-0.5	63.4	0.8612	47.7	1.2348	2.69320	44.4
-1.0	45.0	0.8707	36.3	1.2323	2.72816	31.5
-1.5	33.7	0.8796	28.7	1.2322	2.78081	23.6
-2.0	26.6	0.8813	23.2	1.2293	2.80162	18.6
-3.0	18.4	0.8669	16.0	1.2119	2.68893	12.9
-4.0	14.0	0.8522	11.9	1.1971	2.58991	9.8
-5.0	11.3	0.8391	9.7	1.1788	2.49874	7.9

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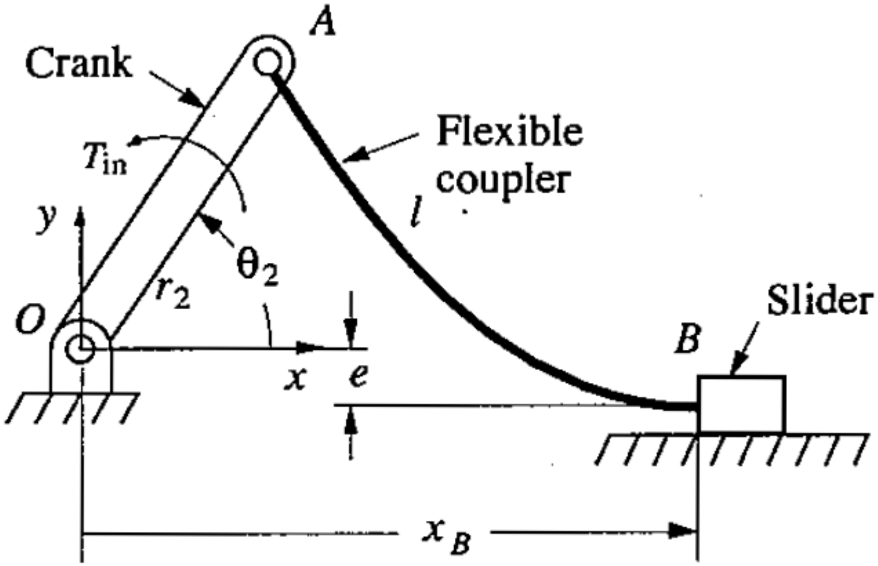
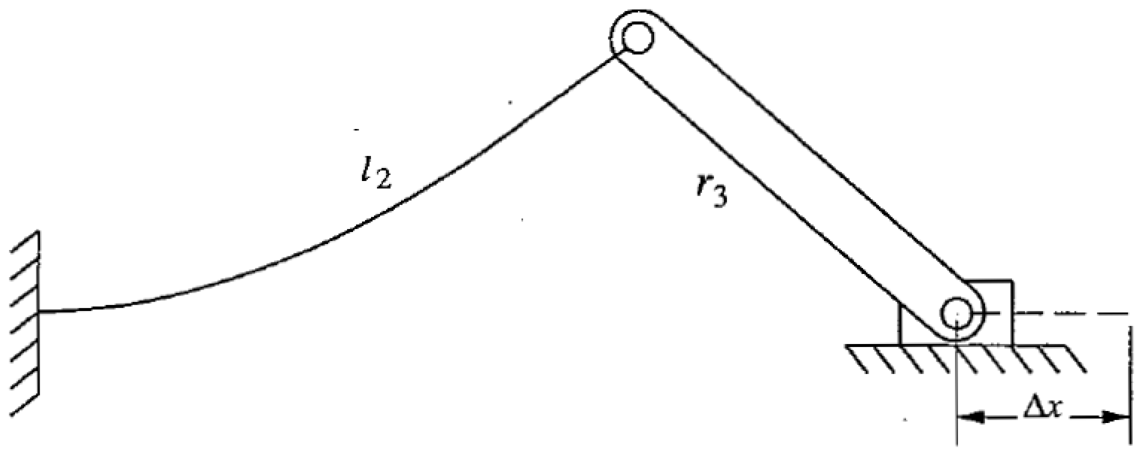
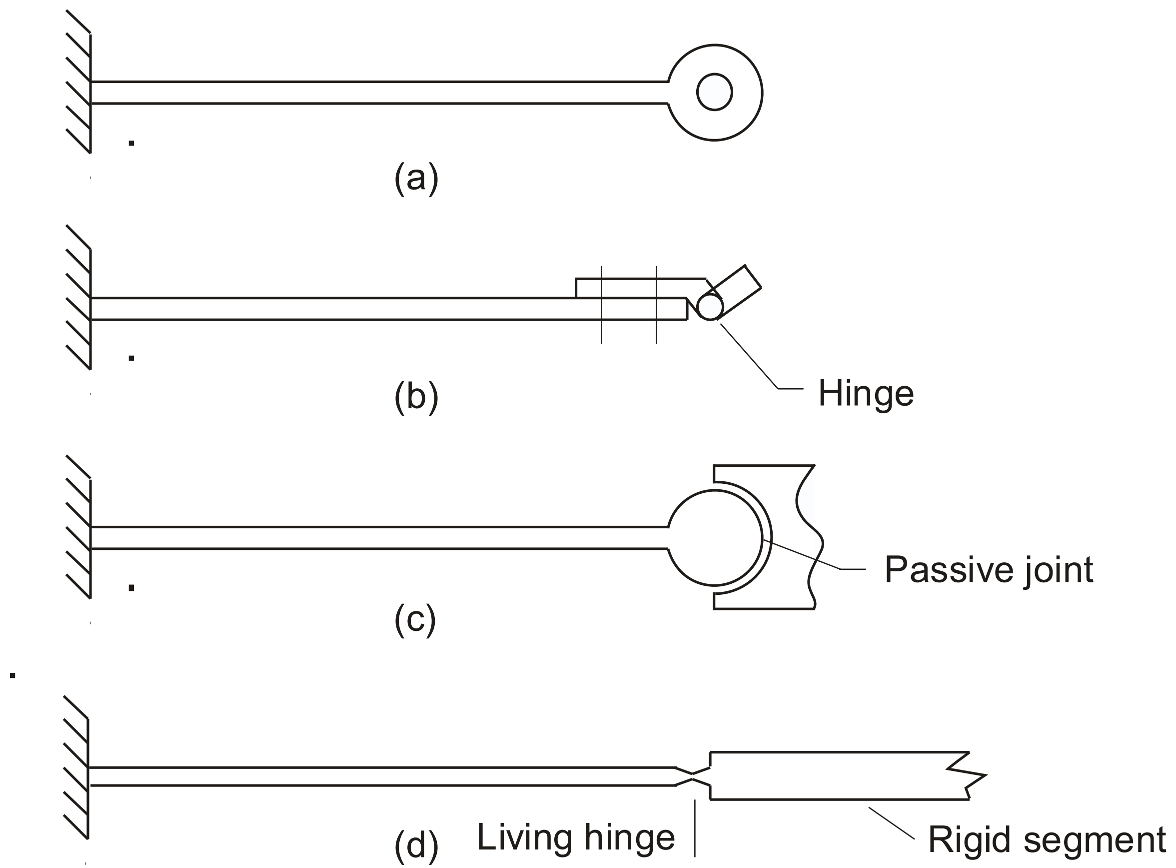
Fixed-Pinned Beam Examples



