ME 3751: Kinematics and Mechanism Design

Kinematic Synthesis of 4-Bar Linkages: A Modern Approach using Geometric Constraint Programming (GCP)

Designing Crank-Rocker Mechanisms for Specified Motion and Performance

From Analysis to Synthesis: The Two Sides of Kinematics

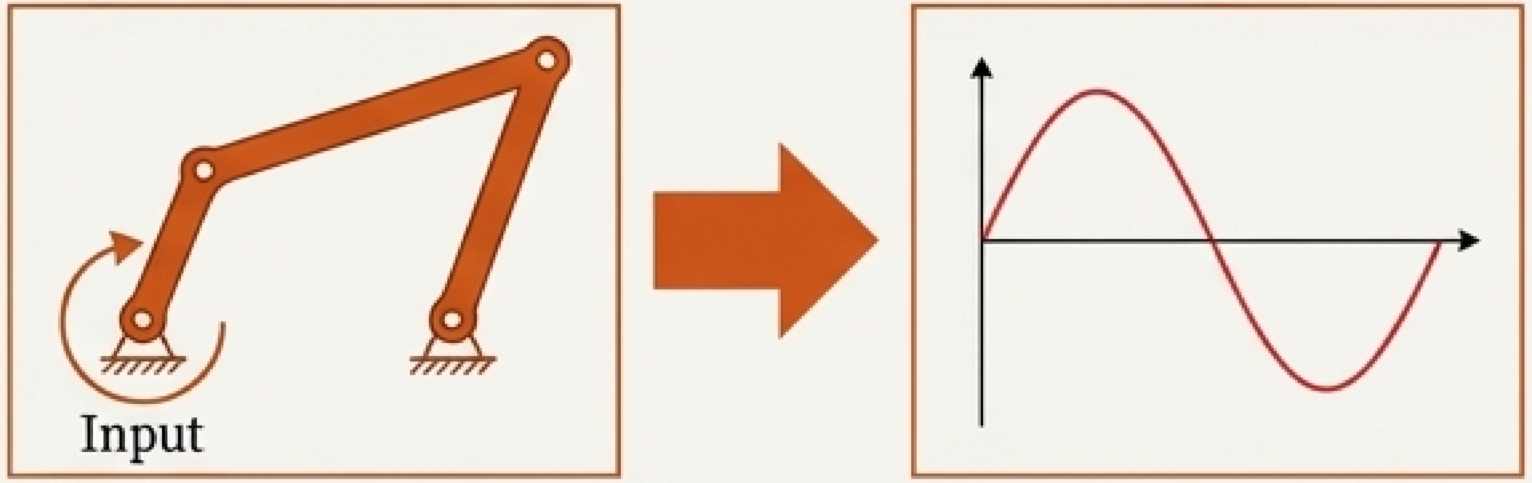
Analysis (Evaluating a Design)

Given a mechanism, find its output motion versus its input motion.

You are given the Mechanism and the Input Motion, and you solve for the Output Motion. A unique, predictable solution.

You are evaluating a design against requirements.

[Mechanism + Input] [? Output]



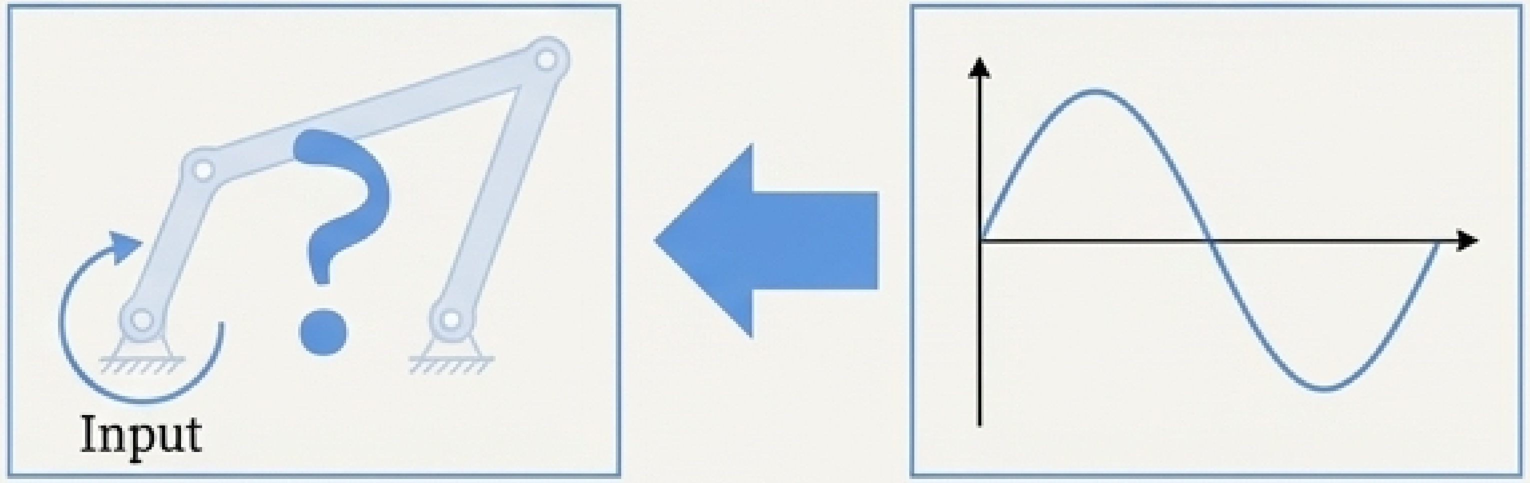
Synthesis (Creating a Design)

Given a desired output motion, assume an input motion and find the mechanism(s) that can produce it.

You are given the desired Output Motion and the Input Motion, and you solve for the Mechanism itself.

