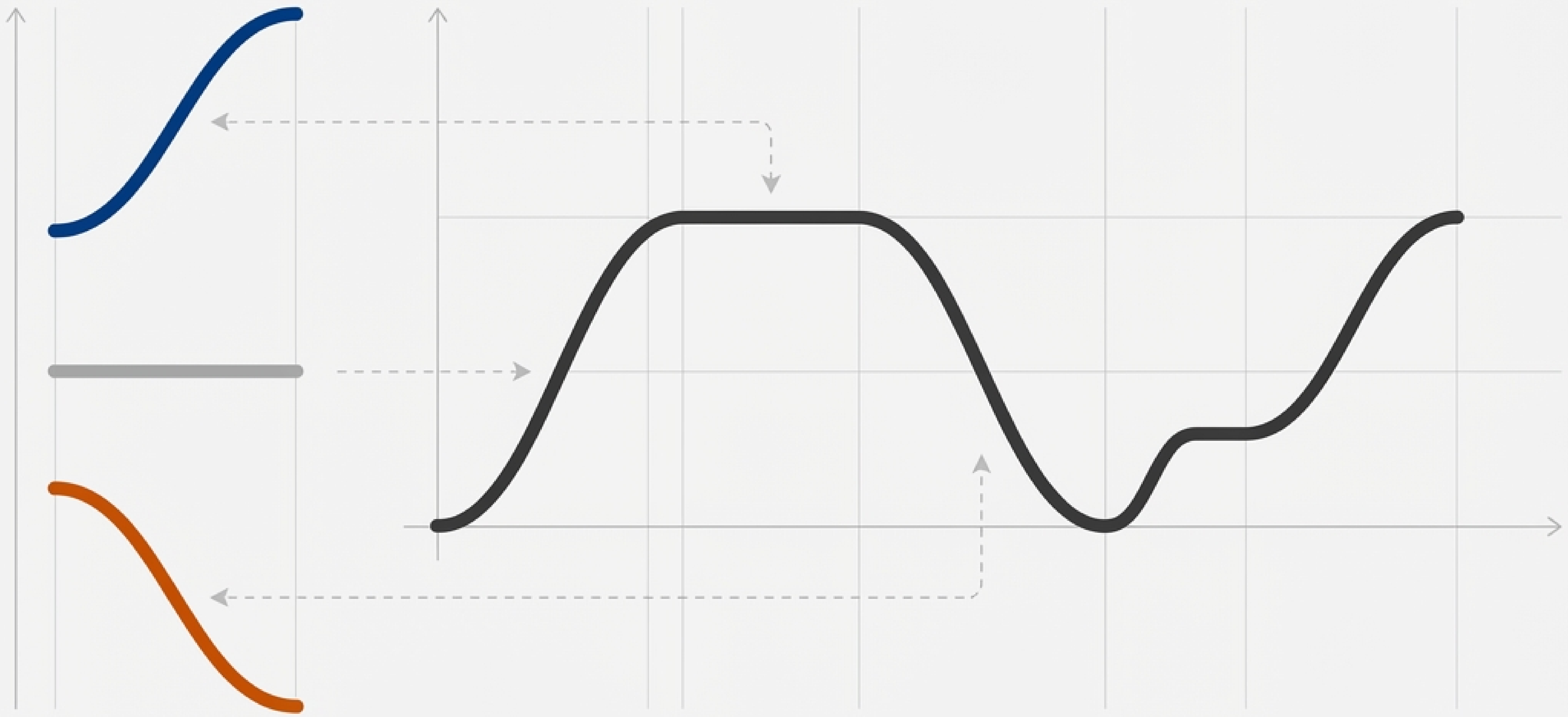
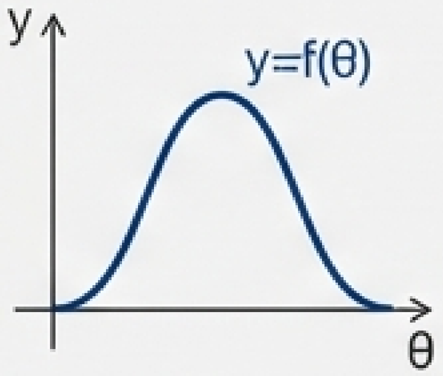
Follower Motion Synthesis

Adapting Standard Functions via Kinematic Coordinate Transformation



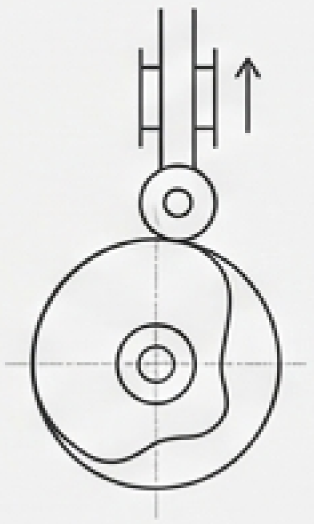
Cam Design: From Analytical Blueprint to Physical Profile

Cam design is a two-stage process. The success of the final component depends on precision in both stages.



Stage 1: Synthesis of Follower Motion Program

This is the analytical design stage. We define the follower's displacement, velocity, and acceleration mathematically to ensure desired dynamic performance (e.g., finite jerk). **This presentation focuses exclusively on this stage.**

