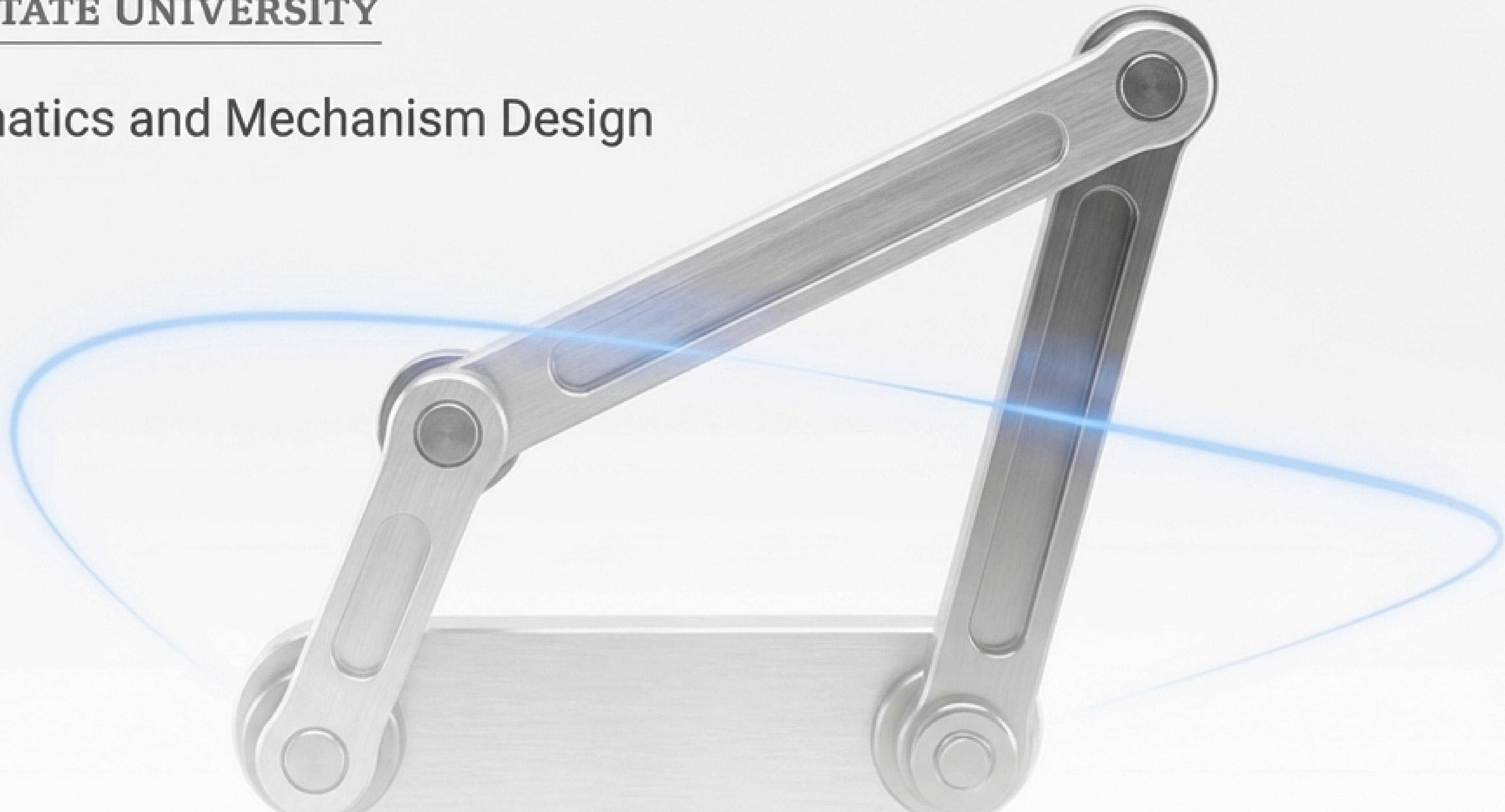
THE OHIO STATE UNIVERSITY

ME 3751: Kinematics and Mechanism Design



Kinematic Synthesis of 4-Bar Linkages

A Practical Guide to Double and Triple Rocker Synthesis using Geometric Constraint Programming (GCP)

From Analysis to Synthesis: The Two Sides of Kinematics

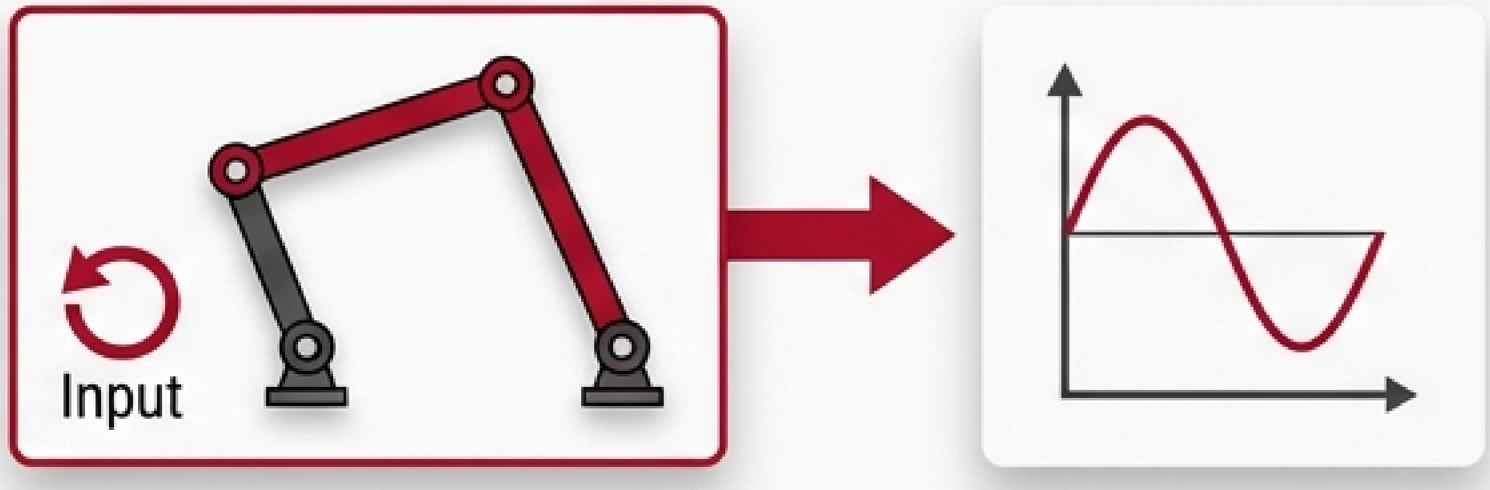
Analysis (Evaluating a Design)

Given a mechanism, find its output motion versus its input motion.

You are given the Mechanism and the Input Motion, and you solve for the Output Motion.

A unique, predictable solution. You are evaluating a design against requirements.