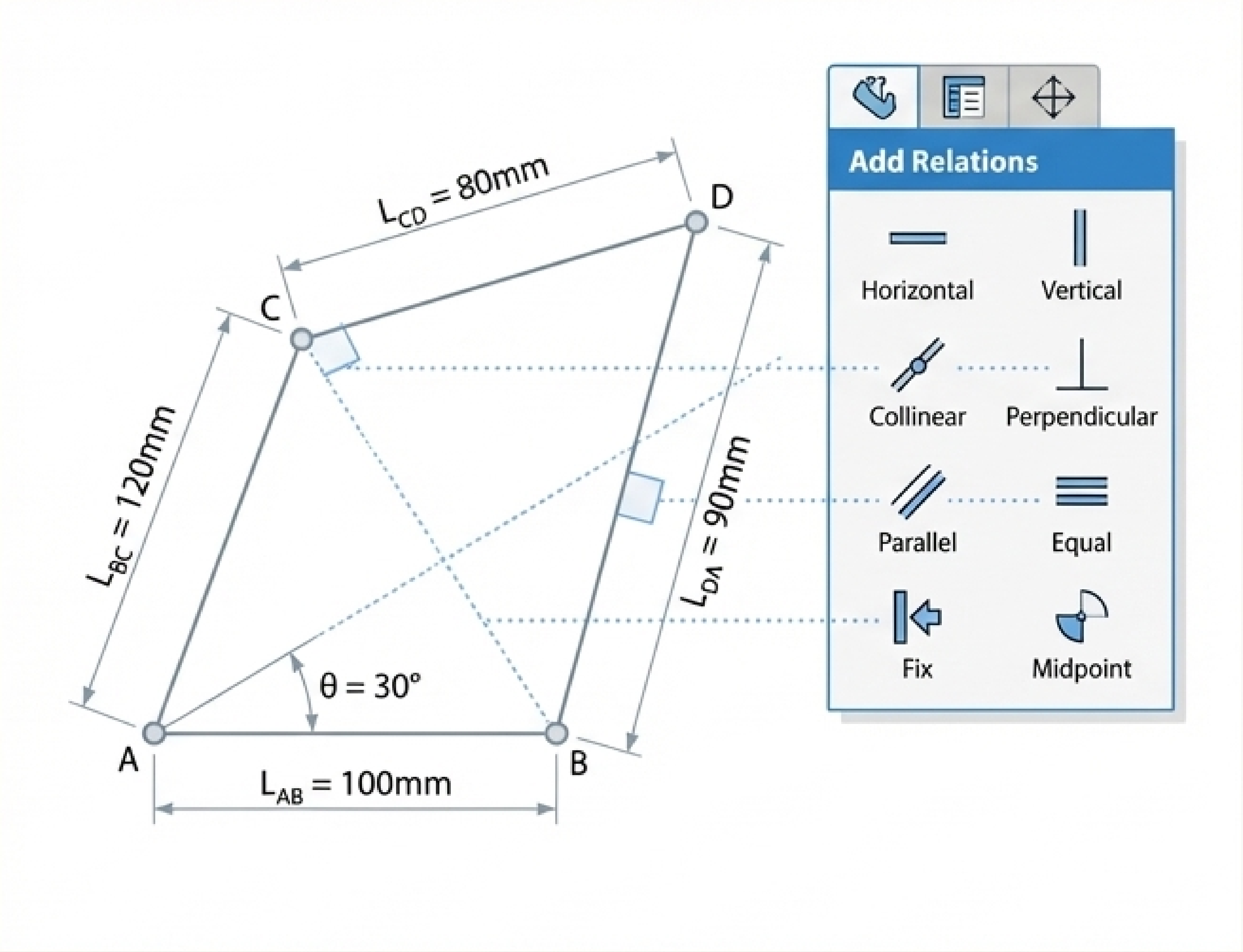
This is the heart of innovation and creative thinking. It often yields multiple solutions, requiring the evaluation of many design candidates.

[? Mechanism + Input] [Output]



The Modern Toolkit: Geometric Constraint Programming (GCP)

Modern CAD programs provide a powerful framework for solving complex synthesis problems with high precision. We will use SolidWorks to apply these principles.

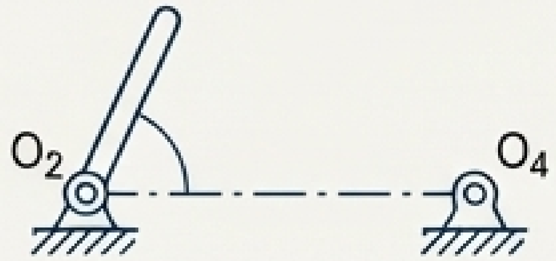


How it Works

- Parametric design programs represent geometry as **systems of equations.** When you change a dimension, the system re-solves.
- **Constraints** are additional equations that define geometric conditions like parallelism, perpendicularity, coincidence, and equality.
- These programs contain powerful non-linear equation solvers that can handle large, complex systems to define geometry with digital accuracy.

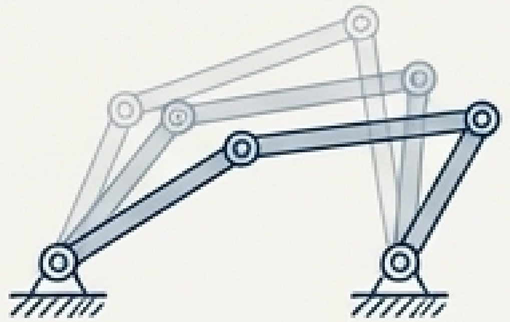
The Kinematic Synthesis Workflow: A Four-Step Guide

A repeatable methodology for designing linkages to meet specific motion requirements using GCP:

1 

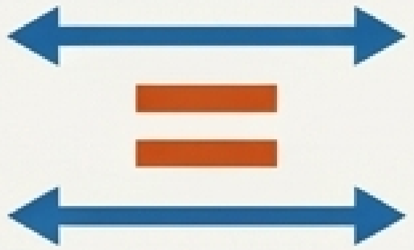
Define Knowns & Ground

Establish your fixed pivots (ground link). Graphically construct all specified positions of the known links (e.g., input and/or output rockers at each required angle).

2 

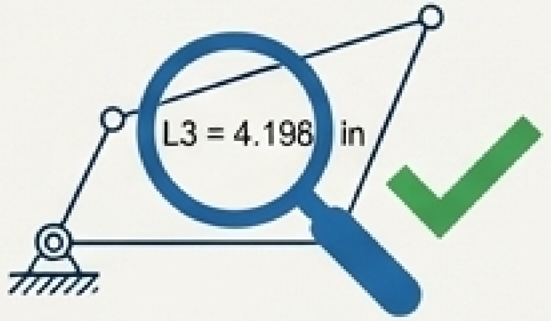
Construct the Linkage Instances

Draw the complete linkage for each specified position. Some link lengths will be unknown.

3 

Apply Equality Constraints

This is the core of synthesis. Constrain all corresponding links to be equal in length across all positions. Let the CAD solver find the solution.

