Homework 1

Total: 20 points
Due: Wed 1 Oct 2014 at the beginning of class

If a question is taken from the textbook, the refence is given on the right of the page.

1. **REVIEW**
   (a) Compute the following derivatives:
      i. $\frac{d}{dx}\, 3e^{-5x}$
      ii. $\frac{d}{dx}\, \frac{x\arctan(x)}{\tan(x)}$
   (b) Compute the following indefinite integrals:
      i. $\int x^{-\frac{2}{3}}\, dx$
      ii. $\int \frac{3x}{x^2+1}\, dx$
      iii. $\int xe^x\, dx$
   (c) Compute the following definite integral using partial fractions. Keep your answer exact, and simplify as much as possible.
   $$\int_{-4}^{0} \frac{x-2}{x^2-3x-10}\, dx$$
   (d) Sketch rough graphs of the following functions, including points where the curves intercept the axes, minima and maxima:
      i. $y = 3\cos\left(2x - \frac{\pi}{4}\right)$
      ii. $y = xe^{-x^2}$

2. **SEPARABLE EQUATIONS**
   (a) Find the general solution for the following differential equations. Solve for y if possible:
      i. $y' = \frac{x^2}{y(1+x^3)}$ Boyce 2.2 Q2
      ii. $y' = \frac{3x^2-1}{3+2y}$ Q4
      iii. $xy' = (1-y^2)^{1/2}$ Q6
   (b) Consider the initial value problem Q9
   $$y' - y^2 + 2xy^2 = 0, \quad y(0) = -\frac{1}{6}$$
      i. Find the solution to the differential equation in explicit form.
      ii. Plot the graph of the solution.
      iii. Determine the interval in which the solution is defined.
   (c) Consider the initial value problem Q13
   $$y' = \frac{2x}{y + x^2 y}, \quad y(0) = -2$$
      i. Find the solution to the differential equation in explicit form.
      ii. Plot the graph of the solution.
      iii. Determine the interval in which the solution is defined.
   (d) Solve the initial value problem and determine where the solution attains its minimum value: Q23
   $$y' = 2y^2 + xy^2, \quad y(0) = 1$$

   NB: More questions overleaf!

3. **METHOD OF INTEGRATING FACTORS**

Find the solutions to the initial value problems below:

(a) $y' - y = 2te^{2t}, \quad y(0) = 1$ — Boyce 2.1 Q13

(b) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \; t > 0$ — Q15

(c) $y' + 2\frac{y}{t} = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0, \; t > 0$ — Q16

(d) $ty' + 2y = \sin(t), \quad y(\frac{\pi}{2}) = 1, \; t > 0$ — Q18

(e) $ty' + (t+1)y = t, \quad y(\ln 2) = 1, \; t > 0$ — Q20