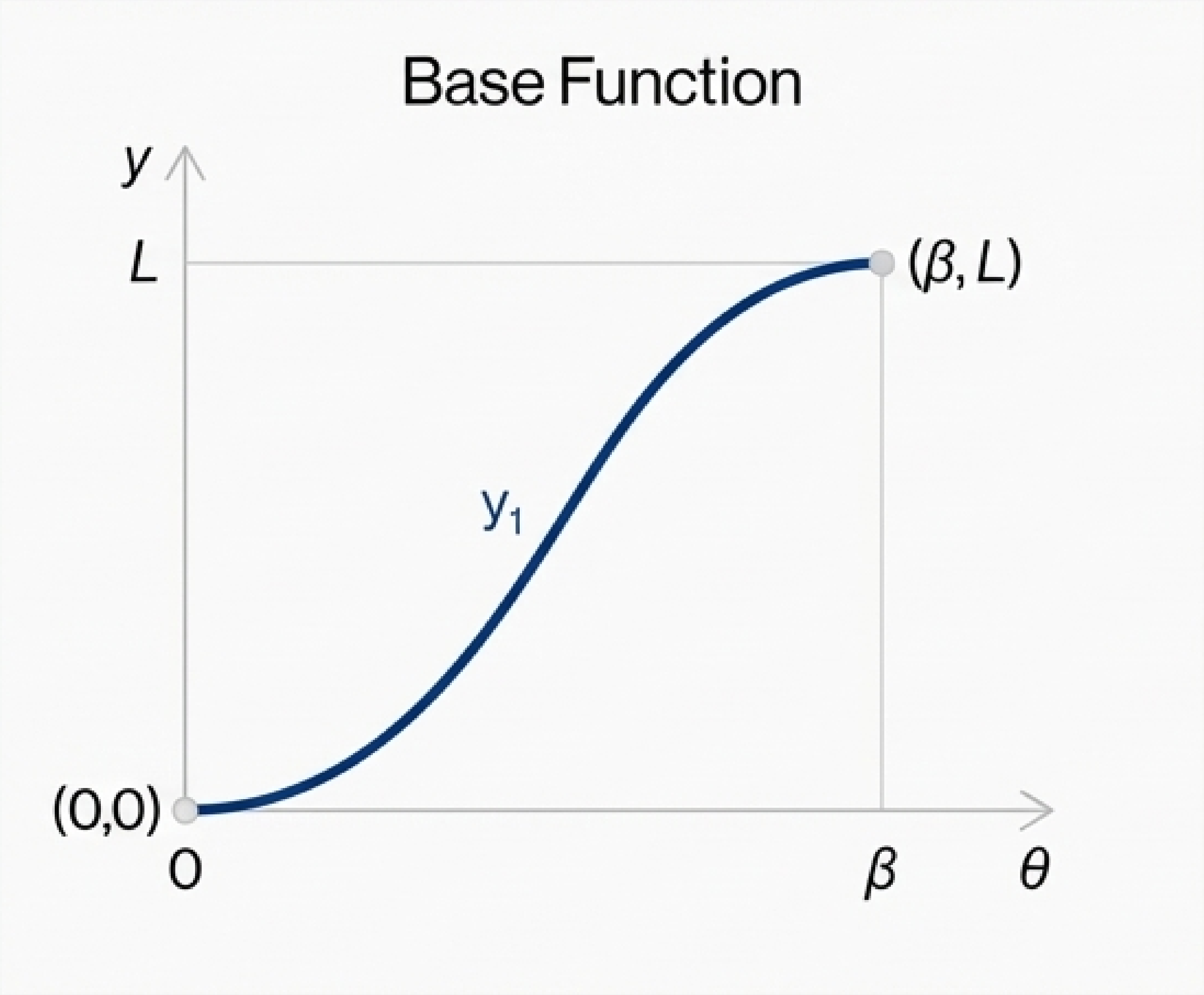


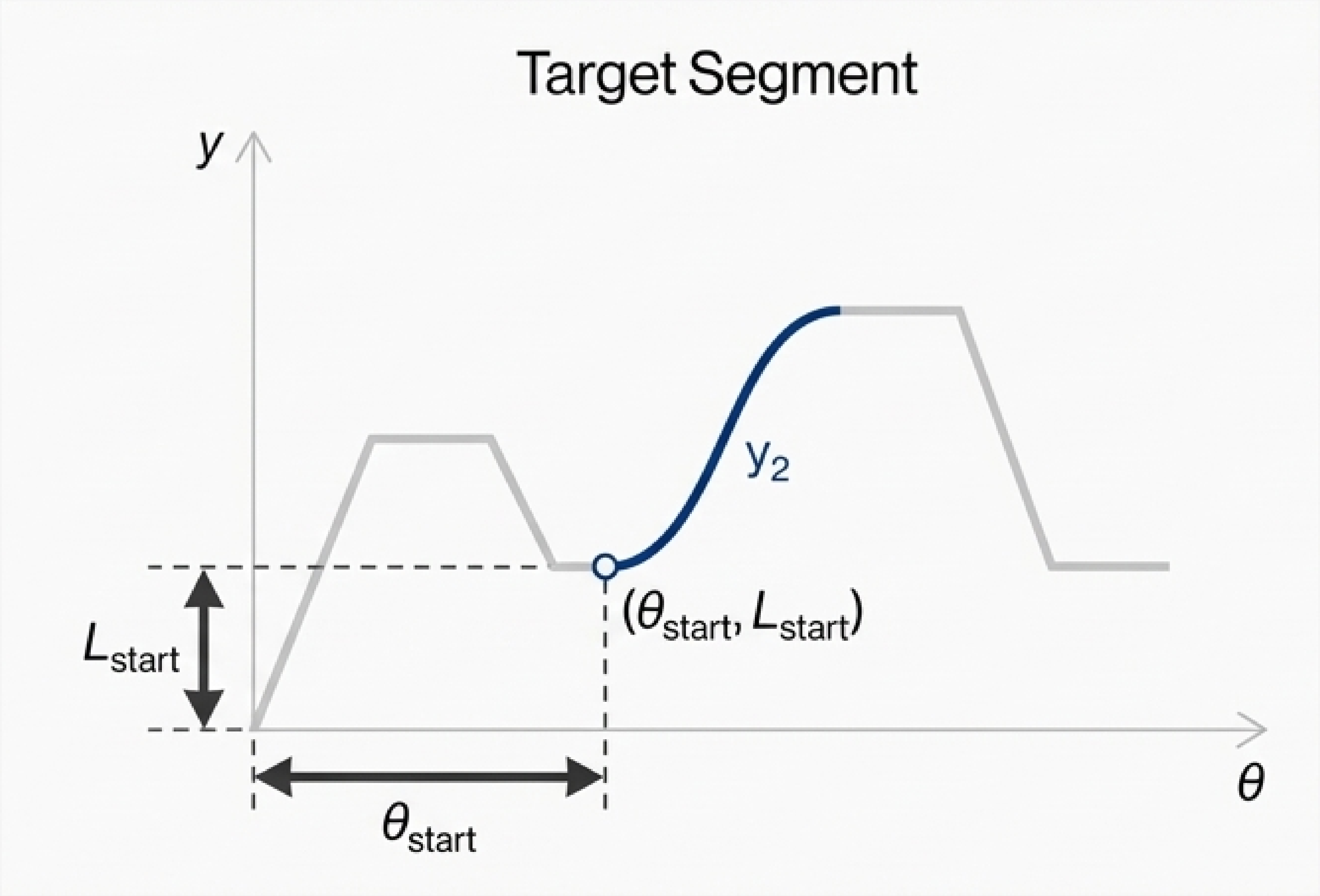
Stage 2: Generation of Cam Profile

This is the geometric stage. The motion program is translated into the physical shape of the cam, a process often done via kinematic inversion.

The Core Challenge: Standard Functions vs. Real-World Segments

Standard follower motion equations are derived in an idealized coordinate system, typically for a rise from (0,0) over a duration β. However, real-world cam profiles are complex assemblies of segments starting at arbitrary angular positions (θ_{start}) and displacement levels (L_{start}).