4 

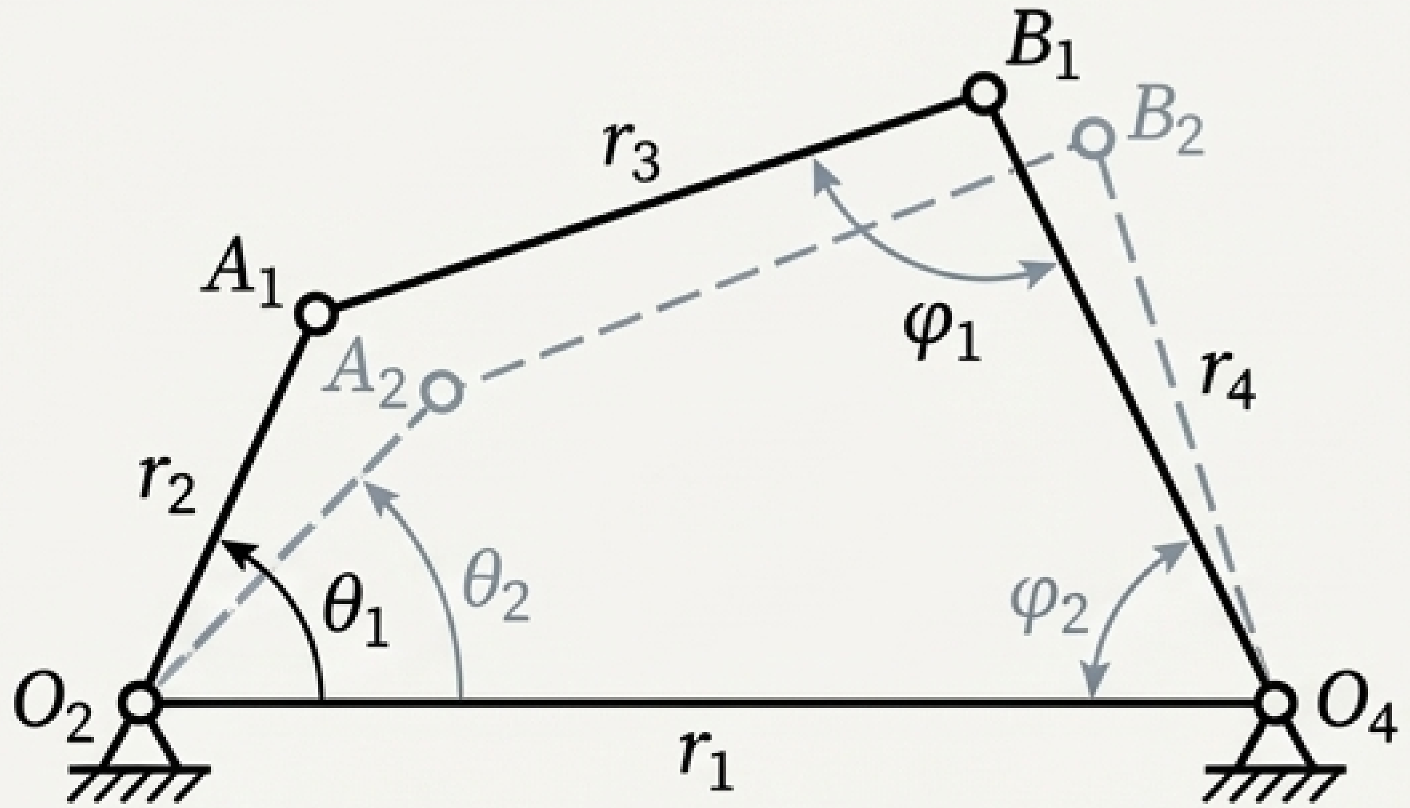
Extract & Verify

Read the now fully-defined dimensions from your sketch. Build a new, separate linkage using these dimensions to test and verify the motion.

Foundation: Two-Position Double Rocker Synthesis

The Design Problem

- **Given:** Two angular positions of an input link (θ_1 and θ_2) and the two corresponding positions of the output link (φ_1 and φ_2). We are also often given the length of one link (e.g., the ground link).
- **Find:** The lengths of the other three links (r_2, r_3, r_4) that satisfy this motion requirement.



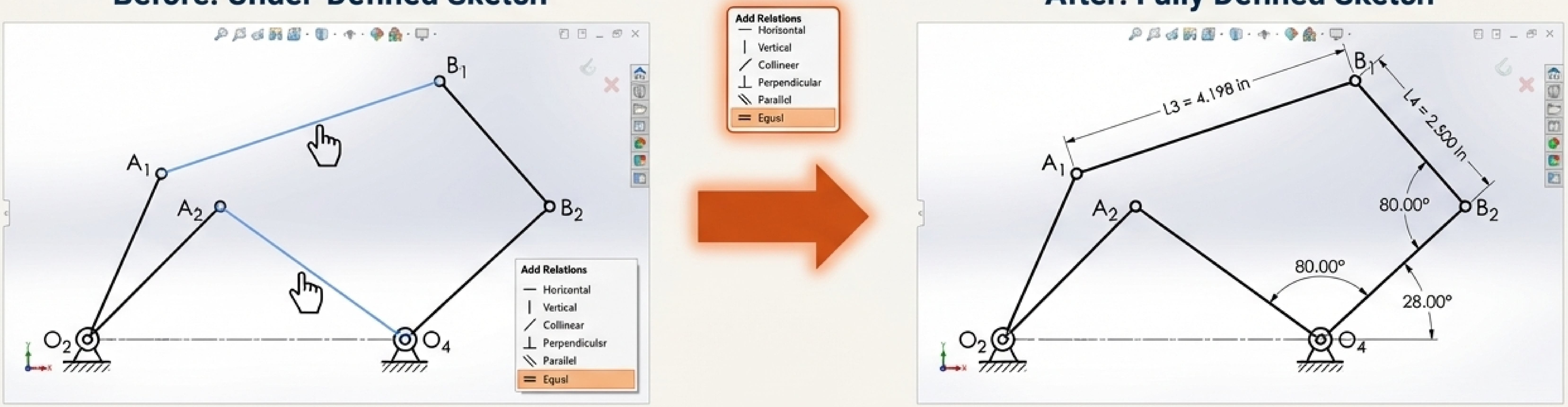
Real-World Application: The complex motion of a backhoe bucket is often governed by a 4-bar linkage designed using these principles.

Two-Position Synthesis: The GCP Solution

Two-Position Synthesis: A rigid-body linkage does not change. In defined run the two-position, tanitiy form constraint, connecting its **input** hur bar keys in rat chage on the equality constraint.

Before: Under-Defined Sketch

After: Fully Defined Sketch



The Key Insight

A rigid-body linkage does not change its dimensions during motion. Therefore, the **coupler link must have the same length in both positions**. This is the core constraint that allows us to solve for the unknown geometry.

Procedure in SolidWorks

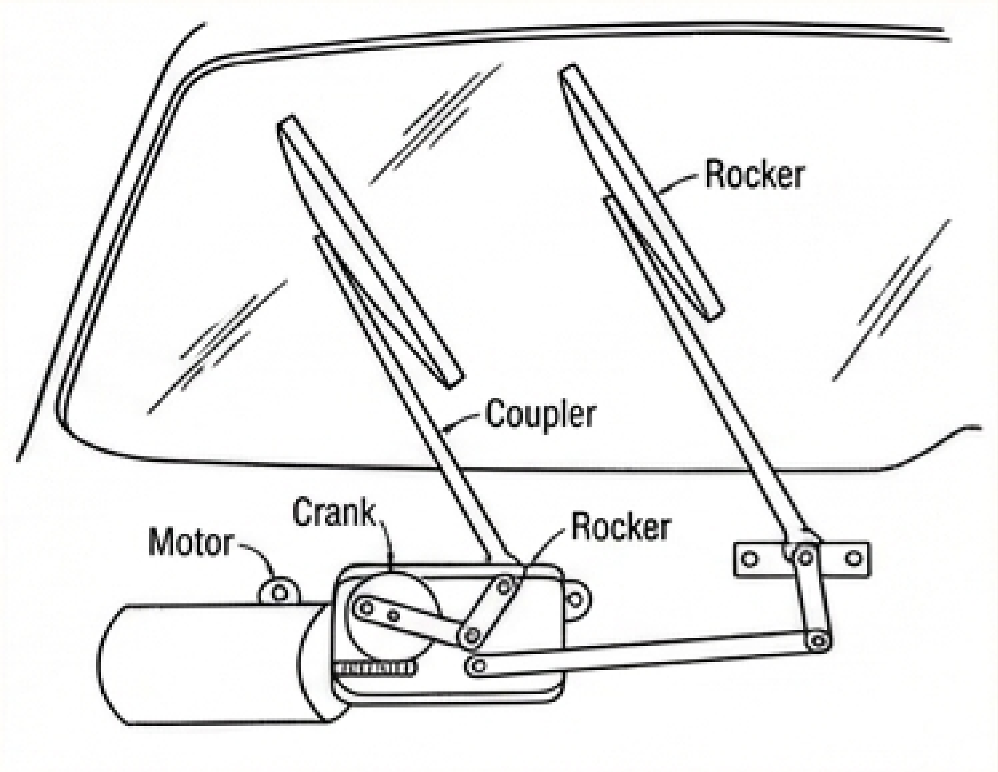
1. Construct the known ground pivots and the input/output links at both specified positions.
2. Draw the coupler link connecting the input and output for Position 1 (A_1B_1) and Position 2 (A_2B_2). The sketch is under-defined.
3. Select both coupler lines and apply the **Equal** constraint.