[Mechanism + Input] [? Output]



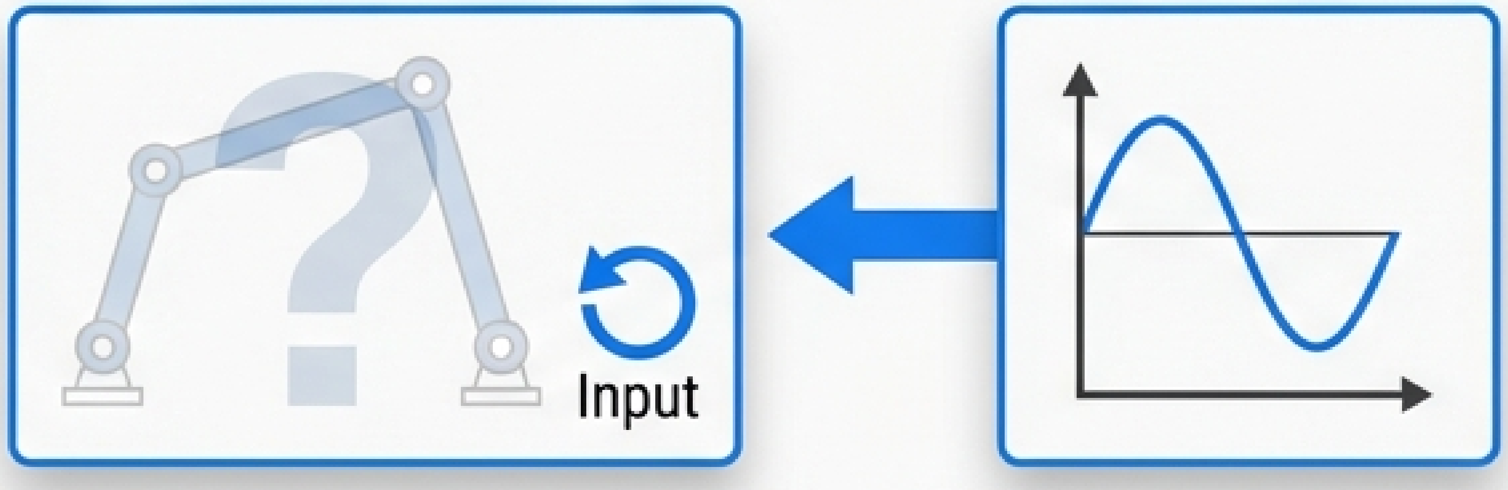
Synthesis (Creating a Design)

Given a desired output motion, assume an input motion and find the mechanism(s) that can produce it.

You are given the desired Output Motion and the Input Motion, and you solve for the Mechanism itself.

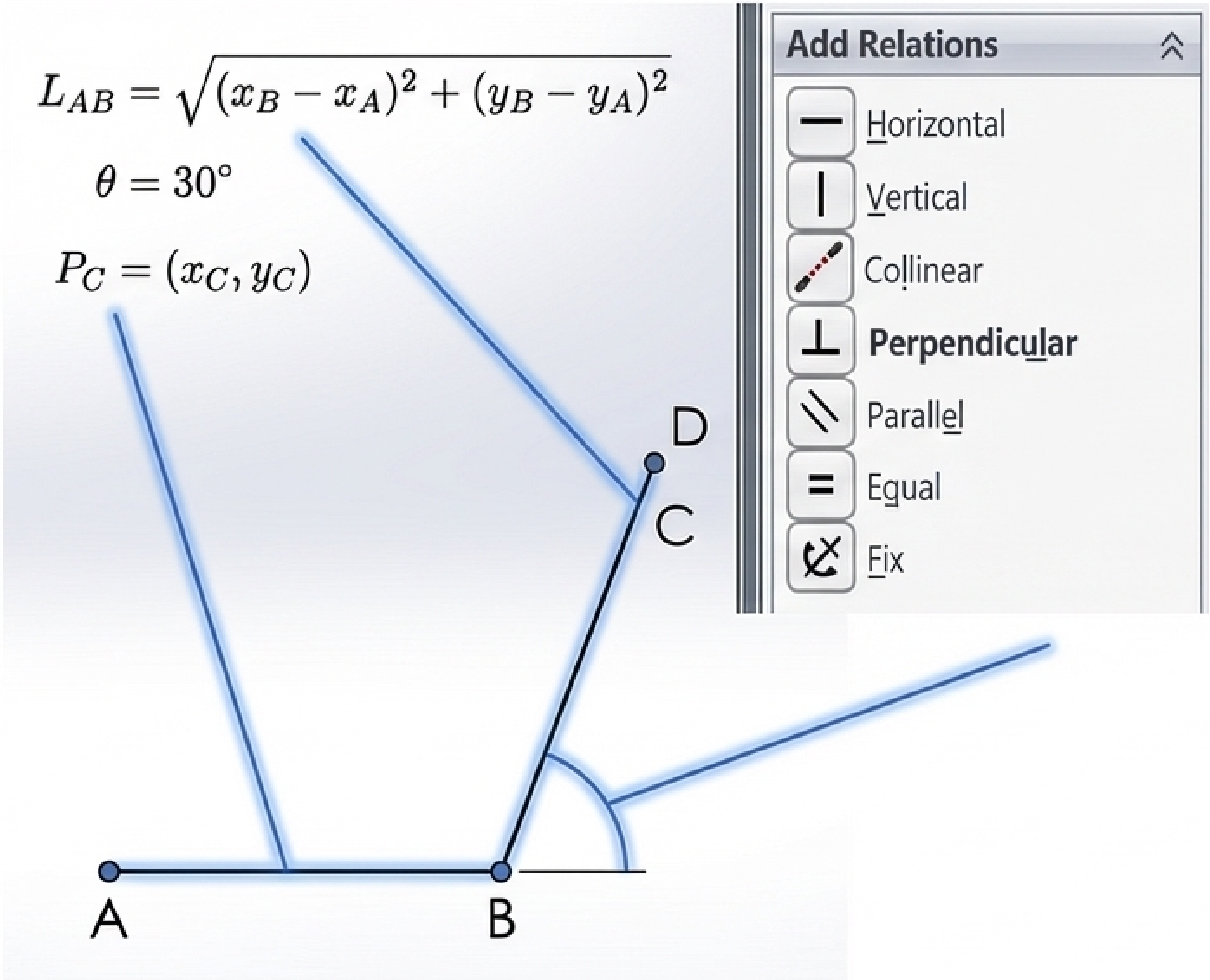
This is the heart of innovation and creative thinking. It often yields multiple solutions, requiring the evaluation of many design candidates.

[? Mechanism + Input] [Output]



The Modern Toolkit: Geometric Constraint Programming (GCP)

Modern CAD programs provide a powerful framework for solving complex synthesis problems with high precision.



