

AN INTERMEDIATE ARTICLE

YOUR NAME.

ABSTRACT. In this note, we prove that our L^AT_EX is at an intermediate level.

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Challenge 1 (citing a 1656 paper). $\binom{2n}{n} \sim 4^n / \sqrt{\pi n}$ follows from Wallis's product formula [Wal56].

Challenge 2 (Fatou's lemma). If $f_n \rightarrow f$ a.s., then

$$\int f \, d\mu \leq \liminf_{n \rightarrow \infty} \int f_n \, d\mu.$$

Challenge 3 (a summation).

$$\sum'_{x \in A} f(x) = \sum_{\substack{x \in A \\ x \neq 0}} f(x).$$

Challenge 4 (a superfluous statement). Consider real-valued functions f_1, f_2, \dots , all defined on

$$R :=]-\infty, 0[\times \{0, 1\} \equiv \{ (x, y) \mid 0 < x < 1 \text{ and } y \in \{0, 1\} \}.$$

If $f_n = f_m$ a.e. for all $n, m \in \mathbb{N}$, then $f_n \rightarrow f$ a.e. for some f .

Challenge 5 (the harmonic series). The harmonic series diverges.

Proof.

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n} &= \frac{1}{1} + \underbrace{\frac{1}{2} + \frac{1}{3}}_2 + \underbrace{\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}}_4 \\ &\quad + \underbrace{\frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15}}_8 + \dots \\ &\geq \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots = \infty. \end{aligned} \quad \square$$

Challenge 6 (a symbol that doesn't exist). $X \perp\!\!\!\perp Y$.

Challenge 7 (Knuth's up-arrow notation).

$$2 \uparrow\uparrow k \stackrel{\text{def}}{=} 2^{2^{\cdot^{\cdot^{\cdot^2}}}} \Big\}^k.$$

Challenge 8 (true/false questions). Answer the following with T or F.

1. There exists a well-order \prec on \mathbf{R} such that for any $x \in \mathbf{R}$ the set $\{y \in \mathbf{R} \mid y \prec x\}$ is countable. Ans: _____

2. Apéry's constant Ans: _____

$$\zeta(3) := \sum_{n=1}^{\infty} \frac{1}{n^3}$$

is transcendental.

3. There exists a polynomial-time algorithm that determines if a given graph is 3-colorable. Ans: _____

Challenge 10¹ (a popular number). For each $n \in \mathbf{N}$, let $D(n)$ be the sum of all positive divisors of n . What is $\sum_{n=1}^9 D(n)$?

Answer:

Challenge 11 (countdown). A blade is aiming for the top.

- (3) Three!
- (2) Two!
- (1) One!
- (0) Go ... shoot!

Counting down is not the cup of tea of the other blade.

- (2) Three!
- (1) Two!
- (0) Oh, shoot.

Challenge 12 (a cyclic order). In $dx\,dy$, $dy\,dz$, and $dz\,dx$, the basic 1-forms dx , dy , and dz always appear in cyclic pairs. See Figure 1.

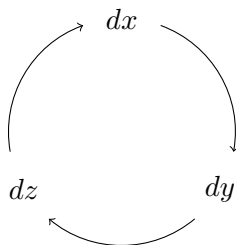


FIGURE 1. The cyclic order of dx , dy , and dz .

¹There is no iPhone 9 or Windows 9. Why should we have one?

Challenge 13 (Riemann sums). Some Riemann sums of $f(x) := x^2/3$ over $[0, 3]$ are shown in Figure 2.

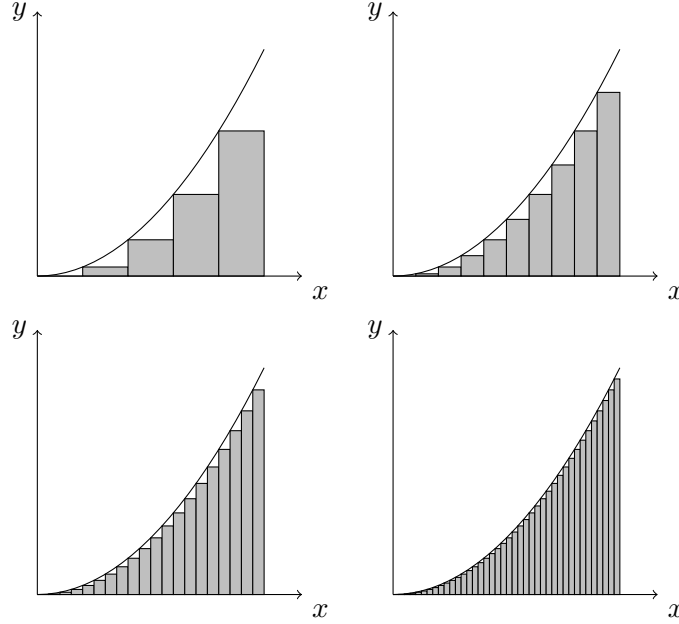


FIGURE 2. The Riemann sums for $n = 5, 10, 20$, and 40 .

REFERENCES

- [Wal56] Wallis, J. (1656). *Arithmetica Infinitorum*. Oxford, England. Available at: archiv.org/details/ArithmeticalInfinitorium/page/n5 Available in English as *The Arithmetic of Infinitesimals* (2004). (Stedall, J. A., trans.) New York, NY: Springer-Verlag.

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