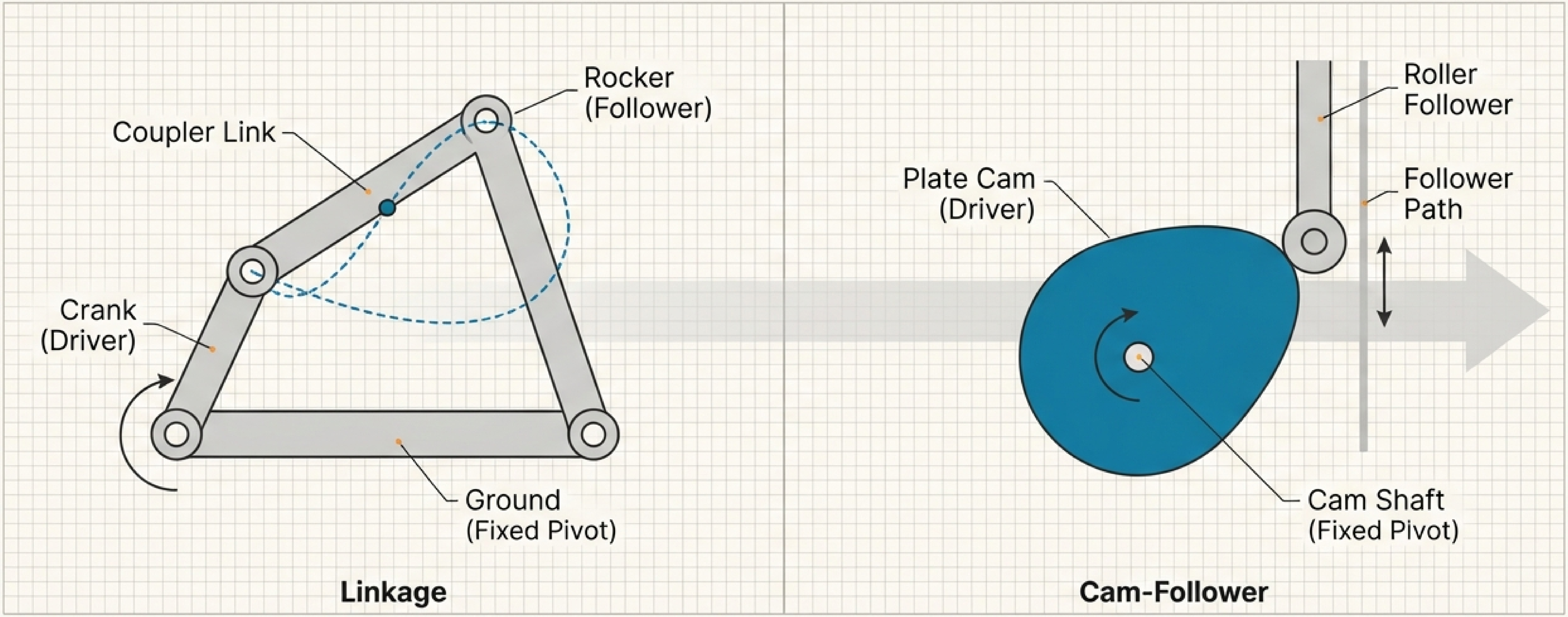
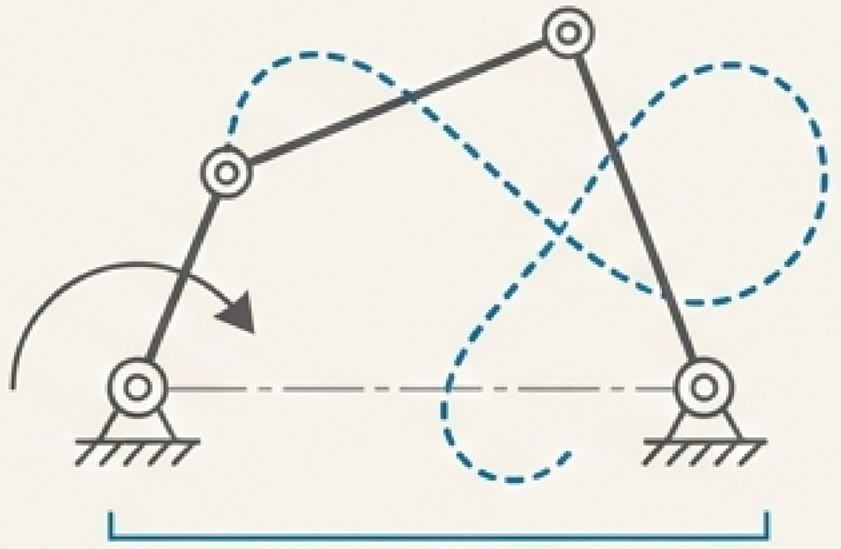
ME 3751: Cam Design – Precision Motion Beyond Linkages

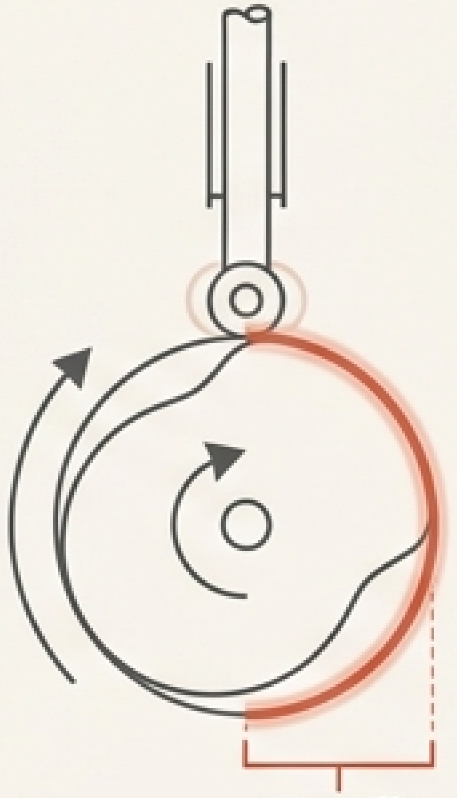
From an Abstract Motion Requirement to a Physical Component



The Engineer's Choice: Cams vs. Linkages

Feature	Linkages	Cams
Motion	Robust and reliable for simpler paths.	Can achieve highly complex and precise motions, including extended dwells.
Design	Can be difficult to design for complex motion paths.	Design process is more direct and can exactly reproduce a desired motion program.
Manufacturing	Inexpensive (e.g., stamping for large lots).	Hard/expensive to manufacture, especially for small lots (CNC machining) or high-speed (grinding).
Performance	Generally lower maintenance.	High contact stresses require hardened materials. Subject to wear, noise, and follower bounce at high speeds.





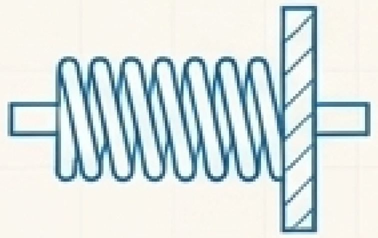
Key Takeaway

Linkages are often the preferred solution for their robustness and cost-effectiveness. However, when the design requires **tight motion specifications**—especially precise velocity control or extended dwells (periods of no output motion)—cams offer a superior and more direct design path.

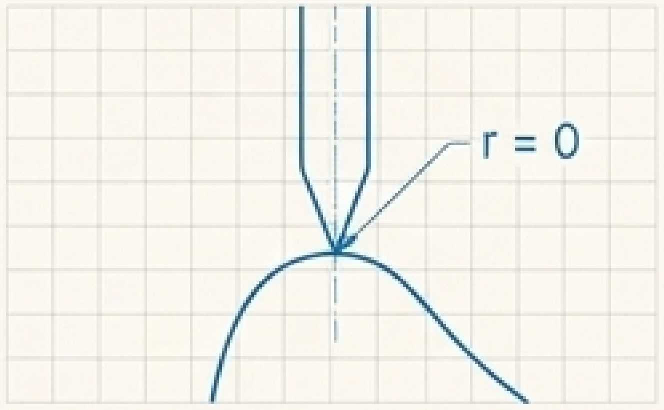
Anatomy of a Cam-Follower System

Core Components

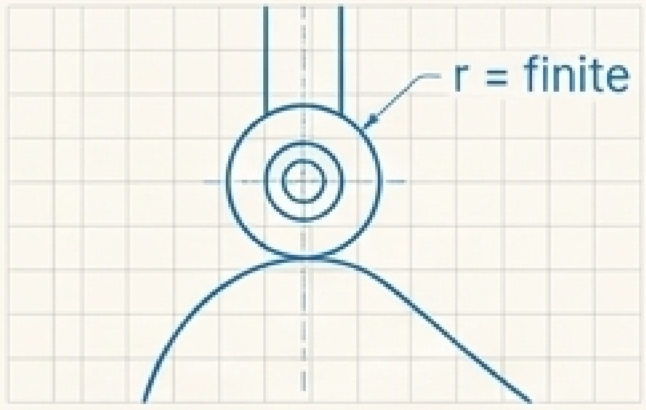
- **Cam**
  The driving link, whose shape dictates the output motion. Generates irregular motion.
- **Follower**
  The driven link, which follows the cam profile. Characterized by its motion (translating or oscillating) and its face shape.
- **Force Closure**
  A mechanism to keep the follower in contact with the cam.
  - **Spring/Gravity**: Most common method.
  - **Groove/Conjugate Cam**: Constrains motion geometrically; expensive but necessary for positive control.