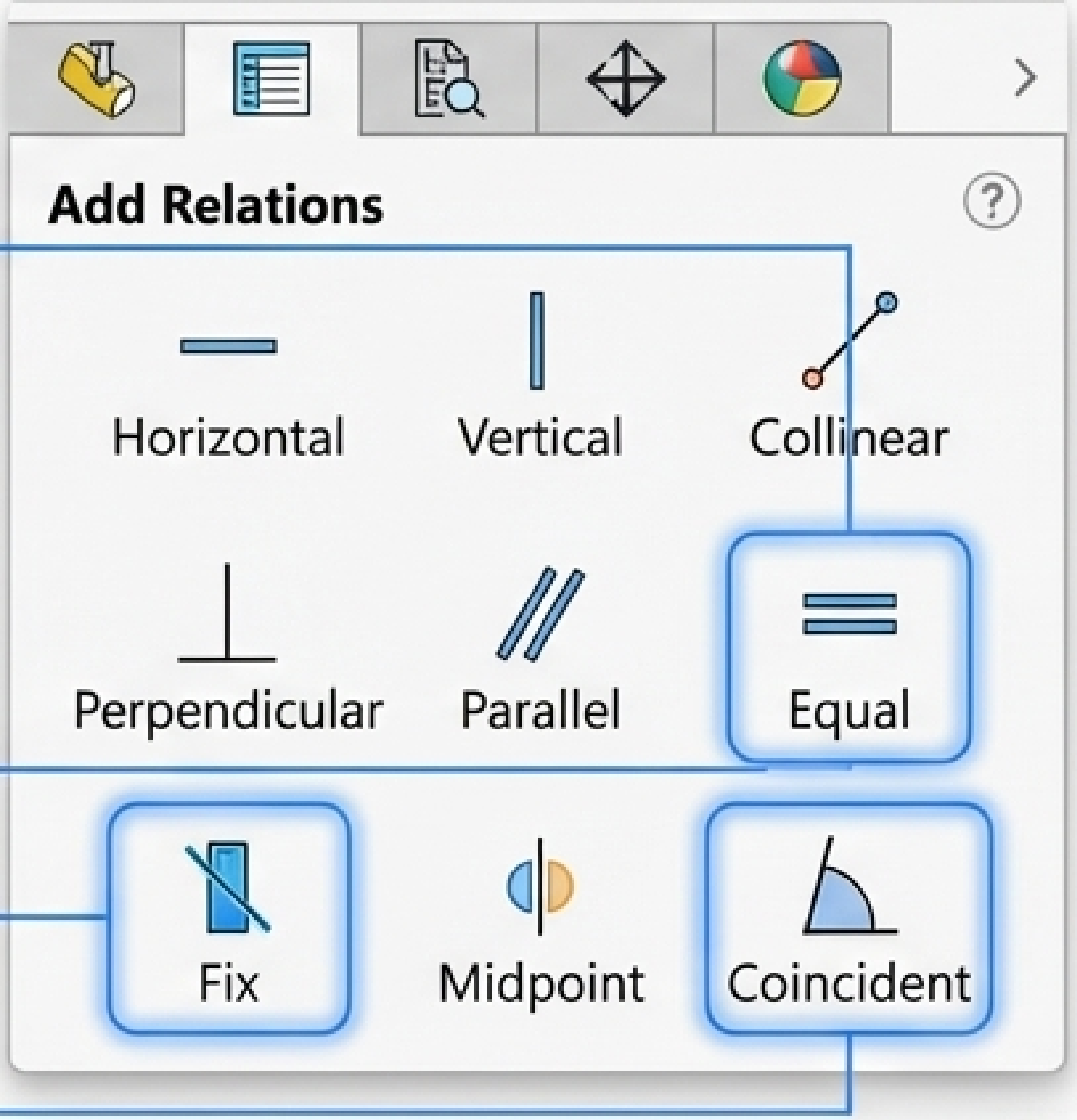
How it Works

- Parametric design programs represent geometry as **systems of equations**. When you change a dimension, the system re-solves.
- **Constraints** are additional equations that define geometric conditions like parallelism, perpendicularity, coincidence, and equality.
- These programs contain powerful non-linear equation solvers that can handle large, complex systems to define geometry with digital accuracy.
- For this course, we will use SolidWorks to apply these principles.

Key Constraints for Kinematic Synthesis in SolidWorks

To perform synthesis, we operate entirely in the **2D Sketch mode** of a 3D Part file. The following relations are our primary tools.

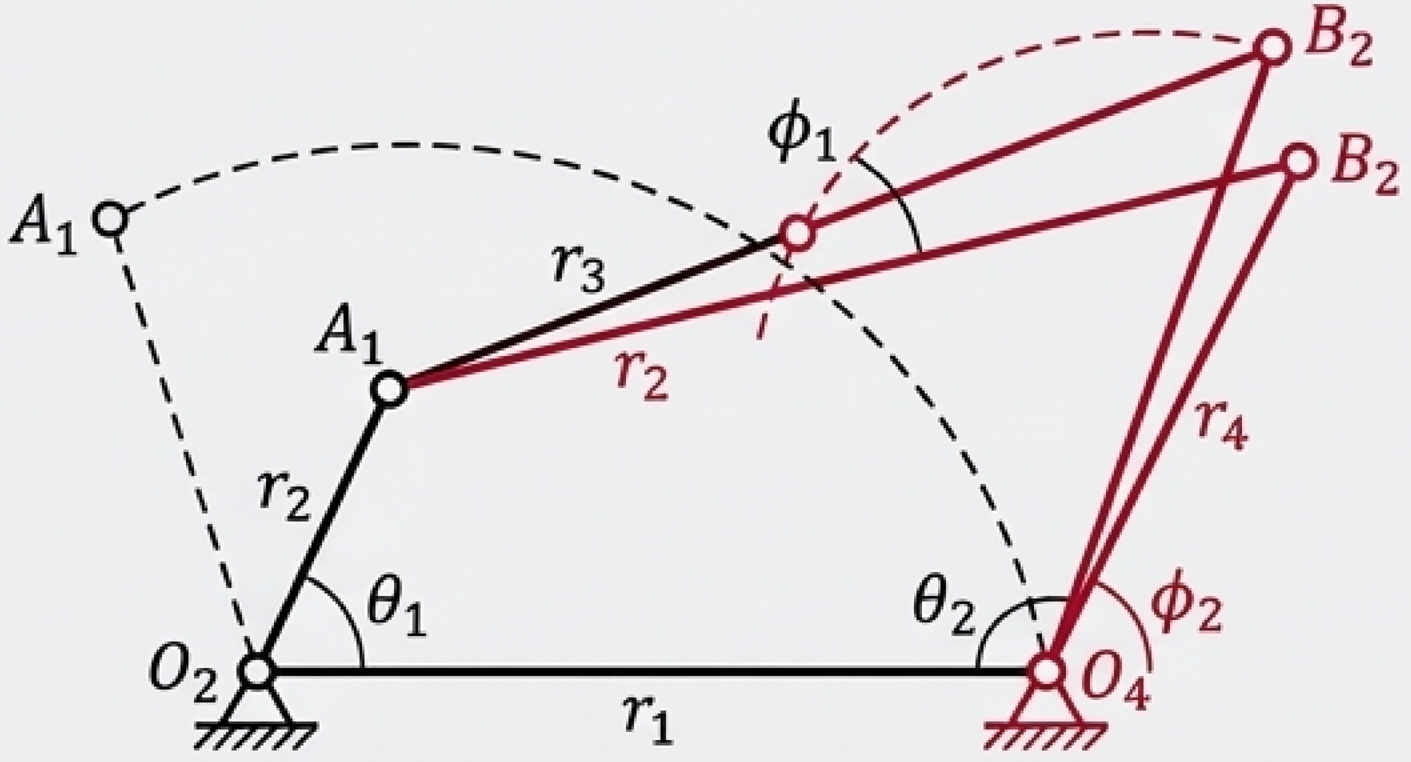
Merge/Coincident: Forces two points to coincide (for revolute joints) or a point to lie on a line.
Dimension Lock: Sets a specific value for a line length or angle. Can be a *driving* dimension (a constraint) or a *driven* dimension (a measurement).
Position Lock (Fix): Fixes a point or line in space, establishing a ground reference.
Equality: This is critical for synthesis. Constrains two or more lines or arcs to be equal in length or radius.
Parallelism / Perpendicularity: Enforces geometric orientation between two lines.



