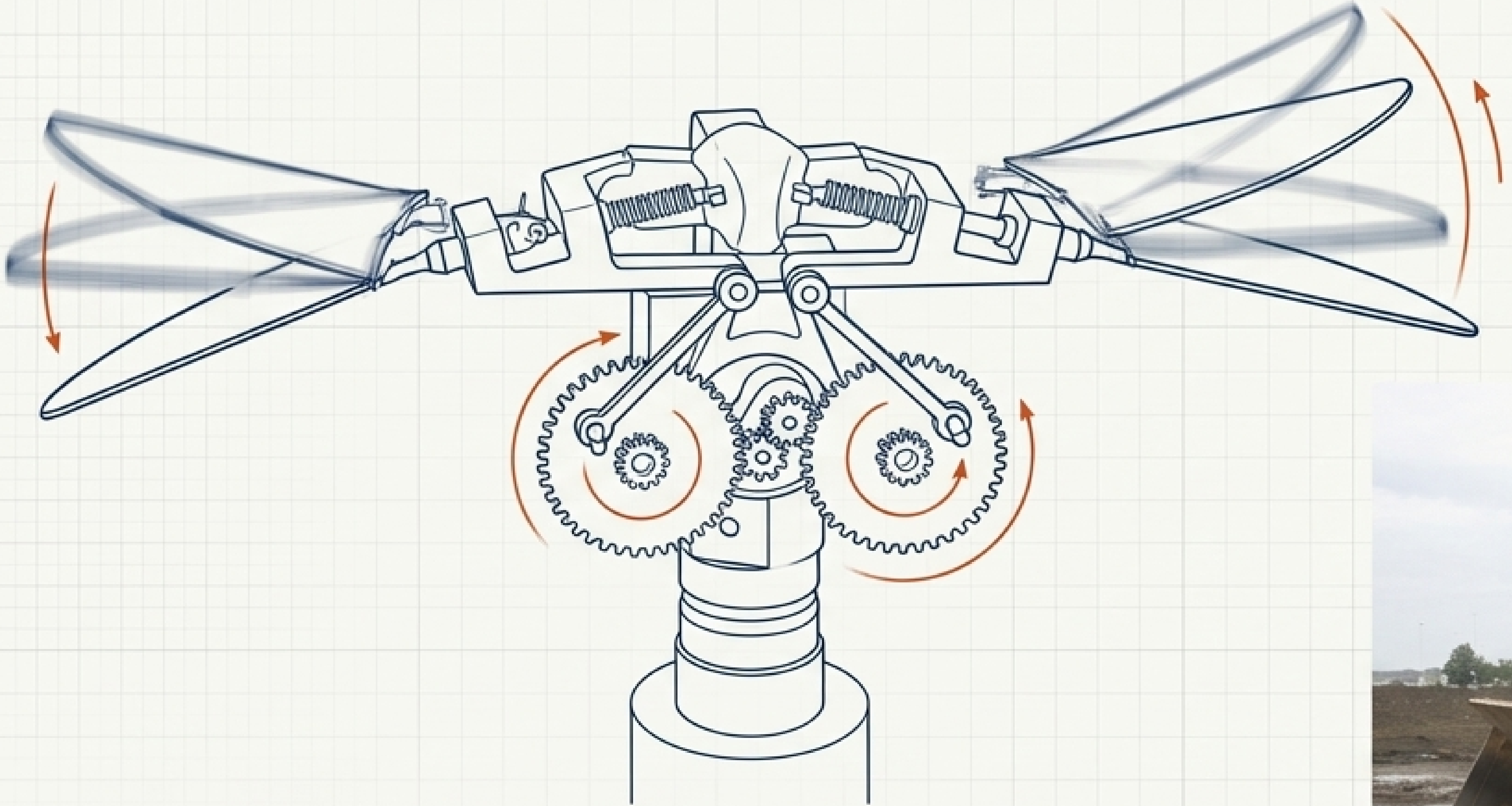
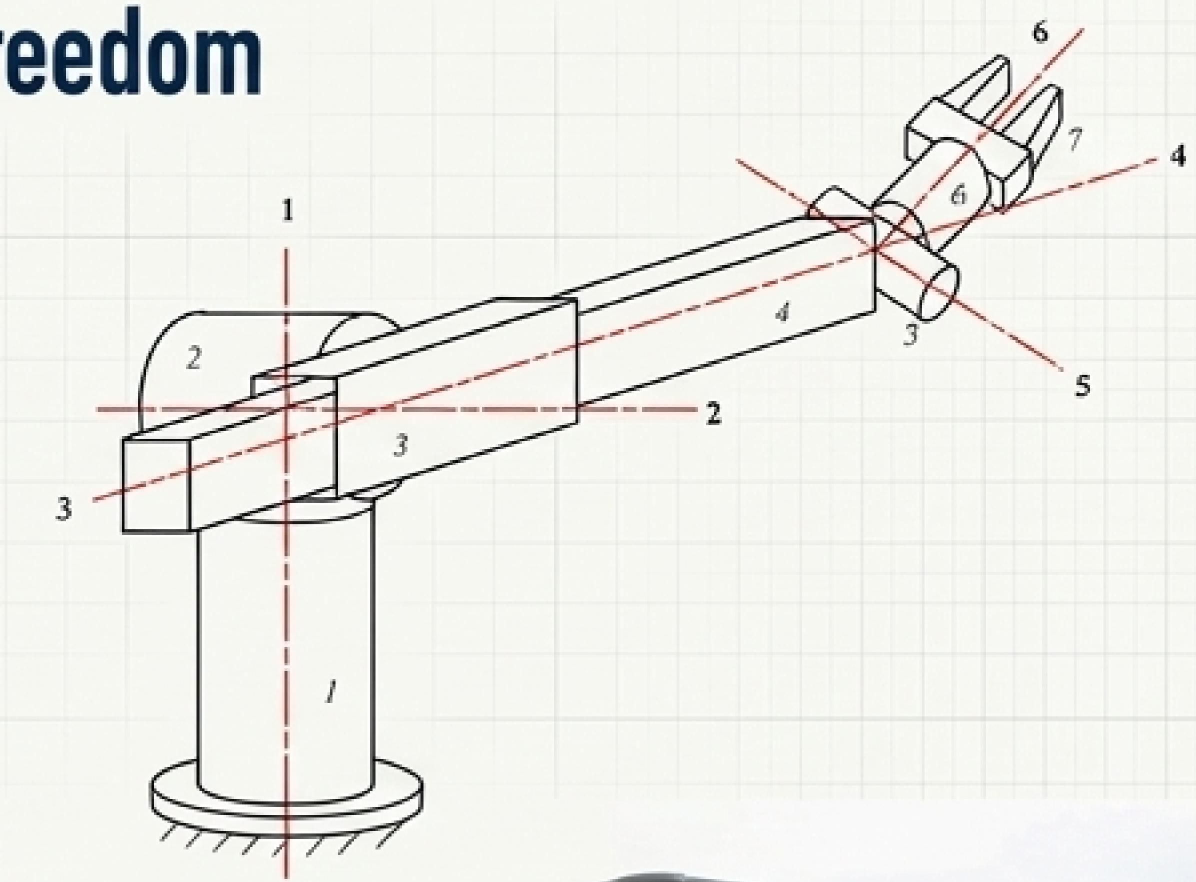
ME 3751: Fundamental Kinematics & Mechanisms