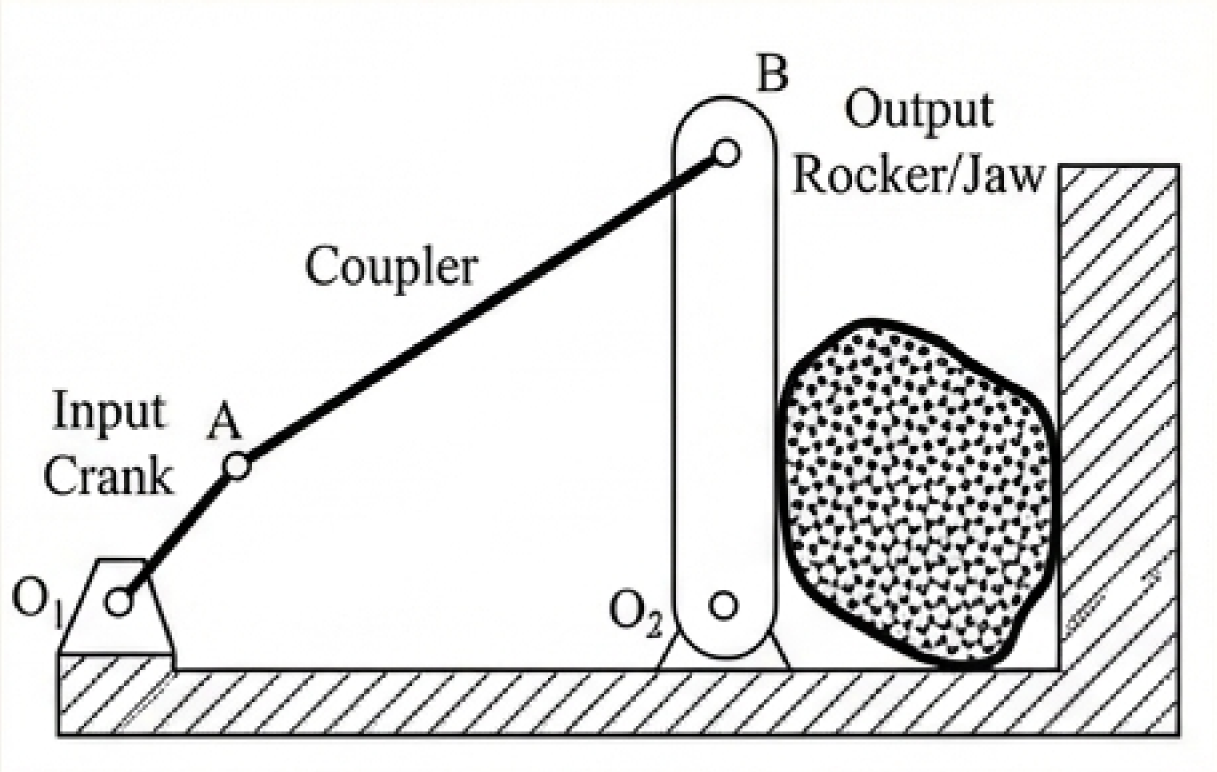
The Main Quest: Crank-Rocker Synthesis

Problem Definition

We often need to design a mechanism where a continuously rotating input (the crank) produces a specific, controlled oscillation in the output (the rocker).

- **Design Goals**
  - **Rocker Amplitude (θ):** The total angle of oscillation for the output link.
  - **Time Ratio (Q):** The ratio of time for the forward stroke to the time for the return stroke, which is critical for applications like quick-return mechanisms.



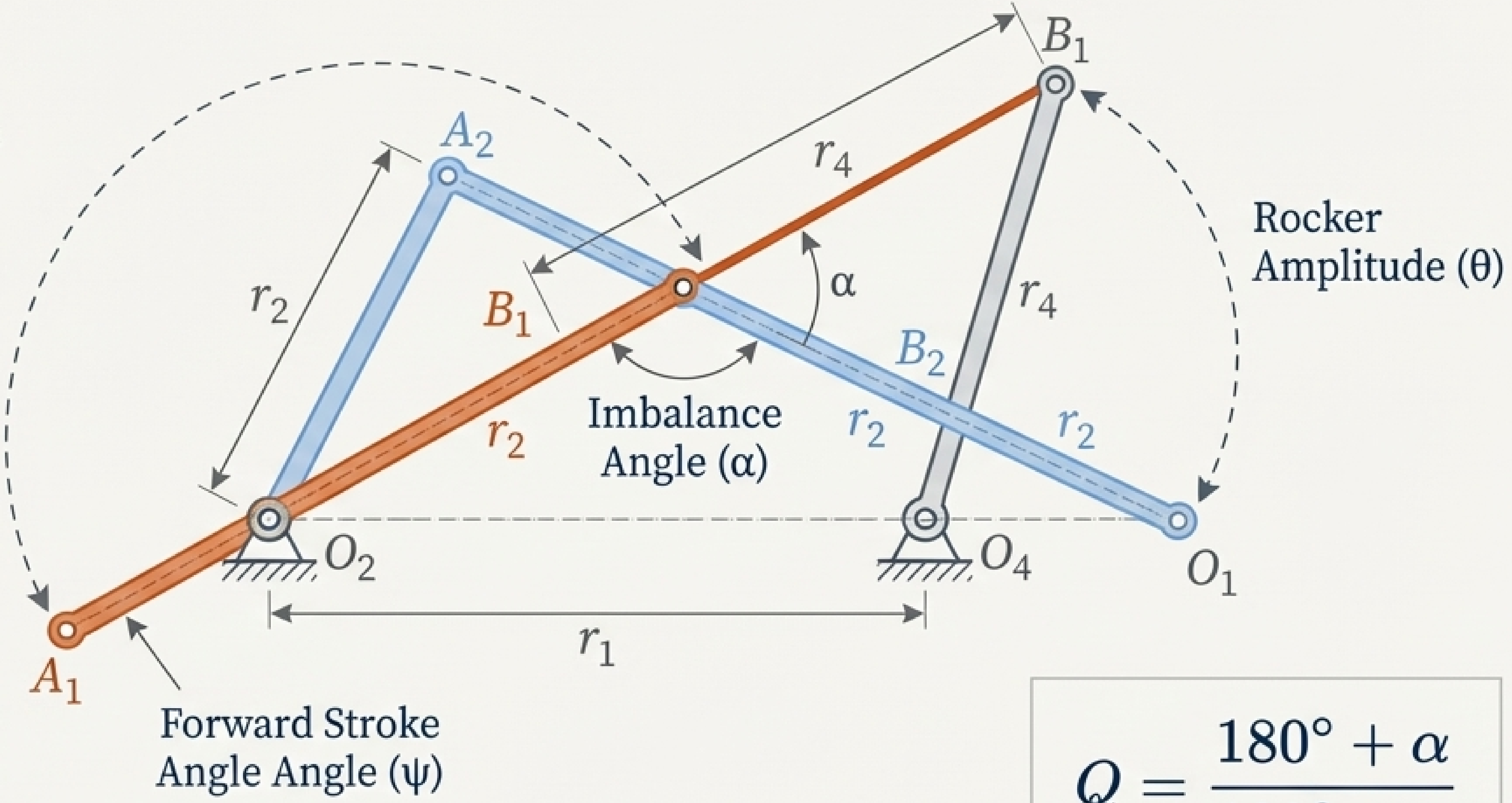


Applications: From windshield wipers to industrial machinery, controlling oscillation timing and angle is a fundamental design task.

