Your Name

Student ID #

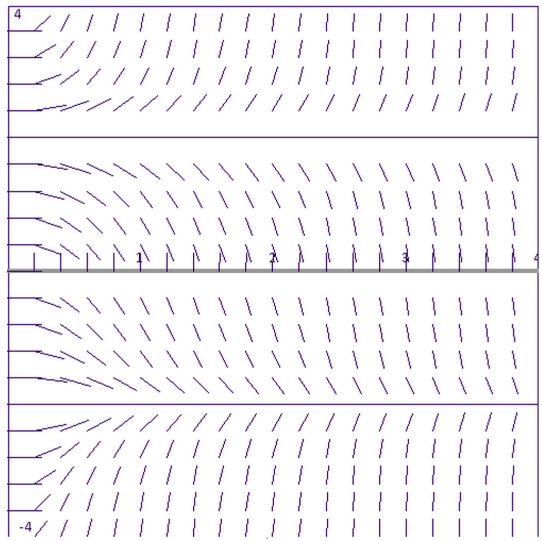
- Cellphones off please!
- Please box all of your answers.
- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show all of your work** unless explicitly stated otherwise by the question. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- If you need more room, use the backs of the pages, and indicate on the front of the page that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Solve the following initial value problem explicitly. Your answer should be a function in the form $y = g(x)$, where there is no undetermined constant in g.

$$\frac{dy}{dx} = \frac{(x^2+1)(y^2+1)}{xy}, \quad y(1) = -1.$$

2. (10 total points)
The slope field to the differential equation $\frac{dy}{dx} = f(x,y)$ is plotted below for $0 \leq x \leq 4, -4 \leq y \leq 4$:



(a) (4 points) Circle the differential equation that corresponds to the above slope field (you do not need to show your working to receive full grade for this part of the question).

$$\frac{dy}{dx} = (x^2-4)y \qquad \frac{dy}{dx} = x(y^2-4) \qquad \frac{dy}{dx} = -x(y^2-4) \qquad \frac{dy}{dx} = -(y^2-4)$$

(b) (6 points) Let $y = h(x)$ be the solution to the differential equation you circled above that satisfies the initial condition $y(1) = 1$. Use Euler's method with a step size of $h = 0.5$ to estimate the value of the solution at $x = 2$. You may use decimal approximations in your final answer (but keep at least 4 digits precision at all points).

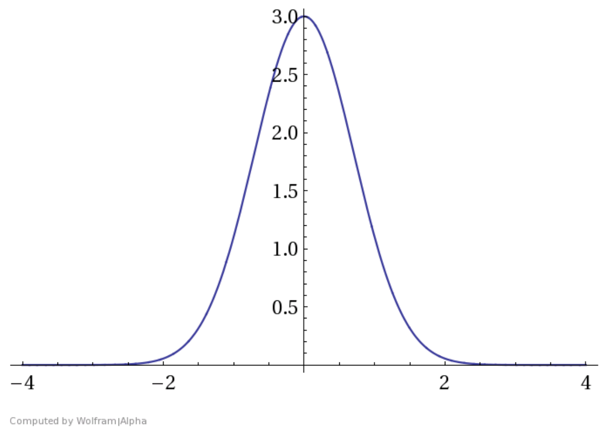
3. (10 total points)

   (a) (2 points) Using the existence and uniqueness theorem for linear first-order differential equations, state the minimum t-interval for which a unique solution is guaranteed to exist for the following initial value problem. **You do not need to solve the DE to answer this question**.

$$t \cdot \frac{dy}{dt} = t \tan(t) \cdot y - \cos(t), \qquad y(-1) = 0$$

   (b) (8 points) The temperature in my kitchen is 21°C and the temperature inside my fridge is 4°C. I pull a cold soda from the fridge and place it on the kitchen counter. I don't like my drinks too cold, so I decide to wait for it to warm up a bit before drinking it. Suppose that the rate of heating of the drink is proportional to the difference between its temperature and that of the kitchen; furthermore suppose I have ascertained that the proportionality constant involved is $k = \frac{1}{20}$ min^{-1}. How long must I wait until my drink warms to 15°C? You may provide your answer in exact form or as a decimal for this question.

4. (10 total points) The graph of $h(y) = 3e^{-y^2}$ looks as follows:



The graph is always positive, as $e^{-y^2} > 0$ for any value of y.

Now consider the autonomous differential equation

$$\frac{dy}{dt} = h(y) + \beta = 3e^{-y^2} + \beta,$$

where β is a constant.

(a) (4 points) For what value(s) of β does the DE have two different equilibrium solutions? Classify each such solution according to whether it stable, unstable or semistable.

[Question continued on the next page.]

(b) (2 points) For what value(s) of β does the DE have only one equilibrium solution? Classify each such solution according to whether it stable, unstable or semistable.

(c) (4 points) Suppose we are given the initial value problem

$$\frac{dy}{dt} = 3e^{-y^2} - 1, \quad y(0) = 1.$$

Without solving the DE, compute the limiting value of the solution as $t \to \infty$. [That is, compute $\lim_{t\to\infty} h(t)$.]

5. (10 total points + 3 bonus points) A skydiver with a total mass of 100kg jumps from a plane, falling vertically downward. Gravity acts on the skydiver – you may take gravitational acceleration to be a constant $g = 10\ \mathrm{ms}^{-2}$. Air resistance also acts on the skydiver with a force proportional (and opposite in direction) to the skydiver's velocity.

   At time $t = 1$ second into the jump the skydiver's downward velocity is $9\ \mathrm{ms}^{-1}$. However, at this time the skydiver starts progressively tucking in her arms, effectively reducing the force of air resistance acting on her as time goes on. The drag coefficient k for the skydiver is therefore no longer constant, and is instead given by

   $$k = \frac{25}{t}\ \mathrm{kg\ s}^{-1},$$

   where t is in seconds since the skydive began.

   (a) (10 points) Establish an initial value problem and solve it to find an explicit formula for the velocity of the the skydiver at any point after she start tucking in her arms.

   (b) (3 bonus points) Estimate how long it takes after jumping for the skydiver's speed to reach 60 ms^{-1}. You may provide an approximate answer, but be sure to justify any such approximation.