

Homework 1

Total: 20 points

Due: Wed 1 Oct 2014 at the beginning of class

If a question is taken from the textbook, the reference is given on the right of the page.

1. REVIEW

(a) Compute the following derivatives:

- i. $\frac{d}{dx} 3e^{-5x}$
- ii. $\frac{d}{dx} \frac{x \arctan(x)}{\tan(x)}$

(b) Compute the following indefinite integrals:

- i. $\int x^{-\frac{2}{3}} dx$
- ii. $\int \frac{3x}{x^2+1} dx$
- iii. $\int xe^x dx$

(c) Compute the following definite integral using partial fractions. Keep your answer exact, and simplify as much as possible.

$$\int_{-4}^0 \frac{x+2}{x^2-3x-10} dx$$

(d) Sketch rough graphs of the following functions, including points where the curves intercept the axes, minima and maxima:

- i. $y = 3 \cos\left(2x - \frac{\pi}{4}\right)$
- ii. $y = xe^{-x^2}$

2. SEPARABLE EQUATIONS

(a) Find the general solution for the following differential equations. Solve for y if possible:

- i. $y' = \frac{x^2}{y(1+x^3)}$ Boyce 2.2 Q2
- ii. $y' = \frac{3x^2-1}{3+2y}$ Q4
- iii. $xy' = (1-y^2)^{1/2}$ Q6

(b) Consider the initial value problem Q9

$$y' - y^2 + 2xy^2 = 0, \quad y(0) = -\frac{1}{6}$$

- i. Find the solution to the differential equation in explicit form.
- ii. Plot the graph of the solution.
- iii. Determine the interval in which the solution is defined.

(c) Consider the initial value problem Q13

$$y' = \frac{2x}{y+x^2y}, \quad y(0) = -2$$

- i. Find the solution to the differential equation in explicit form.
- ii. Plot the graph of the solution.
- iii. Determine the interval in which the solution is defined.

(d) Solve the initial value problem and determine where the solution attains its minimum value: Q23

$$y' = 2y^2 + xy^2, \quad y(0) = 1$$

NB: More questions overleaf!

3. METHOD OF INTEGRATING FACTORS

Find the solutions to the initial value problems below:

- | | |
|--|---------------|
| (a) $y' - y = 2te^{2t}, \quad y(0) = 1$ | Boyce 2.1 Q13 |
| (b) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \quad t > 0$ | Q15 |
| (c) $y' + 2\frac{y}{t} = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0, \quad t > 0$ | Q16 |
| (d) $ty' + 2y = \sin(t), \quad y(\frac{\pi}{2}) = 1, \quad t > 0$ | Q18 |
| (e) $ty' + (t+1)y = t, \quad y(\ln 2) = 1, \quad t > 0$ | Q20 |