NotebookLM

Understanding Crank-Rocker Motion: Limit Positions & Time Ratio

Key Geometric Insight:
For a crank-rocker linkage, the rocker reaches its two extreme (limit) positions when the **crank (r_2) and coupler (r_3) become collinear**.



Time Ratio (Q):
Defined as the ratio of time for the forward stroke to the time fora return stroke ($Q = t_{forward} / t_{return}$). If the crank rotates at a **constant speed**, Q is determined by the **imbalance angle (α)**.

$$Q = \frac{180° + \alpha}{180° - \alpha}$$

[SOURCE_IMAGE_111], [SOURCE_IMAGE_113]

Design Brief: Crank-Rocker with Specified Amplitude & Time Ratio

Problem Statement

Design a crank-rocker mechanism with the following specifications:

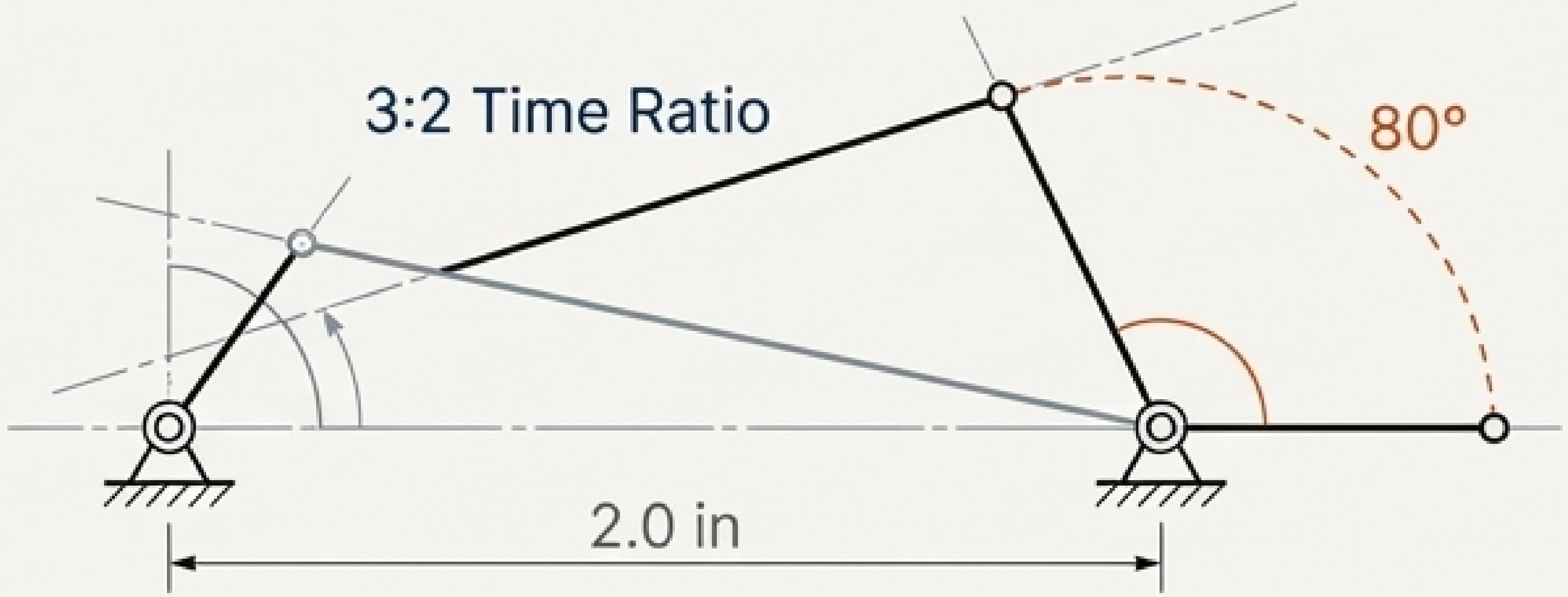
- **Rocker Oscillation (θ):** 80°
- **Time Ratio (Q):** 3:2 (for a quick-return motion)
- **Ground Link Length (r_1):** 2.0 inches
- **Ground Link Length (r_1):** 2.0 inches

Calculation

First, we must calculate the required imbalance angle α from the Time Ratio Q.

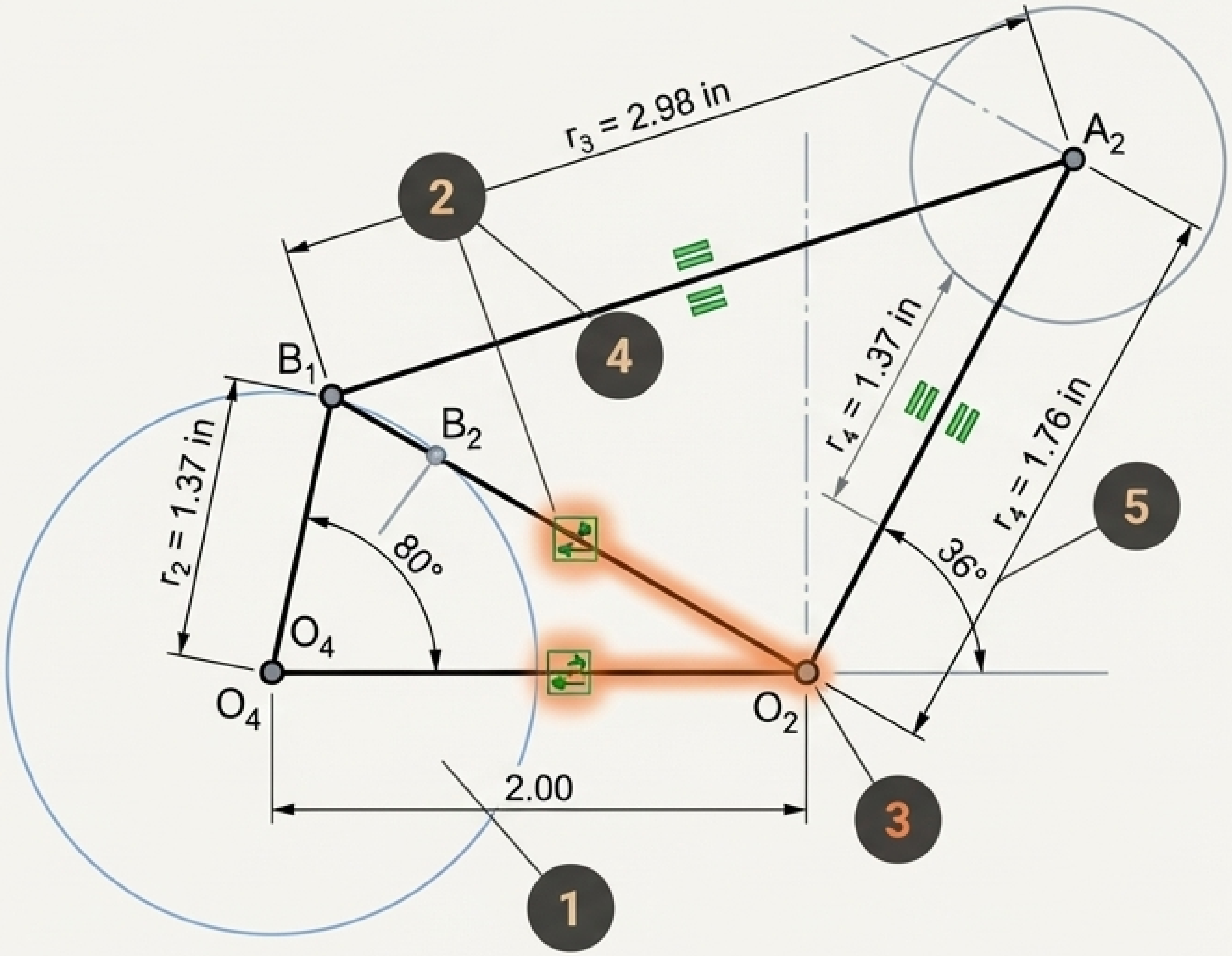
$$\boxed{\alpha = 180^\circ * \frac{Q-1}{Q+1}}$$

