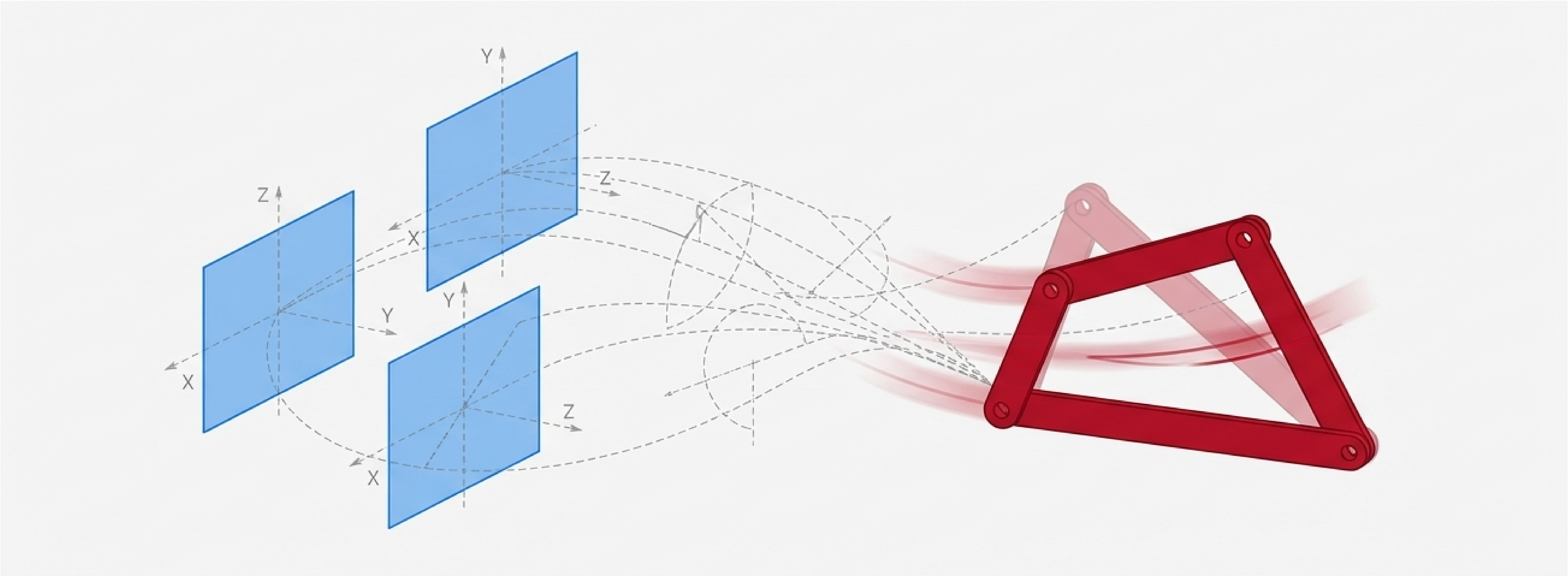


From Goal to Motion: Designing 4-Bar Linkages with Kinematic Synthesis

ME 3751: Kinematics and Mechanism Design

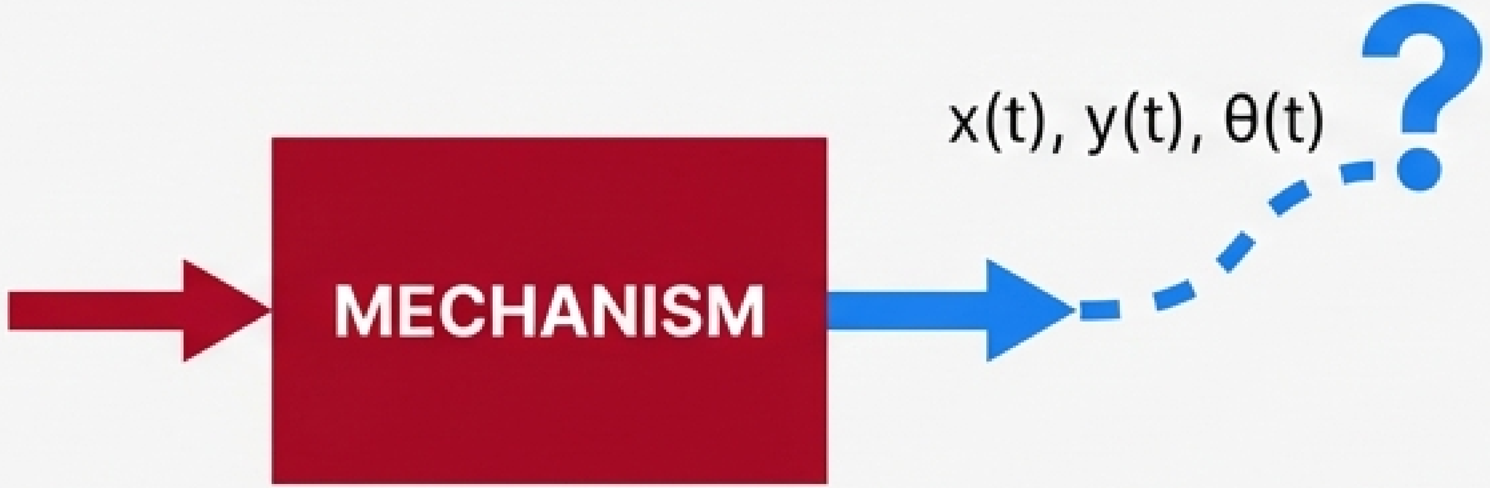
The Two Sides of Kinematics: Analysis vs. Synthesis

ANALYSIS

Given a mechanism, what is its motion?

Evaluate a known design against requirements.

A single, unique solution.

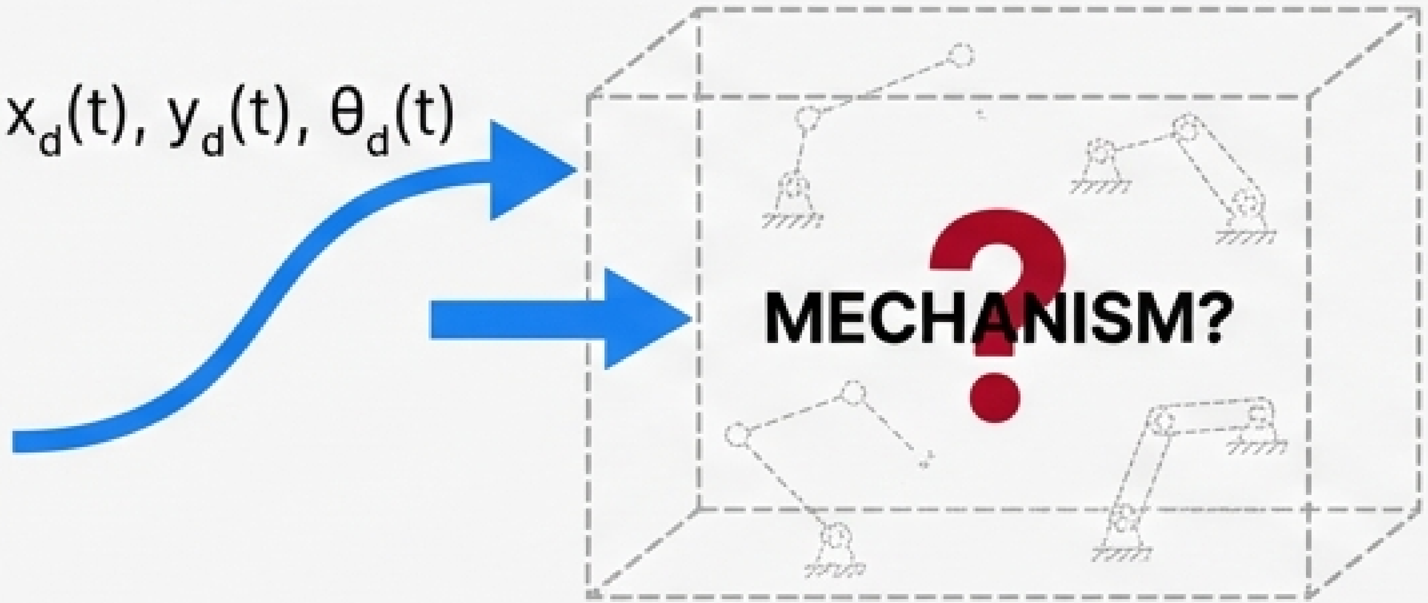


SYNTHESIS

Given a desired motion, what is the mechanism?