Lecture 1-2: Mobility Analysis – Quantifying Degrees of Freedom





University of Engineering & Technology
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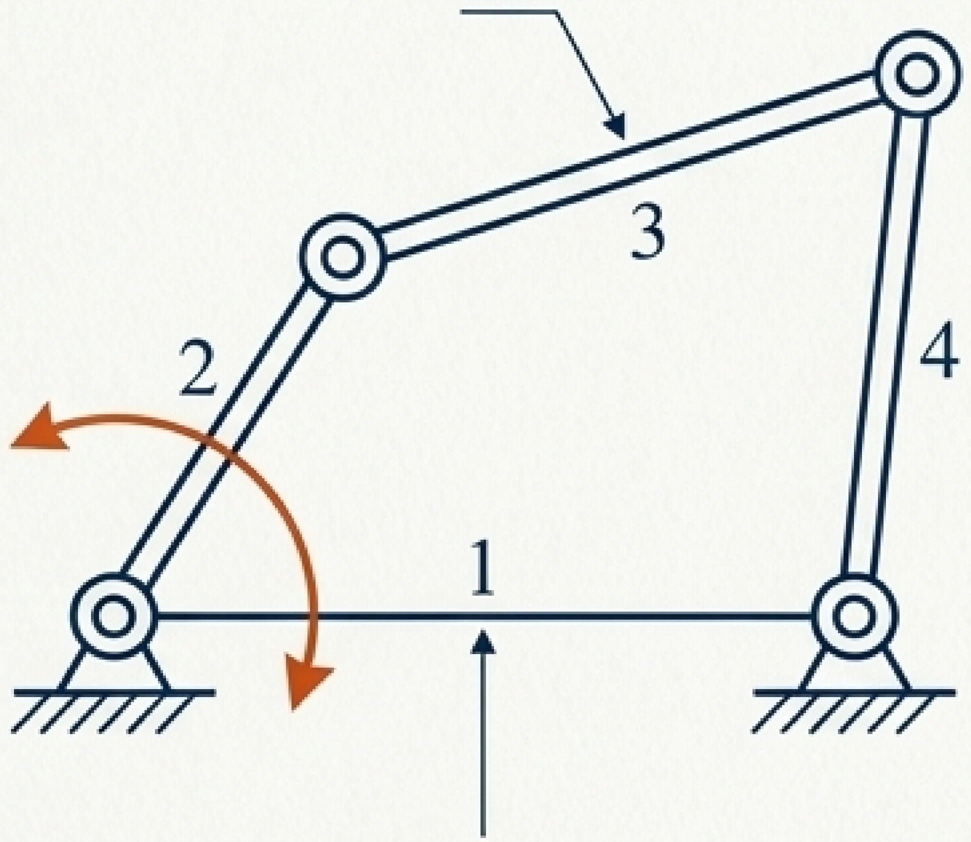
The First Question: Will It Move? How Many Inputs Does It Need?

