Your Name

Student ID #

- This exam is closed books. No aids are allowed for this exam. You can use the information on the note sheet on the last three pages without justification, unless otherwise noted.
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet and the note sheet(s). Please make sure that your exam is complete.
- You have 110 minutes to complete the exam.

Question	Points	Score
1	8	
2	6	
3	6	
4	5	
5	5	
Total	30	

1. (8 points) A mass of 100 kg stretches a (very big) spring by 2 m. Suppose that the mass is in a medium that exerts a viscous resistance of 100 N when the mass has a velocity of $\frac{1}{2}$ m/s. An external force of $2500t + 1000\cos(t)$ N is applied to the mass. Suppose that, initially, the mass is in its equilibrium position, and that its initial velocity is zero.

   Find the position of the mass for $t > 0$. You can take the standard gravity to be $g = 10\,\text{m/s}^2$.

space for problem 1

2. (6 total points) On Thanksgiving day, you take the turkey out of the oven in your kitchen. At that time, the temperature of the Turkey is 240° C. The temperature in the kitchen is kept constant at 20° C. After one hour, the turkey is put into the refrigerator which is set to a temperature of 10° C. Finally, it is known that the cooling constant of a Turkey equals 4.

   (a) (4 points) Use the Laplace transform to determine the temperature of the turkey for any time $t > 0$.

(b) (1 point) What is the temperature of the Turkey after two hours?

(c) (1 point) Assume that you want to serve the turkey precisely when it is 40° C warm. At what time does that happen? (*Hint:* It happens within one hour after you took the turkey out of the oven).

3. (6 points) A series circuit has a resistor of $4\,\Omega$, a capacitor of $\frac{1}{5}$ F and an inductor of 1 H. The initial charge on the capacitor is zero. Assume that up to time zero, the circuit is closed, so that the current at time $t = 0$ is zero, too. At time $t = 0$, an electromotive force is connected to the circuit which impresses a voltage of $8\cos(t)$.

   Use the Laplace transform to determine the charge at time $t > 0$.

space for problem 3

4. (5 points) You want to launch a missile straight up into the sky. Assume you start the missile at time $t = 1$, at that the missile is in rest at that time. You know that due to the consumption of fuel, the mass of the missile decays, and that its mass can be well approximated by the function $\frac{1}{t}$ for times $t > 1$. Moreover, you know that the force due to air resistance equals $2|v|$ where v is the velocity of the missile. The force due to the missile's drive propulsion is constant equal to 30 N. You can take the standard gravity to be $g = 10\,\mathrm{m/s^2}$.

   Determine the velocity of the missile for all times $t > 1$ in terms of the function

   $$F(t) = \int_1^t e^{s^2}\,\mathrm{d}s.$$

   Hint: By the fundamental theorem of calculus, $F'(t) = e^{t^2}$.

5. (5 points) Initially, there are 2000 lions in the savanna. At that time, 1000 antelopes are released into the savanna. Based on previous experience, it is known that while antelopes are present, the population of the lions increases proportionally to the number of antelopes, while the population of antelopes declines proportionally to the number of lions.

   Determine the number of lions in the savanna at the moment when the antelopes have disappeared. Simplify your answer; your final answer should only involve very simple functions such as powers or roots. *Note:* Your answer will depend on certain unknown parameters.

space for problem 5