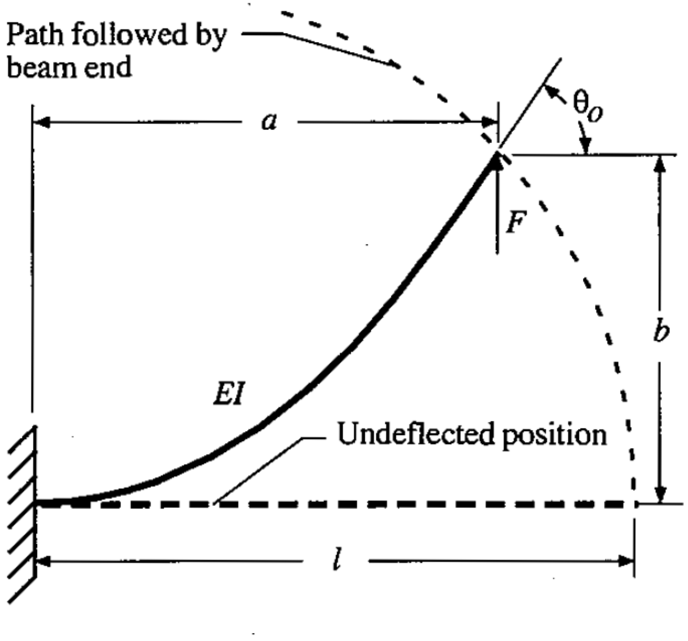
(a)