Innovate and create a new design to meet a goal.

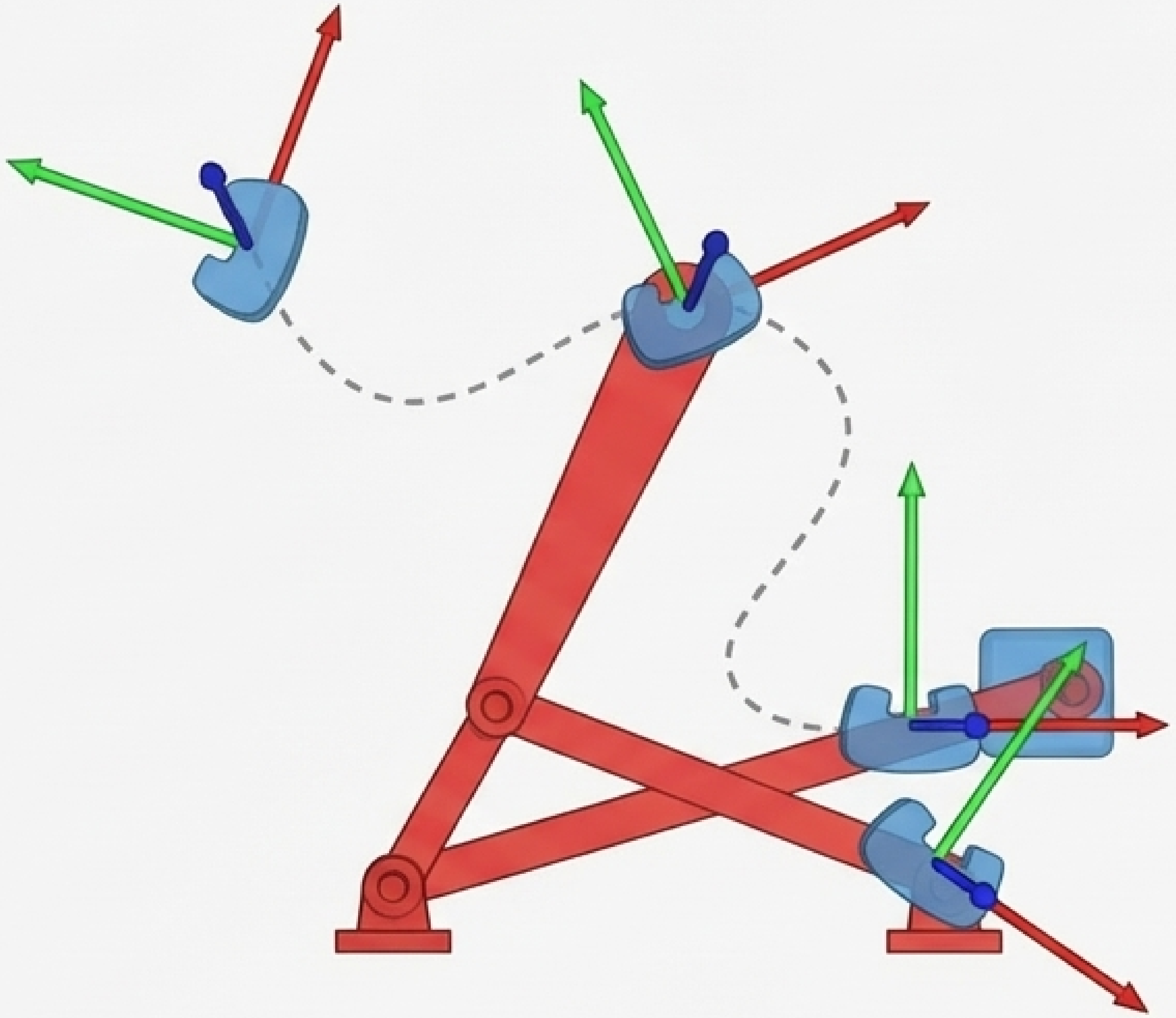
Multiple potential solutions; requires evaluating many candidates.



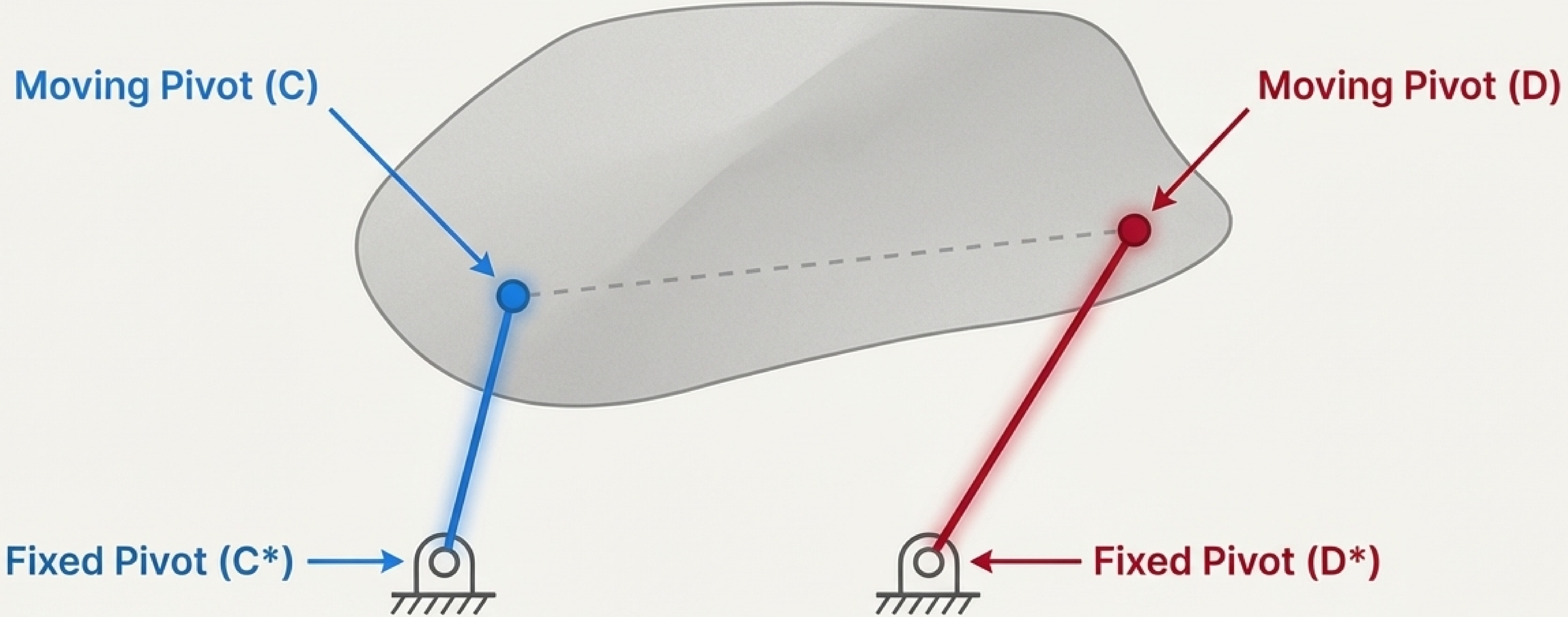
Our Design Quest: Rigid-Body Guidance

We want to design a mechanism that guides a coupler link through a series of pre-defined positions.