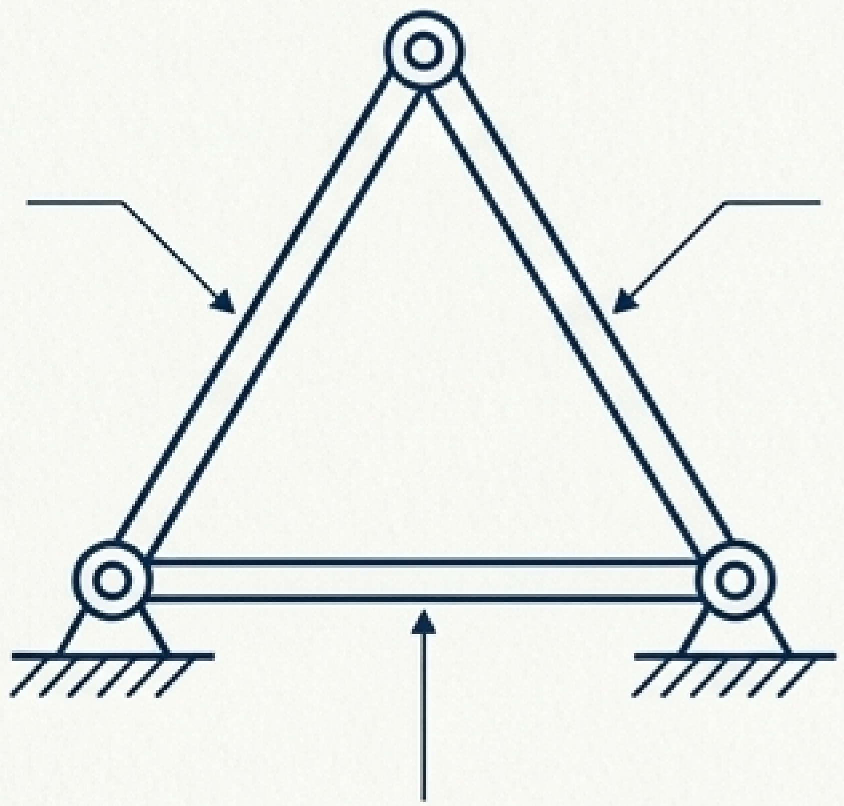
Before we analyze the *how* of motion (velocity, acceleration), we must first answer the *if* and *how many*. This is the role of **mobility analysis**.

Mobility (M): The number of independent inputs (or actuators) required to precisely control the position of every link in a mechanism. It is the system's Degrees of Freedom (DOF).

Why It Matters:
- **M ≥ 1**: It's a **mechanism,** capable of controlled motion.
- **M = 0**: It's a **structure,** statically determinate and rigid.
- **M < 0**: It's a pre-loaded or **statically indeterminate structure**.



Mechanism (M=1)



Structure (M=0)

The Engineer's Tool: The Planar Mobility Equation

For any planar linkage, the mobility (M) can be calculated with a simple formula, often called the Gruebler-Chebyshev equation.

Mobility or Degrees of Freedom (DOF)

Total number of joints

$$M = 3(n - 1 - j) + \sum f_i$$

Total number of links (including the fixed ground link)

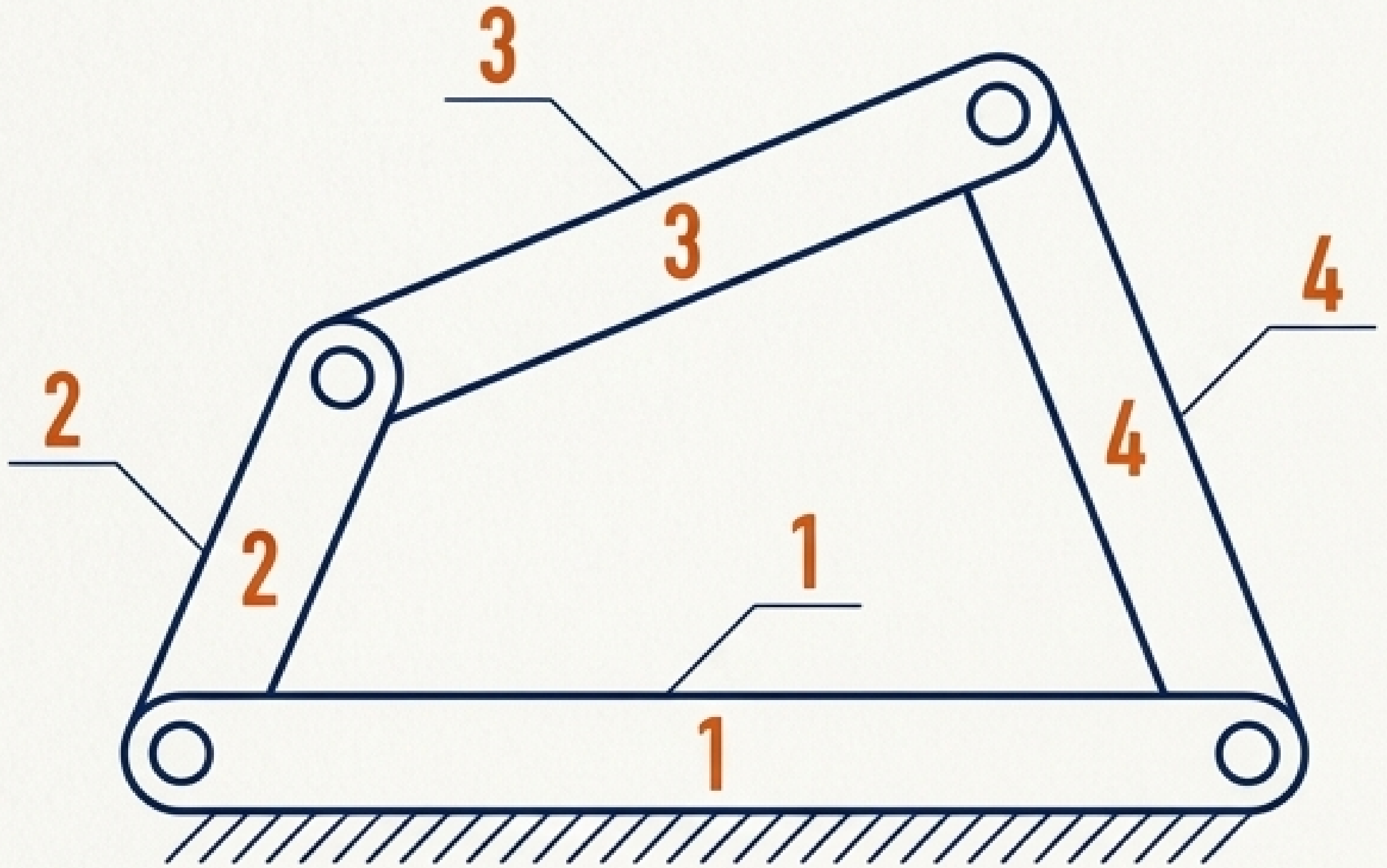
Connectivity (degrees of freedom) of the i-th joint.