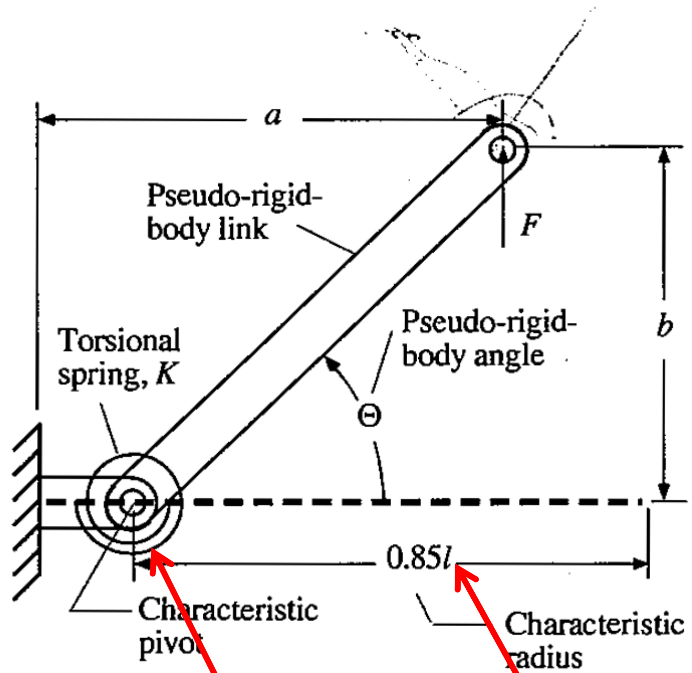


Practical Implementation of Fixed-Pinned Beams



Special Case: Fixed-Pinned Beams with Vertical Forces (ϕ=90°)





$$K = 2.25\frac{EI}{l} \qquad \gamma = 0.85$$

Kinematics and Statics

$$\theta_o = 1.24\Theta$$

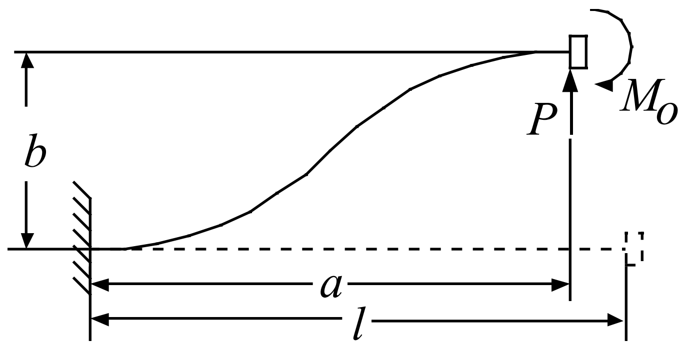
$$a = l[1-\gamma(1-\cos\Theta)]$$

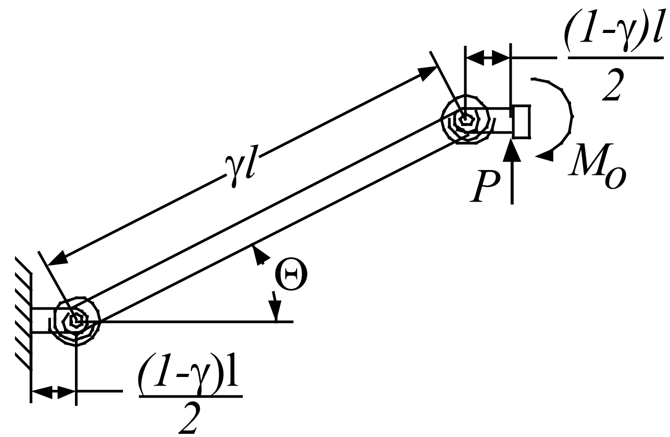
$$b = \gamma l \sin\Theta$$

$$\sigma_{max} = \frac{Pa(h/2)}{I}$$

Use n=0 or ϕ**=90°** from the general case. See Table E.1.