Core Project: Two-Position Double Rocker Synthesis

The Design Problem

- This is one of the most common kinematic design challenges.
- **Given:** Two angular positions of an input link (θ_1 and θ_2) and the two corresponding positions of the output link (φ_1 and φ_2). We are also often given the length of one link (e.g., the ground link).
- **Find:** The lengths of the other three links (r_2, r_3, r_4) that satisfy this motion requirement.



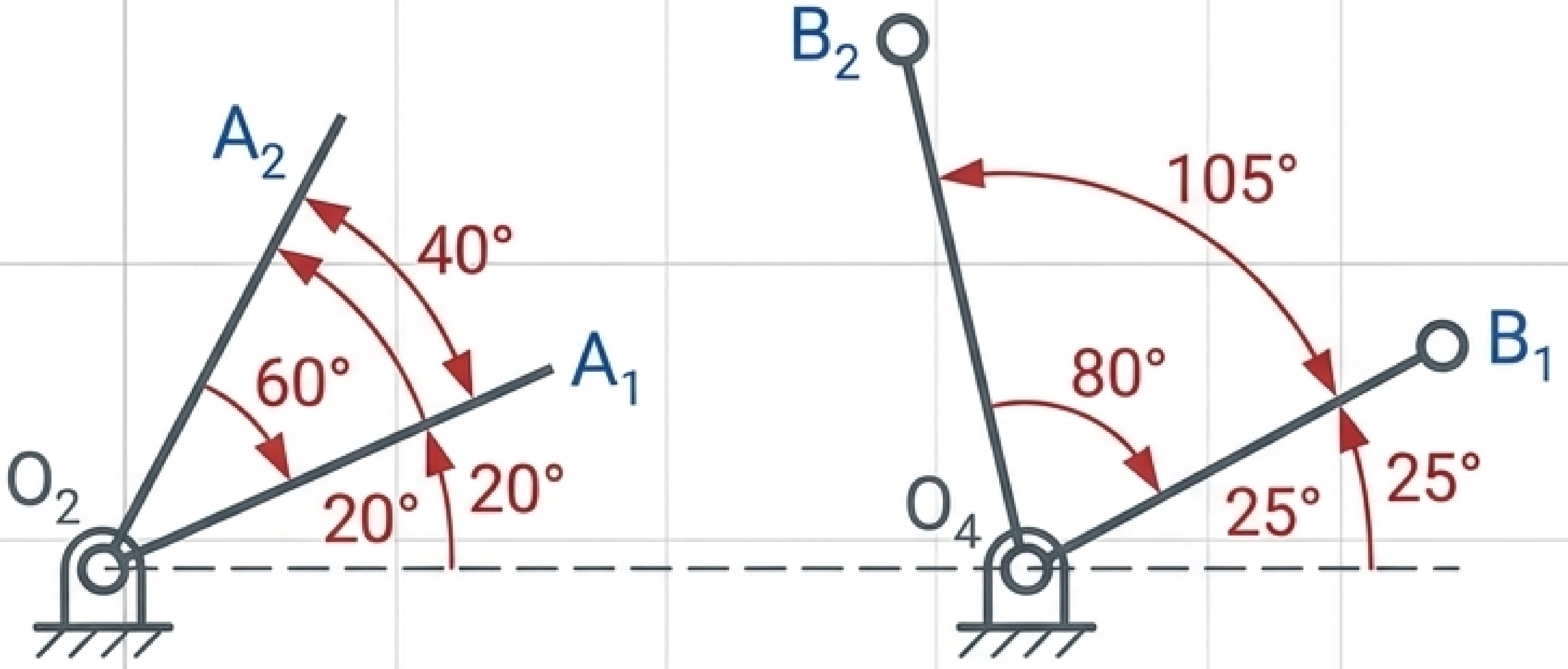
Real-World Application: The complex motion of a backhoe bucket is often governed by a 4-bar linkage designed using these principles.

Our Design Brief: A Double Rocker Mechanism

Problem Statement: Design a double rocker, four-bar linkage with the following specifications:

- **Ground Link (r_1):** 4.0 inches long.
- **Output Link (r_4):** 2.0 inches long.
- **Input Motion:** The input link turns counter-clockwise (CCW) by 40°. Its initial angle (θ_1) is 20° from the horizontal.
- **Output Motion:** The output link turns CCW by 80°. Its initial angle (ϕ_1) is 25° from the horizontal.

Goal: Determine the lengths of the input link (r_2) and the coupler link (r_3).