- Each position is defined by both **location** and **orientation**.
- The method guarantees the coupler passes *through* these "task positions," but the **path** between them is not specified.
- For a 4-bar linkage, we can specify a maximum of 5 task positions. We will focus on the common case of 3 positions.



The Key Insight: A 4-Bar Linkage is Two Independent Dyads



The entire synthesis process hinges on this idea: We can design the input and output dyads completely independently and then connect them with the coupler link.

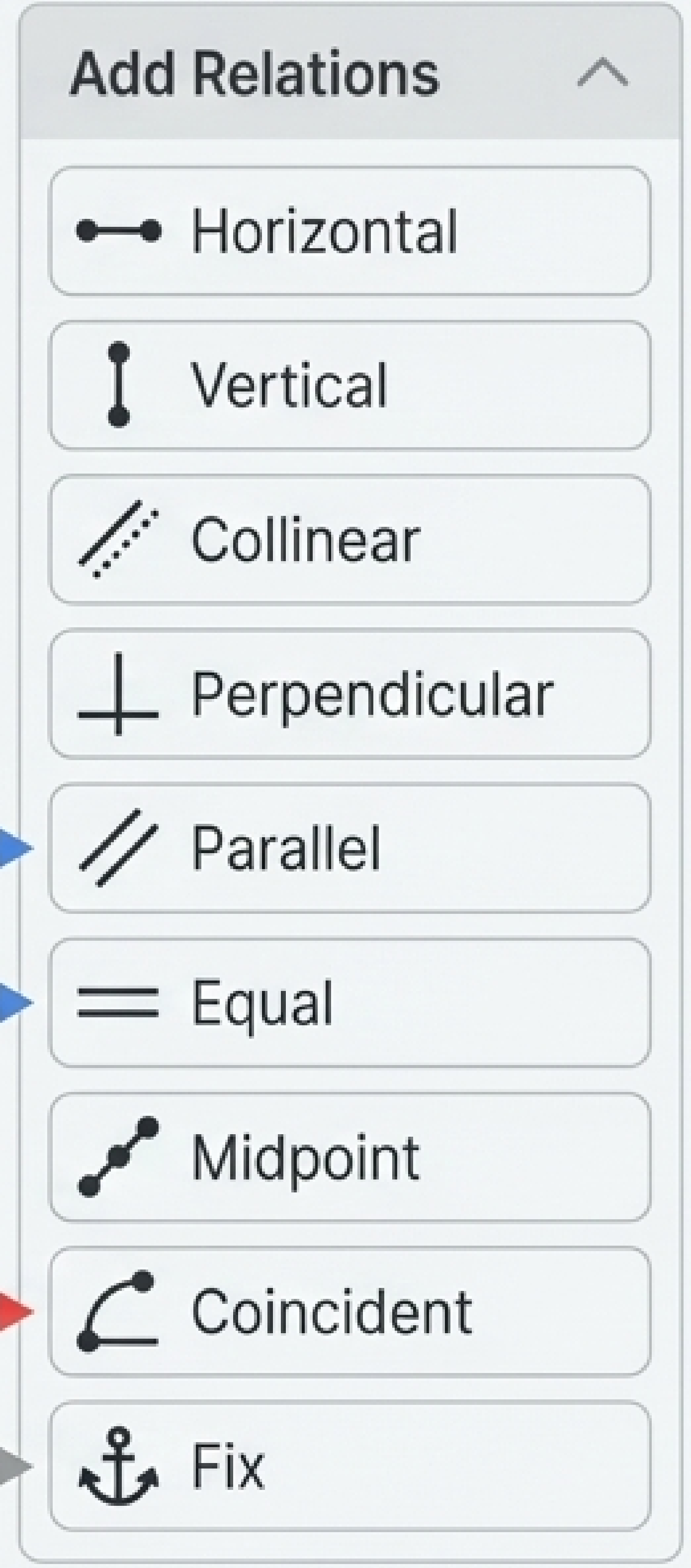
The Toolkit: Geometric Constraint Programming (GCP)

Modern CAD programs provide a powerful framework for solving complex geometry problems with high precision.

- We work in a 2D sketch mode.
- Geometric objects (lines, circles, points) are represented by parametric equations.
- **Constraints** are additional equations that define geometric relationships.

Key Constraints We Will Use:

- **Coincident:** A point is required to lie on a line or arc.
- **Equal:** Two or more lines are constrained to be the same length.
- **Fix:** A point or line is locked in position.
- **Parallel / Perpendicular:** Defines the orientation between two lines.



The Blueprint: A 3-Position Synthesis Problem

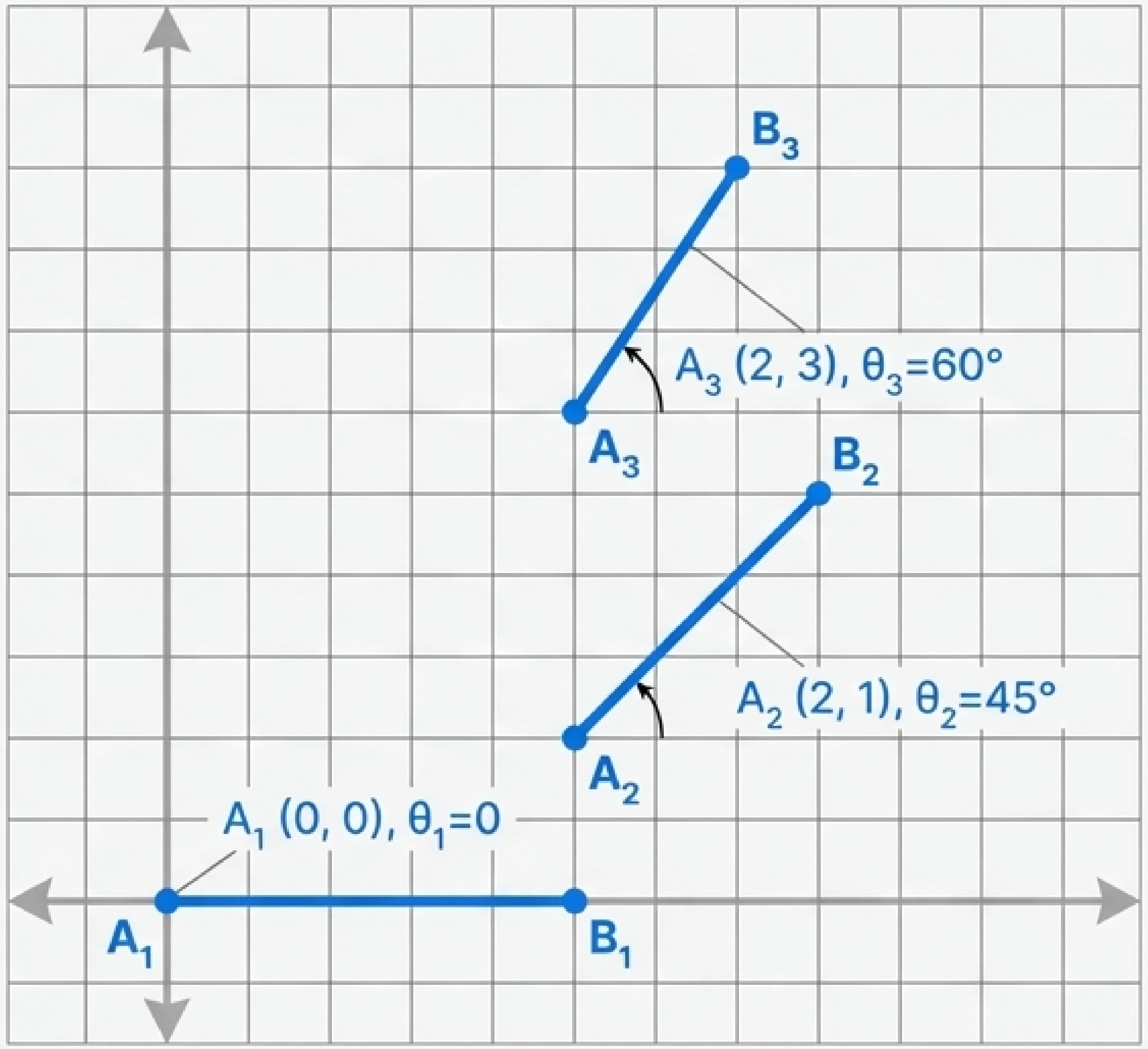
Design a four-bar linkage to guide a **coupler link** (represented by the line segment AB) through the three positions shown.