Common Planar Joint Connectivities

Revolute (Pin) Joint: $f = 1$
Prismatic (Sliding) Joint: $f = 1$
Pin-in-Slot (Higher Pair): $f = 2$

"This is our primary tool... It's a budget of degrees of freedom. We start with the total possible freedom of all moving links, $3(n-1)$, and then subtract the constraints imposed by each joint."

Planar Example 1: The Classic Four-Bar Linkage



1. Identify & Count Links (n):

- Links are numbered 1 (ground), 2, 3, 4.
- **n = 4**

2. Identify & Count Joints (j) and their Connectivities (f_i):

- There are 4 Revolute joints (pins). All have f=1.
- **j = 4**
- **$\Sigma f_i = 1+1+1+1 = 4$**

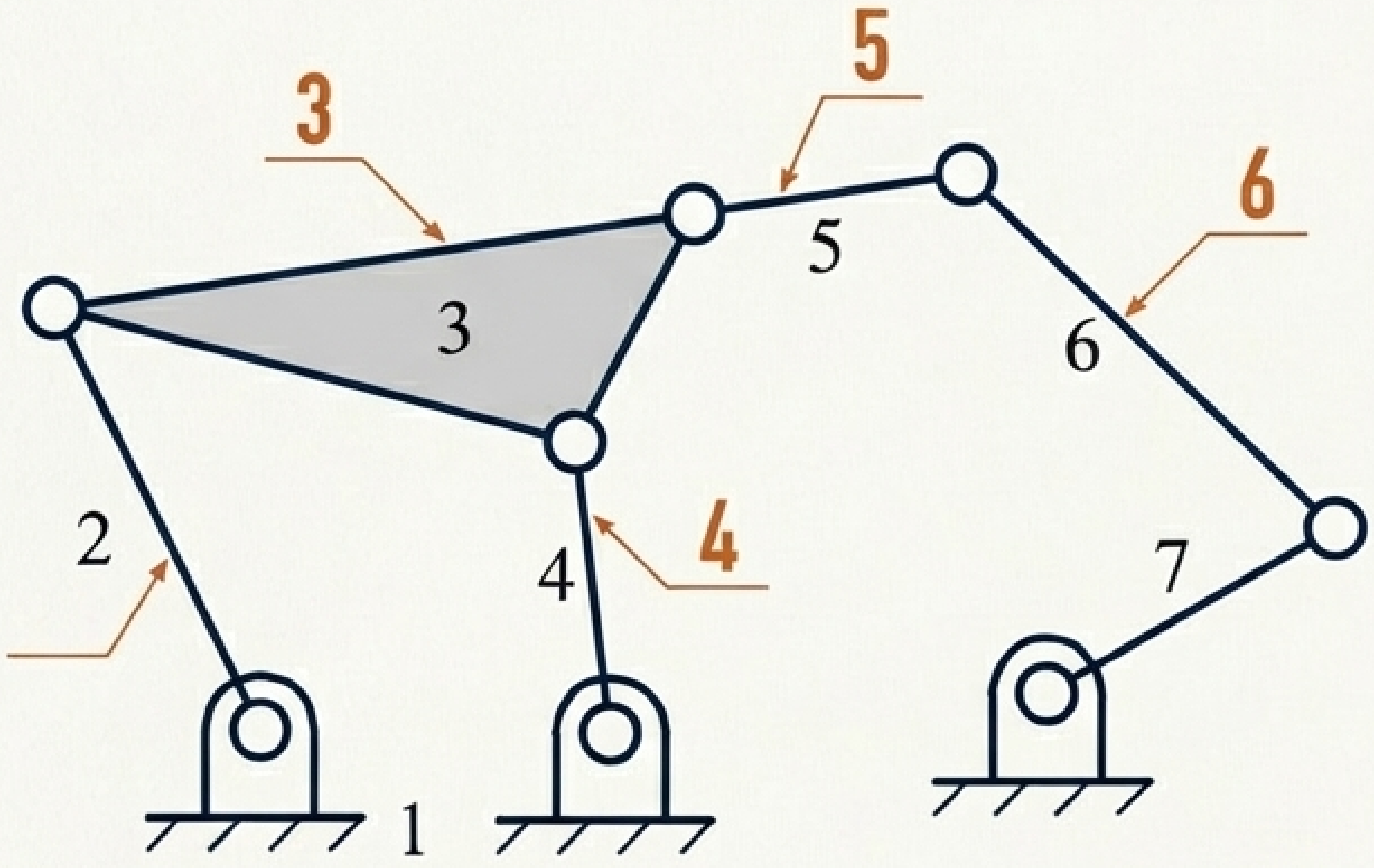
3. Calculate Mobility (M):

- $M = 3(n - 1 - j) + \Sigma f_i$
- $M = 3(4 - 1 - 4) + 4$
- $M = 3(-1) + 4 = 1$

Conclusion:

M = 1. This is a single degree-of-freedom mechanism. It requires only one input (e.g., rotating link 2) to fully define the position of all other links.

