# Homework 7

Total: 20 points
Due: Monday 1 December 2014 at the beginning of class [Note the different hand-in date!]

Remember to show all steps in your working. If a question is taken from the textbook, the reference is given on the right of the page.

1. **LAPLACE TRANSFORMS**

   In each of the following problems $a$ is a a real constant. Compute the Laplace tranform of the given function. Your answer should be in the form of a function $F(s)$:

   (a) $f(t) = \cosh(at)$ — Boyce 6.1 Q7

   (b) $f(t) = \sinh(at)$ — Boyce 6.1 Q8

   (c) $f(t) = te^{at}$ — Boyce 6.1 Q15

   (d) $f(t) = t\sin(at)$ — Boyce 6.1 Q15

2. **SOLVING DEs WITH LAPLACE**

   (a) In each of the following problems use the rules of Laplace transforms that you know and the lookup table in Boyce to find the inverse Laplace tranform of the given function. Your answer should be in the form of a function $f(t)$:

   i. $F(s) = \frac{3}{s^2+4}$ — Boyce 6.2 Q1

   ii. $F(s) = \frac{4}{(s-1)^3}$ — Boyce 6.2 Q2

   iii. $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$ — Boyce 6.2 Q8

   iv. $F(s) = \frac{2s-3}{s^2+2s+10}$ — Boyce 6.1 Q15

   (b) Use the Laplace transform to solve the given initial value problems:

   i. $y'' - y' - 6y = 0, \;\; y(0) = 1, \;\; y'(0) = -1$ — Boyce 6.2 Q11

   ii. $y'' - 2y' + 2y = 0, \;\; y(0) = 0, \;\; y'(0) = 1$ — Boyce 6.2 Q13

   iii. $y'' - 2y' + 2y = e^{-t}, \;\; y(0) = 0, \;\; y'(0) = 1$ — Boyce 6.2 Q22

   iv. $y'' + 2y' + y = 4e^{-t}, \;\; y(0) = 2, \;\; y'(0) = -1$ — Boyce 6.2 Q23

3. **STEP FUNCTIONS**

   (a) Sketch the graph of the function — Boyce 6.3 Q1

   $$g(t) = u_1(t) + 2u_3(t) - 6u_4(t)$$

   for $t \geq 0$.

[More problems on the next page!]

(b) In each of the following problems, find the Laplace transform of the given function:

i. $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$ Boyce 6.3 Q13

ii. $f(t) = \begin{cases} 0, & t < \pi \\ t - \pi, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$ Boyce 6.3 Q15

iii. $f(t) = (t-3)u_2(t) - (t-2)u_3(t)$ Boyce 6.3 Q17

(c) In each of the following problems, find the inverse Laplace transform of the given function:

i. $F(s) = \frac{e^{-2s}}{s^2+s-2}$ Boyce 6.3 Q20

ii. $F(s) = \frac{2e^{-2s}}{s^2-4}$ Boyce 6.3 Q22

iii. $F(s) = \frac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s}$ Boyce 6.3 Q24

4. **DEs WITH DISCONTINUOUS FORCING FUNCTIONS**
THIS QUESTION IS NOT FOR POINTS, BUT YOU SHOULD ATTEMPT IT ANYWAY

In each of the following, find the solution to the given initial value problem:

(a) $y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \quad f(t) = \begin{cases} 1, & 0 \leq t < 3\pi \\ 0, & 3\pi \leq t < \infty \end{cases}$ Boyce 6.4 Q1 (a)

(b) $y'' + 3y' + 2y = f(t), \quad y(0) = 0, \quad y'(0) = 0 \quad f(t) = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & t \geq 10 \end{cases}$ Boyce 6.4 Q5 (a)

(c) $y'' + 4y = u_\pi(t) - u_{3\pi}(t), \quad y(0) = 0, \quad y'(0) = 0$ Boyce 6.4 Q11 (a)