`Coupler reference length AB = 1.25 cm`

Position 1*: A_1 at (0, 0), $\theta_1 = 0°$

Position 2*: A_2 at (2, 1), $\theta_2 = 45°$

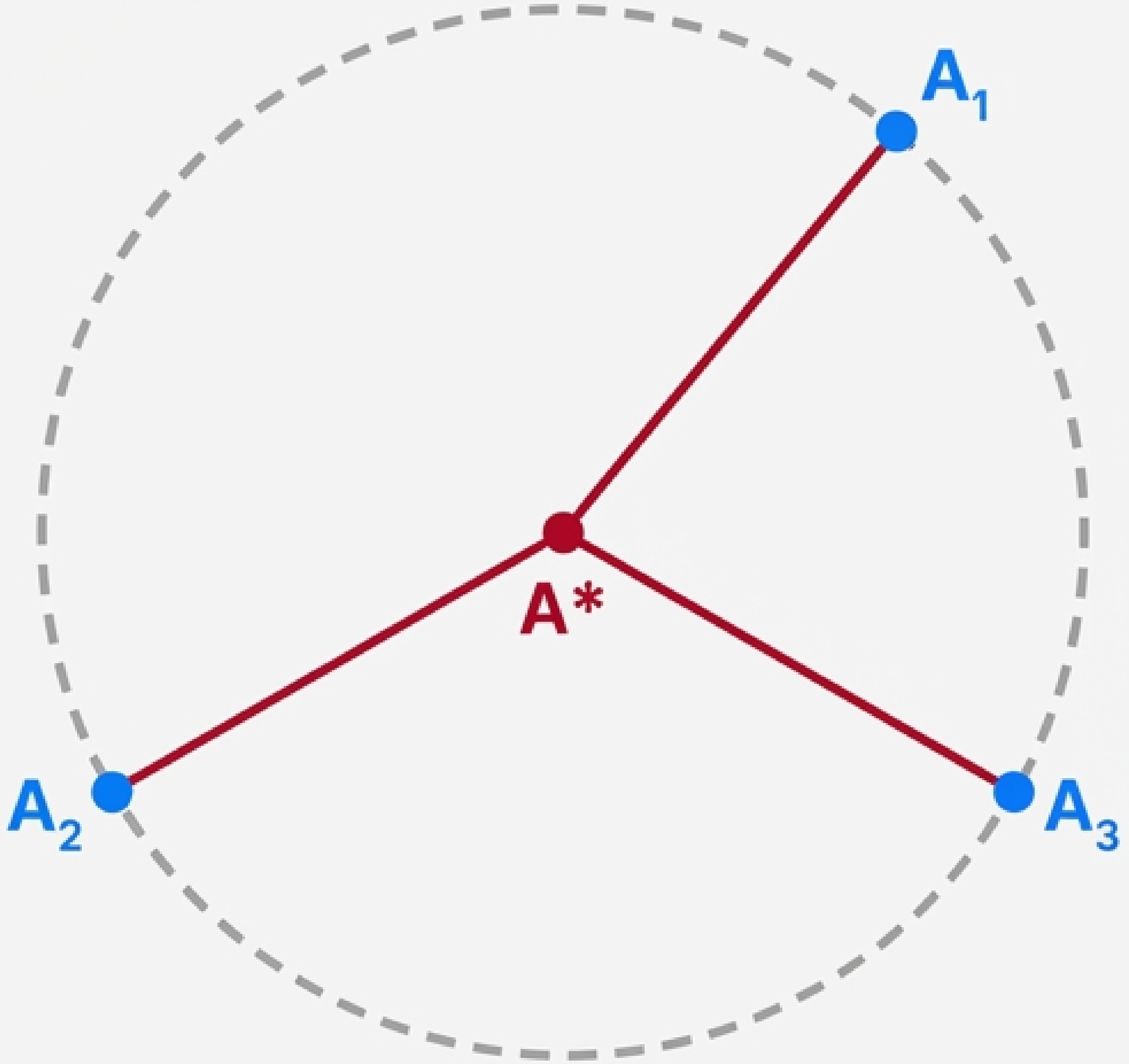
Position 3*: A_3 at (2, 3), $\theta_3 = 60°$



Phase 1: Synthesizing the Input Dyad

The Logic

1. We need to find a fixed pivot **A*** such that the input link **A*A** can guide the coupler.
2. The moving pivot **A** is a point on the coupler. In our three positions, it is at A_1, A_2, and A_3.
3. Since the input link **A*A** has a constant length, the distance from the fixed pivot **A*** to each of the moving pivot's positions must be the same: $|A^*A_1| = |A^*A_2| = |A^*A_3|$.
4. Geometrically, this means the points A_1, A_2, and A_3 must all lie on a circle. The center of this circle is our fixed pivot, **A***.