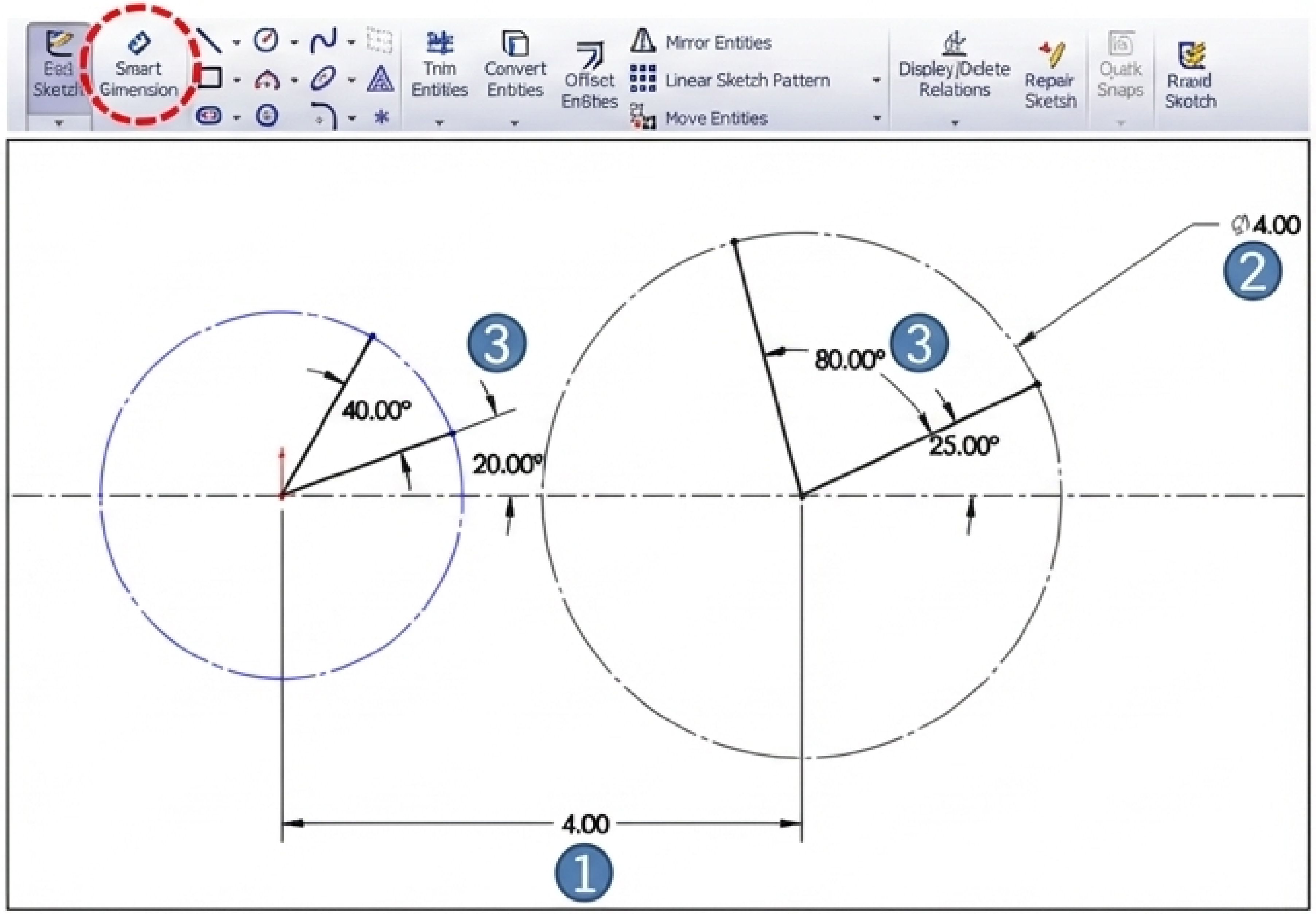


Step 1: Constructing the Known Positions

Objective: To graphically represent the two specified positions of the input and output links.

Procedure in SolidWorks (Sketch Mode):

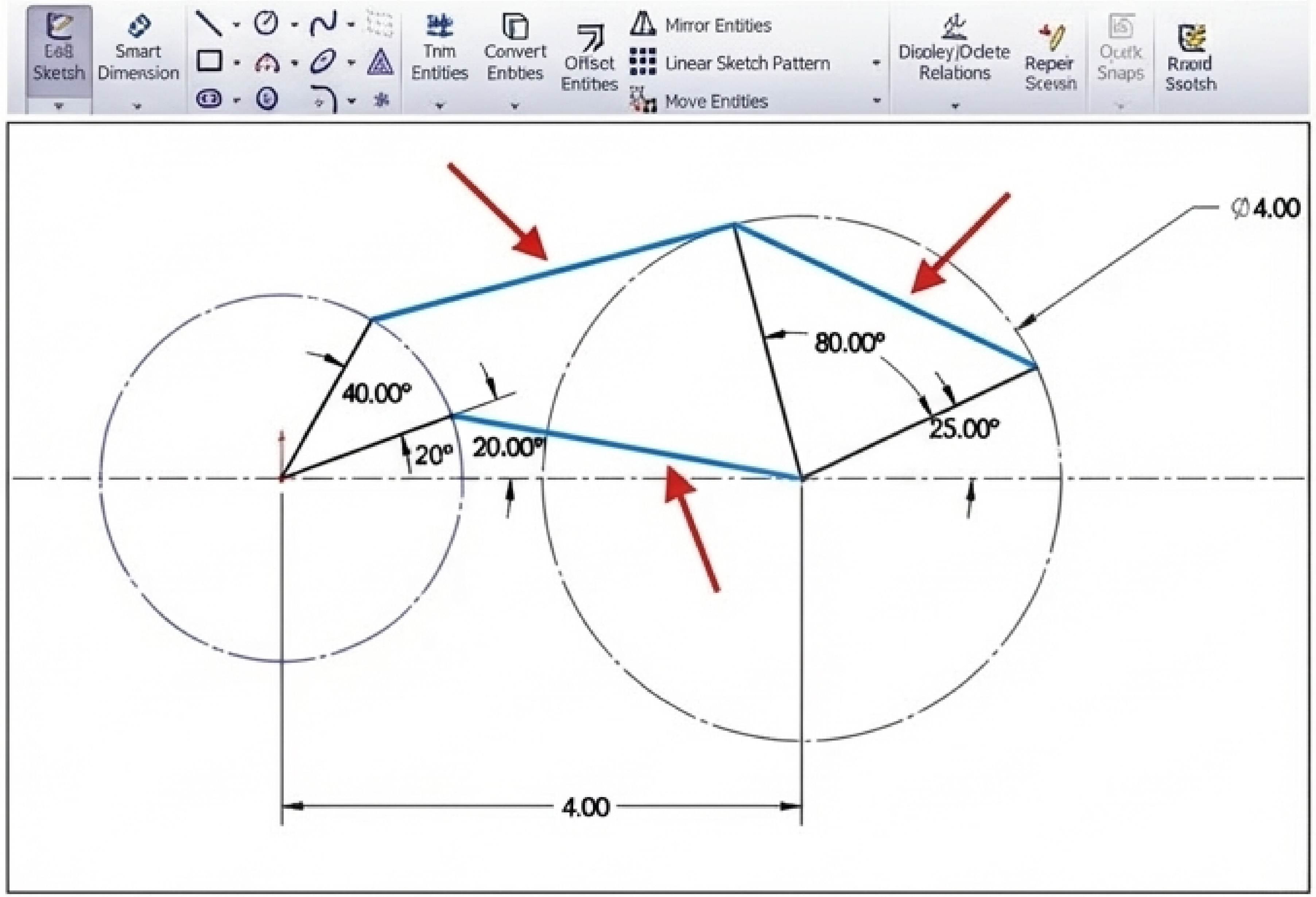
1. **Establish Ground:** Draw an infinite horizontal construction line. Place the input pivot (O_2) at the origin and the output pivot (O_4) on the line. Use Smart Dimension to set the distance between them to 4.00 inches.
2. **Define Output Rocker:** At pivot O_4, draw two lines representing the output link at its two positions. Dimension them to 2.00 inches long (or use a construction circle of 4.00" diameter). Dimension the angles relative to the horizontal line: 25° for the first position, and an 80° angle between the two lines.
3. **Define Input Rocker Positions:** At pivot O_2, draw two lines representing the input link positions. Dimension the angles: 20° for the first position, and a 40° angle between the two lines. The length is currently unknown, so leave them un-dimensioned for now.



NotebookLM

Step 2: Drawing the Coupler in Both Positions

Objective: To represent the coupler link connecting the input and output rockers at each of the two specified positions.



Procedure in SolidWorks:

1. **Draw Coupler at Position 1:** Draw a solid line connecting the endpoint of the first input link (A_1) to the endpoint of the first output link (B_1).
2. **Draw Coupler at Position 2:** Draw a second solid line connecting the endpoint of the second input link (A_2) to the endpoint of the second output link (B_2).

- **Current State:** The sketch is still under-defined because the lutch is still under-defined because the length of the input link (and thus the coupler) is not yet known. The system has one degree of freedom.