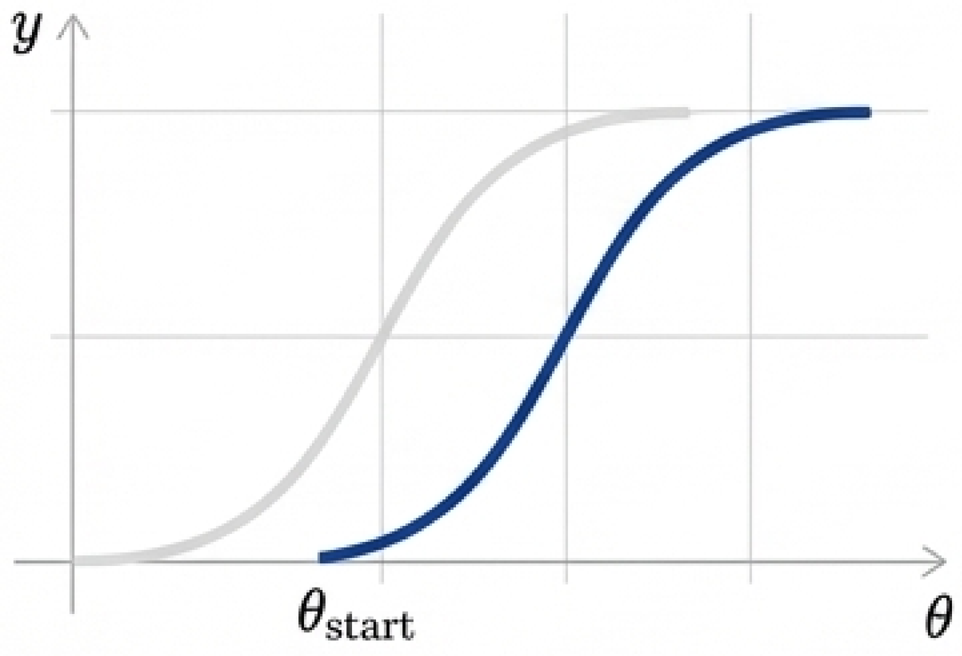


The Analytical Toolkit: Three Fundamental Transformations

Angular Shift (θ-Axis)

To shift a function's start from $\theta = 0$ to θ_{start}, substitute θ with $(\theta - \theta_{\text{start}})$.

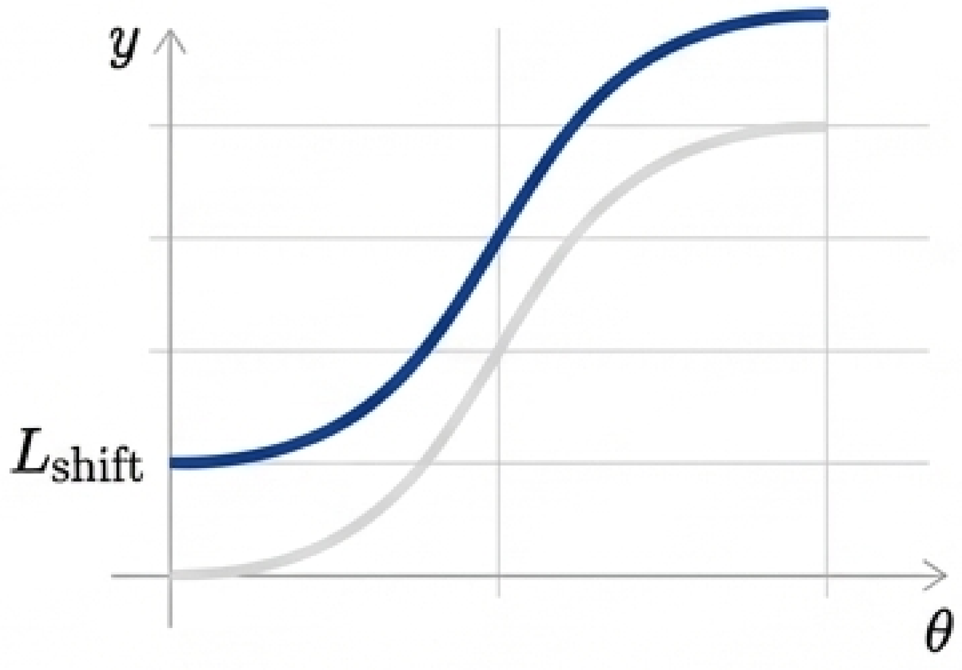
$$\theta \to (\theta - \theta_{\text{start}})$$



Vertical Shift (y-Axis)

To shift a function's baseline up by L_{shift}, add the offset to the function.

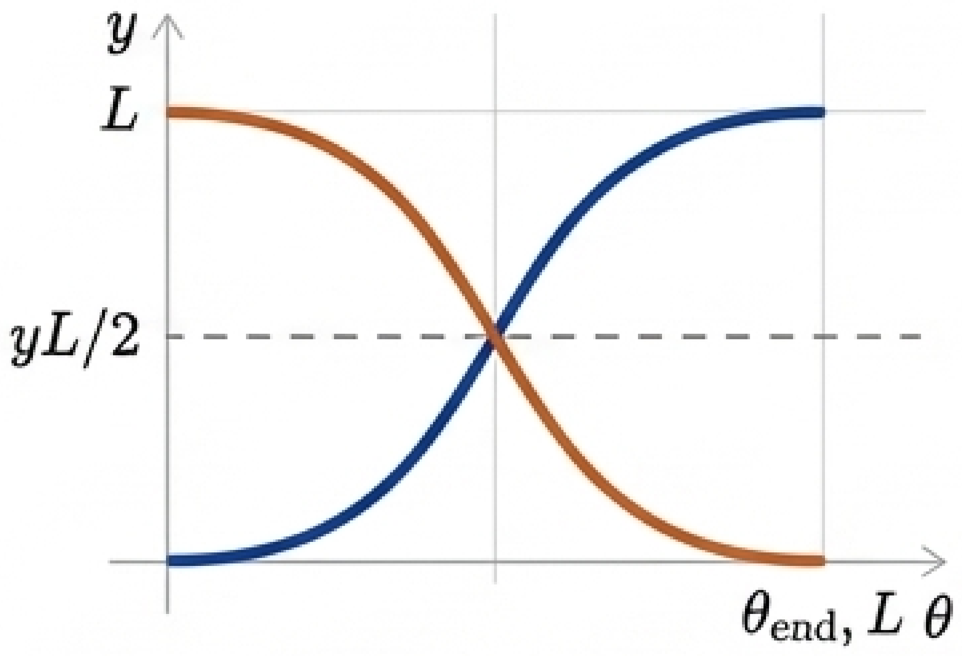
$$y \to y + L_{\text{shift}}$$



Inversion (Return Motion)

To create a symmetric return from a rise function of total lift L, use the inversion $L - y_{\text{rise}}$.