Planar Example 2: A Two-Loop, Seven-Bar Linkage



1. Count Links (n):

- 1 (ground), 2, 3, 4, 5, 6, 7.
- **n = 7**

2. Count Joints (j) & Connectivities (f_i):

- There are 8 Revolute joints. All have f=1.
- **j = 8**
- **$\Sigma f_i = 8$**

3. Calculate Mobility (M):

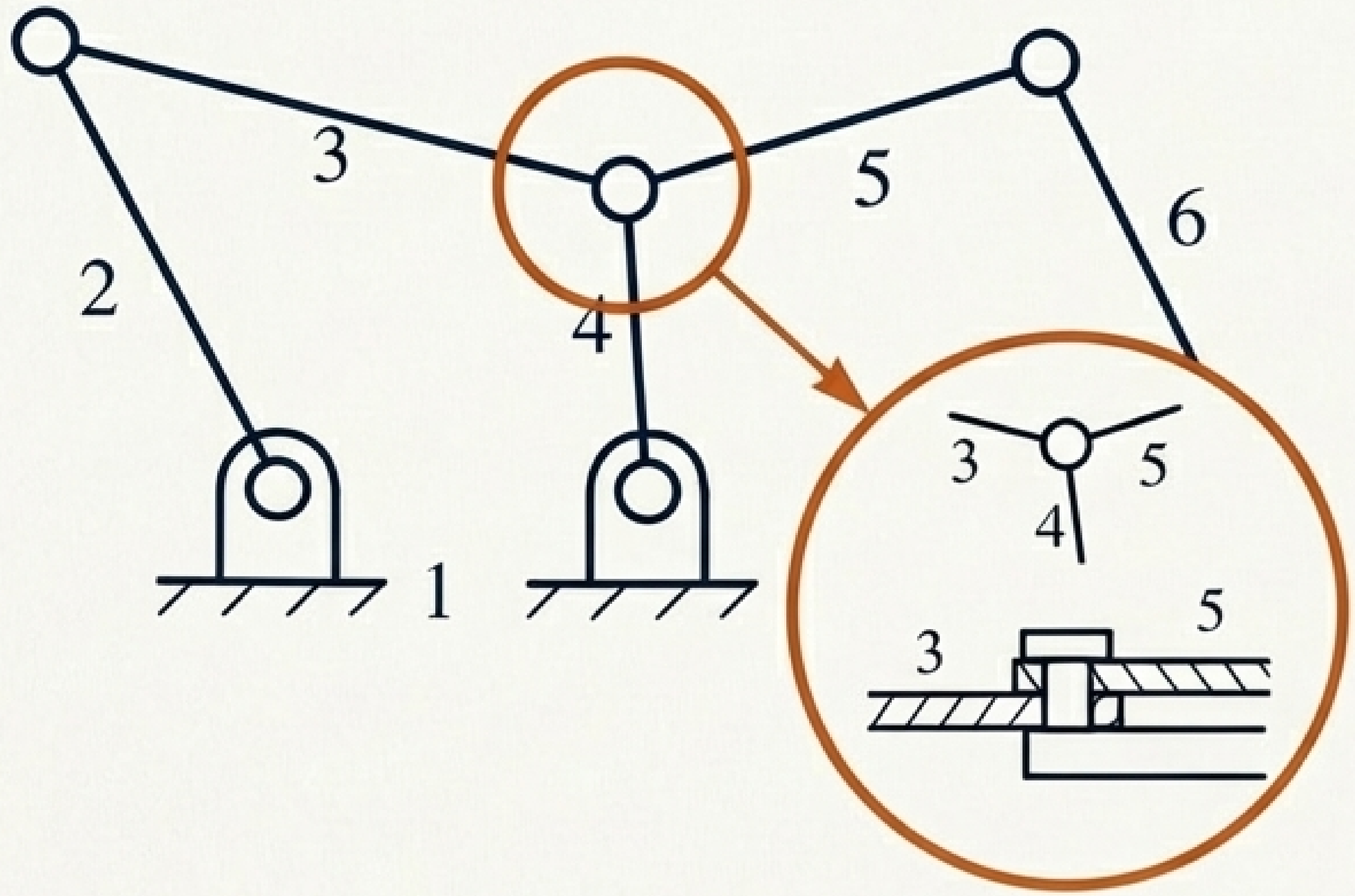
- $M = 3(n - 1 - j) + \Sigma f_i$
- $M = 3(7 - 1 - 8) + 8$
- $M = 3(-2) + 8 = 2$

Conclusion:

M = 2. This linkage requires two independent inputs to be fully constrained. For example, the angles of links 2 and 7 would need to be defined to know the position of all other links.

Planar Example 3: The Multiple Joint Rule

When p links connect at a single point, it is counted as $(p - 1)$ joints of that type.