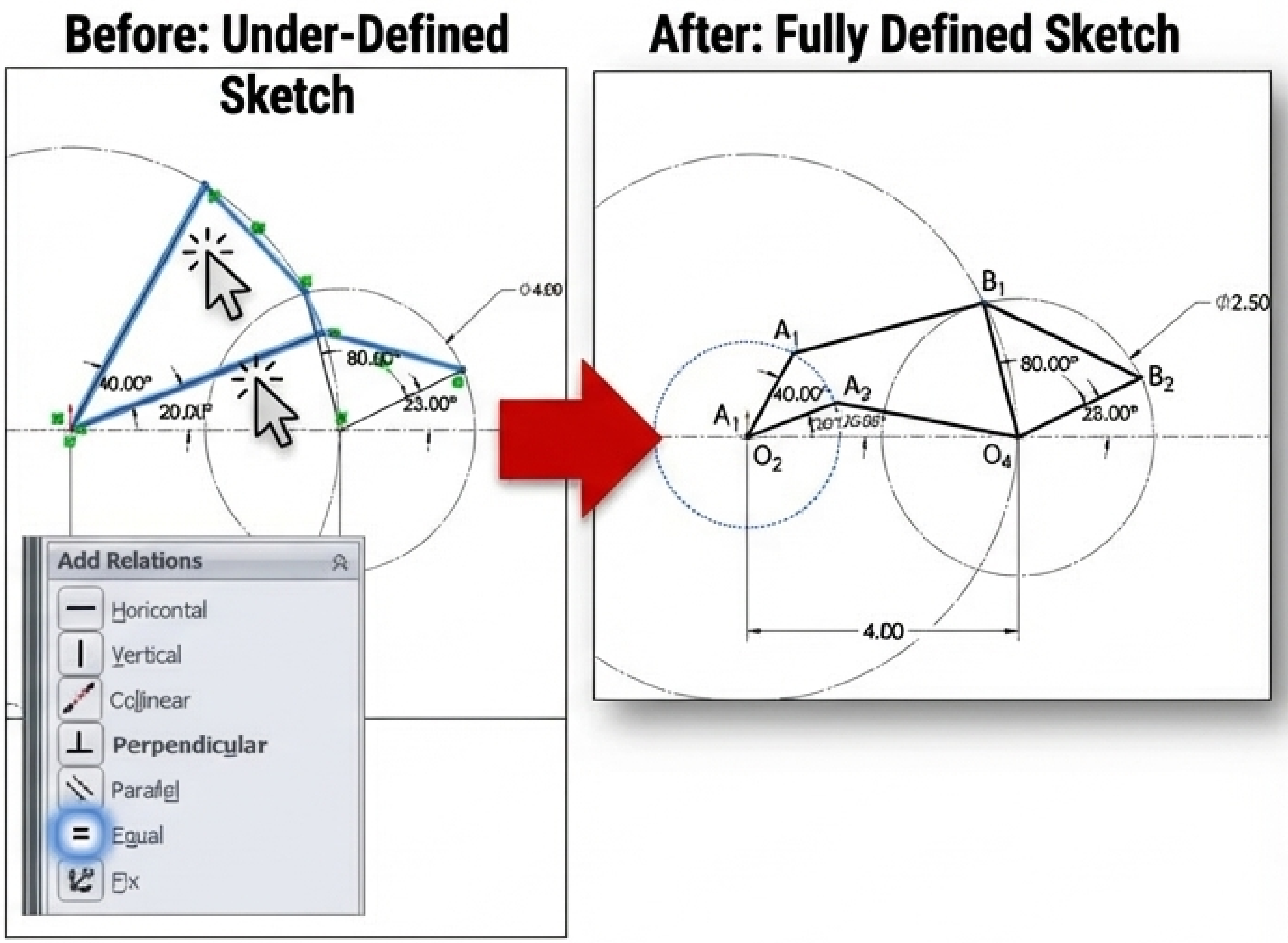
Step 3: The Synthesis Solution—Applying the Equality Constraint

The Key Insight
A rigid-body linkage does not change its dimensions during motion. Therefore, the coupler link must have the **same length in both positions**. This is the **core constraint** that allows us to solve for the unknown geometry.

Procedure in SolidWorks:

1. Select the line representing the coupler in Position 1.
2. Hold CTRL and select the line representing the coupler in Position 2.
3. In the "Add Relations" panel, select the Equal constraint.

Result
The SolidWorks solver calculates the only possible geometry where both coupler links are equal. The entire sketch becomes fully defined, and the unknown input link length is now solved.

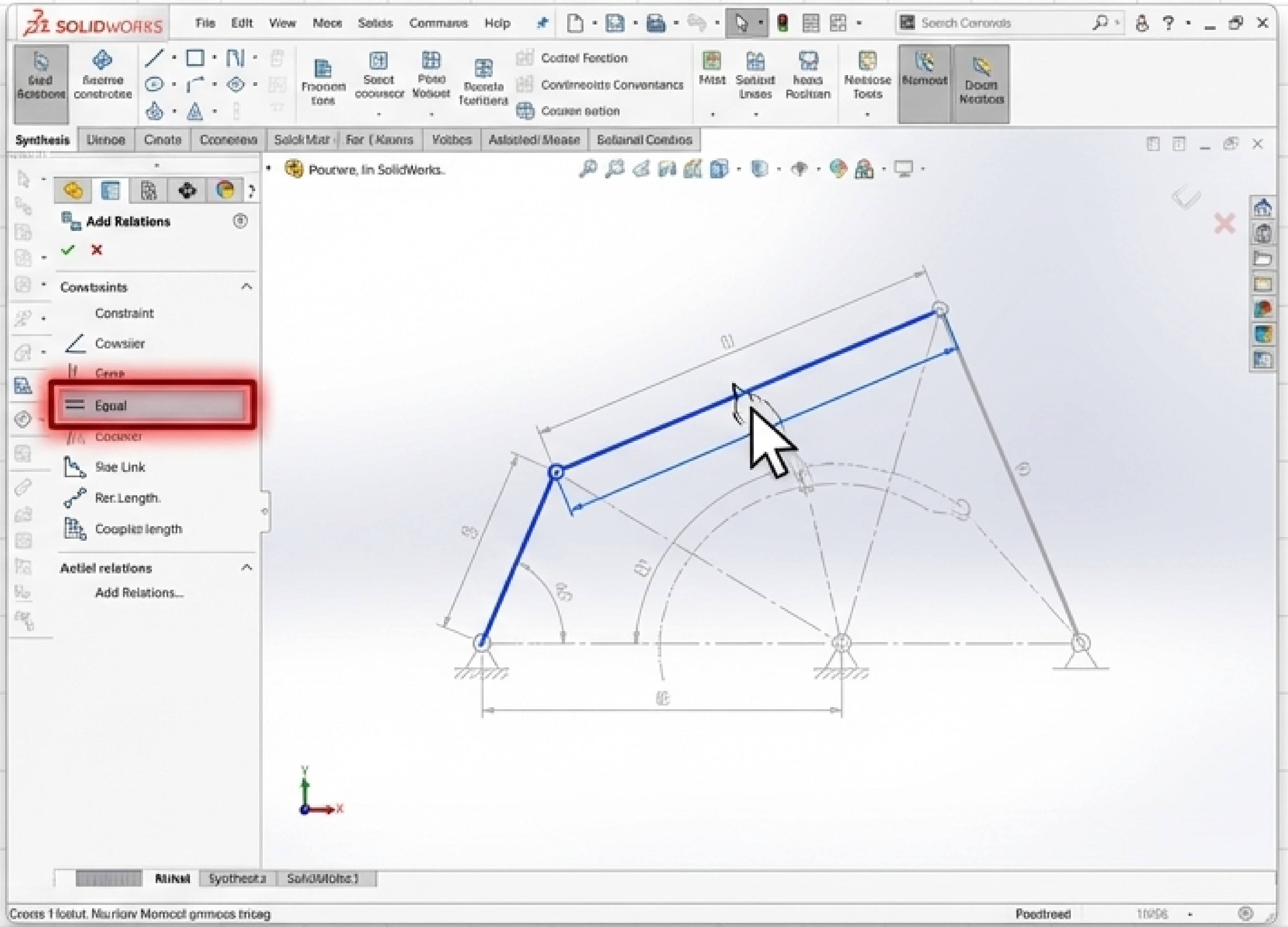


Step 4: Verifying the Synthesized Linkage

Objective: To confirm that the calculated link lengths produce the required motion.

