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Sketches can be categorized as personal, shared, persuasive, and handover sketches. Depending on each category, their level of ambiguity also varies. The applications of sketches include conceptual design, eliciting user preferences, shape retrieval, and sketch-based modeling (SBM). There is a need for quantification of uncertainty in sketches in mapping of sketches to 3D models in sketch-based modeling, in eliciting user preferences, and in tuning the level of uncertainty in sketches at the conceptual design stage. This paper investigates the role of probability of importance in quantifying the level of uncertainty in sketches by raising the following three research questions: How are the features in a sketch ranked? What is the probability of importance of features in a sketch? What is the level of uncertainty in a sketch? This paper presents an improved framework for uncertainty quantification in sketches. The framework is capable of identifying and ranking the features in the sketch, determining their probability of importance, and finally quantifying the level of uncertainty in the sketch. Ranking the features of a sketch is performed by a hierarchical approach, whereas probability of importance is determined by assessing the probability of likeliness using a shape matching approach and a probability transformation. Quantification of uncertainty is accomplished by using the principle of normalization of entropy. A case study of a bicycle sketch is used to demonstrate that the framework eliminates the need of expert input in assessment of uncertainty in sketches and, hence, can be used by design practitioners with limited experience. [DOI: 10.1115/1.4035867]

Introduction

Types of Sketches. Sketches can be cast in four groups, namely, thinking sketches, talking sketches, prescriptive sketches, and storing sketches [1]. Here, the talking sketches are used to support group interaction and foster communication among different stakeholders during the design process. For many designers, the most fundamental role of sketches is for communication [2]. Pei et al. [3] developed a taxonomy for sketches; they grouped sketches as personal sketches, shared sketches, persuasive sketches, and handover sketches. Personal sketches are created specifically for private use for the purposes of externalizing thoughts quickly, developing and investigating the appearance and visual impact of the idea, and constructing a visual understanding. Shared sketches are used in a design team to communicate ideas clearly. They should not only express ideas correctly, they should also foster discussion. On the other hand, persuasive sketches are realistic representations which are created by rendering and, hence, have a low level of ambiguity. Handover sketches are prescriptive sketches which are used to transmit a detail design information for the purpose of constructing a 3D model or prototype.

Sketches in Conceptual Engineering Design Stage. Experts in design describe sketches as the language of design cognition, and thus mention that the performance of the design process could be highly impacted by the level of proficiency with which designers are able to manage that language [4]. In the same context, experts also emphasize the importance of visual representations in design as "essential" in order to achieve the thorough communication of engineering design information between people. The extensive use of sketches in engineering design makes them are the most widely used type of visual representation, and thus requires designers to have a certain level of skill in their use [5]. The importance of sketches is based on their benefits, such as: reduced production time and minimum amount of resources, often just a pencil and a piece of paper. This makes sketches into great exploratory tools during the early ideation stages of design [6]. Furthermore, in the conceptual engineering design stage, sketches play a key role due to their inherent ambiguity, which arises as a result of their vague and/or incomplete information. This inherent ambiguity allows sketches to serve as a source of creativity for designers by eliciting the development, transformation, and emergence of new design ideas [7,8]. It has also been shown that cognitive uncertainty in sketches, i.e., ambiguity in sketches, seems to inspire expert designers in developing a larger amount of design ideas during the conceptual design stage [9].

Sketches in Eliciting User Preference. In an attempt to speed up the introduction of new products to the market, recently it has become common to elicit feedback from users using provisional design representations that are often in the form of sketches [10]. Several factors have been shown to influence a product's perception on user assessment. Two key aspects of user perception have been shown to be fidelity and representation mode. In this context, fidelity is defined as the level of realism or detail of the design representation. Macomber and Yang [11] showed that users have ranked high fidelity sketches the over the low fidelity sketches. Other researchers used experiments to show that computer sketches provided sufficient information for users to respond reliably [12].

Sketch-Based Shape Retrieval (SBSR). In areas such as computer science, sketches have been a relevant subject of study for quite some time now. Researchers have discovered that computers enhance the interaction of people with sketches in ways that are