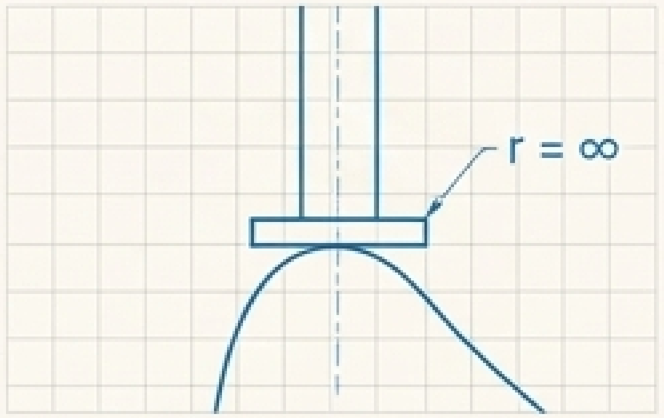
A Playlist of Follower Faces



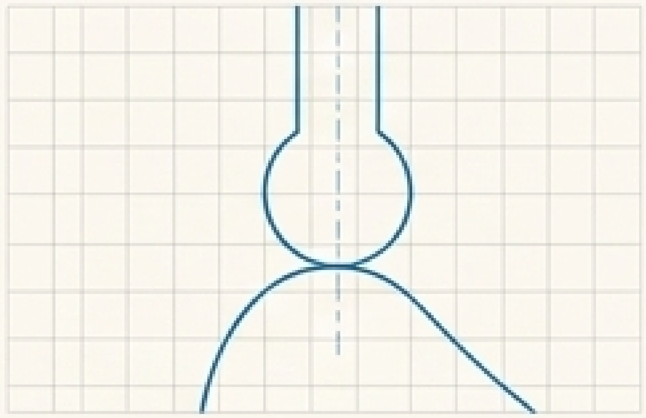
Knife-Edge: $r = 0$. Simple kinematically, but causes very high stress. Used only in low-load applications.



Roller: $r = \text{finite}$. Low friction and stress due to rolling contact. The most common type.



Flat-Faced: $r = \infty$. Lower stress than cylindrical followers, but requires a larger cam and must maintain a convex profile.



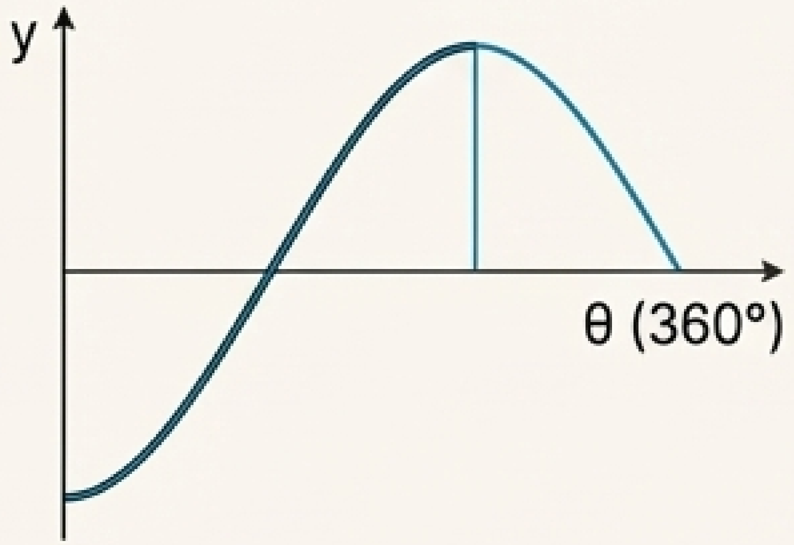
Cylindrical-Faced: Kinematically identical to a roller follower, but with higher wear due to sliding contact.

A Two-Stage Process: From Blueprint to Build

Cam design is a two-step translation from a motion requirement into a physical part.

Stage 1

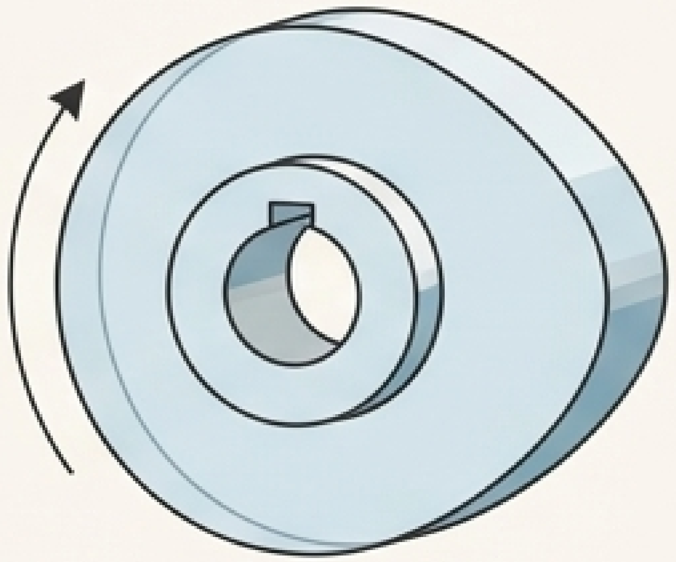
Synthesize the Follower Motion Program (The Blueprint)



- **Goal:** Define the follower's displacement, velocity, and acceleration over one full revolution of the cam (360°).
- **Output:** A set of displacement-vs-angle graphs (y vs. θ), also known as SVAJ plots. This is the "soul" of the motion, independent of the physical cam size.

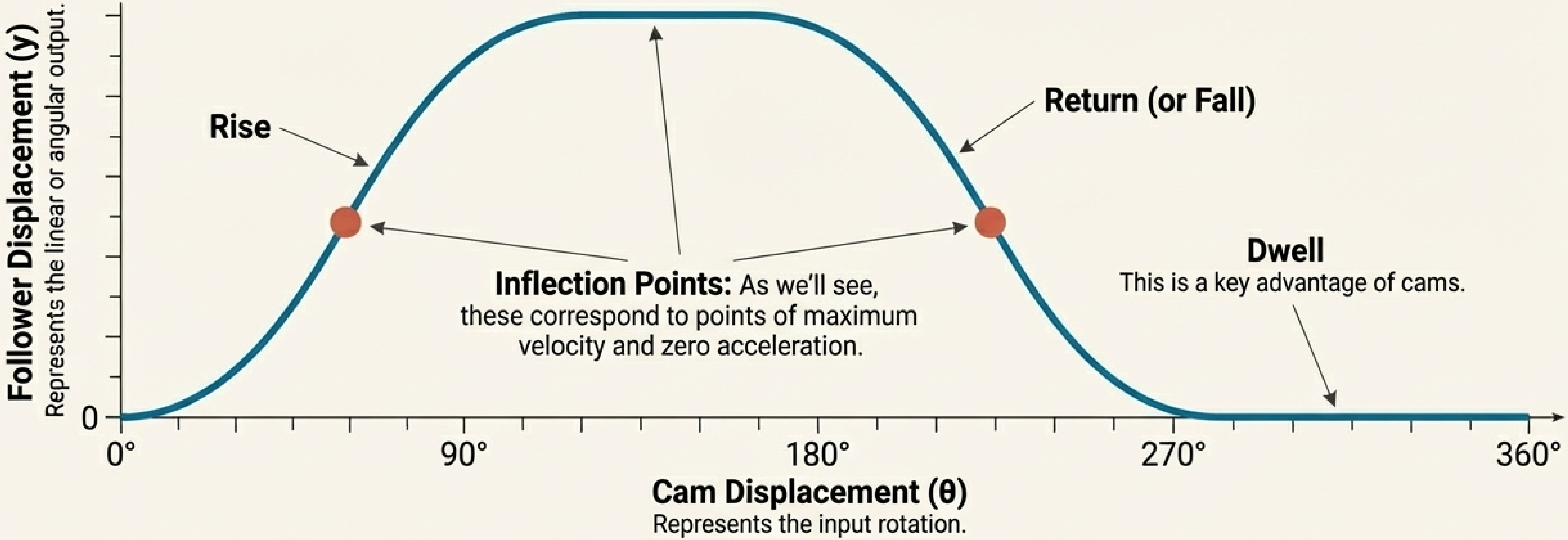
Stage 2

Generate the Cam Profile (The Build)



- **Goal:** Use the motion program to determine the physical shape of the cam.
- **Method:** We will use **kinematic inversion**—fixing the cam and imagining the follower moving around it—to construct the profile graphically. This gives the "soul" its "body".