The Method

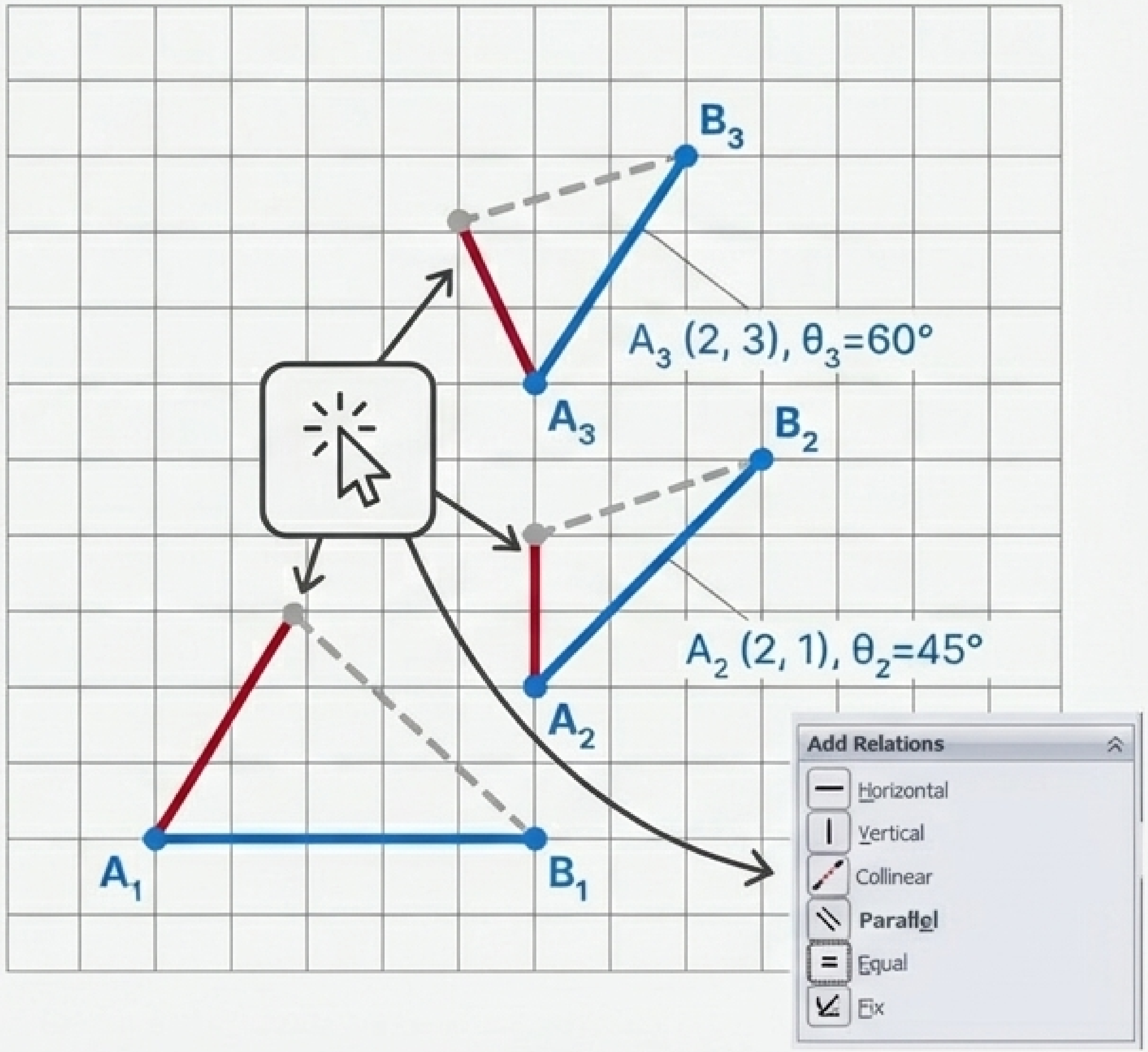
We will use a geometric construction to find the center of a circle that passes through three points.

Step 1A: Construct Congruent Triangles

In a new sketch, for each of the three defined coupler positions (A_1B_1, A_2B_2, A_3B_3), we construct an identical triangle. The vertex of this triangle represents the location of the moving pivot we are trying to define.

1. Draw three triangles, one attached to each of the coupler positions. For this dyad, we will orient them ‘below’ the coupler lines.
2. Select the corresponding sides of the three triangles (e.g., all three left sides).
3. Apply an ‘Equal’ constraint to make the side lengths identical.
4. Repeat for the other corresponding sides.

Key Check: Ensure the triangles have the same orientation and are not mirrored images.



NotebookLM

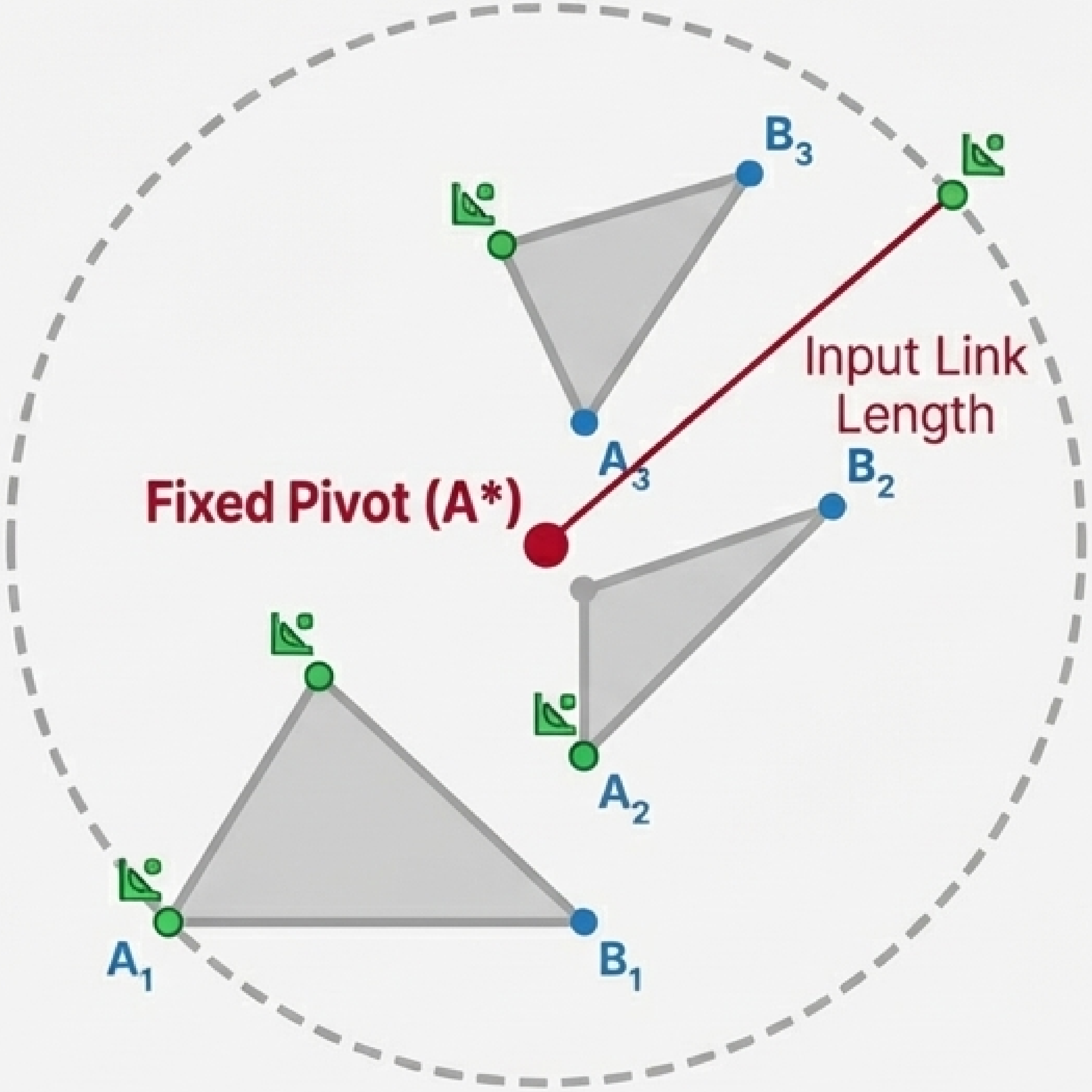
Step 1B: Find the Input Circle

Now that we have the three positions of our moving pivot (the vertices of the triangles), we can find the fixed pivot.