PRBM 3: Fixed-Guided





$$\theta_o = 0$$

$$a = l[1 - \gamma(1 - \cos\Theta)]$$

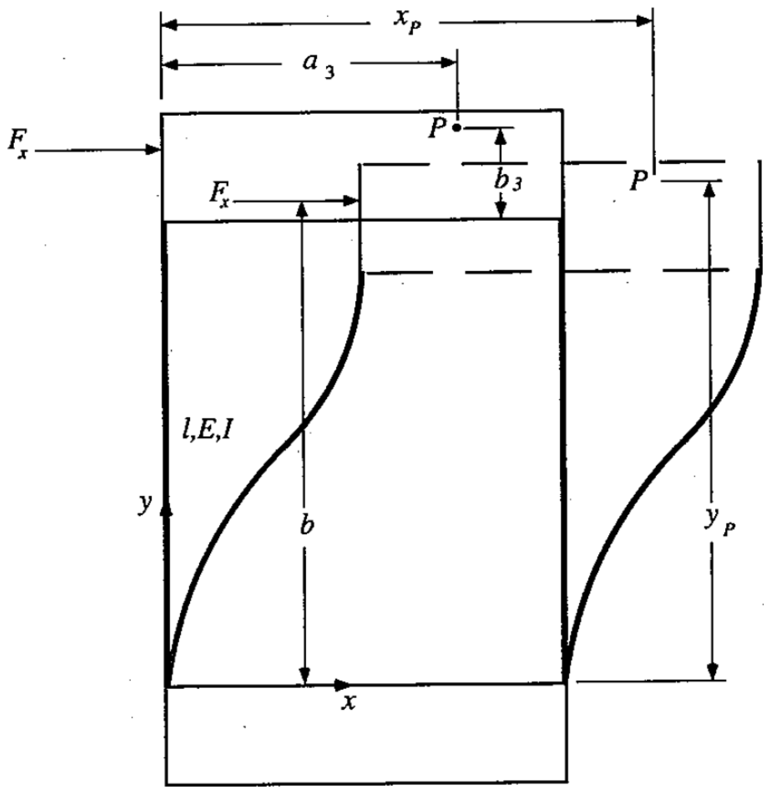
$$b = \gamma l \sin\Theta$$

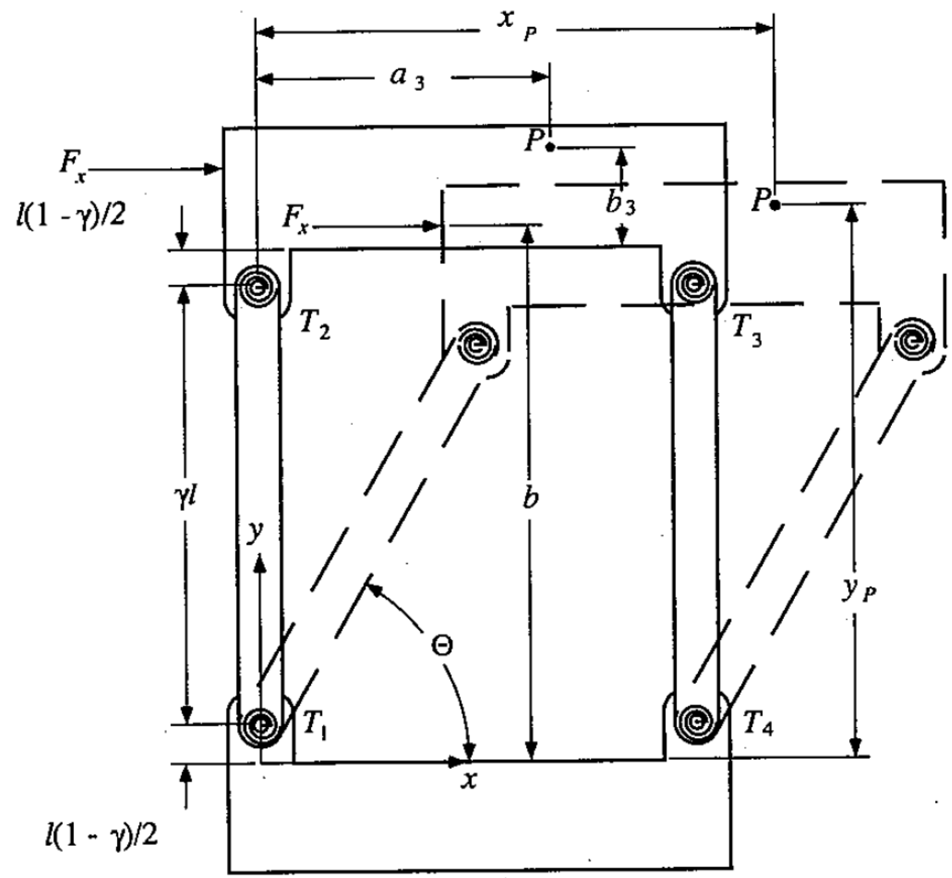
$$K = 2\gamma K_\Theta \frac{EI}{l}$$

