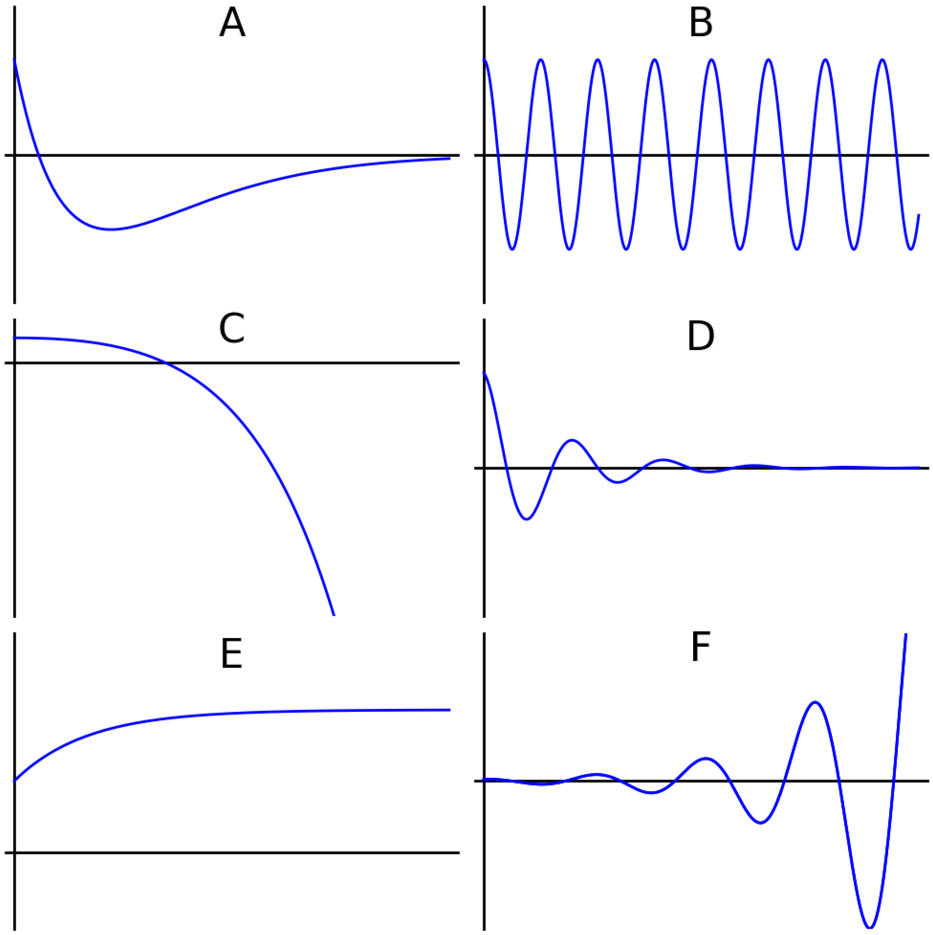
Your Name

Student ID #

- Cellphones off please!
- Please box all of your answers.
- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show your working** unless explicitly stated otherwise by the question. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may quote and use any formula you have seen in class to save time, but be sure to indicate with a word or two when you are doing so.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- If you need more room, use the backs of the pages, and indicate on the front of the page that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Below are the graphs of six functions $y(t)$, with t and y being the horizontal and vertical axes respectively. The graphs are labeled A through F. The graphs are **not** all drawn to the same scale, and axis markings have been purposely omitted.



Each of the functions graphed above is a solution to exactly one of the six differential equations below. By analyzing the form of the equations' general solutions, write the letter of the graph next to the differential equation for which it is the solution. You do not need to show your work in this question to receive full credit.

$$y'' - 3y' + 2y = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

$$y'' + 16y = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

$$y'' - y' + \frac{3}{2}y = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

$$y'' + y' + \frac{3}{2}y = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

$$y'' + y' + \frac{1}{4}y = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

$$y'' + 2y' = 0 \quad \_\_\_\_\_\_\_\_\_\_$$

2. (10 points) Solve the following initial value problem:

$$y'' - 6y' + 9y = 36t, \qquad y(0) = 1,\ y'(0) = 0.$$

3. (10 total points) A certain vibrating system satisfies the differential equation

$$0.5y'' + 0.1y' + 2y = 3\cos(\omega_0 t)$$

where ω_0 is the natural frequency of the system.

(a) (5 points) Compute the amplitude of the system's steady-state solution.

(b) (5 points) Suppose the forcing function's frequency is doubled to $2\omega_0$, but everything else remains the same. What does the amplitude of the steady-state solution now become?

4. (10 total points) Consider the initial value problem

$$(\alpha - 2)y'' + (3\alpha)y' + (2\alpha + 1)y = 0, \qquad y(0) = 1, y'(0) = 0$$

for a given constant α.

(a) (5 points) Find the values of α for which the solution to the IVP exhibits oscillatory behavior. For these values will the solution's oscillations be damped, constant in amplitude or exponentially growing?

(b) (5 points) Let α be the value which maximizes the solution's quasi-frequency, and let $y(t)$ be the solution to the IVP for this value of α. Find a time t_0 beyond which the amplitude of y never exceeds 0.1, i.e. for which $|y(t)| \leq 0.1$ for all $t > t_0$.

5. (10 total points + 3 bonus points) An object of unknown mass is placed on a flat surface and attached to a horizontal spring with spring constant 2.5 kg/s^2. The damping constant in the system is precisely 1 kg/s. When the object is pulled to the right of its equilibrium position and released, the damped oscillations of its subsequent motion are observed to have a quasi-period of $\frac{20}{7}\pi$ seconds.

   (a) (10 points) Suppose we know the object weighs more than 1 kg. What is the mass of the object? Justify your answer numerically.

   (b) (Bonus: 3 points) Compute the Wronskian of the system (i.e. compute the Wronskian of $y_1(t)$ and $y_2(t)$, where y_1 and y_2 are a fundamental basis of solutions to the differential equation obeyed by the object above).