Stage 1: The Displacement Diagram - Charting the Follower's Journey



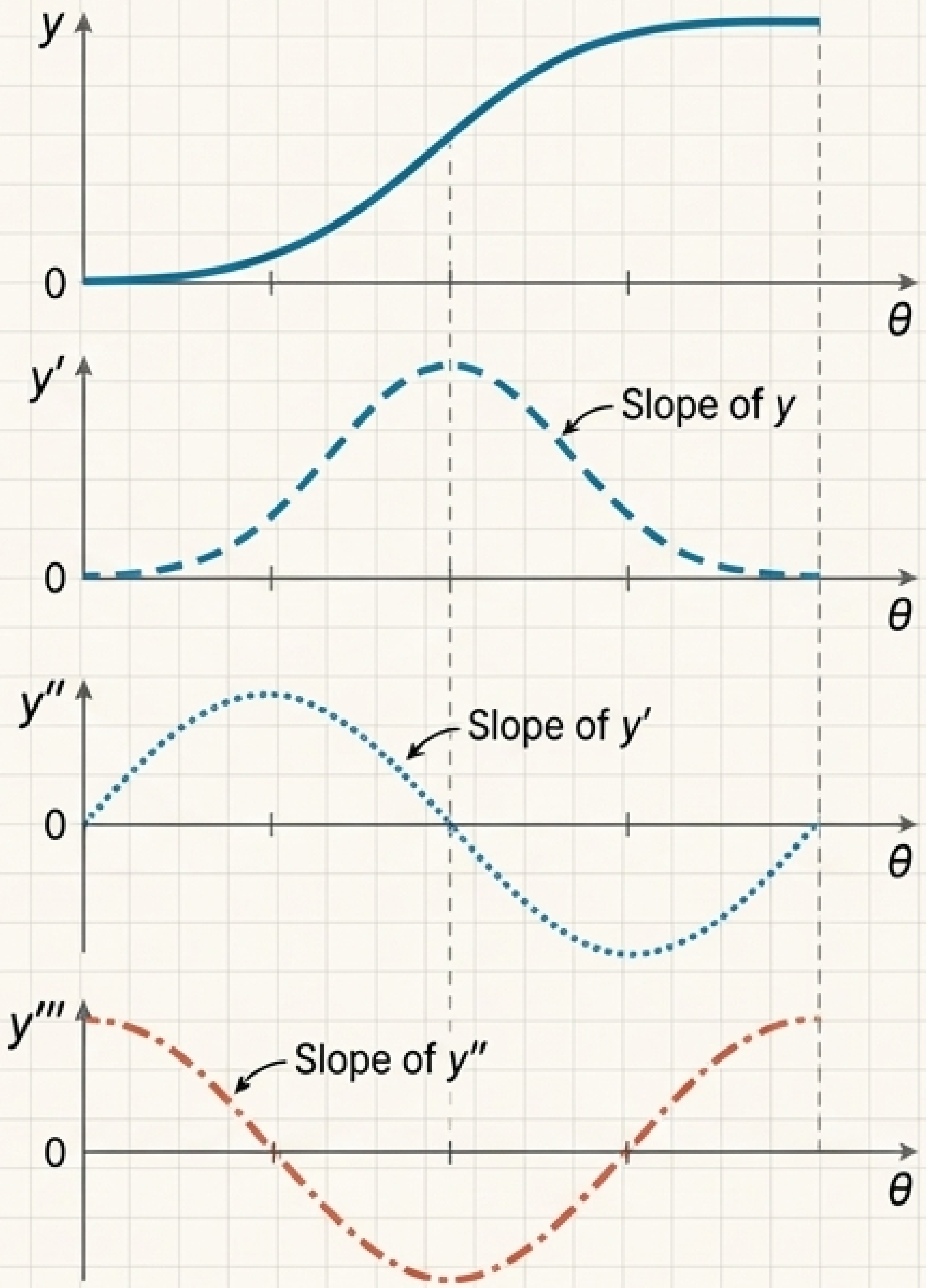
The first step is to plot the required motion. Often, only certain parts are specified (e.g., "dwell at 1.0 inch from 110° to 150°"). Our job is to synthesize the "in-between" parts—the rises and returns—using mathematical functions that yield good dynamic performance.

Beyond Displacement: The Importance of Smoothness (SVAJ)

For a cam rotating at a constant angular velocity (ω), the time derivatives of displacement are directly proportional to the derivatives with respect to cam angle (θ). This allows us to analyze the dynamics by just looking at the shape of our displacement curve.

- $\dot{y} = y' * \omega$ (Velocity)
- $\ddot{y} = y'' * \omega^2$ (Acceleration)
- $\ddot{y}' = y''' * \omega^3$ (Jerk - the rate of change of acceleration)

We call these the **SVAJ curves** (Displacement, Velocity, Acceleration, Jerk).



The Fundamental Law of Cam Design

To avoid infinite forces and ensure smooth operation, the cam function must be **continuous through its first (velocity) and second (acceleration) derivatives** across the entire 360° interval.

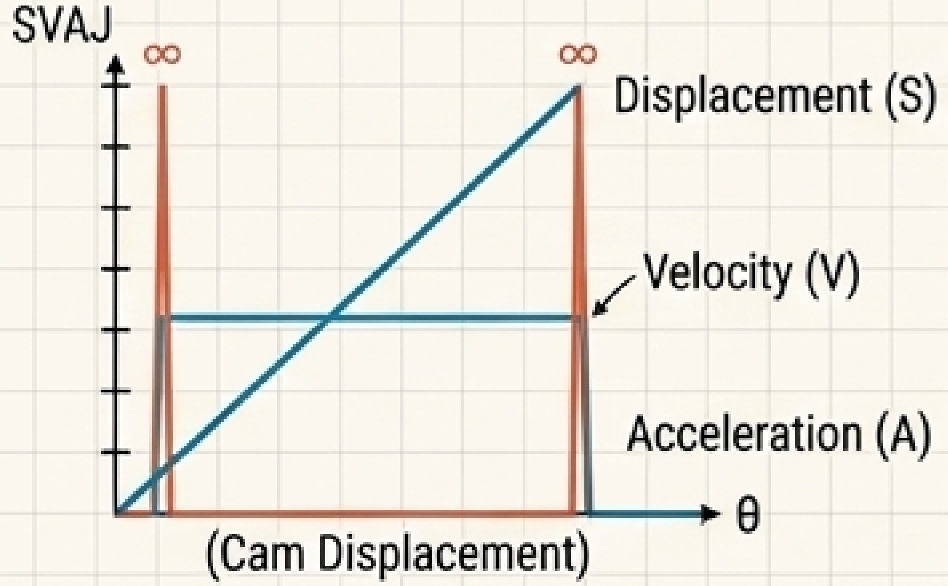
Corollary: The jerk function must be **finite** across the entire interval.

Takeaway: Abrupt changes in the displacement diagram lead to spikes in acceleration and jerk, causing vibration, noise, and wear. Our goal is to choose transition curves that are as 'smooth' as possible through their derivatives.

A Comparison of Standard Motion Programs

Let's examine how different mathematical functions perform against the Fundamental Law.

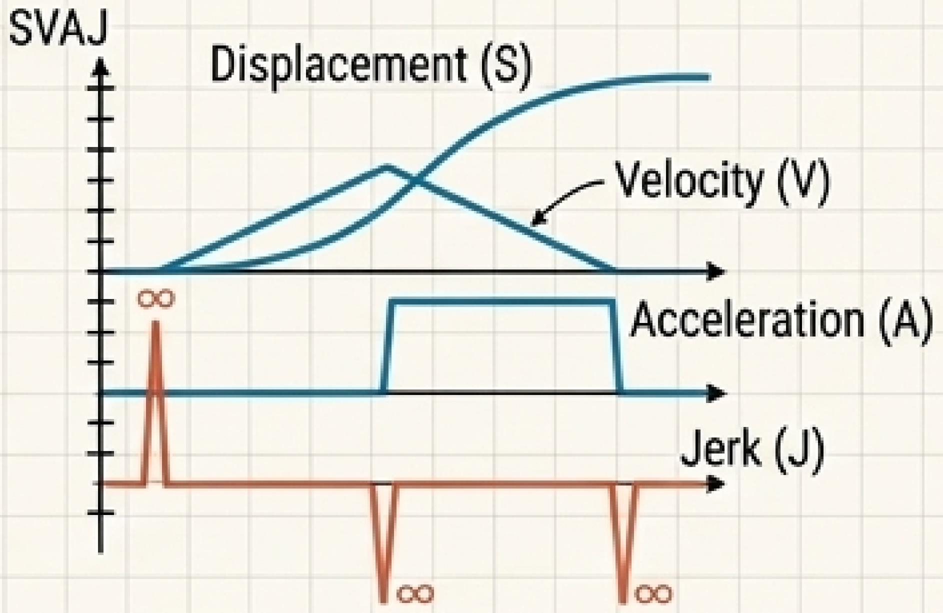
Uniform (Linear) Motion



Analysis

Discontinuous velocity leads to **infinite acceleration**. Almost never used.