$$\sigma_{\max} = \frac{M_{\max}(h/2)}{I} = \frac{Pa(h/2)}{I}$$

$$\gamma = 0.85, K_\Theta = 2.676$$

Fixed-Guided Examples





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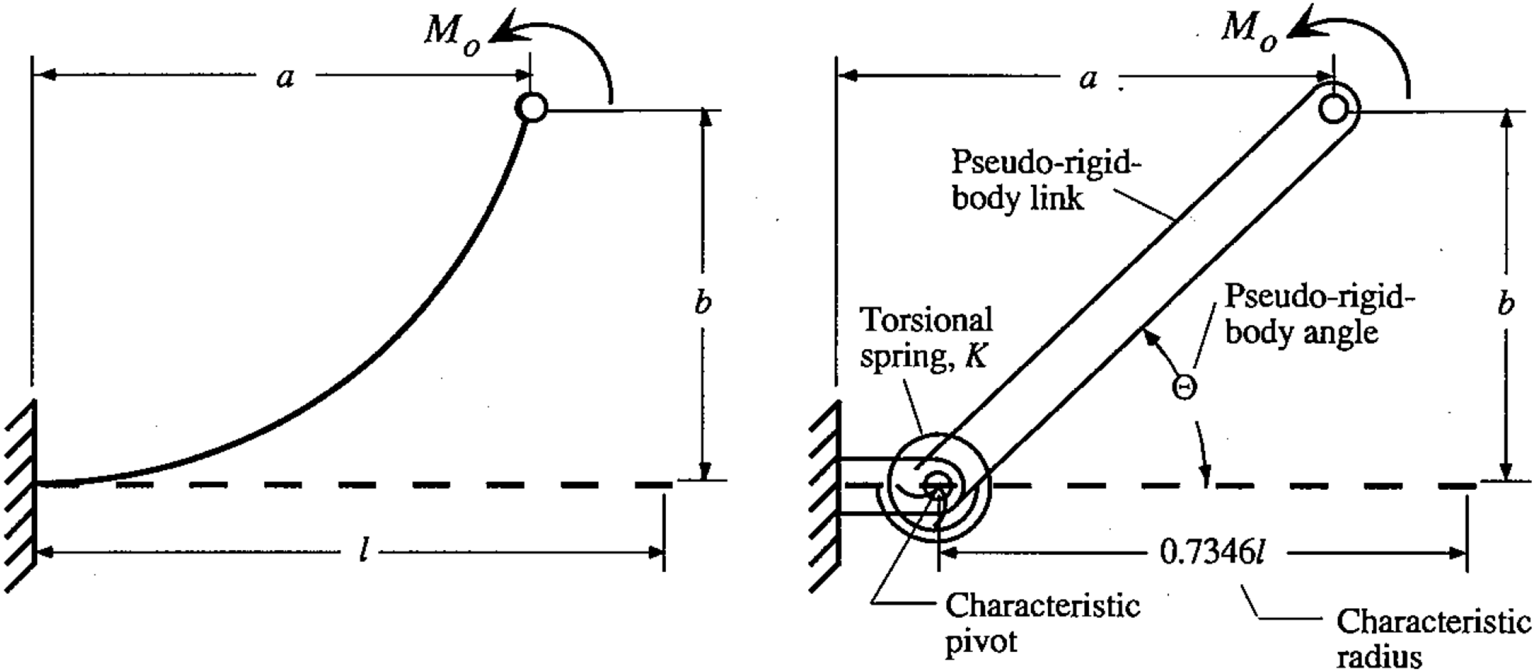
PRBM 4: End-Moment Loading