$$\boxed{\alpha = 180^\circ * \frac{1.5-1}{1.5+1} = 36^\circ}$$



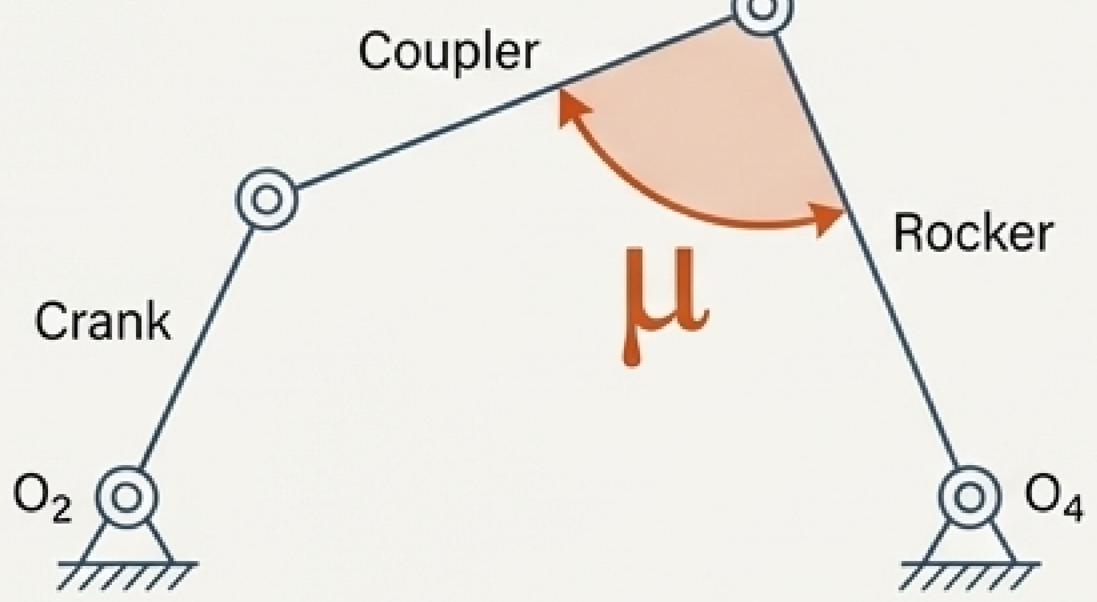
Crank-Rocker Synthesis: The GCP Construction

1. **Define Rocker Limits**
   Draw the ground link O_2-O_4 (2.0 in). At pivot O_4, draw the rocker link in its two limit positions, separated by the specified amplitude of 80°.
2. **Construct Coupler Instances**
   Draw two lines representing the coupler. Each line starts at an endpoint of the rocker (B_1 and B_2) and ends on the path of the crank pivot A.
3. **Apply Collinearity (The Key Step!)**
   Constrain the first coupler instance to be **collinear** with the crank pivot O_2 and the rocker endpoint B_1. Do the same for the second coupler instance.
4. **Apply Equality**
   Constrain the two coupler instances to be **equal** in length.
5. **Apply Time Ratio**
   Set the angle between the two collinear coupler lines to be the calculated imbalance angle, $\boldsymbol{\alpha = 36°}$.



The Next Level: Adding Advanced Constraints

Real-world design involves more than just achieving a desired motion. Performance metrics are critical.



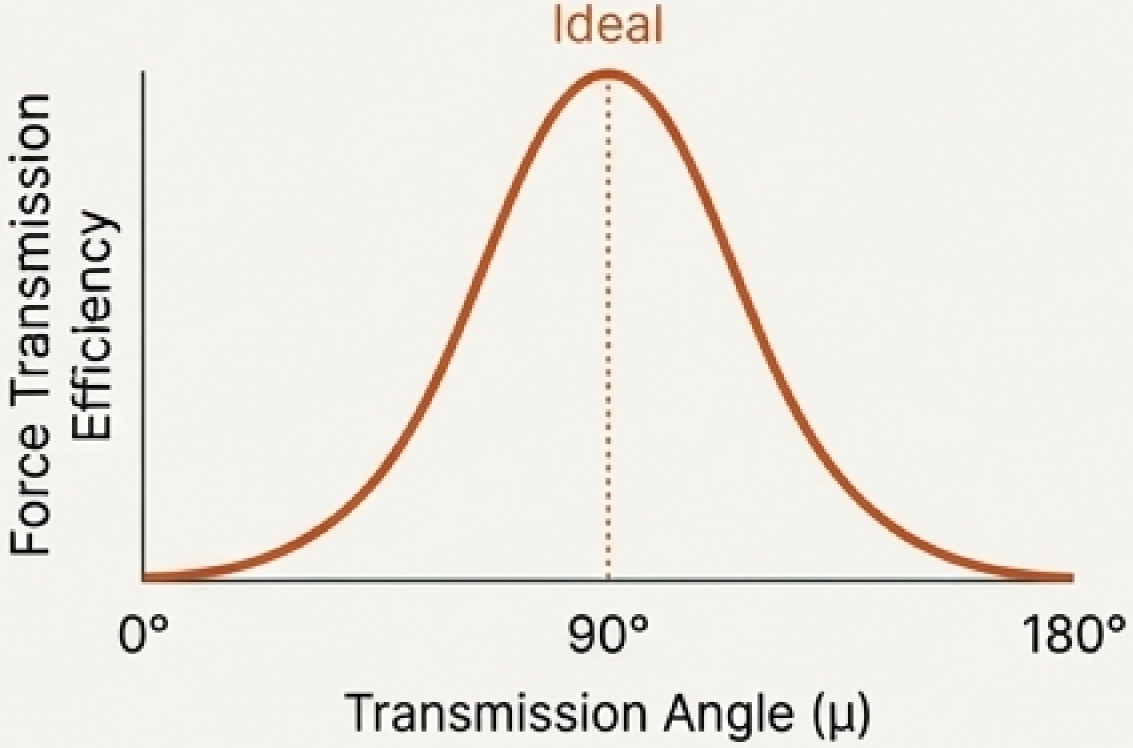
Introducing the Transmission Angle (μ)

The angle between the output link (rocker) and the coupler.