Analysis Walkthrough

1. Count Links (n):

- 1 (ground), 2, 3, 4, 5, 6.
- **n = 6**

2. Count Joints (j):

- There are 5 single revolute joints.
- There is 1 multiple joint where p=3 links (3, 4, 5) meet. This counts as (3-1) = 2 joints.
- **Total j = 5 + 2 = 7**
- $\boldsymbol{\Sigma f_i = 7}$ (all joints are revolute with f=1)

3. Calculate Mobility (M):

- $M = 3(n - 1 - j) + \Sigma f_i$
- $M = 3(6 - 1 - 7) + 7$
- $M = 3(-2) + 7 = 1$

Conclusion:

M = 1. Applying the rule correctly reveals this is a single-DOF mechanism. A joint is a connection between *two* bodies.

Expanding to 3D Space: The Kutzbach Criterion

In 3D space, a free rigid body has 6 Degrees of Freedom (3 translational, 3 rotational). This changes the basis of our mobility calculation.

$$M = 6(n - 1 - j) + \sum f_i$$

M, n, j, f_i: Definitions remain the same.

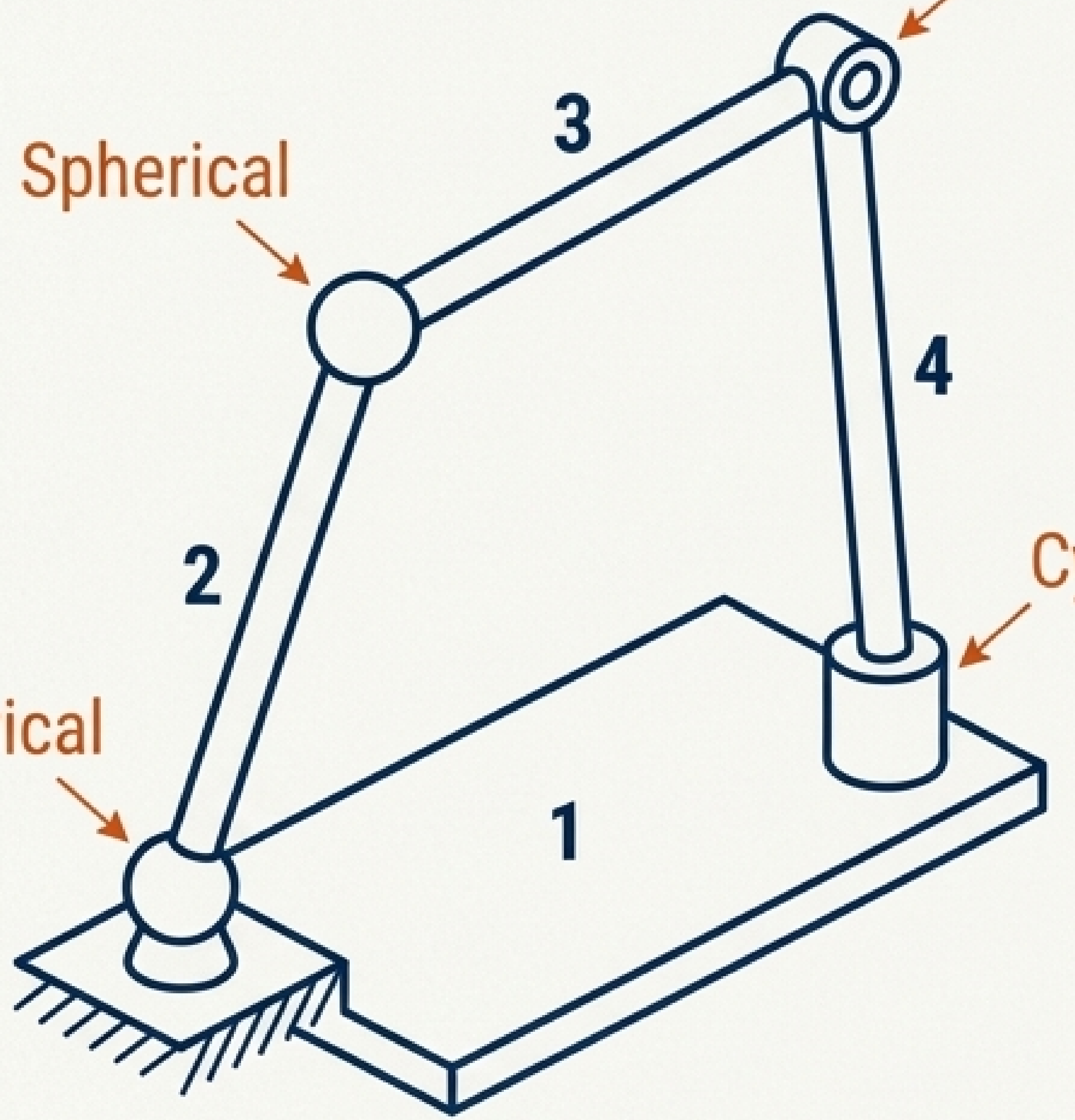
Common Spatial Joint Connectivities (f)

- Revolute (R): f = 1
- Prismatic (P): f = 1
- Cylindrical (C): f = 2
- Spherical (S) or Ball Joint: f = 3
- Planar (P_L): f = 3

(Reference: Table 1.1)

The key challenge in spatial analysis is correctly identifying the joint types and their associated connectivities.

