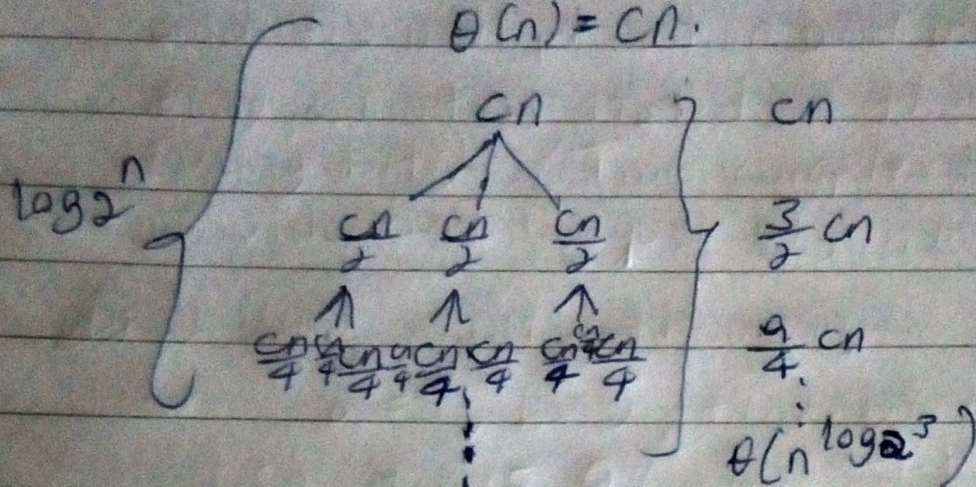


Problem 5.2

d) (Bonus)

$$T(n) = 3T(n/2) + \theta(n)$$

$$\theta(n) = cn.$$



$$T(n) = \sum_{k=0}^{\log_2 n - 1} \left(\frac{3}{2}\right)^k cn + \theta(n \log_2 3)$$

$$= cn \left(\frac{\left(\frac{3}{2}\right)^{\log_2 n} - 1}{\frac{1}{2}} \right) + \theta(n \log_2 3)$$

$$= 2cn (3^{\log_2 n} - 1) + \theta(n \log_2 3)$$

To prove $T(n) = \theta(n \log_2 3)$, ~~we~~ prove or show that $3^{\log_2 n} = \theta(n \log_2 3)$

$$3^{\log_2 n} \leq \theta(n \log_2 3) \cdot c$$

let $x = c' / \log_2 3$ and then take \ln of

$$\log_2 n \ln(3) \leq \log_2 3 \ln(xn) \quad \text{the inequality}$$

$$\Rightarrow \log_3 n \leq \log_3 xn \quad \text{for large } x$$

$$\text{so } T(n) = \theta(n \log_2 3).$$