$$y_{\text{return}} \to L - y_{\text{rise}}$$



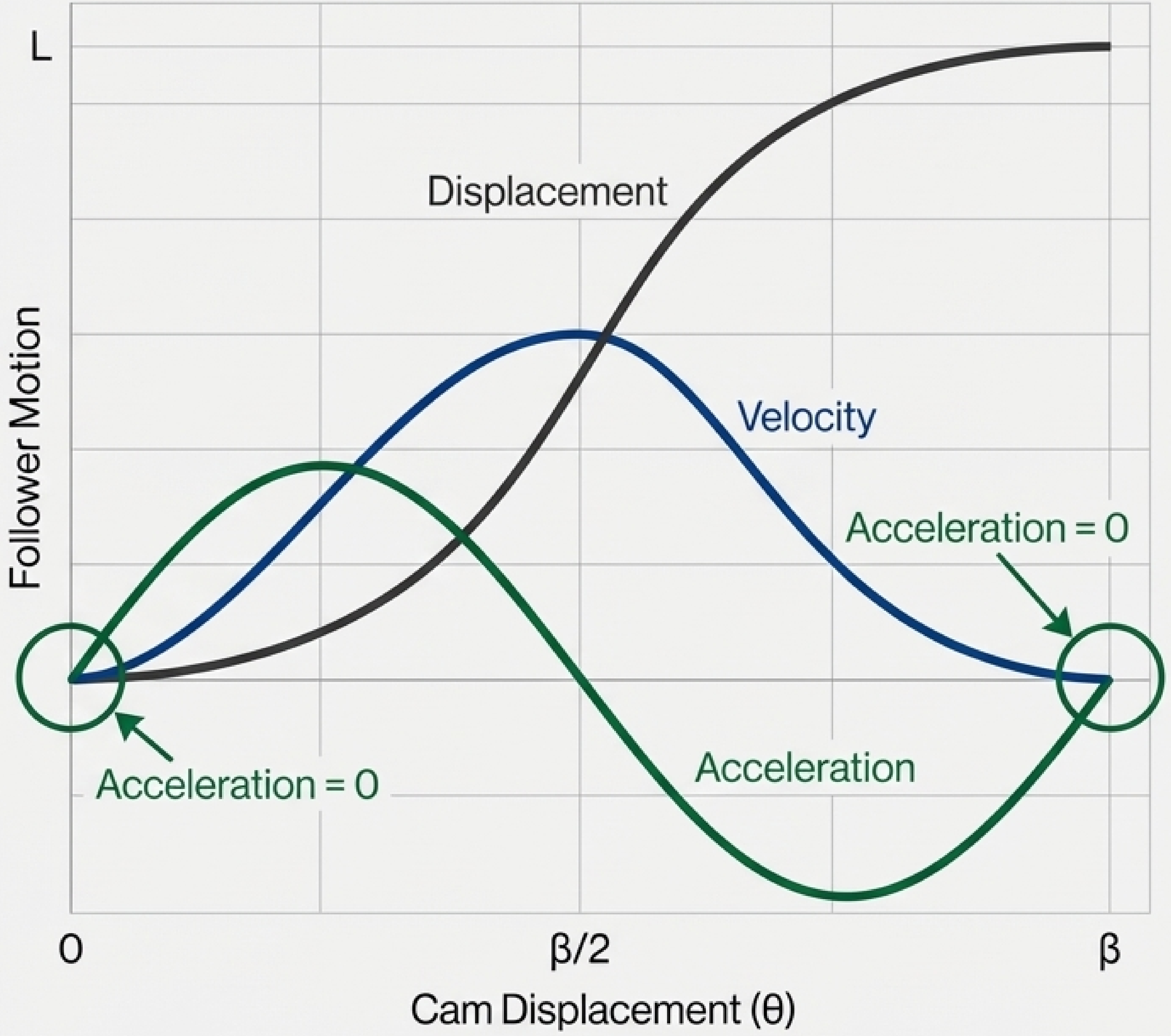
Base Component Spec Sheet: Cycloidal Motion

Key Characteristics

Cycloidal motion is a preferred profile for high-speed applications. Its primary advantage is providing **zero acceleration** at the transitions to and from dwells.

Dynamic Consequence

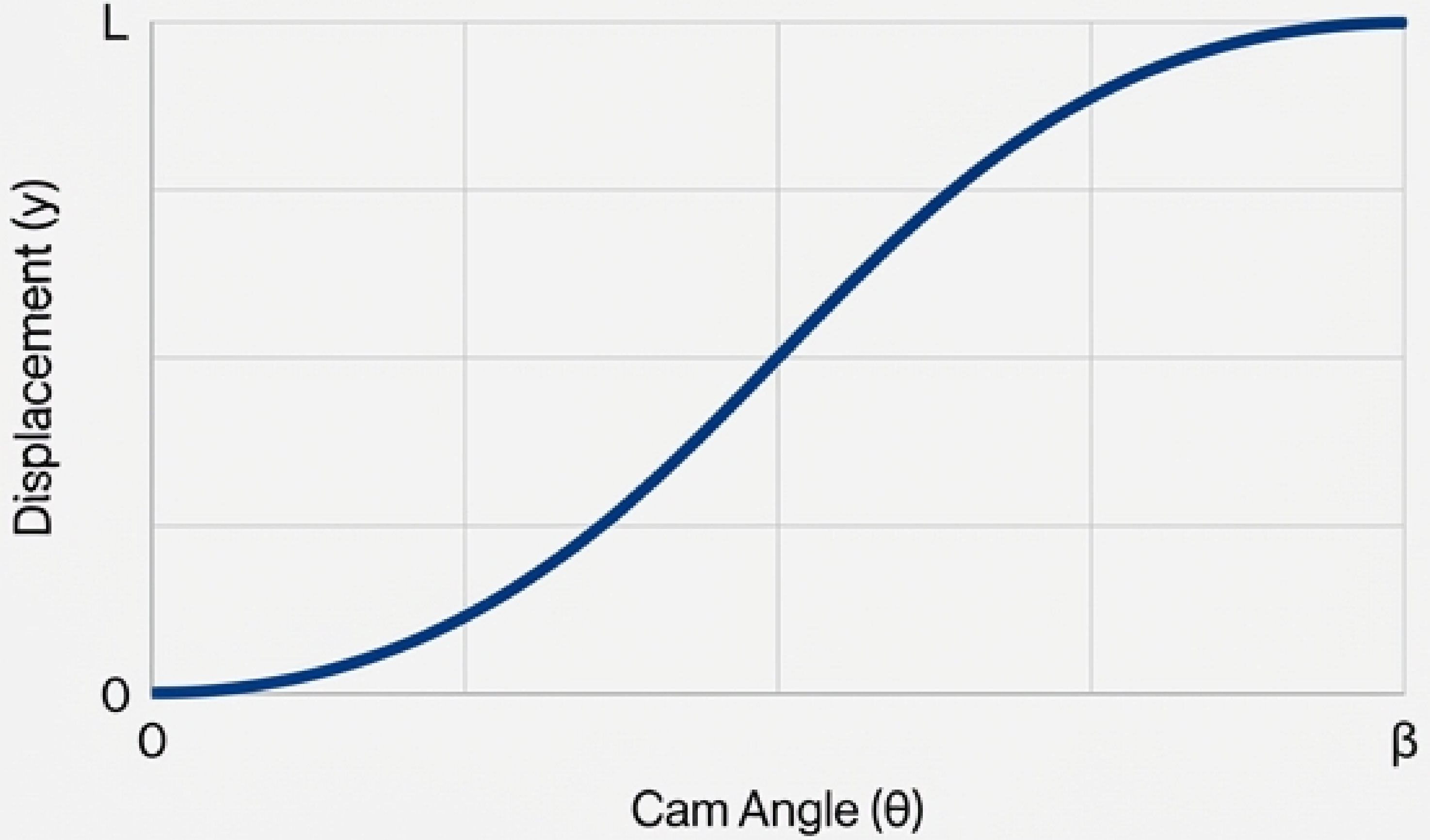
This ensures the jerk (the rate of change of acceleration) remains **finite** throughout the motion. The result is minimized shock and vibration, which is critical in high-speed systems. The acceleration curve is sinusoidal.



