→ tightest upper bound

$T(n) = O(n^4)$ for $c \geq 1$

tightest lower bound

$T(n) = \Omega(n^3)$

$T(n) = c12\left(\frac{n}{2}\right)^3 + n^2 \lg n$

$= \frac{12}{8} cn^3 + n^2 \lg n$

$= \frac{3cn^3}{2} + n^2 \lg n \geq cn^3$

for $c \geq 1$

So, tightest lower bound is

$T(n) = \Omega(n^3)$

$n^{3.6} > n^2 \lg n$

so $n^2 \lg n = O(n^{\log_2 12 - \varepsilon})$
$(f(n))$

and so

$T(n) = \Theta(n^{\log_2 12})$

for $\varepsilon = 0.5 > 0$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$

⇒ tightest upper bound

$T(n) = O(n^3)$

$T(n) = 3c(n/5)^3 + c(n/2)^3 + 2^n$

$= \frac{3cn^3}{125} + \frac{cn^3}{8} + 2^n = \frac{149}{1000} cn^3 + 2^n$

$= 0.149cn^3 + 2^n \leq cn^3$

→ so, tightest upper bound is

upper ← $T(n) = O(n^3)$