$$K = 1.5164 \frac{EI}{l}$$

$$\theta_o = 1.5164\Theta$$

$$\gamma = 0.7346$$

$$\sigma_{\max} = \frac{M_{\max}(h/2)}{I}$$

PRBM 5: combined end force and moment

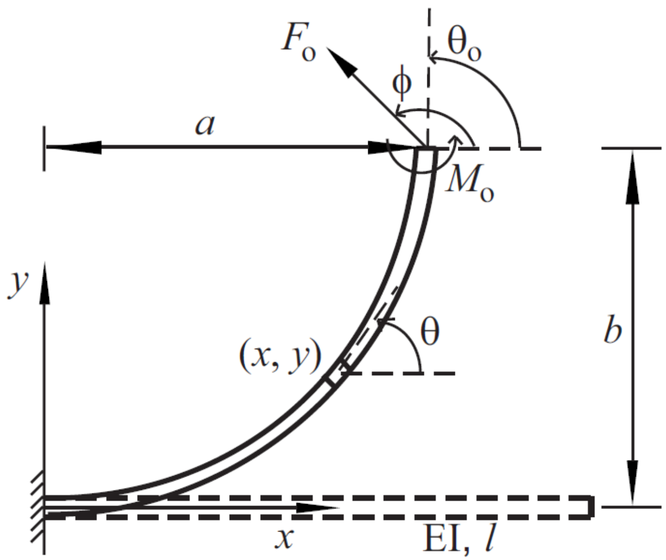
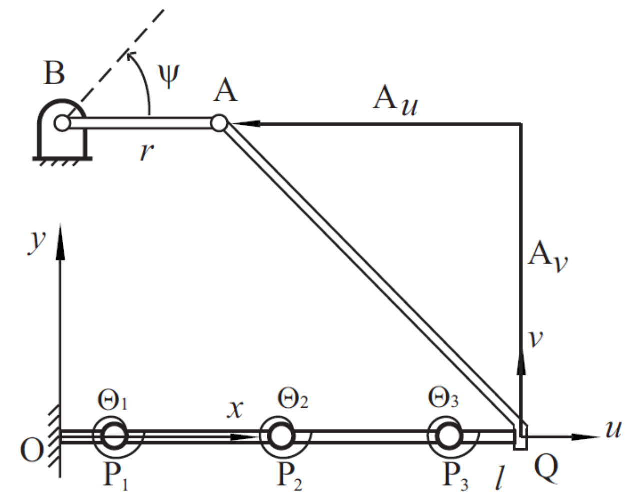


Fig. 1 Large deflection of a cantilever beam subject to a combined end force and moment



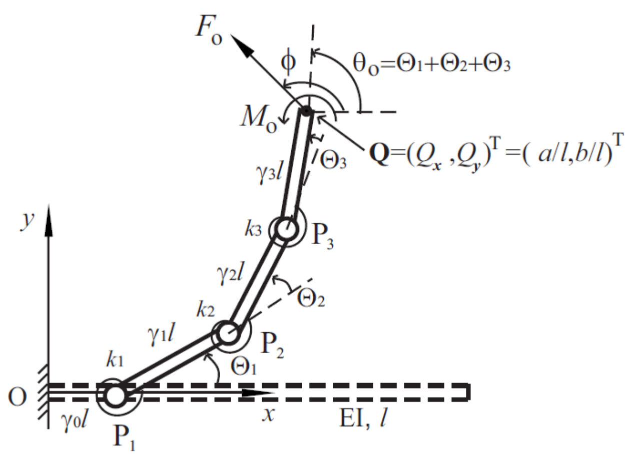
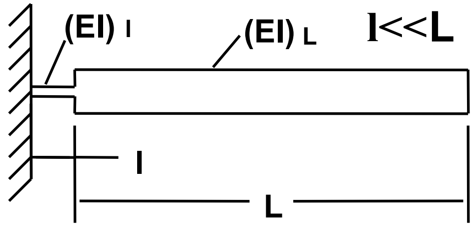
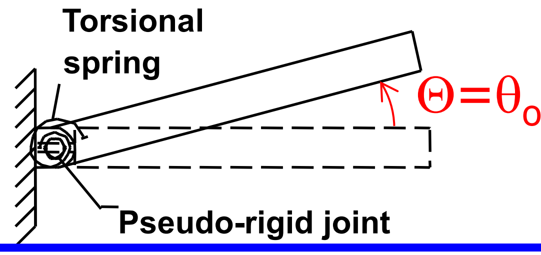


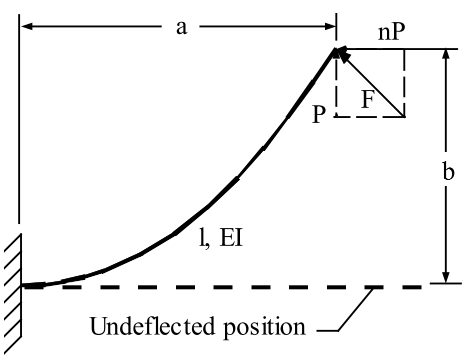
Fig. 6 A pseudorigid-body 3R model for a cantilever beam subject to a combined end force and moment