The Cycloidal Blueprint: Base Rise Formula

This is the standard, non-transformed equation for a follower displacement y rising from 0 to a total lift L over a cam rotation angle of β.

$$y_{\text{rise}}(\theta) = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\beta}\right)\right)$$



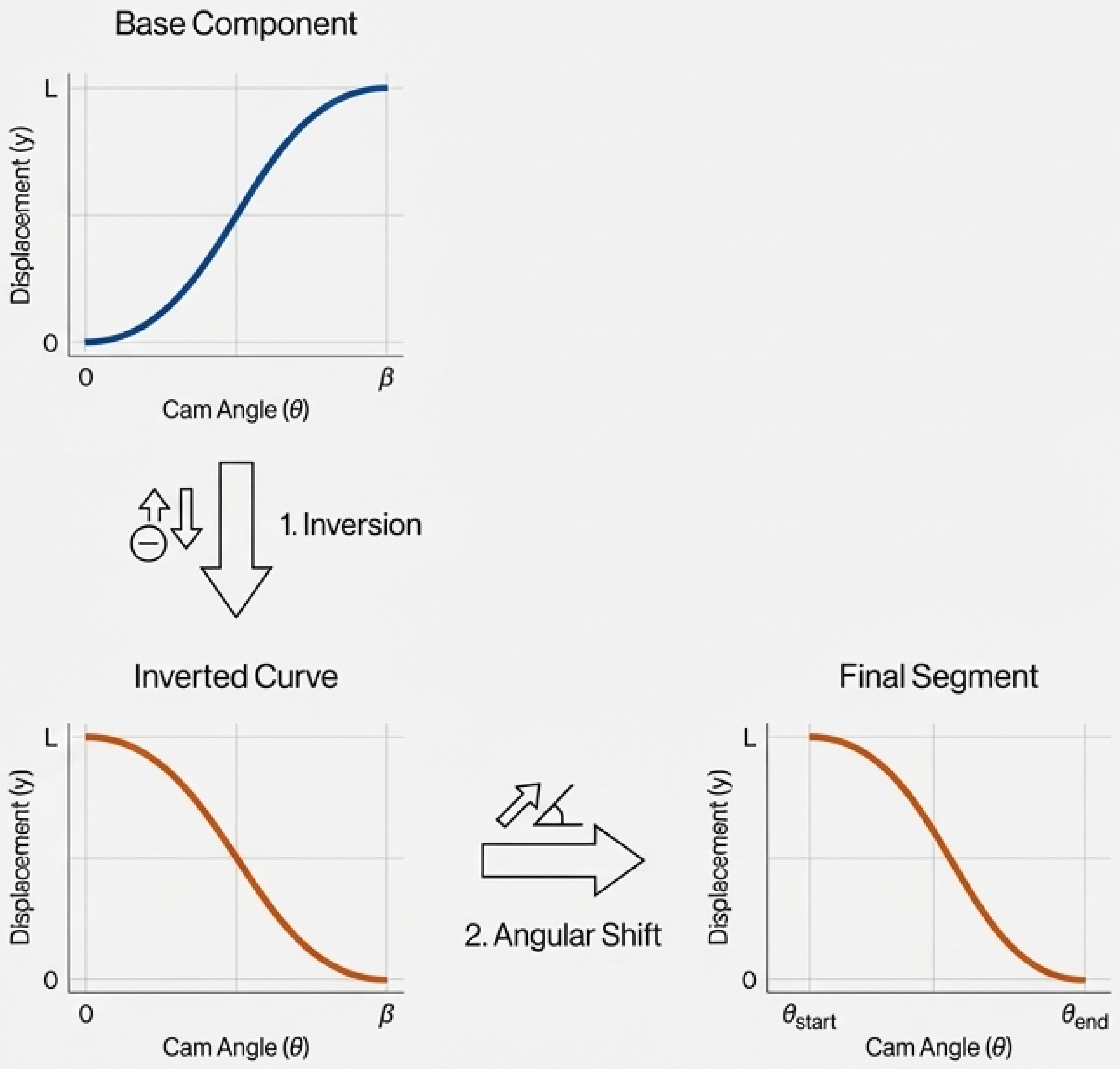
Application 1: Synthesizing a Shifted Cycloidal Return

Scenario

We need to create a cycloidal return segment of total lift L. The segment must start at ae cam angle θ_{start} and end at y=0. The duration is $\beta = \theta_{\text{end}} - \theta_{\text{start}}$.

Derivation Steps

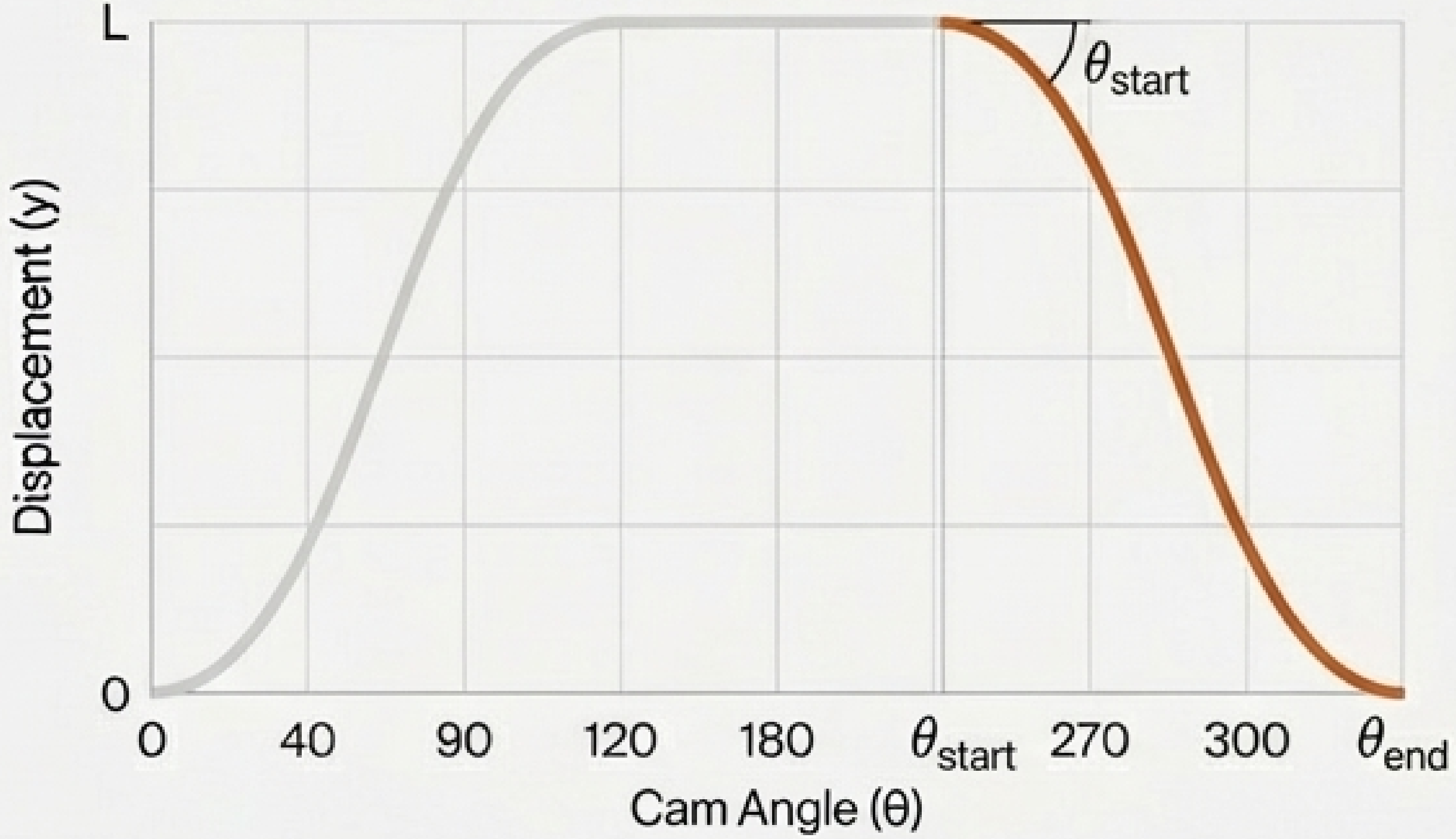
- **Select Tools:** We will use the Inversion tool for the return motion and the Angular Shift tool to place it correctly.
- **Apply Inversion:** The base y_rise function is transformed into a return function: L – y_rise(...).
- **Apply Angular Shift:** The angle variable θ inside the function is replaced with the relative angle: $(\theta - \theta_{\text{start}})$.