Spatial Example 1: An RSSC-Type Linkage



Analysis Walkthrough

1. Count Links (n):

* 1 (ground), 2, 3, 4.
* $n = 4$

2. Count Joints (j) & Connectivities (f_i):

* There are $j = 4$ joints.
* Joint Types: 2 Spherical (f=3), 1 Revolute (f=1), 1 Cylindrical (f=2).
* $\Sigma f_i = (2 \times 3) + (1 \times 1) + (1 \times 2) = 6 + 1 + 2 = 9$

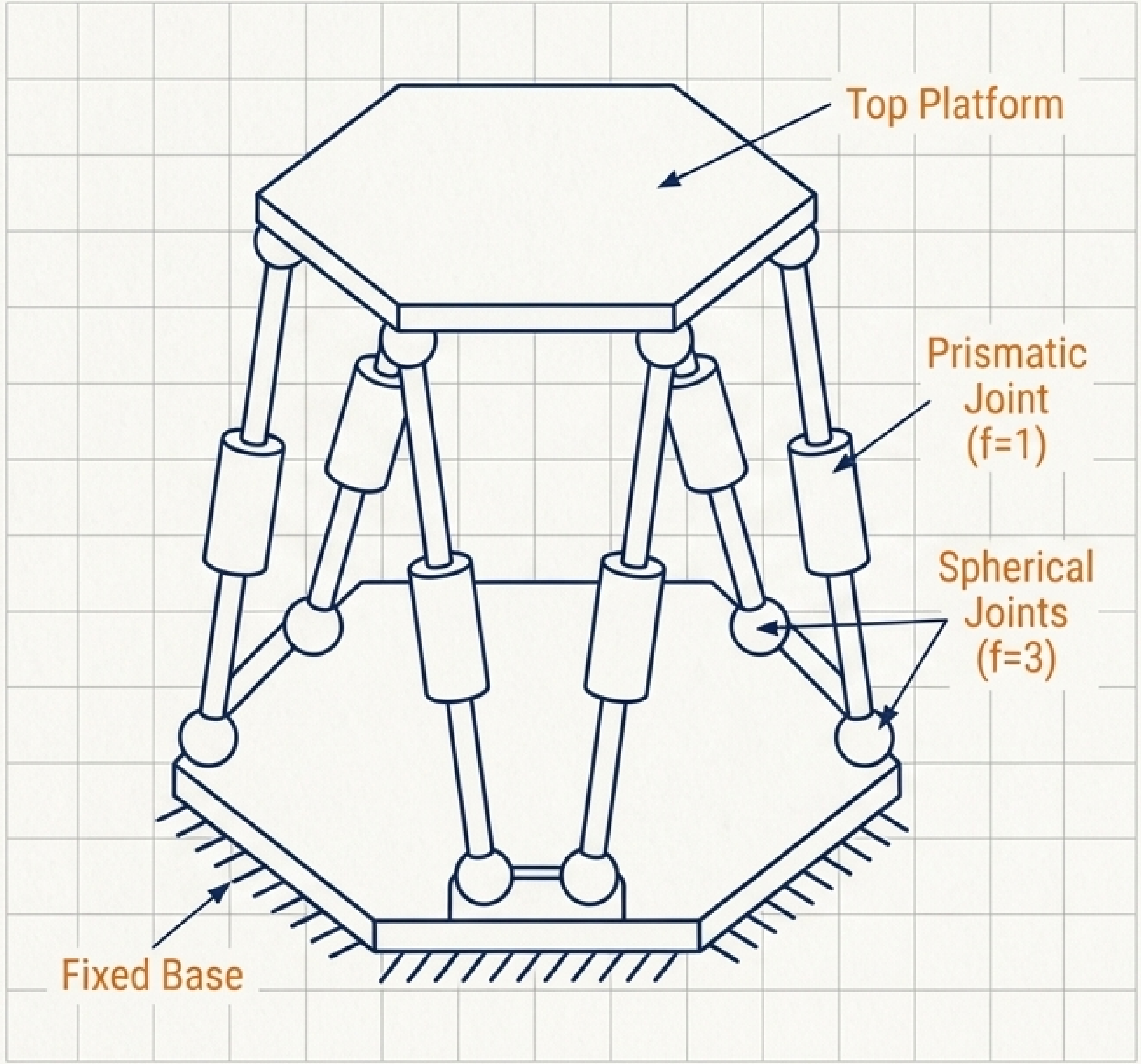
3. Calculate Mobility (M):

* $M = 6(n - 1 - j) + \Sigma f_i$
* $M = 6(4 - 1 - 4) + 9$
* $M = 6(-1) + 9 = 3$

Conclusion:

$M = 3$. This mechanism requires three independent inputs to fully control the position of all its links.

Spatial Example 2: The Stewart-Gough Platform



Analysis Walkthrough

1. Count Links (n):

- 1 (base) + 1 (platform) + 6 legs (each leg is 2 links) = 1 + 1 + 12 = 14
- **n = 14**

2. Count Joints (j) & Connectivities (f_i):

- 12 Spherical joints (f=3), two per leg.
- 6 Prismatic joints (f=1), one per leg.
- **j = 12 + 6 = 18**
- **$\sum f_i = (12 \times 3) + (6 \times 1) = 36 + 6 = 42$**

3. Calculate Mobility (M):

- $M = 6(n - 1 - j) + \sum f_i$
- $M = 6(14 - 1 - 18) + 42$
- $M = 6(-5) + 42 = 12$

Conclusion:

The calculated mobility is **M = 12**.
Does this match the reality of a platform designed for 6-DOF motion control (x, y, z, roll, pitch, yaw)?
The formula can sometimes be misleading.