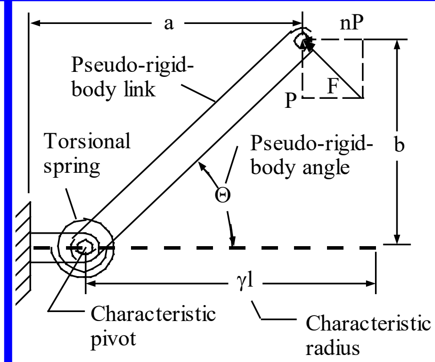
PRBM Summary





$$K = \frac{(EI)_l}{l} \qquad \theta_o = \Theta$$

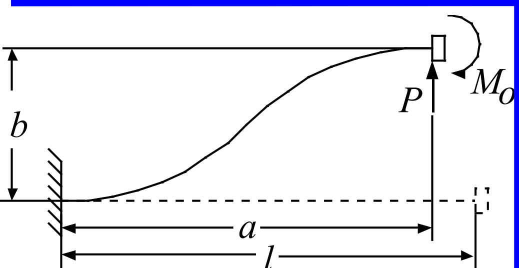


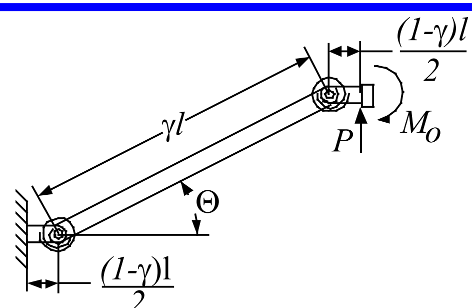


$$K = \gamma K_\Theta \frac{EI}{l}$$

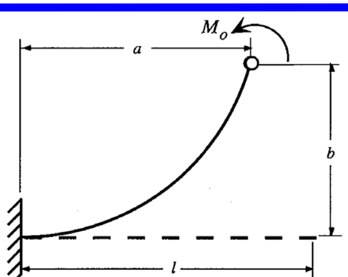
$$\theta_o = c_\theta \Theta$$
$$a = l\left[1-\gamma(1-\cos\Theta)\right]$$
$$b = \gamma l \sin\Theta$$

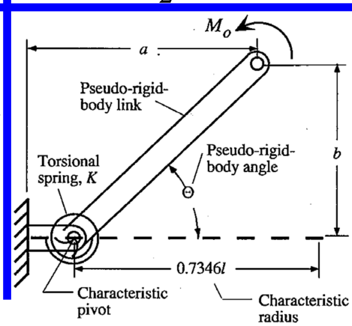
See Table E.1 for values of γ, c_Θ K_Θ





$$\theta_o = 0$$
$$a = l\left[1-\gamma(1-\cos\Theta)\right]$$
$$b = \gamma l \sin\Theta$$
$$K = 2\gamma K_\Theta \frac{EI}{l} \qquad \gamma = 0.85, K_\Theta = 2.676$$





$$\theta_o = 1.5164\Theta$$
$$\gamma = 0.7346$$
$$K = 1.5164\frac{EI}{l}$$

PRBM Examples

- Identify the loading cases and
- Draw the pseudo-rigid-body model of the following compliant mechanisms