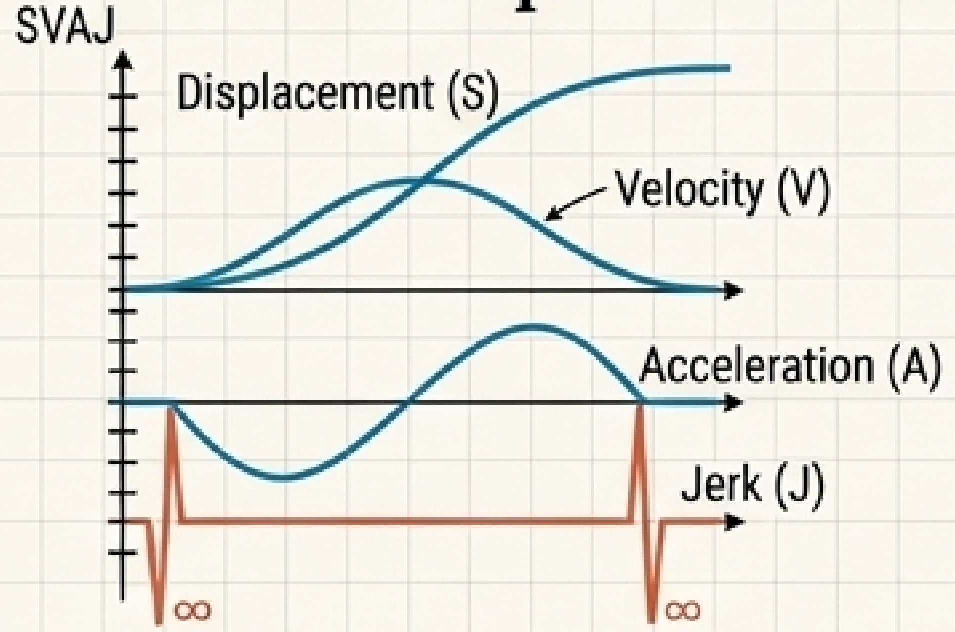
Parabolic Motion



Analysis

Continuous velocity, but discontinuous acceleration. This results in **infinite jerk**. Bad for high-speed applications.

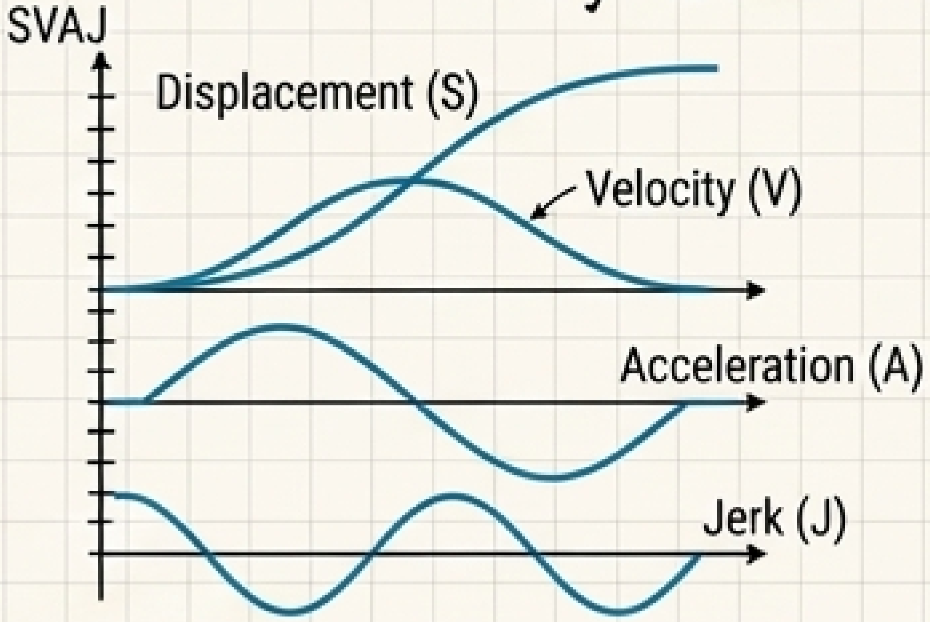
Simple Harmonic Motion



Analysis

Continuous velocity, but discontinuous acceleration at the ends. Also results in **infinite jerk**.

Cycloidal Motion



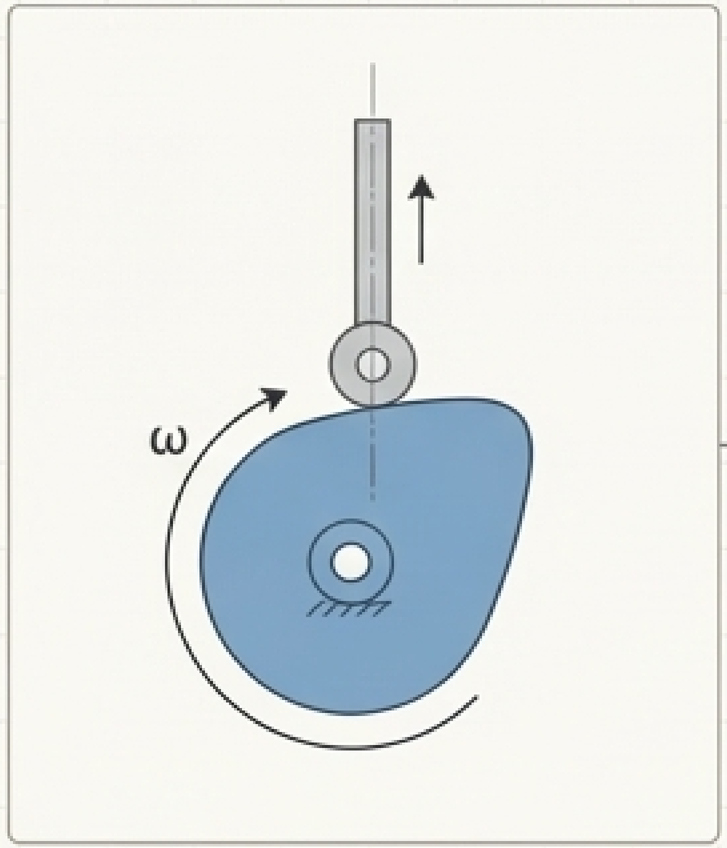
Analysis

Continuous velocity AND acceleration. **Finite jerk**. Zero acceleration at the start and end of the rise makes it excellent for high-speed applications.

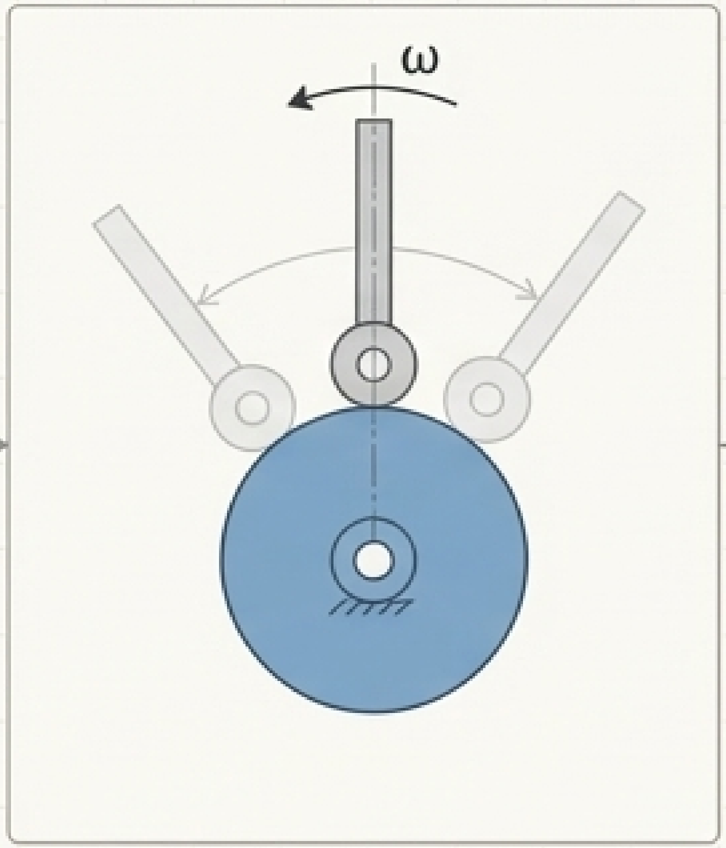
For even greater control, **Polynomial functions** (e.g., 3-4-5 polynomial) can be used to ensure continuity through jerk and higher derivatives, but they require extreme manufacturing precision.

Stage 2: Giving the Motion a Physical Form

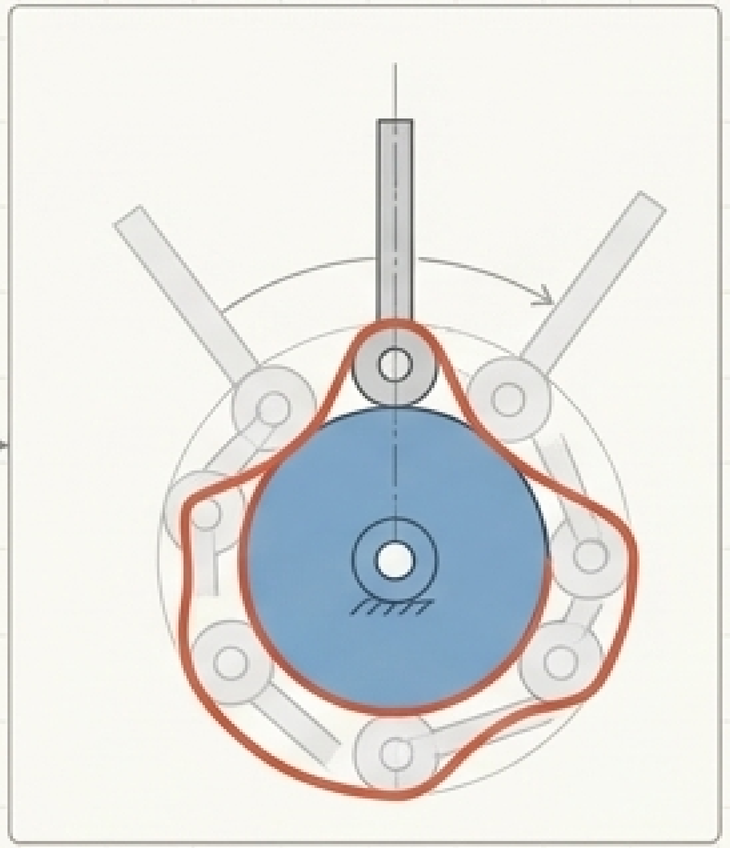
Now that we have our "blueprint"—the follower displacement program that defines the desired motion—we need to translate it into a physical cam profile.



