Your Name

Student ID #

- Cellphones off please!
- Please box all of your answers.
- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show your working** unless explicitly stated otherwise by the question. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may quote and use any formula you have seen in class to save time, but be sure to indicate with a word or two when you are doing so.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- If you need more room, use the backs of the pages, and indicate on the front of the page that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Find the general solution to the following second-order differential equation:

$$y'' + 16y = e^{-t}.$$

Your answer should be a function with two undetermined constants in it.

2. (10 points) In each part of this question you are given a function $y(t)$ which is the general solution to a constant-coefficient homogeneous 2nd-order differential equation. Write down the differential equation that that function satisfies. Your answer should be a DE in the form $ay'' + by' + cy = 0$ for some values of a, b and c.

   Each part is worth 2.5 points. You don't need to show your working to get full credit for this question.

   (a) $y(t) = c_1 e^{4t} + c_2 e^{-t}$

   (b) $y(t) = c_1 e^{\frac{t}{2}} \cos(t) + c_2 e^{\frac{t}{2}} \sin(t)$

   (c) $y(t) = c_1 + c_2 t$

   (d) $y(t) = c_1 e^{-3t} + c_2 t e^{-3t}$

3. (10 total points) A 5 kg mass is placed on a fairly smooth horizontal surface and attached to a horizontal spring. The spring exerts a force of $-\frac{y}{20}$ Newtons on the mass, where y is the displacement in meters of the mass relative to its equilibrium position. The surface subjects the mass to a frictional force of $-\frac{y'}{10}$ Newtons, where y' is the mass's velocity in ms^{-1}. Furthermore, the mass is subjected to an external force of $2\cos(\omega t)$ Newtons, where t is time measured in seconds, and ω is a constant.

   (a) (2 points) Write down a differential equation that describes the position of the mass at time t.

   (b) (4 points) For which of the four values of ω below will the amplitude of the steady-state response be greatest? Circle your answer, and write a sentence or two justifying your choice.

$$\omega = 0 \qquad \omega = \frac{1}{10} \qquad \omega = 1 \qquad \omega = 10$$

   (c) (4 points) For the value of ω you found above, compute the amplitude R of the steady-state response.

4. (10 total points) Consider the differential equation

$$t^2y'' - 6ty' + 6y = 0, \quad t > 0$$

One can check that $y_1(t) = t$ is a solution to this DE.

(a) (10 points) Use the method of reduction of order or any other method of your choosing to find a second solution $y_2(t)$ to the DE that is linearly independent from $y_1(t)$. Your answer should be a function with no undetermined constants in it.

5. (10 total points + 3 bonus points) A series circuit contains an inductor of inductance 0.2 henrys and a capacitor of capacitance 2×10^{-3} Farads. Resistance in the circuit is negligible, and the both current in the circuit and charge on the capacitor are initially zero. Starting at time $t = 0$ an external voltage of $0.49\cos(48t)$ volts is applied on the circuit.

   (a) (10 points) The capacitor is rated to hold a maximum charge of 0.03 Coulombs. Is the circuit safe, or will the capacitor fail? Be sure to justify your conclusion numerically.

   (b) (Bonus: 3 points) If the capacitor could hold any amount of charge (i.e. it never fails), compute or estimate the maximum amount of current that occurs in the circuit given the above setup.