Procedure in SolidWorks:

1. **Create a New Sketch:** On the same plane, create a new sketch. The synthesis construction will be visible as a gray template.
2. **Build the Linkage:** Draw a new 4-bar linkage (input, coupler, output) on top of one of the solved positions (e.g., Position 1).
3. **Constrain Lengths:** For the input and output links, make them equal in length to the corresponding links in the underlying synthesis sketch.
4. Crucially, constrain the new coupler's length to be equal to the solved coupler length from the synthesis sketch.

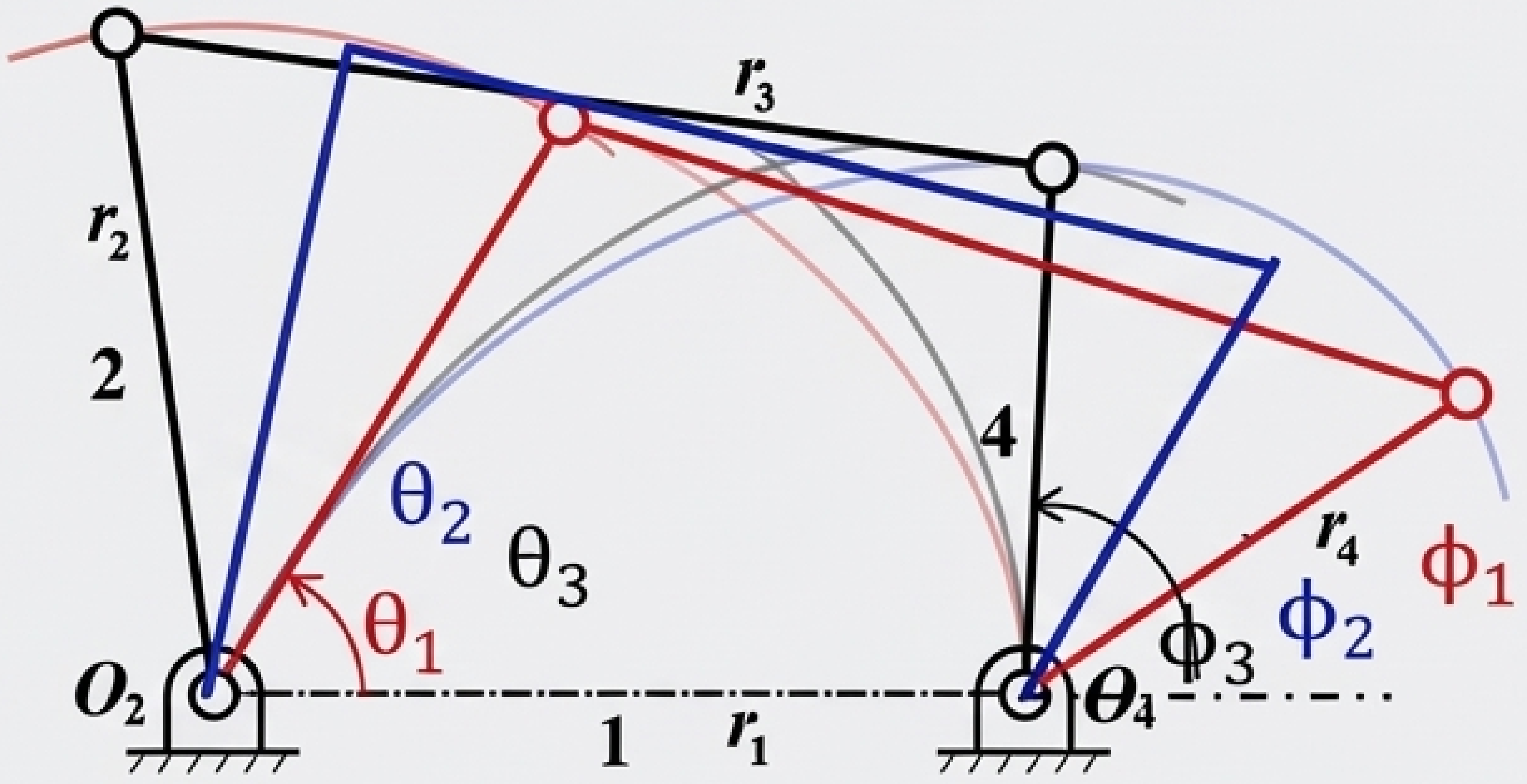


Confirmation: The new linkage is now a functional model. You can drag the input link with your mouse, and you will see that it moves precisely between the two defined start and end points of the synthesis sketch.

Advanced Challenge: Three-Position Triple Rocker Synthesis

The Next Level of Precision

- Often, two positions are not enough to fully define a required motion path. Three-position synthesis provides greater control over the mechanism's behavior.
- **Given**: Three positions of the input link ($\theta_1, \theta_2, \theta_3$) and the three corresponding positions of the output link (ϕ_1, ϕ_2, ϕ_3).
- **Find**: The lengths of all four links (r_1, r_2, r_3, r_4) that pass through all three specified positions.



The Method

The GCP approach remains the same. We will construct the linkage in all three positions and apply equality constraints.

Design Brief: A Triple Rocker Mechanism

Design a four-bar linkage such that the input and output links pass through the following three specified positions. The base link length must be r_1 = 4.5 inches.

Input Link Angles:

- $\theta_1 = 35.02°$
- $\theta_2 = 67.50°$
- $\theta_3 = 100.0°$

Output Link Angles:

- $\phi_1 = 91.21°$
- $\phi_2 = 101.79°$
- $\phi_3 = 117.19°$

Goal: Determine the lengths of the input link (r_2), coupler (r_3), and output link (r_4).

