# Homework 5

Total: 20 points
Due: Wed 5 November 2014 at the beginning of class

Remember to show all steps in your working. If a question is taken from the textbook, the refence is given on the right of the page.

1. **CHARACTERISTIC EQUATIONS WITH EQUAL ROOTS**

   (a) Find the general solution to the following differential equations:
      i. $9y'' + 6y' + y = 0$ Boyce 3.4 Q2
      ii. $y'' - 6y' + 9y = 0$ Boyce 3.4 Q6

   (b) Find the solution to each of the following initial value problems. Sketch the graph of the solution and describe its behaviour for increasing $t$:
      i. $9y'' - 12y' + 4y = 0, \quad y(0) = 2, y'(0) = -1$ Boyce 3.4 Q11
      ii. $y'' + 4y' + 4y = 0, \quad y(-1) = 2, y'(-1) = 1$ Boyce 3.4 Q14

   (c) Boyce 3.4 Q16
   Consider the initial value problem

   $$y'' - y' + \frac{1}{4}y = 0, \qquad y(0) = 2, \quad y'(0) = b,$$

   where $b$ is a constant. Find the solution as a function of $t$ for a given $b$, and determine the critical value of $b$ that separates solutions that grow positively from those that eventually grow negatively.

   (d) Boyce 3.4 Q24
   Consider the Euler equation

   $$t^2y'' + 2ty' - 2y = 0, \qquad t > 0.$$

   We may use the method detailed in the previous homework (substituting $x = \ln(t)$) to find the general solution to this equation. However, if we already know a specific solution we may instead use the method of reduction in order to find the DE's general solution.

   Given that $y_1(t) = t$ is a solution to this differential equation, use the method of reduction of order to find the general solution to this DE.

2. **NONHOMOGENEOUS EQUATIONS**

   (a) Find the general solution to the following differential equations:
      i. $y'' - 2y' - 3y = 3e^{2t}$ Boyce 3.5 Q1
      ii. $y'' + 2y' + 5y = 3\sin(2t)$ Boyce 3.5 Q2
      iii. $y'' + 2y' + y = 2e^{-t}$ Boyce 3.5 Q8
      iv. $y'' + y' + 4y = 2\sinh t$ [Hint: $\sinh t = \frac{1}{2}(e^t - e^{-t})$] Boyce 3.5 Q13

   (b) Boyce 3.5 Q20
   Solve the initial value problem

   $$y'' + 2y' + 5y = 4e^{-t}\cos(2t), \qquad y(0) = 1, \quad y'(0) = 0.$$

[Questions continued on the next page!]

(c) Consider the following initial value problem:

$$y'' + 4y' + 4y = \cos(t) + e^{-2t}, \qquad y(0) = 1, \quad y'(0) = 0.$$

The function $g(t) = \cos(t) + e^{-2t}$ is often called the *forcing function* in this DE, and the corresponding solution $y(t)$ to the IVP is called the *response*.

i. Find the solution to the above IVP.

ii. The limiting behaviour of the response, after exponential terms have decayed, can be written in the form $R\cos(t-\delta)$, where $R$ and $\delta$ are called the *amplitude* and *phase shift* of the response respectively. Find $R$ and $\delta$.

iii. Plot a graph of the response as a function of time. Also include on the graph a plot of $\cos(t)$, the oscillating part of the forcing function. Be sure to indicate on the graph the amplitude and phase shift of the response's limiting behaviour vs. that of $\cos(t)$.