Importance: It governs how effectively force is transmitted from the coupler to the output link.

Ideal Value: The best force transmission occurs when $\mu = 90°$.

Problematic Value: When μ approaches 0° or 180°, the linkage can bind or "lock up." Poor transmission angles lead to high joint forces and inefficient operation.



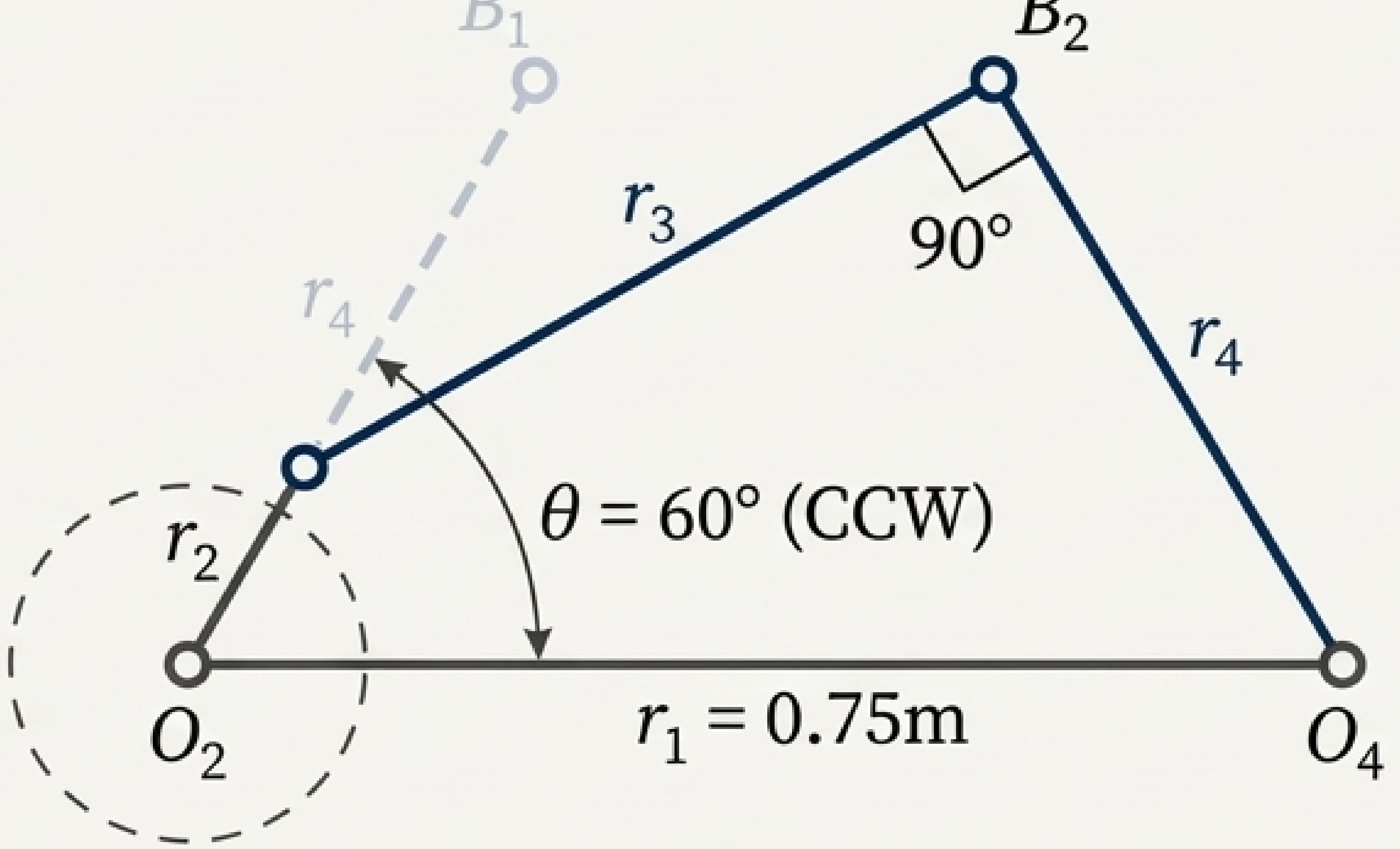
Design Brief 2: Crank-Rocker with Transmission Angle Constraint

Problem Statement

- **Rocker Amplitude (θ):** 60° (CCW)
- **Ground Link Length (r_1):** 0.75m
- **Key Constraint:** The transmission angle at the second limit position must be 90°.
- (Note: The time ratio is not specified, so α is a free variable the solver will determine).

Goal

Determine the lengths of the other three links (r_2, r_3, r_4).