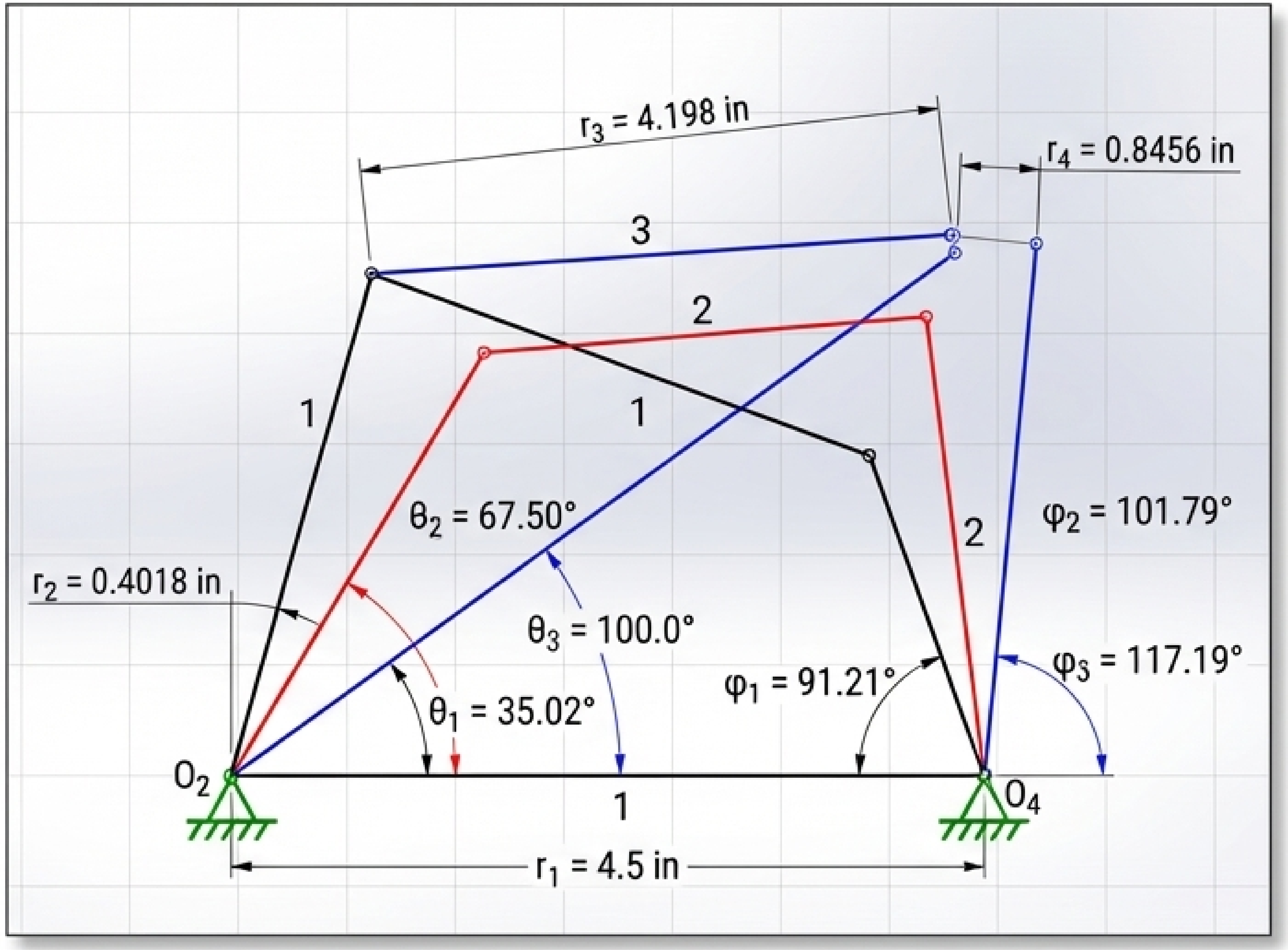
Solving for Three Positions with GCP

The Process is Identical

1. **Construct**
   Draw the ground link (r_1 = 4.5 in). Construct the input and output links at all three specified angular positions (θ_1, θ_2, θ_3 and φ_1, φ_2, φ_3).
2. **Connect**
   Draw three separate coupler links, connecting the corresponding input/output pairs for each position.
3. **Constrain**
   Apply **Equal** length constraints to all three input links, all three output links, and all three coupler links.

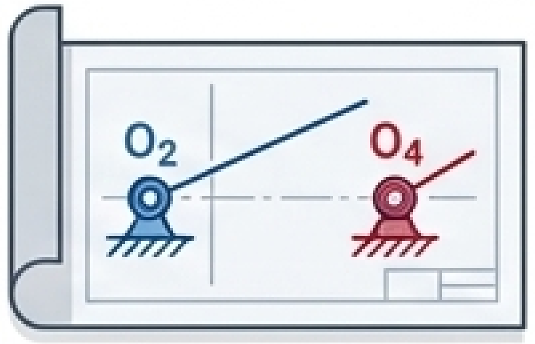
The Solution

The SolidWorks solver will simultaneously solve this larger system of equations, yielding a single geometric solution that satisfies all three position requirements.



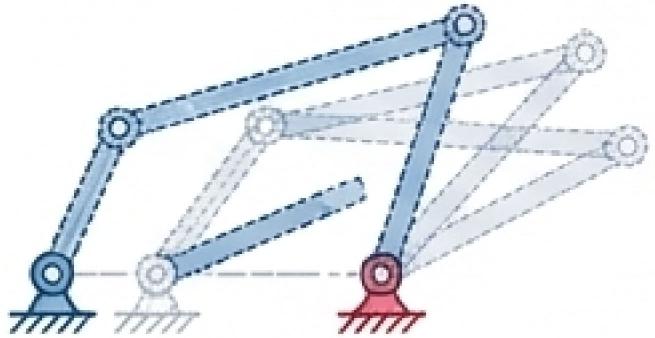
The Kinematic Synthesis Workflow: A Four-Step Guide

A repeatable methodology for designing linkages to meet specific motion requirements using GCP:



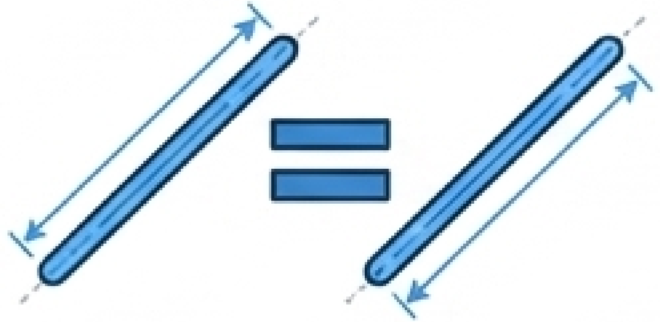
Step 1: Define Knowns & Ground

Establish your fixed pivots (ground link). Graphically construct all specified positions of the known links (e.g., input and/or output rockers at each required angle).



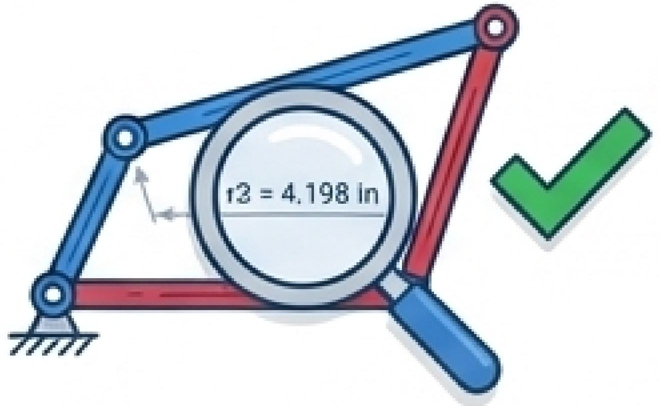
Step 2: Construct the Linkage Instances

Draw the complete linkage for each specified position. Some link lengths will be unknown.



Step 3: Apply Equality Constraints

This is the core of synthesis. Constrain all corresponding links to be equal in length across all positions. Let the CAD solver find the solution.



Step 4: Extract & Verify

Read the now fully-defined dimensions from your sketch. Build a new, separate linkage using these dimensions to test and verify the motion.

Synthesis: From Abstract Requirements to Functional Design

Main Takeaway: Kinematic synthesis is the creative process of inventing mechanisms.

Your New Skill: By mastering Geometric Constraint Programming, you have moved beyond simply analyzing existing mechanisms. You can now systematically translate a set of desired motions—the core of a design problem—into a fully-defined, functional CAD model ready for prototyping and analysis. This is ational skill for modern mechanical design and innovation.