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impossible with paper alone [13]. For example, it has been shown that although a sketch might be “rough,” i.e., unfinished, ambiguous, or uncertain, algorithms can still be quantified and assessed its information content. Therefore, based on the quantification and assessment of this information from a sketch, e.g., perceptual information [14], the information content of a sketch is determined using one of the main visual attributes of an object: its shape. As a result, the field of sketch-based shape retrieval (SBSR) has grown dramatically [15]. SBSR has become an extensive field of research which is categorized according to the method describing the shape of a sketch: contour-based methods and region-based methods. Contour-based methods are the most widely used methods which include contour curvature, shape context, shock graphs, and Fourier descriptors [16]. Thus, using the information content of a sketch based on its shape, tasks such as retrieval of information (e.g., information retrieval based on sketch queries instead of written queries), recognition (e.g., determining object similarity), and classification (e.g., determining the most representative object of a class) are now being developed [16]. These studies, in turn, pave the path for the development of novel applications such as sketch-based 3D shape retrieval [17] and sketch-based 3D shape modeling [18] by using rough 2D hand-drawn sketches. Three-dimensional objects can be recovered from 3D object repositories or 3D rendered models can be created without the need of parametrized CAD models. Hence, such studies contribute to the understanding and management of quantification of information content in sketches.

Sketch-Based Modeling. Sketch-based modeling (SBM) is an application of SBSR where 2D hand-drawn sketches are used to construct 3D rendered models. Because of its wide applicability, SBM has gained enormous attention in various areas such as (i) engineering design, where models of engineering parts are usually described using 2D orthogonal view drawings [19]; (ii) animation, where sketch-based interface animators are used to create complete 3D models using 2D sketches [20]; and (iii) complete 3D scenarios based on a small set of 2D sketched objects [17]. Entem et al. [18] were able to create 3D models of various animals by using a set of a priori smoothness, structural assumptions, and 2D contour side-view sketches of the animals. Olsen et al. [21] mentioned the need to extend the research efforts of SBM toward the development of sketch-based interfaces for modeling (SBIM), as research in cognitive sciences showed that sketching was an easy way for humans to communicate meaning. The authors also reported that the ultimate goal of research on SBIM should be to successfully develop modeling systems that integrate the expressive power and control of window, icon, menu, and pointer-based systems with the expeditious and natural interaction of sketching. However, it was emphasized that one of the greatest challenges in achieving the ultimate goal of SBIM was the mapping of 2D sketches to 3D models due to the prevalence of ambiguity in 2D sketches.

Shape Characterization. The shape of an object contains some basic features that are important to perform image characterization [22,23]. Typically, the shape of an object is represented by its external boundaries or contour. Since contours are single-closed lines which do not have holes or internal details, these lines can be parametrized by using arc lengths [24]. In general, a shape descriptor is a set of numbers produced to describe the mathematical signature of a given shape. A good descriptor is able to describe mathematically any shape feature according to the human perception or according to task-specific requirements [16]. A large number of different shape descriptors have been proposed: shape signatures, signature histograms, shape invariants, moments, curvature, shape context, shape matrix, and spectral features among others. One common method to evaluate the accuracy of these shape descriptors is to compare their performance to retrieve similar shapes from a designated database [22]. Among the above mentioned shape descriptors, histograms have been widely used for representing, analyzing, and characterizing 2D and 3D shapes. The main advantage of using histograms on shape characterization is that they mathematically represent probability density distribution of the underlying random variable [25]. One desirable characteristic of a shape descriptor is its low computation complexity, i.e., its reduced uncertainty related to the factors involved in the computational process [22]. Compactness, simplicity, and robustness are additional characteristics that are desirable on shape descriptors to correctly represent information content [16]. In general, shape representation and description techniques fall into two main classes of methods depending on where the shape features are extracted [16,26]. The first class, which uses only the shape contour information to describe the object, is called contour-based method; the second class, which uses the shape interior information to characterize the object, includes the region-based methods. Some authors also include a third class referred to as skeleton-based methods, where the features are extracted from internal axis of the shape, which are considered more suitable to represent shapes with occlusions and articulations [23].

Elements of Sketch. van de Panne and Sharon [27] proposed a recognition of a sketch based on an assumption that the sketch consists of a core set of possibly disconnected features that have a relative location to each other. In essence, this approach perceives a sketch as a constellation model comprised of local features, where each feature has a geometry model that depicts the relative location to other features and an appearance model. It is noted that this approach may benefit from adapting some of the Gestalt principles, which may speed up the recognition of the sketches. Recently, the Gestalt principles [28] were used as first step to group elements of a sketch into features, and the following steps were then used to objectively quantify the adherence to the principles of proximity, continuity, closure, symmetry, parallelism, and similarity. There are cases where the breakdown of a sketch into features was aided by decomposition or breaking points strategically located at the contour boundary of each feature often characterized by points of discontinuity [29]. Sketches can also be analyzed based on their geometric properties using shape rules, shape grammars, and subshapes [30]. Shape rules have a potential of constructing valuable information not embedded in sketches which could be used in generating alternative concepts. Extracting of subshapes from sketches provides a mechanism of manipulating shapes and generating new shape alternatives. Ranscombe et al. [31] proposed a strategy for decomposing sketches into constituent esthetic features. Using a sketch of a car as an example, the authors proposed the following types of esthetic features: outline, daylight opening, muscles, graphics, and explicit detail. It was shown through a series of experiments that this decomposition strategy provides a less subjective means of reasoning and decision making regarding brand recognition from sketches.