Solving with Advanced Constraints: The GCP Solution

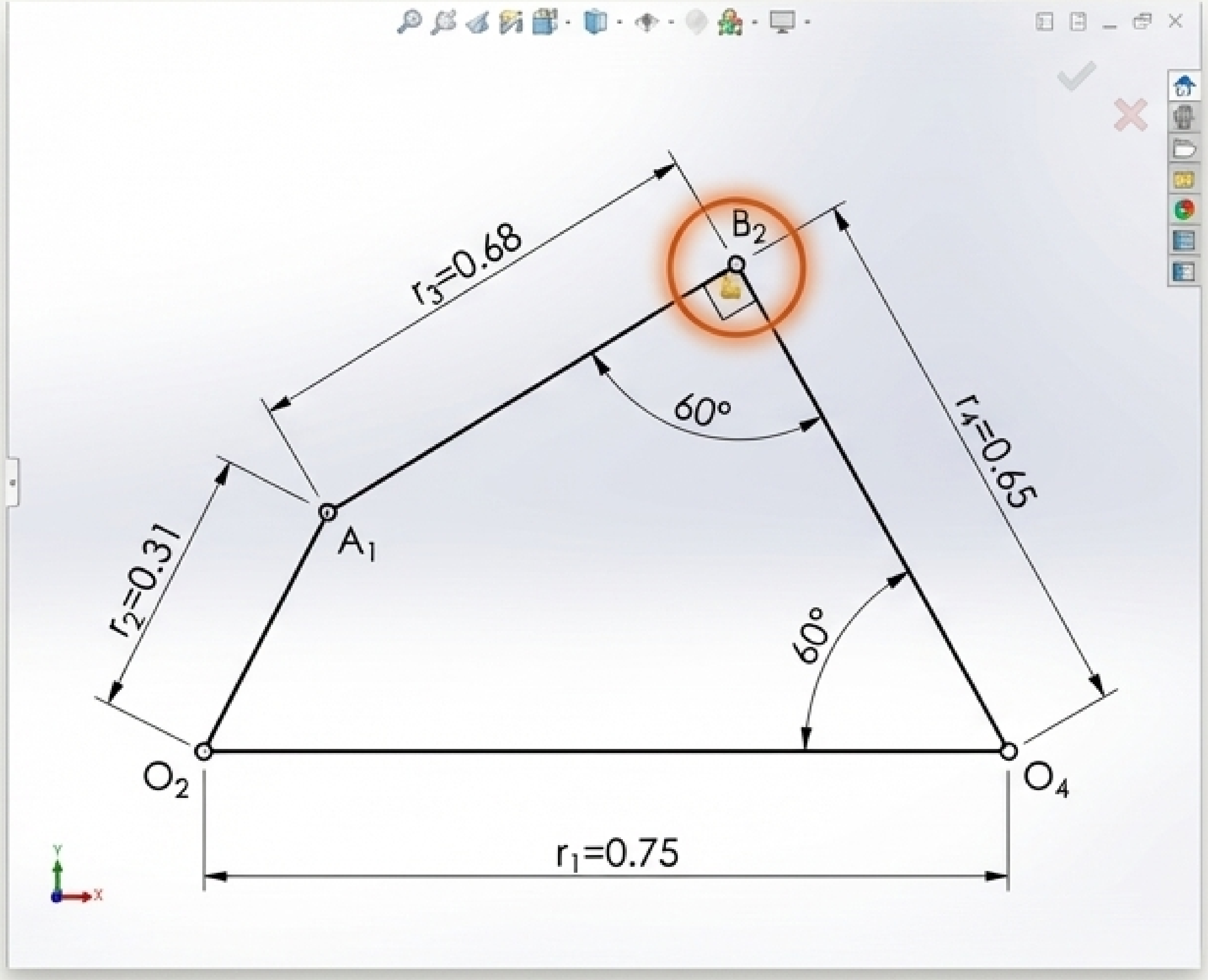
Modified Construction

The workflow is nearly identical to the previous crank-rocker synthesis, with one crucial addition.

1. Construct the rocker at its two limit positions, separated by 60°.
2. Draw the two coupler instances.
3. Apply the collinearity and equality constraints as before.
4. **Add the New Constraint:** Select the coupler and the rocker at the second limit position (A_2B_2 and O_4B_2) and apply a **Perpendicular** (90°) constraint.

The Result

The SolidWorks solver finds the unique geometry that satisfies all constraints simultaneously, including the transmission angle requirement.



Verification: From Synthesis Sketch to Functional Model

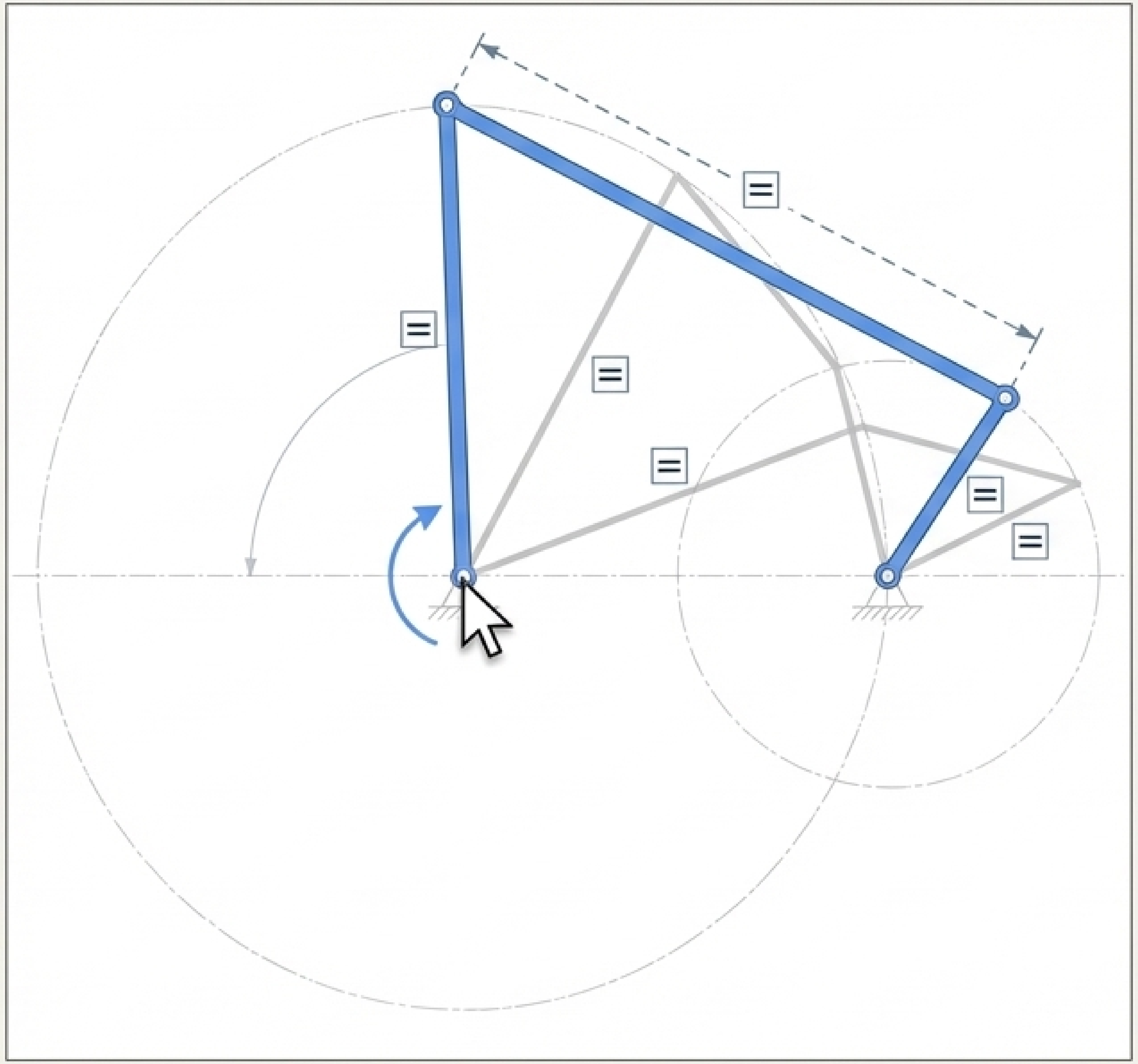
Objective: To confirm that the calculated link lengths produce the required motion.

Key Principle: Your synthesis sketch is a static "calculator." You must build a separate, functional model to test its dynamic behavior.

Procedure in SolidWorks:

1. **Create a New Sketch** on the same plane. The synthesis construction will be visible as a gray template.
2. **Build the Linkage:** Draw a new, clean 4-bar linkage on top of one of the solved positions.
3. **Constrain Lengths:** For each new link, make it equal in length to the corresponding link in the underlying synthesis sketch.

Confirmation: The new linkage is now a functional, movable model. You can drag the input link and verify that the output rocker moves between the precisely defined start and end points.



Your New Superpower: From Problem to Prototype

Main Takeaway: Kinematic synthesis is the creative process of inventing mechanisms.

Your New Skill: By mastering Geometric Constraint Programming, you have moved beyond simply analyzing existing mechanisms. You can now systematically translate a set of desired motions and performance criteria—the core of a design problem—into a fully-defined, functional