Math 307 A - Fall 2011
Pactice Mid-Term Exam
October 31, 2011

Name: ______________________ Student number: ______________

1	12	
2	9	
3	10	
4	9	
5	10	
Total	50	

- Complete all questions.
- You may use a scientific calculator during this examination. Other electronic devices (e.g. cell phones) are not allowed, and should be turned off for the duration of the exam.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 60 minutes to complete the exam.

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1. Match each differential equation with the most immediately effective method for solving it. You do *not* need to find the solutions.

1) Separate Variables	2) Substitute $u = \frac{y}{x}$	3) Find an integrating factor
4) Find the potential function	5) Substitue $u = x + y$	6) None of the these

(a) (2 points) $$y' = \frac{\ln(y) + 1 - \ln(x)}{y/x + 1}$$ Ans:_____

(b) (2 points) $$(y^2 - xye^{xy}) + (2xy - x^2e^{x^2y})y' = 0$$ Ans:_____

(c) (2 points) $$y' = \frac{y}{x^2}$$ Ans:_____

(d) (2 points) $$y' = e^{x-1} \cdot e^y - 1$$ Ans:_____

(e) (2 points) $$(y^2 - 2xye^{x^2y}) + (2xy - x^2e^{x^2y})y' = 0$$ Ans:_____

(f) (2 points) $$y' = \frac{y}{x} - x$$ Ans:_____

2. (a) (3 points) Find the general solution to the linear equation by finding an integrating factor

$$\frac{dy}{dx} = \frac{1}{x}y - xe^x.$$

(b) (3 points) Find the general solution to the homogeneous equation using the substitution $u = y/x$

$$xy\frac{dy}{dx} = (y+x)^2, \quad x, y > 0.$$

(c) (3 points) Find the general solution to the exact equation by finding the potential function

$$y(y+1) + x(2y+1)y' = 0.$$

3. (a) (3 points) Right now, there are 100 bunnies in your neighborhood. Assume, in the absence of other factors, their population doubles every week. However, there are hawks, owls, wolves, bobcats, and other predators which hunt the bunnies. If $P(t)$ is the population of rabbits after t weeks, denote by $K(t)$ the number of bunnies which the predators kill each week [in general this should depend on t]. Write down, but not solve, a differential equation modeling $P(t)$.

   (b) (5 points) For simplicity, assume that $K(t)$ is a constant, K. Find such a constant K so that in the long run, the bunnies neither go extinct nor have a population explosion.

   (c) (2 points) If K is this critical value, is the population of bunnies stable?
   Hint: This is asking about a stable equilibrium of an autonomous equation.

4. Solve the following second-order differential equations:

(a) (3 points) $y'' - 2y' = 0.$

(b) (3 points) $\frac{1}{4}y'' + 3y' + 9y = 0.$

(c) (3 points) $y'' - y' + \frac{5}{4}y = 0,$

5. (10 points) Consider the linear homogeneous differential equation

$$y'' - \frac{t}{t^2\ln(t) - t^2}y' + \frac{1}{t^2\ln(t) - t^2}y = 0, \qquad t > 0.$$

(a) (2 points) Verify directly that $y_1(t) = t$ is a solution.

(b) (8 points) Find another independent solution using the method of reduction of order, and write down the general solution to the differential equation.