1. A rotating cam drives a translating follower.



2. **Invert**: Fix the cam and rotate the follower's path in the opposite direction.



3. The envelope of follower positions defines the cam surface.

The Core Principle: Kinematic Inversion

Instead of thinking about a rotating cam moving a stationary follower, we will invert the mechanism.

1. **Fix the Cam:** Imagine the cam is held stationary.
2. **Move the Follower:** Imagine the follower's pivot or path rotating around the fixed cam in the *opposite* direction.
3. **Trace the Profile:** The resulting envelope of the follower's positions defines the cam surface.

This technique simplifies the geometry and allows for a direct graphical construction. We will demonstrate this for a **translating radial roller follower**.

The Foundation: Base, Prime, and Pitch Curves

Base Circle (radius r_b): The smallest circle tangent to the cam profile, centered at the cam's axis of rotation. Represents the position of the follower at zero displacement.

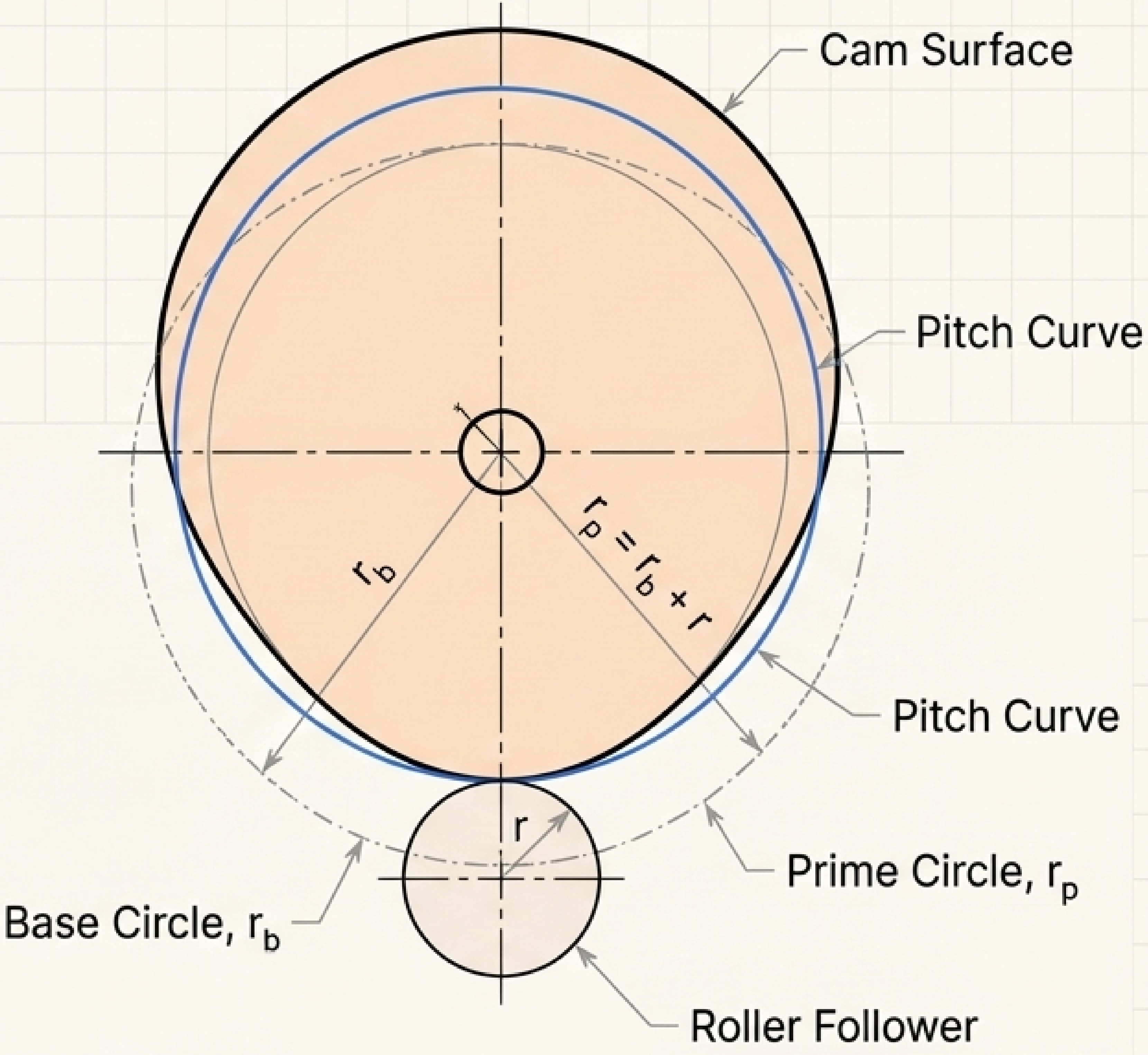
Rule of Thumb: r_b should be 2 to 3 times the maximum follower lift to avoid design issues.

Roller Radius (r): The radius of the roller follower.

Prime Circle (radius r_p): A circle defined by $r_p = r_b + r$. It represents the path of the *center* of the roller follower at zero displacement.

Pitch Curve: The path traced by the center of the roller follower as it moves around the inverted cam. We construct this first.

Cam Surface: The final physical profile, which is the envelope of the roller positions as its center follows the pitch curve.



NotebookLM

Graphical Construction in 6 Steps (Part 1/3)

Step 1: Draw Foundational Circles

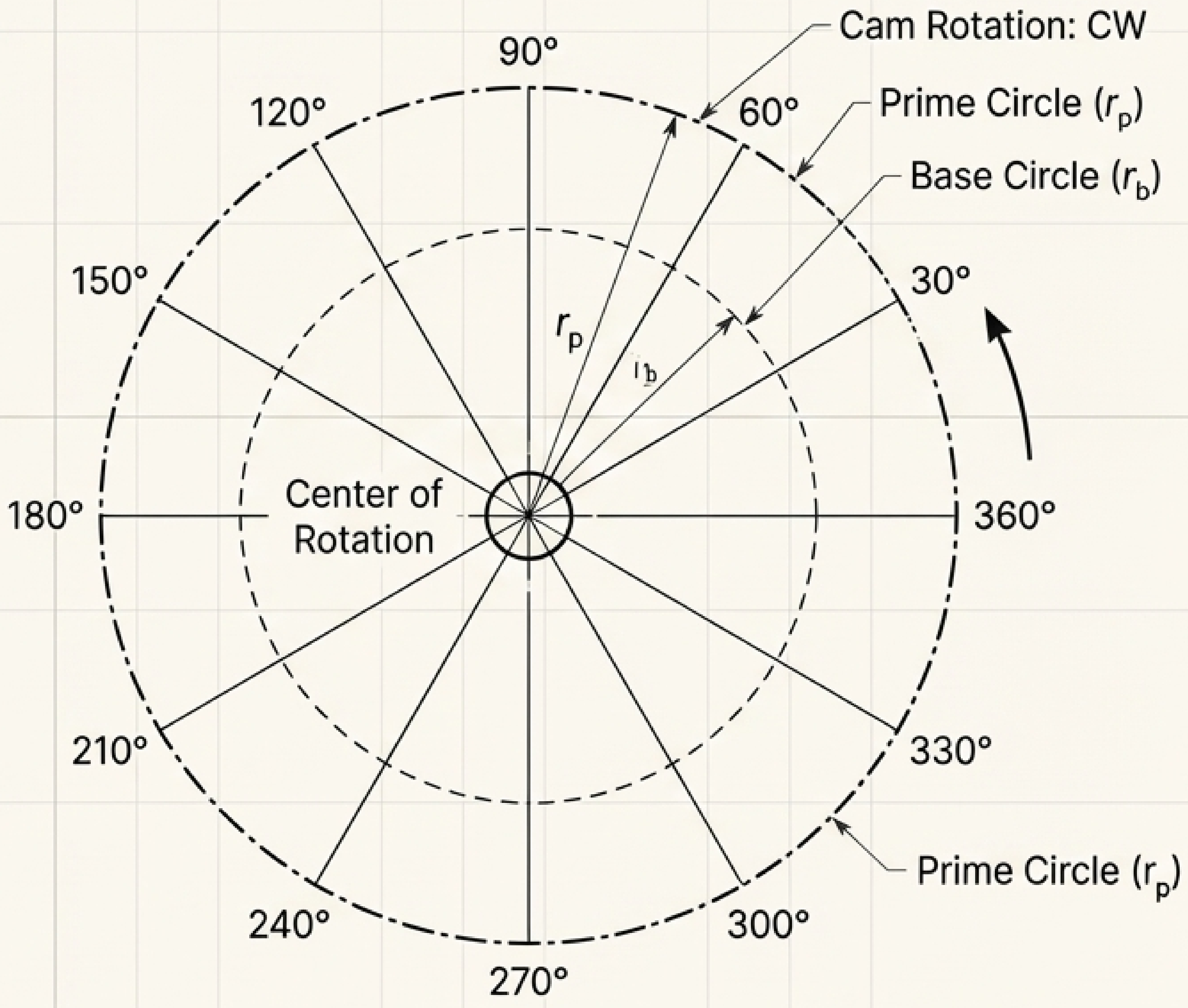
- Choose a base circle radius, r_b.
- Choose a roller follower radius, r.
- Calculate the prime circle radius, $r_p = r_b + r$.
- Draw both the base and prime circles concentric about the cam's center of rotation.