1. Draw a construction circle of arbitrary size and location.
2. Select the circle and the vertex of the first triangle. Apply a **'Coincident'** constraint.
3. Repeat for the second and third triangle vertices. The CAD solver will automatically resize and relocate the circle so that all three points lie on its circumference.

Result

- The **center** of this circle is the fixed pivot **A***.
- The **radius** of this circle is the length of the input link.



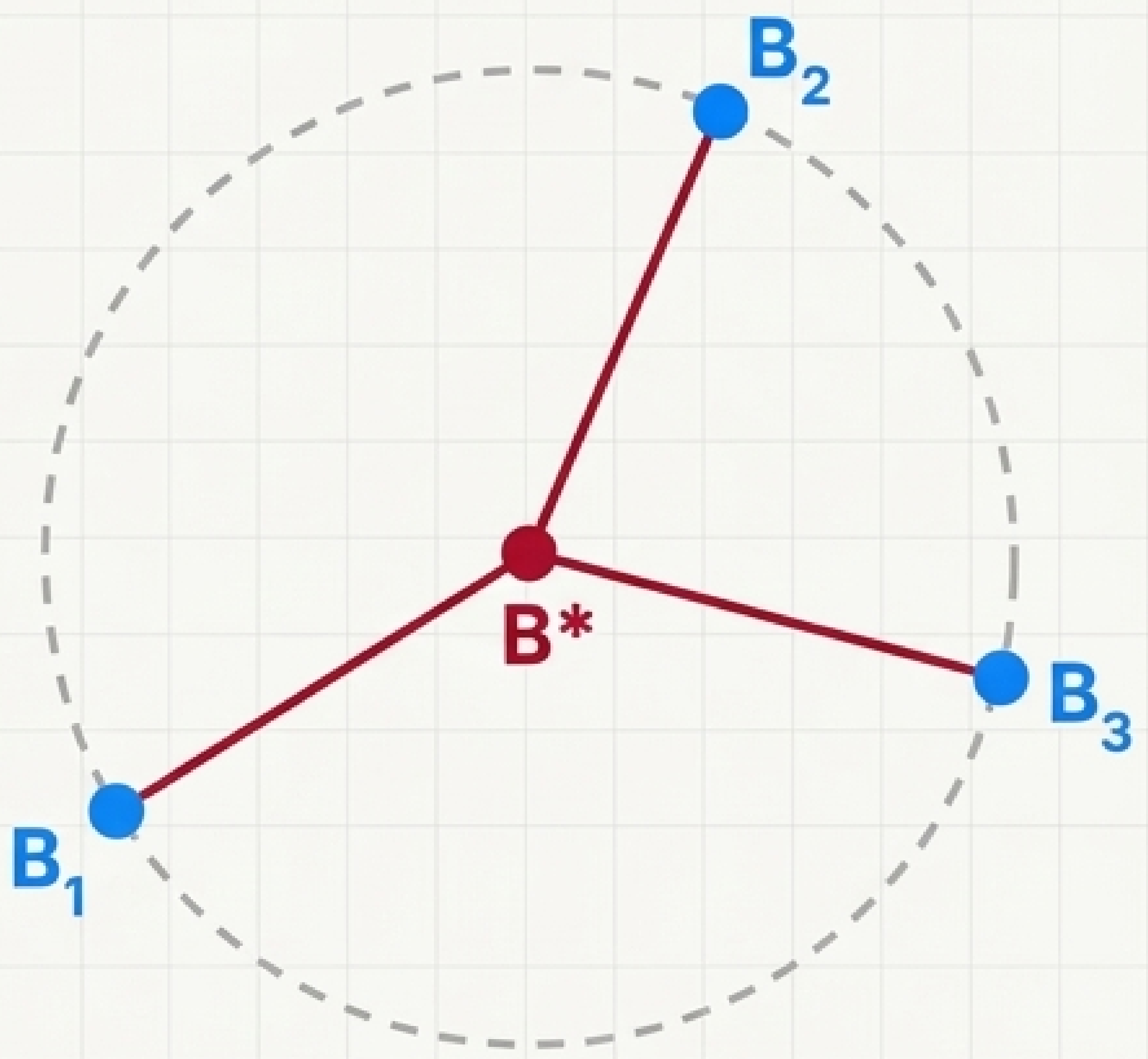
Phase 2: Synthesizing the Output Dyad

The Logic

The process is identical to the input side, but now we find the fixed pivot B* for the moving pivot B.

1. The moving pivot B is at positions B_1, B_2, and B_3.
2. The output link B*B must have a constant length, so: $|B^*B_1| = |B^*B_2| = |B^*B_3|$.
3. Therefore, B_1, B_2, and B_3 must lie on a circle whose center is the fixed pivot, B*.

We will again use the congruent triangle construction, but this time connected to points B_1, B_2, and B_3, to find the center of the output circle.