Final Equation for a Shifted Cycloidal Return

Combining the inversion and angular shift transformations on the base cycloidal function yields the complete analytical equation for the target segment.

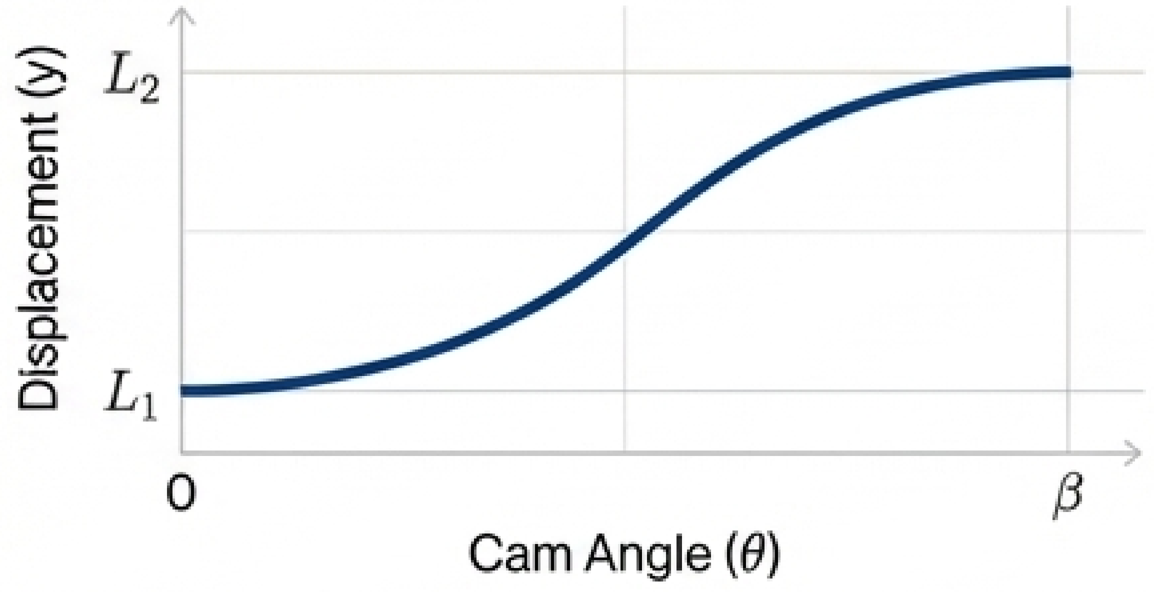
$$y_{\text{return}}(\theta) = L - L\left(\frac{(\theta - \theta_{\text{start}})}{\beta} - \frac{1}{2\pi}\sin\left(\frac{2\pi(\theta - \theta_{\text{start}})}{\beta}\right)\right)$$



Application 2: Connecting Two Non-Zero Dwells

Scenario

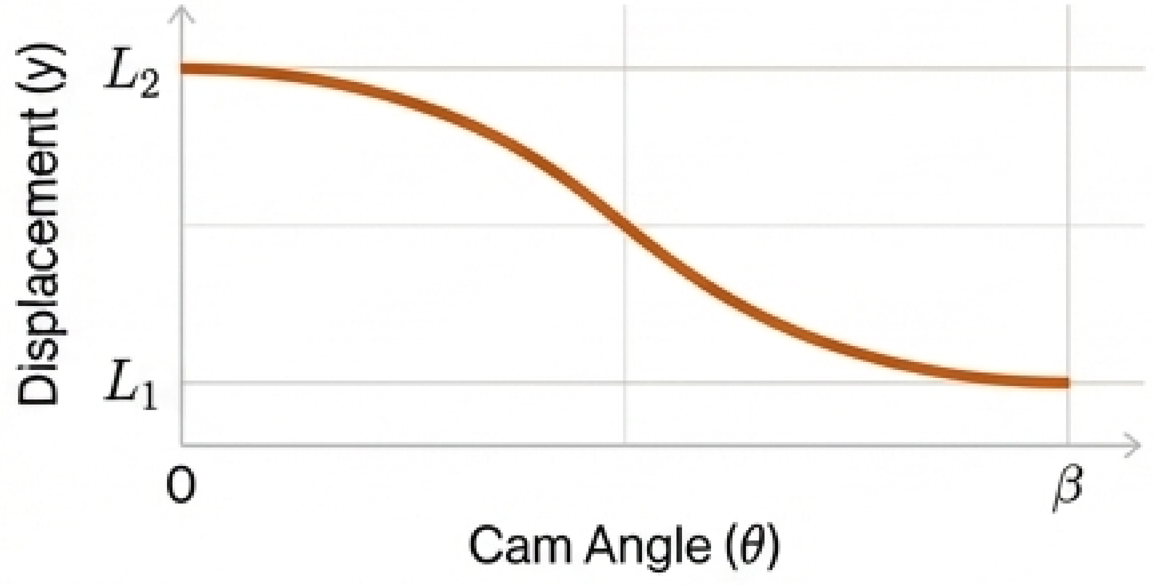
We now adapt the cycloidal function to connect two elevated dwells, starting at L_1 and ending at L_2. This requires the **Vertical Shift** tool. The amplitude of the motion is $\Delta L = L_2 - L_1$.