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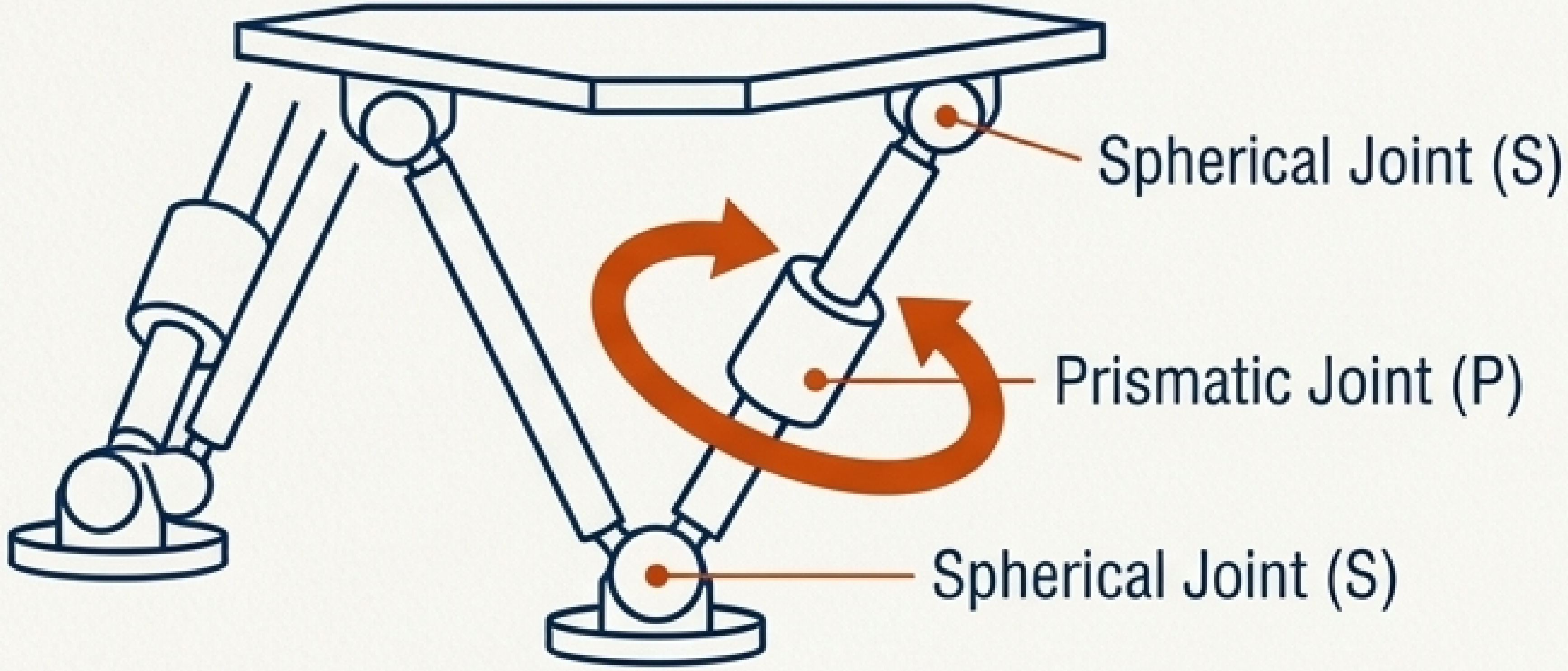
When the Formula Needs Interpretation: Idle Degrees of Freedom

An **Idle Degree of Freedom** is a motion of a link that does not affect the input-output relationship of the mechanism.



Explanation:

- The Kutzbach criterion calculates the *total* mobility of every link.
- We often only care about the *effective* mobility between the primary input and output links.
- **Effective Mobility = M_calculated - M_idle**

Each of the 6 legs can spin freely about its own axis. This spinning motion doesn't change the position or orientation of the top platform. These are 6 idle degrees of freedom.'

Recalculation:

Effective M = 12 (calculated) - 6 (idle freedoms) = 6

The Stewart–Gough platform has 6 effective degrees of freedom, matching its function perfectly. Mobility analysis requires engineering insight.

Your Mobility Analysis Toolkit

Core Concepts

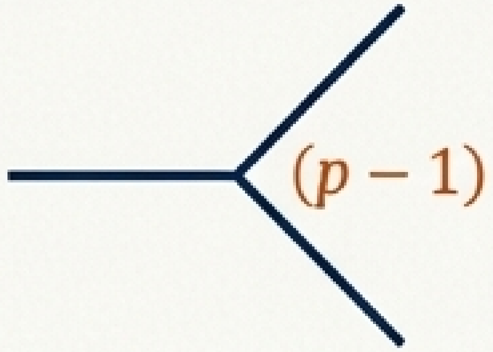
Purpose: Determine the Degrees of Freedom (DOF) of any linkage to understand if and how it will move, and how many actuators are needed.

Planar Formula: $M = 3(n - 1 - j) + \sum f_i$

Spatial Formula: $M = 6(n - 1 - j) + \sum f_i$

1.

Always **count the ground link** when determining *n*.



2.

A **connection of *p* links** at one point equals **$(p - 1)$ joints.**

3.

Always apply **engineering judgment.** Check for idle degrees of freedom that don't contribute to the mechanism's primary function.

Next Steps & Further Study

Next Lecture:

Position Analysis - Now that we know *if* a mechanism moves, we will learn how to calculate *where* it moves.

Upcoming Topics:

- Overconstrained Linkages (when the mobility formula gives an answer that seems 'too low').
- Grashof's Rules for four-bar linkages.

Assignments:

- **Reading:** Chapter 1, Sections 1.10 - 1.15
- **Homework Problems:** 1.12, 1.13, 1.29, 1.30