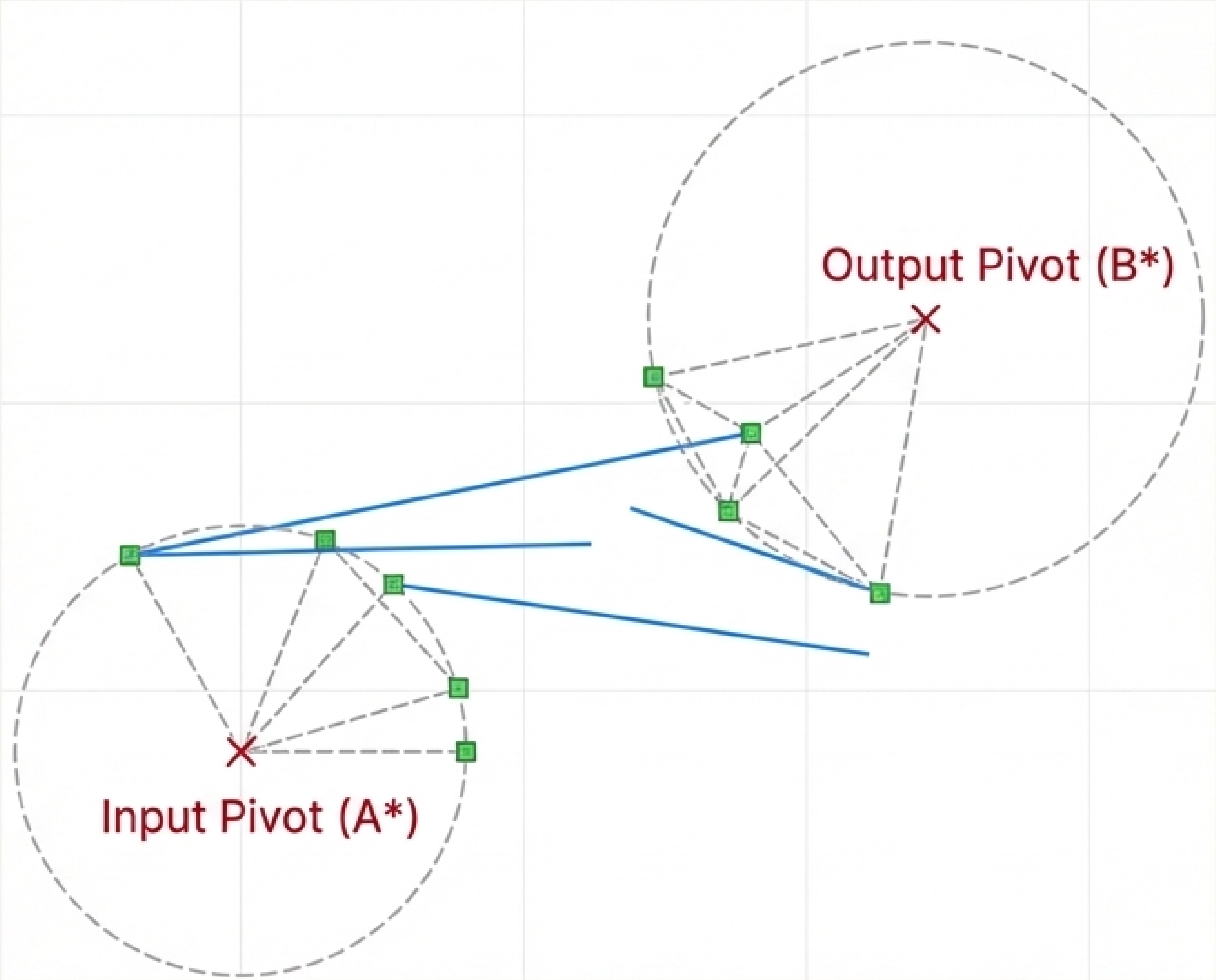
Step 2 in Practice: Finding Both Pivots

Action: Repeat the construction process (congruent triangles -> coincident circle) for the B-side of the coupler.

- **Note:** The triangles for the output side can be drawn with a different orientation (e.g., “above” the coupler lines) to distinguish them.

Result: The sketch now contains two circles.

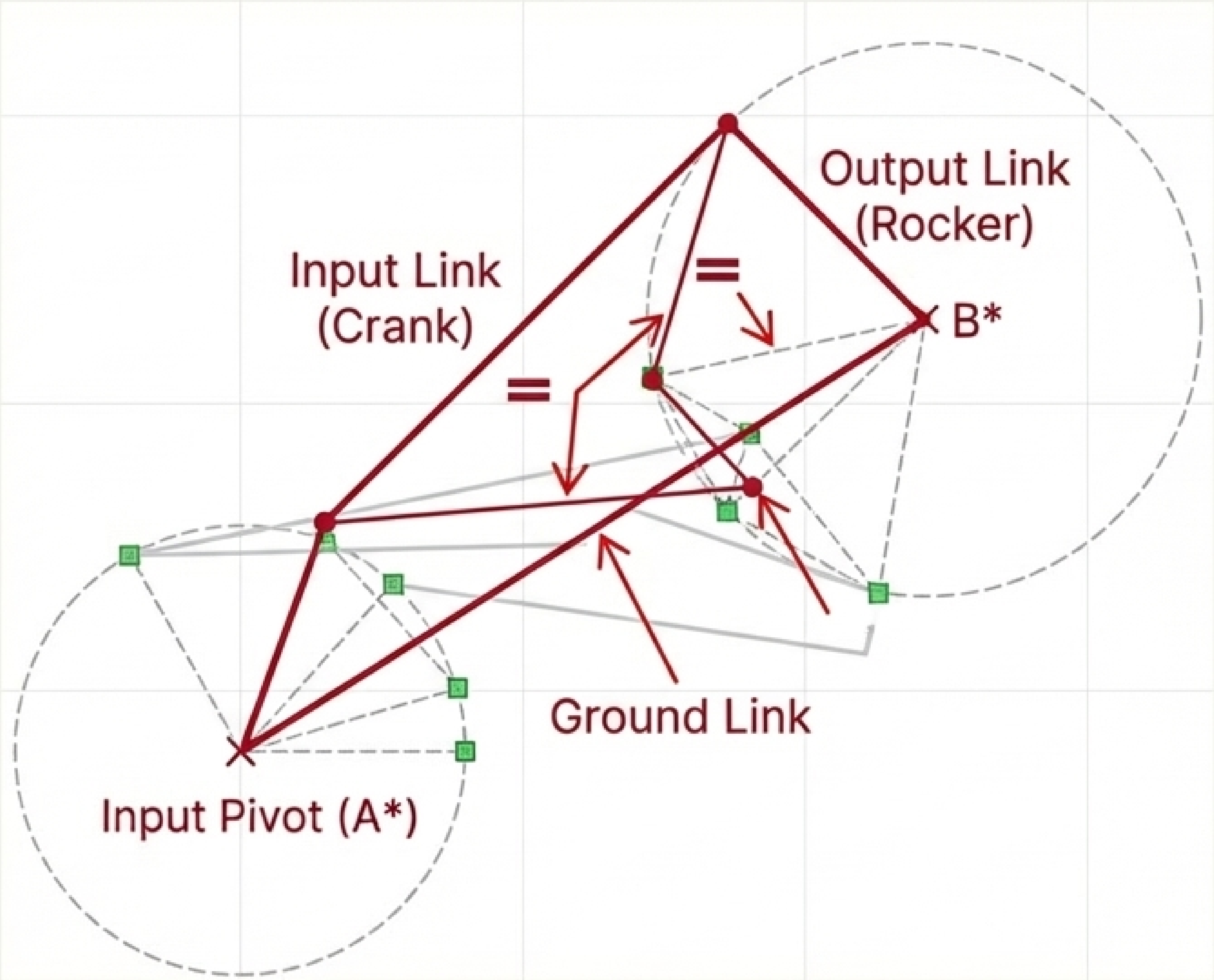
- **Input Circle**: Center is the input fixed pivot **A***.
- **Output Circle**: Center is the output fixed pivot **B***.



Phase 3: Assembling the Final Linkage

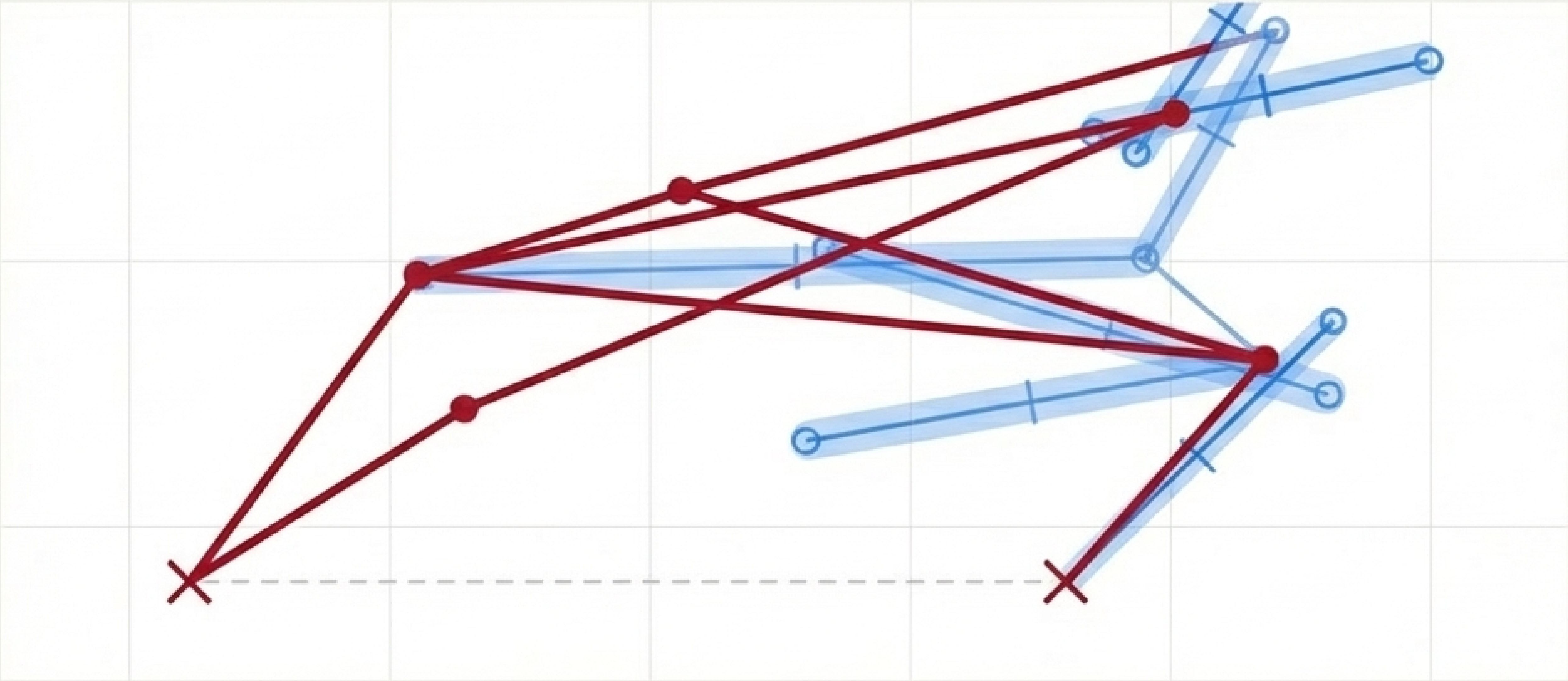
Action: In a new, clean sketch, we draw the mechanism defined by our synthesis.