Formula A: Rise to a Higher Dwell (L_1 to L_2)

A standard cycloidal rise of amplitude ΔL is shifted vertically up by L_1.

$$y_{\text{rise}}(\theta) = L_1 + \Delta L \cdot y_{\text{std}}\left(\frac{\theta}{\beta}\right)$$



Formula B: Return to a Higher Dwell (L_2 to L_1)

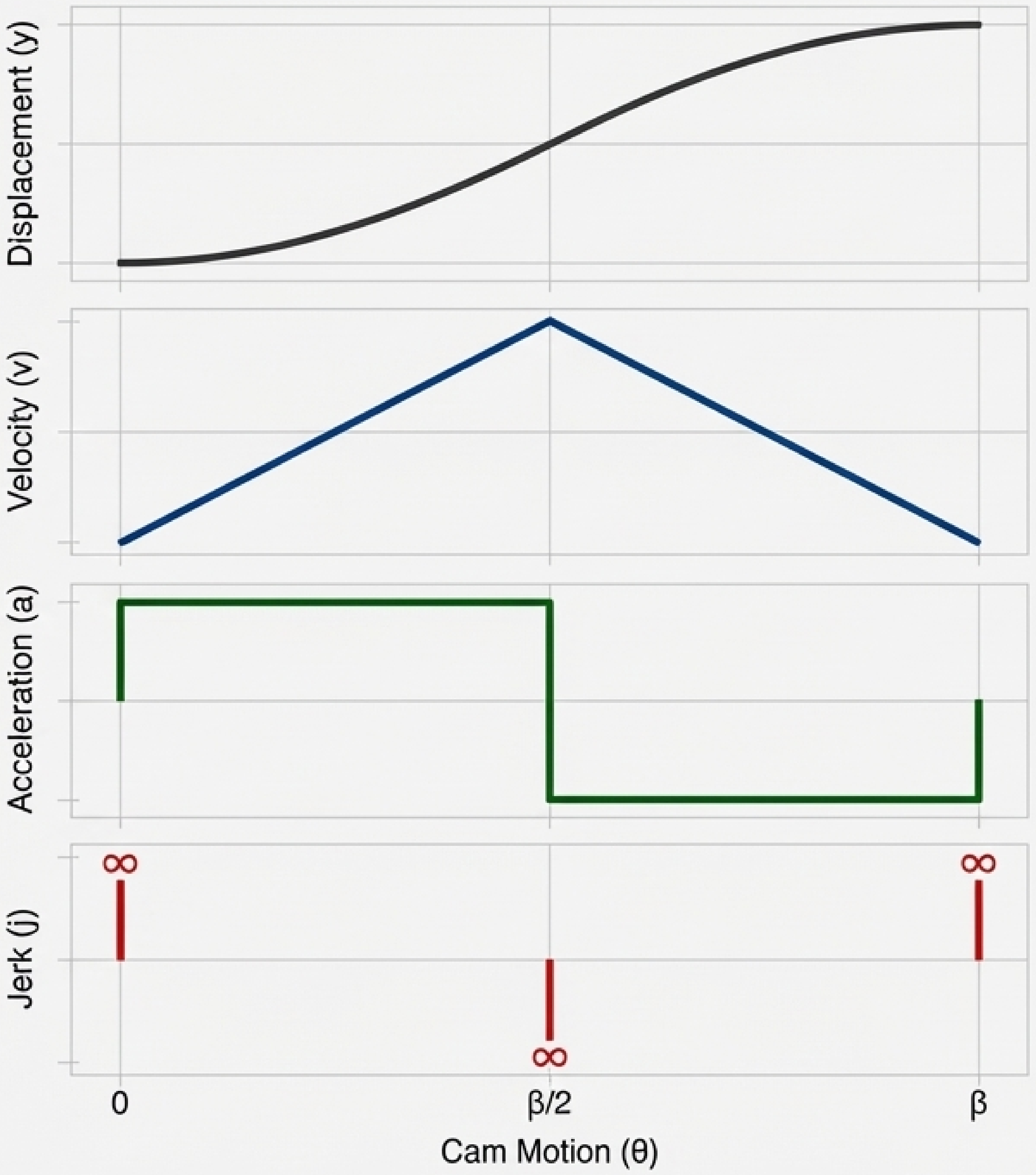
A standard return of amplitude ΔL is created using the inversion rule, then shifted vertically up by L_1.

$$y_{\text{return}}(\theta) = L_1 + (\Delta L - y_{\text{rise}}(\theta))$$

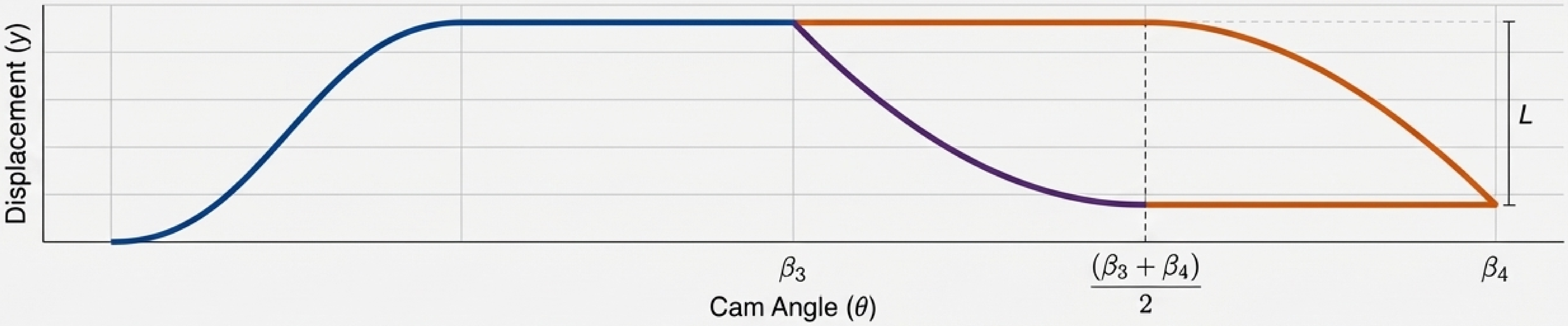
Alternative Component Spec Sheet: Parabolic Motion

Key Characteristics
This profile is synthesized from two piecewise parabolas, resulting in segments of constant acceleration and constant deceleration.

Dynamic Consequence
While velocity is continuous, the acceleration is discontinuous at the transitions ($\theta=0$, $\theta=\beta/2$, and $\theta=\beta$). This discontinuity causes the jerk to become infinite, exciting vibrations and making it unsuitable for high-speed applications.



Application 3: Universality on a Piecewise Function



Derivation Application: The coordinate transformation methodology is universal. Here, we apply the same Inversion and Angular Shift tools to the ***first half*** of a parabolic return segment.

