

→ tightest upper bound

$$T(n) = O(n^4) \text{ for } C \geq 1$$

tightest lower bound

$$T(n) = \Omega(n^3)$$

$$T(n) = a_2 \left(\frac{n}{2}\right)^3 + n^2 \lg n$$

$$= \frac{12}{8} cn^3 + n^2 \lg n$$

$$= \frac{3}{2} cn^3 + n^2 \lg n \geq cn^3$$

$$\textcircled{2} \quad n^{3.6} > n^2 \lg n$$

$$\text{So } n^2 \lg n = O(n^{\log_2 12 - \epsilon})$$

(for)

for  $C \geq 1$

and so

$$T(n) =$$

$$\theta(n^{\log_2 12})$$

So, tightest lower bound is

$$T(n) = \Omega(n^3)$$

for  $\epsilon = 0.570$

$$d) \quad T(n) = 3T(n/5) + T(n/2) + 2^n$$

→ tightest upper bound

$$T(n) = O(n^3)$$

$$T(n) = 3c\left(\frac{n}{5}\right)^3 + c\left(\frac{n}{2}\right)^3 + 2^n$$

$$= \frac{3cn^3}{125} + \frac{cn^3}{8} + 2^n = \frac{149}{1000} cn^3 + 2^n$$

$$= 0.149cn^3 + 2^n \leq cn^3$$

→ so, tightest ~~upper~~ bound is

upper ←  $T(n) = O(n^3)$

*[Handwritten notes at the bottom of the page, including "upper bound" and "lower bound" written multiple times.]*