1. **Ground Link**: Draw a line connecting the fixed pivots, **A*** to **B***.
2. **Input Link (Crank)**: Draw a line from **A*** to a point on the input circle.
3. **Output Link (Rocker)**: Draw a line from **B*** to a point on the output circle.
4. **Coupler Link:** Draw the coupler triangle, connecting the endpoints of the input and output links. The lengths of the coupler sides are defined by the construction triangles from our previous sketches. We use "Equal" constraints to link them back to the original construction geometry.



Verification: Hitting the Target Positions

By rotating the input link, we can check if the coupler passes through the three specified task positions.

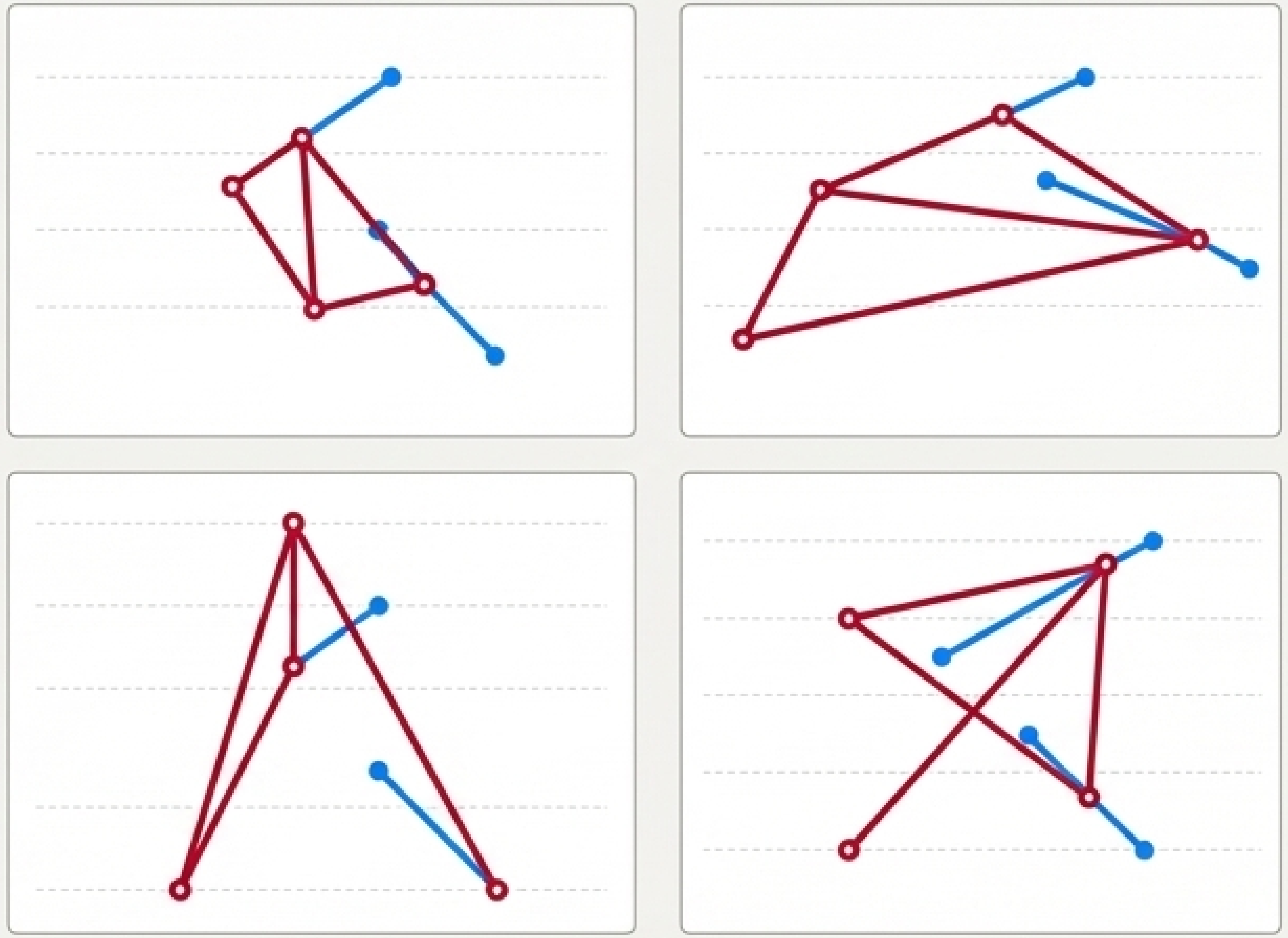


Success! The synthesized linkage guides the coupler through all three required positions and orientations.

Exploring the Multiverse of Solutions

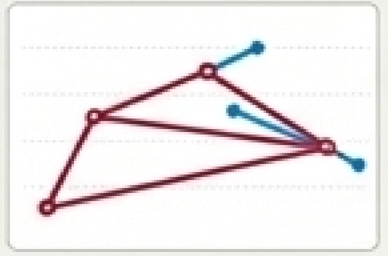
The three-position synthesis does not produce a unique solution. We had free choices in our construction (specifically, the shape of our construction triangles and the location of the output pivot).

- By changing these free parameters, we can generate an infinite number of different 4-bar linkages that all satisfy the same three task positions.
- A designer can explore this “solution space” to find a mechanism that is optimal for other criteria like size, transmission angle, or obstacle avoidance.



Synthesis is the Starting Point of Design

Kinematic synthesis allows you to move from a desired **function** to a physical **form**.



The dyad method provides a systematic, graphical approach to designing 4-bar linkages for motion generation.

Geometric Constraint Programming (GCP) in modern CAD makes this process precise, fast, and interactive.

Most importantly, synthesis reveals a **space of possible solutions**, empowering the engineer to explore trade-offs and select the truly optimal design.