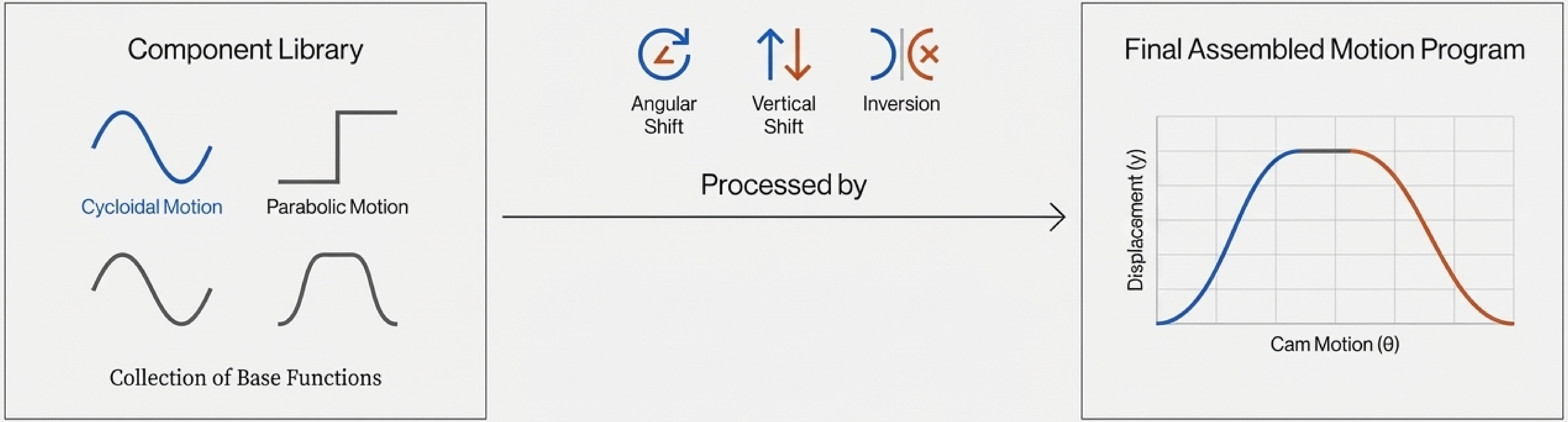
Scenario: The segment starts at angle β_3 and is part of a return with total duration $\beta = \beta_4 - \beta_3$.

Synthesized Parabolic Return Equation (First Half of Return):

$$y_{return} = L\left[1 - 2\left(\frac{\theta - \beta_3}{\beta_4 - \beta_3}\right)^2\right] \quad \text{for } \beta_3 \leq \theta \leq \frac{\beta_3 + \beta_4}{2}$$

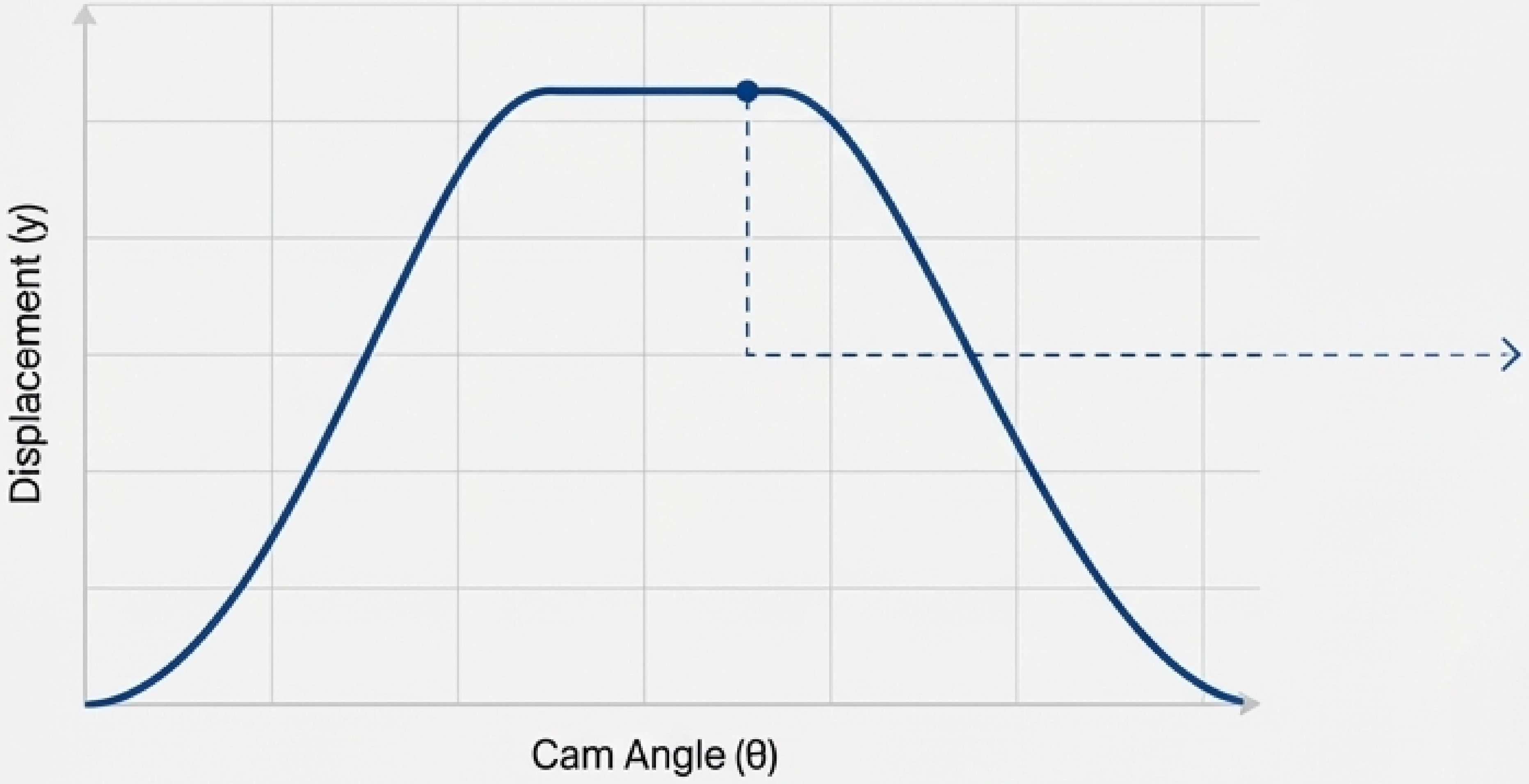
Analytical Synthesis: A Modular and Precise Methodology

The coordinate transformation toolkit is a powerful and universal method. It allows any analytically derived base function to be precisely mapped to any segment of a complex cam profile. This provides engineers with full analytical control over the follower's dynamic performance before any metal is cut.



The Critical Link: From Analytical Blueprint to Physical Reality

The rigorous analytical synthesis (Stage 1) is the blueprint for ideal dynamic performance. However, realizing these calculated benefits—such as the finite jerk of a cycloidal profile—is entirely dependent on the extreme manufacturing accuracy of the physical cam (Stage 2). The elegance of the math is only valuable if it can be realized with physical precision through processes like CNC milling and precision grinding.



Stage 1: Analytical Design

Stage 2: Physical Component