Step 2: Determine Rotation & Invert

- Identify the direction of cam rotation (e.g., Clockwise - CW).

Step 3: Lay Out Angular Increments

- Draw radial lines from the center at regular angular increments (e.g., 10°, 20°, or 30°).
- **Crucially**: Label these lines in the direction **OPPOSITE** to the cam rotation. If the cam rotates CW, you label the lines Counter-Clockwise (CCW).



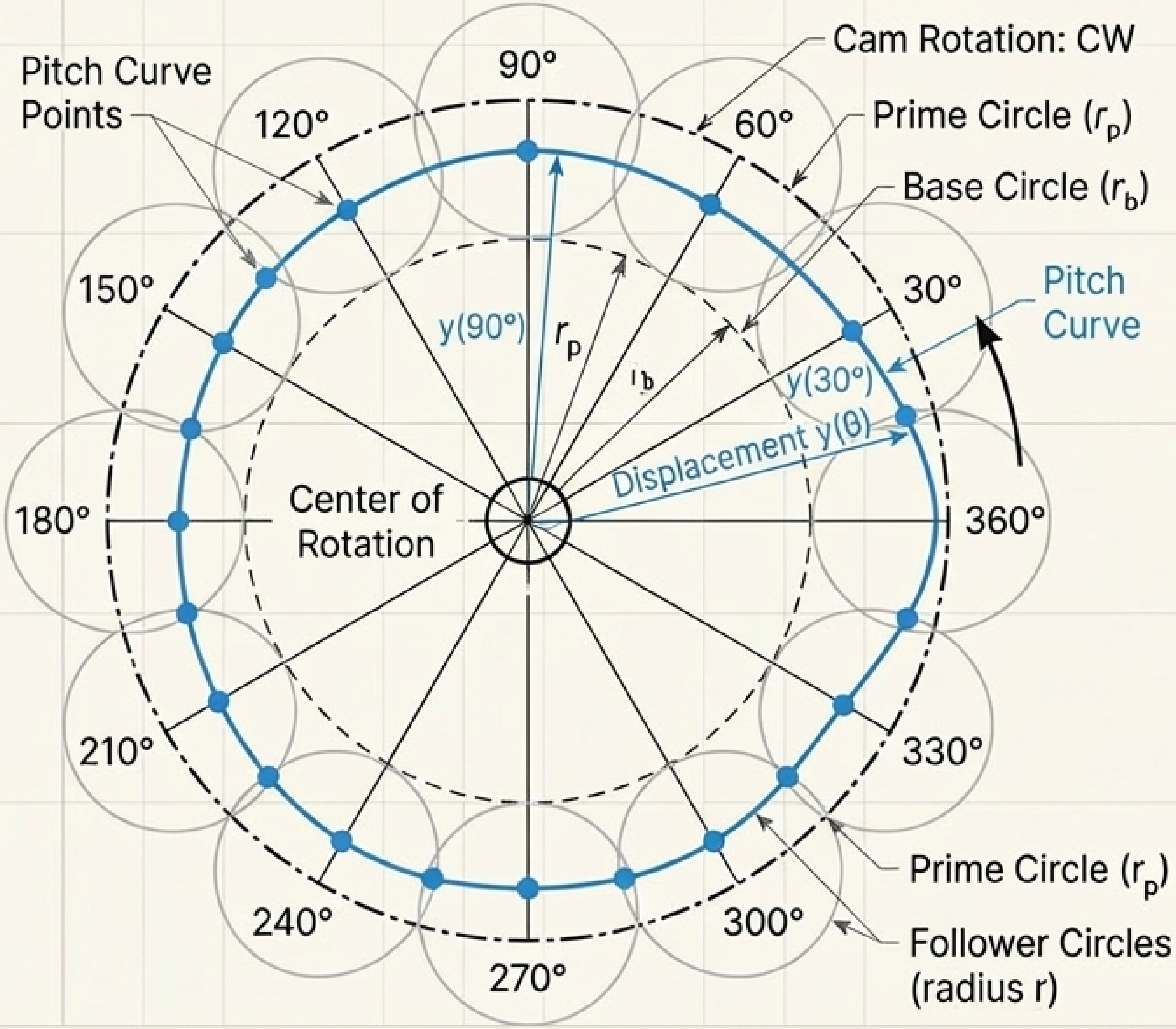
Graphical Construction in 6 Steps (Part 2/3)

Step 4: Plot the Pitch Curve

- For each angular station (θ), find the corresponding follower displacement y(θ) from your motion program table or graph.
- Measure this displacement y(θ) **radially outward from the Prime Circle** along the corresponding radial line.
- Connecting these points defines the **Pitch Curve**, the path of the follower's center.

Step 5: Draw the Follower Positions

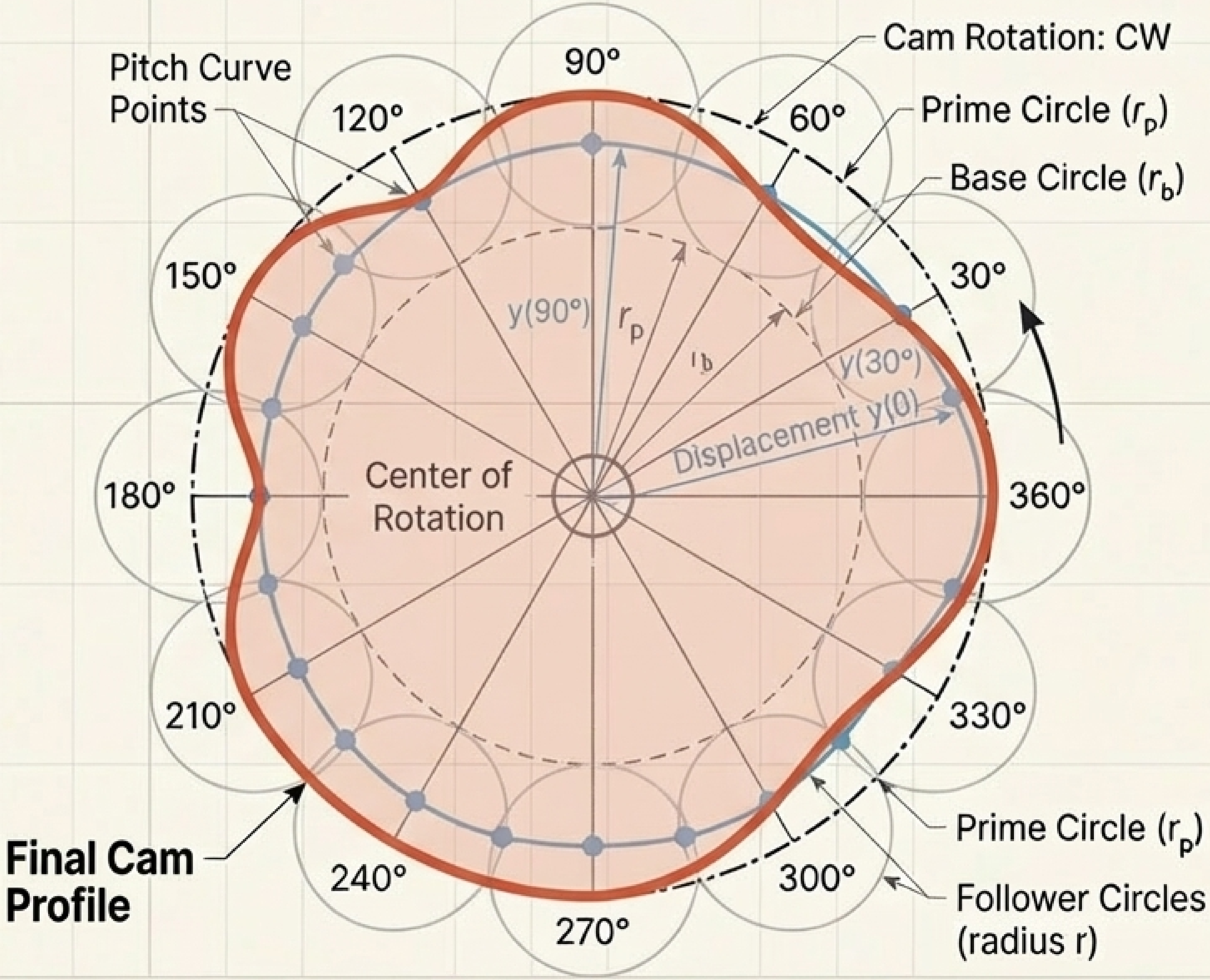
- Using each point on the pitch curve as a center, draw a circle with the roller radius, r.



