$T(n) = \Theta(n)$
[illegible] $\Theta(n)$ [illegible] $T(n)$ [illegible]

e) (bonus) $T(n) = T(2n/5) + T(3n/5) + \theta(n)$

<u>Ans</u>: $\Rightarrow$ tightest upper bound is
$T(n) = O(n \lg n)$
$\rightarrow$ tightest lower $T(n) = \Omega(n \lg n) = \theta(n \lg n)$

<u>Proof</u>:

Constants $a, b, c > 0$ so prove for θ

$$cn + a\left(\frac{2n}{5}\right)\lg\left(\frac{2n}{5}\right) + a\left(\frac{3n}{5}\right)\lg\left(\frac{3n}{5}\right) \le T(n) \le$$
$$b\left(\frac{2n}{5}\right)\lg\left(\frac{2n}{5}\right) + b\left(\frac{3n}{5}\right)\lg\left(\frac{3n}{5}\right) + cn$$

$$cn + an\left(\frac{2}{5}\lg\frac{2}{5} + \frac{3}{5}\lg\frac{3}{5}\right) \le T(n) \le$$
$$bn\left(\frac{2}{5}\lg\frac{2}{5} + \frac{3}{5}\lg\frac{3}{5}\right) + cn$$

$$cn + an(-1.32 + -0.73) \le T(n) \le bn(-1.32 + -0.73) + cn$$

$$cn + an(-2.05) \le T(n) \le bn(-2.05) + cn$$

$$cn - 2.05an \le T(n)$$

and for this to be true,
i.e. $an \log n \le T(n) \le bn \lg n$

$cn + an(-2.05) \le 0 \qquad c - 2.05a \le 0$
$c \le 2.05a \qquad 0 \le -bn2.05 + cn$
$2.05b \le c$

$$2.05b \le c \le 2.05a$$