Homework 5

Total: 20 points
Due: Wed 19 Feb 2014 09:30 in class

Remember to show all steps in your working. If a question is taken from the textbook, the refence is given on the right of the page.

1. **NONHOMOGENEOUS EQUATIONS**

   (a) Solve the following initial value problem.

   $$y'' + 3y' - 4y = 4t - 5e^{-4t}, \qquad y(0) = 1, \quad y'(0) = 0.$$

   [Note: This is the same IVP as the one mentioned at the end of class on Monday Feb 10, except that one of the coefficients in the forcing function on the RHS of the equation has been changed to make the arithmetic a bit nicer.]

   (b) Plot a graph of the solution to the above IVP for $t \geq 0$. What is $\lim_{t\to\infty} y(t)$?

2. **2nd-ORDER MODELING**

   (a) In the following two equations, determine ω_0, R and δ so as to write them in the form $y = R\cos(\omega_0 t - \delta)$:

   i. $y = -\cos(t) + \sqrt{3}\sin(t)$ Boyce 3.7 Q2

   ii. $y = -2\cos(\pi t) - 3\sin(\pi t)$ Boyce 3.7 Q4

   (b) Boyce 3.7 Q5

   A mass weighing 2 lb hangs from a spring, stretching it 6 inches. If the mass is pulled down an additional 3 in and then released, and there is no damping, determine the vertical position y of the mass at any time t. Also find the frequency, period and amplitude of the motion.

   [Hint: You will need to use the fact that that the 2 lb weight stretches the spring 6 inches when at rest to find the spring constant k. This is the point where the downward gravitational force on the mass exactly balances the upwards force imparted by the spring. You may take $g = 32\ ft/s^2$, and remember that in imperial units mass m is given by $m = w/g$, where w is the weight of the object in pounds. If you need extra guidance on this question, Boyce section 3.7 has a number of similar examples, and details the method on how to set up the DE.]

   (c) A 2 kg block is placed on a smooth surface attached to a horizontally-acting spring with spring constant $k = 9\,\frac{1}{16}$ kg/s^2. We have seen in class that the displacement in meters $y(t)$ of the block from its rest position is then modeled by the differential equation

   $$2y'' + \frac{145}{16}\,y = 0.$$

   Suppose that the surface is not actually frictionless, but instead imparts a force of $-\gamma v$ on the block, where v is the horizontal velocity of the block, and γ is a constant.

   i. The block is released from a position of $y = 0.5$ m with zero initial velocity. Find the position of the block for all time $t \geq 0$ if $\gamma = 0.5$ kg/s.

   ii. Plot a graph of the above solution. Find the quasi-period of the oscillations of the block, and a time t_0 for which $|y(t)| < 0.05$ m for all $t > t_0$.

   iii. Given the same starting conditions as above, what is the smallest value of γ which will result in the block never crossing it rest position?

(d) Boyce 3.7 Q8

A series circuit has a capacitor of capacitance 0.25×10^{-6} F, an inductor of inductange 1 H, and negligible resistence. If the initial charge on the capacitor is 10^{-6} C and there is no initial current, find the charge Q on the capacitor at any time t.

(e) Boyce 3.7 Q12

A series circuit contains a capacitor of 10^{-5} F, an inductor of 0.2 H, and a resistor of 3×10^{2} Ω. The initial charge on the capacitor is 10^{-6} C and there is no initial current. Find the charge Q on the capacitor at any time t.

(f) Boyce 3.7 Q13

A certain vibrating system satisfies the differential equation

$$y'' + \gamma y' + y = 0.$$

Find the value of the damping coefficient γ for which the quasi-period of the damped motion is 50% greater than the period of the corresponding undamped motion.

(g) Boyce 3.7 Q24

The position of a vibrating system satisfies the initial value problem

$$\frac{3}{2}y'' + ky = 0, \qquad y(0) = 2, y'(0) = v.$$

If the period and amplitude of the resulting motion are observed to be π and 3 respectively, determine the values of k and v.