Graphical Construction in 6 Steps (Part 3/3)

Step 6: Draw the Cam Surface

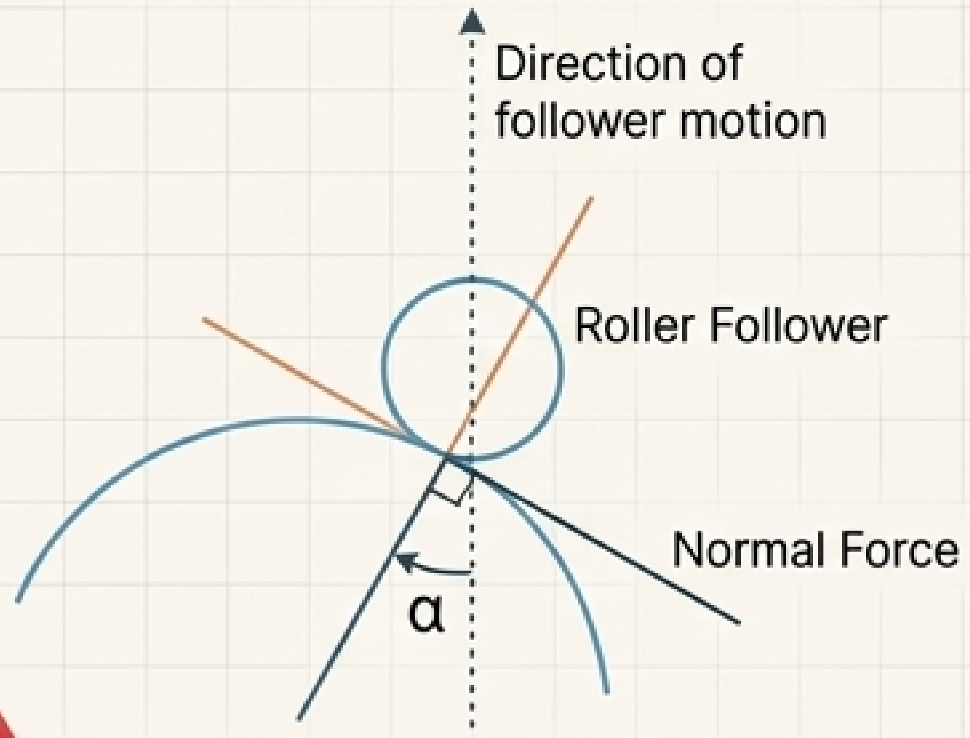
- Draw a smooth curve that is **tangent** to the outside of all the roller circles you drew in Step 5.
- This tangent envelope **is the final cam profile**.



The Reality Check: Is the Design Viable?

A beautiful cam profile is useless if it causes the machine to fail. We must check two critical geometric constraints.

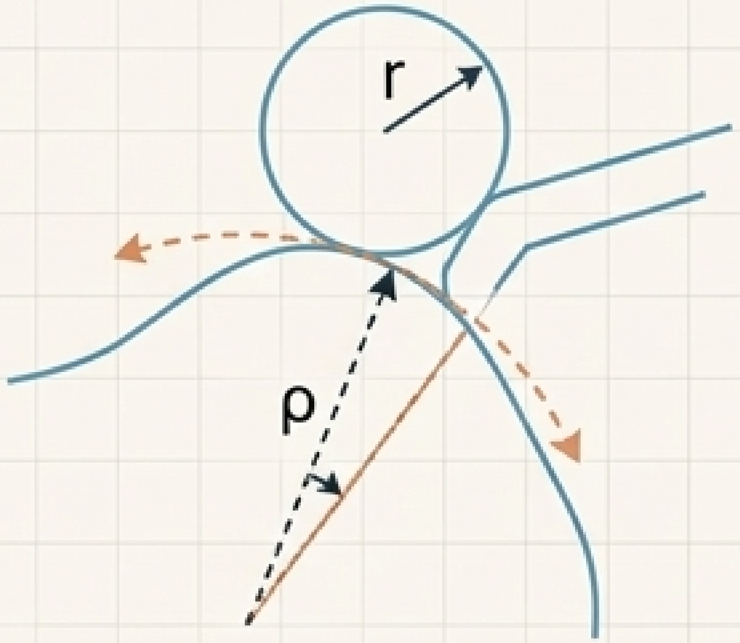
Pressure Angle (α)



$\alpha > 30°$ ⚠

- **What it is:** The angle between the direction of follower motion and the normal force from the cam.
- **Why it Matters:** High pressure angles (> 30°) create large sideways forces on the follower, leading to high friction, binding in bearings, and excessive wear.
- **Where it's Worst:** The maximum pressure angle occurs at the inflection points of the displacement diagram.
- **The Fix:** Increase the base circle radius (r_b). A larger cam results in a smaller pressure angle.

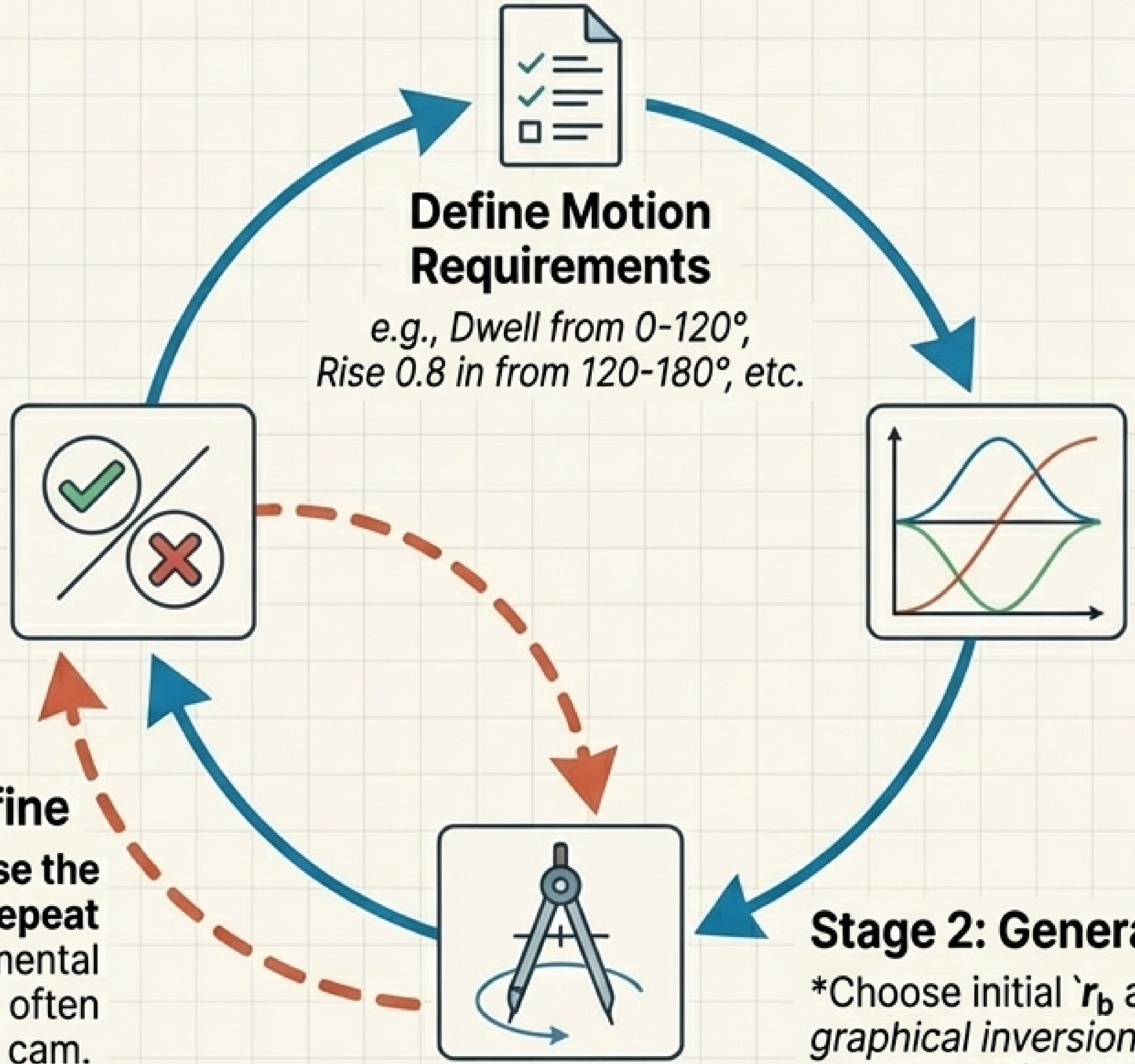
Radius of Curvature (ρ)



$\rho_{min} < r$ ⚠

- **What it is:** The local radius of the cam's surface. Can be convex (+) or concave (-).
- **Why it Matters:**
    - **Roller Follower:** The minimum convex radius of the cam must be larger than the roller radius ($\rho_{min} > r$) to prevent undercutting and sharp points (cusps).
    - **Flat-Faced Follower:** The cam profile must *always* be convex ($\rho > 0$) everywhere.
- **The Fix:** Increase the base circle radius (r_b).