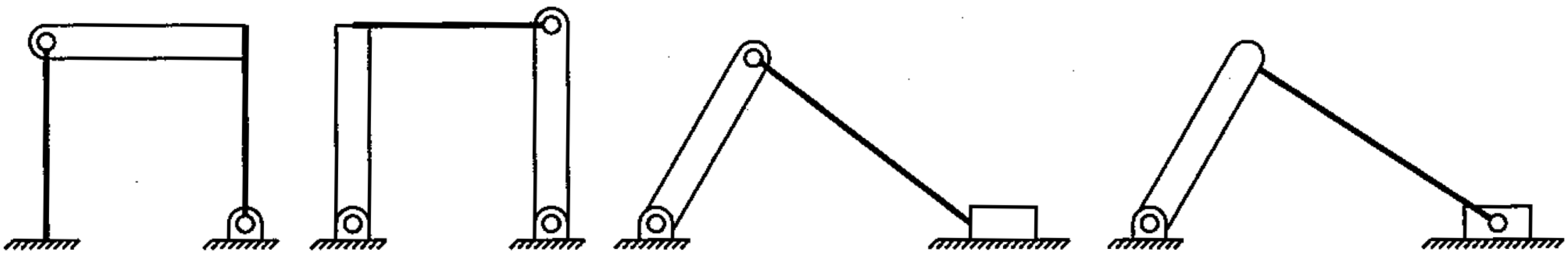


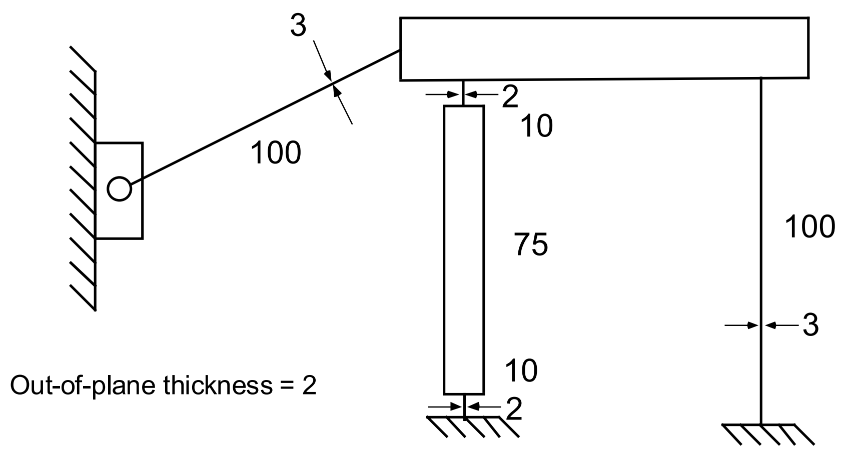
Figure 5.15. Examples of compliant mechanisms with fixed–pinned segments.

Exercise

(a) Sketch the PRBM
(b) Calculate the lengths of the links
(c) Write equations for spring constants symbolically
(d) Calculate numerical values of spring constants

Assume dimensions in mm and material is Aluminum (E=72 GPa)

$I = bh^3/12$



Example 1: Application of PRBM

The small-length flexure pivot is required to have vertical deflection of b=3in. Determine the corresponding horizontal deflection, maximum stress and vertical force required.

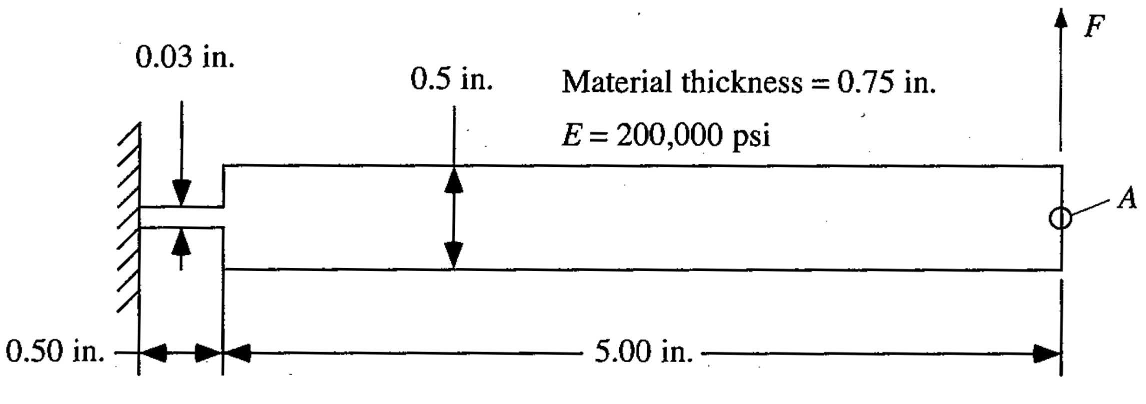


Figure 5.5. Example of a small-length flexural pivot.

Example 2: Application of PRBM

Use of the PRBM for flexible beams that undergo large deflections. A flexible steel beam with modulus E=30x10^6 lb/in^2, length l=20in, width w=1.25in and height h=1/32 in is required to obtain a vertical deflection of b=10in.

a) Calculate the vertical force required to cause this deflection

b) Find the horizontal end coordinate *a*, the beam end angle θ_0 and the maximum stress σ_{max}

c) Solve the problem assuming that the force acts as an angle of ϕ=135°