Homework 1

Total: 20 points
Due: Wed 15 Jan 2014 09:30 in class.

If a question is taken from the textbook, the refence is given on the right of the page.

1. **REVIEW**
   (a) Compute the following derivatives:
      i. $\frac{d}{dx}\ 4e^{-2*x}$
      ii. $\frac{d}{dx}\ \frac{x^2 sin(x)}{\arctan(x)}$
   (b) Compute the following indefinite integrals:
      i. $\int x^{-\frac{1}{2}}\ dx$
      ii. $\int \frac{3x}{x^2+1}\ dx$
      iii. $\int xe^x\ dx$
   (c) Compute the following integral using partial fractions. Note that you will need to use complex numbers:
   $$\int \frac{x+2}{x^2+9}\ dx$$
   (d) Sketch rough graphs of the following functions, including points where the curves intercept the axes, minima and maxima:
      i. $y = 3\cos\left(2x - \frac{\pi}{4}\right)$
      ii. $y = xe^{-x^2}$

2. **SEPARABLE EQUATIONS**
   (a) Find the general solution for the following differential equations. Solve for y if possible:
      i. $y' = \frac{x^2}{y(1+x^3)}$ Boyce 2.2 Q2
      ii. $y' = \frac{3x^2-1}{3+2y}$ Q4
      iii. $xy' = (1-y^2)^{1/2}$ Q6
   (b) Consider the initial value problem Q9
   $$y' - y^2 + 2xy^2 = 0, \quad y(0) = -\frac{1}{6}$$
      i. Find the solution to the differential equation in explicit form.
      ii. Plot the graph of the solution.
      iii. Determine the interval in which the solution is defined.
   (c) Consider the initial value problem Q13
   $$y' = \frac{2x}{y + x^2 y}, \quad y(0) = -2$$
      i. Find the solution to the differential equation in explicit form.
      ii. Plot the graph of the solution.
      iii. Determine the interval in which the solution is defined.
   (d) Solve the initial value problem and determine where the solution attains its minimum value: Q23
   $$y' = 2y^2 + xy^2, \quad y(0) = 1$$

NB: More questions overleaf!

3. METHOD OF INTEGRATING FACTORS

Find the solutions to the initial value problems below:

(a) $y' - y = 2te^{2t}, \quad y(0) = 1$ — Boyce 2.1 Q13

(b) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \;\; t > 0$ — Q15

(c) $y' + 2\frac{y}{t} = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0, \;\; t > 0$ — Q16

(d) $ty' + 2y = \sin(t), \quad y(\frac{\pi}{2}) = 1, \;\; t > 0$ — Q18

(e) $ty' + (t+1)y = t, \quad y(\ln 2) = 1, \;\; t > 0$ — Q20