The Complete Cam Design Loop



Define Motion Requirements

e.g., Dwell from 0-120°, Rise 0.8 in from 120-180°, etc.

Stage 1: Synthesize Motion Program

**Choose a function (e.g., Cycloidal) that provides smooth derivatives (finite jerk) for the required speed.*

Stage 2: Generate Cam Profile

*Choose initial `$r_b$ and `r`. Use the *graphical inversion method* to lay out the cam surface.

Check Constraints

- Is $|\alpha_{max}| < 30°$?
- Is $\rho_{min} > r$?

Iterate and Refine

If constraints are not met, **increase the base circle radius (r_b)** and **repeat** steps 3 and 4. This is the fundamental trade-off: better performance often requires a larger, heavier cam.

From Problem to Profile: A Complete Example

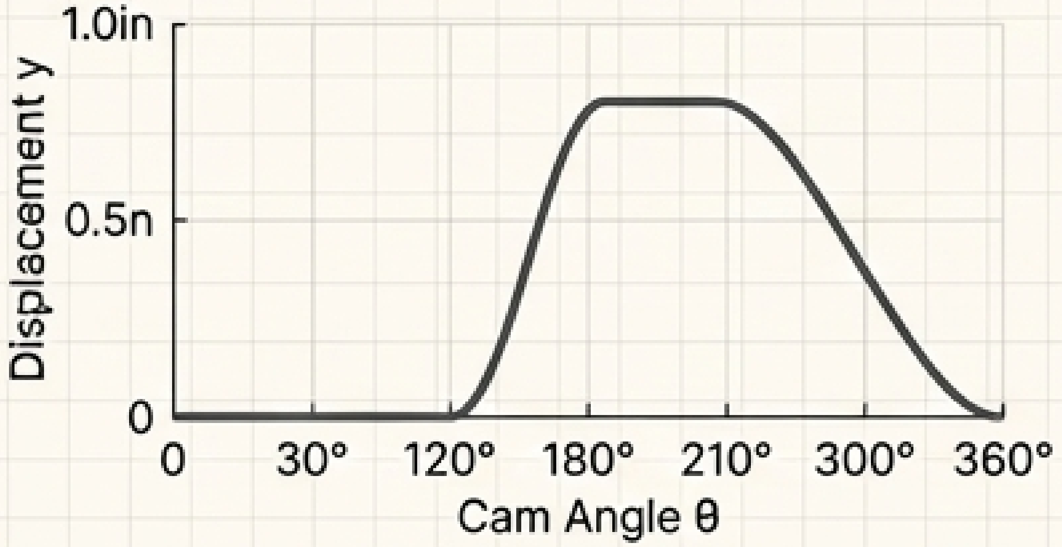
****Problem Statement****: Design a cam for a translating roller follower (r = 0.5in) rotating CW.

- Dwell at zero lift for 1st 120°.
- Rise 0.8 in from 120°-180° using Harmonic motion.
- Dwell at 0.8 in from 180°-210°.
- Return to zero from 210°-360°.
- Assume base circle radius r_b = 1.5in.

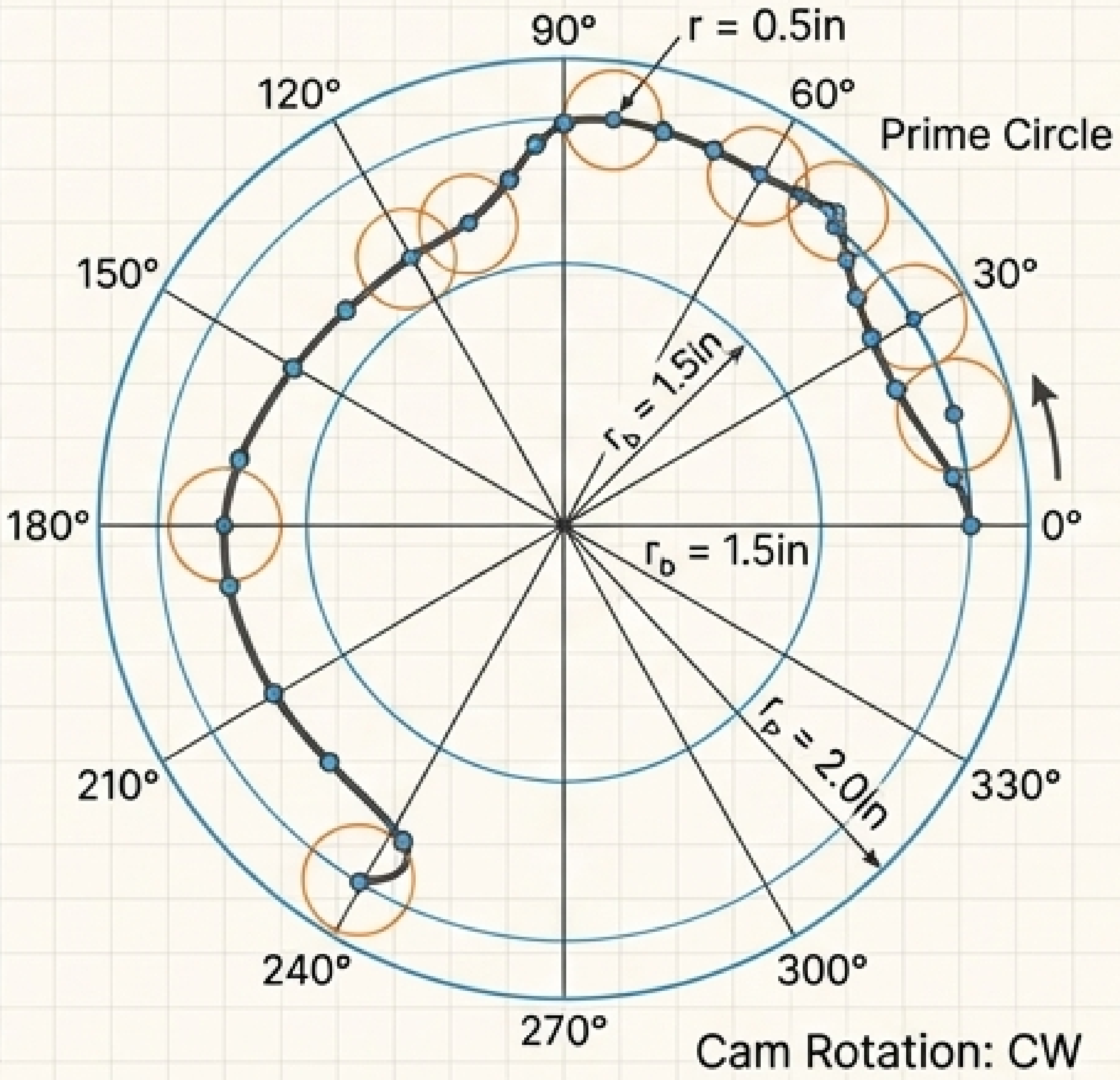
Stage 1 - Motion Program

θ	y
0	0
30	0
60	0
90	0
120	0
150	0.4

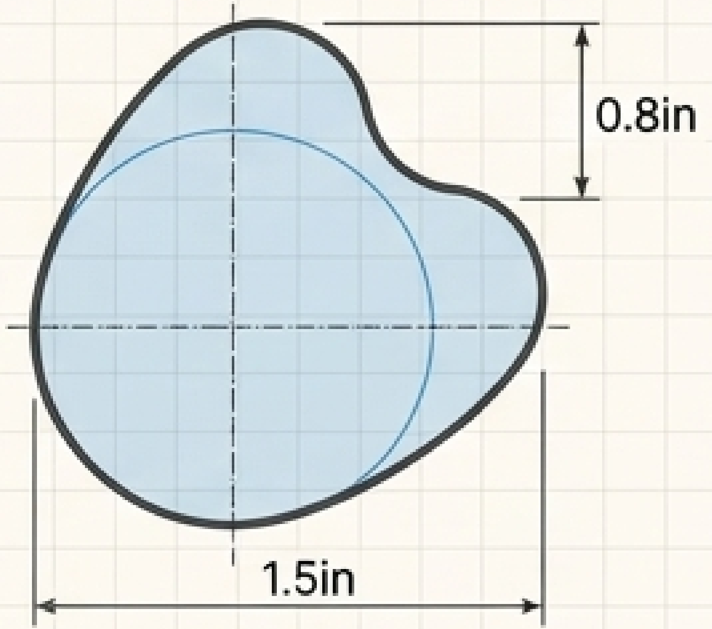
θ	y
180	0.8
210	0.8
240	0.724
270	0.524
300	0.276
330	0.076



Stage 2 - Graphical Construction



Final Checks & Profile



Final Checks

Check Pressure Angle: $\alpha_{max} \approx 28°$.
Status: OK ✓

Check Radius of Curvature: $\rho_{min} \approx 0.75$in > r.
Status: OK ✓

Result: A viable cam design that meets all kinematic and geometric requirements.