Shannon’s Information Theory. In engineering design, communication is a sociotechnical process, which in order to be achieved combines the technical aspects of information transmission with the sociotechnical aspects of design [32], thus enabling design communication. Design communication is a fundamental activity of a design process, which enables the communication of design information among the different stakeholders during the design process. Hence, given its importance, researchers have carried out studies in design communication such as the following: (i) determining the amount of information in different types of design representations, i.e., sketches versus physical prototypes, in order to determine their level of interpretability [33]; (ii) determining the level of recognition of sketches by designers based on their stroke predictability, which is certainty of a sketch’s embodying strokes [34]; and (iii) determining the level of complexity of a design solution based on the ambiguity of the design

AQ5 information used to produce it [35]. In his work, Shannon [36] defined a message as information source which produces a message that is encoded into a signal and is transmitted across a channel. A receiver decodes this signal and a message arrives at the destination. Thus, the design information in design communication, e.g., sketches, physical prototypes, among others, is akin to the transmitters of a message as defined by Shannon. Design information has been defined as a message transmitter, which is later decoded and interpreted by different stakeholders during the design process [2,35]. Therefore, based on Shannon's communication model, several studies have been developed.

Probability of Importance. For many designers, the most fundamental role of sketches is for communication [2]. At the conceptual design stage, sketches are used to communicate to individuals with expertise; during eliciting user preferences, sketches are used to communicate to users; in sketch-based shape retrieval and sketch-based modeling, sketches are used to communicate to computers [2]. A model, based on Shannon's entropy, was proposed for use in quantifying the uncertainty in communication with a sketch [35,37]. The probability of each feature used in the entropy equation was defined as the probability of importance [37–39]. The "importance" attribute was demonstrated to be a result of the ranking or importance of the respective features composing a sketch. In this paper, uncertainty in a sketch is defined as the amount of normalized entropy associated to the communication of the information that is transmitted by the sketch [37].

Motivation and Research Questions. Pei et al. [3] categorized sketches as personal, shared, persuasive, and handover sketches. Although these sketches were classified according to their purpose, they have different levels of ambiguity or uncertainty associated with them. It can be deduced that personal sketches have a very high level of ambiguity followed by shared ambiguity. On the other hand, persuasive sketches have a low level of ambiguity followed by handover sketches which have the lowest ambiguity. The shared sketches have application in the conceptual design stage, and the persuasive sketches have application in eliciting user preferences, sketch-based shape retrieval, and sketch-based modeling.

The motivations of this paper are:

(1) In sketch-based modeling, the challenge is the mapping of 2D sketches to 3D models due to the prevalence of ambiguity in 2D sketches [21]. There is a need to quantify the uncertainty in order to facilitate the conversion of 2D sketches to 3D models.
(2) In eliciting user preference, the choices made by the users have been shown to be impacted by the level of fidelity of sketches [11]. There is a need to quantify the uncertainty in order to manage the fidelity in sketches.
(3) In sketches in conceptual design stage, it has been shown that uncertainty is essential for the creative process [2]. There is a need to quantify uncertainty so that it can be tuned for the benefit of the creative process at various stages.

In order to determine the role of probability of importance in constructing the level of uncertainty in a sketch, three research questions were framed. (1) How are the features in a sketch ranked? (2) What is the probability of importance of features in a sketch? (3) What is the level of uncertainty in a sketch?

Methodology

Ranking of Features. The ranking of the features of a sketch is a major focus in determining the uncertainty in communication based on a sketch and is directly associated to the probability of importance and the level of uncertainty in the sketch. The ranking of the features is driven by the concept of nested hierarchy states that the hierarchy of the embodying components of a product can be determined by quantifying their pleiotropy score based on the impact this components have toward the completion of the particular service characteristics of a product [40]. This gives rise to the concept of design hierarchy, which arises from the merger of the concept of dominant design and the concept of nested hierarchy. Design hierarchy allows to determine and prioritize the embodying components of a product based on the importance of each component on the completion of products' particular functions, which are also referred to as service characteristics [41]. Using visual significance hierarchy for unique design elements on externally visible design attributes such as those contained in a sketch, researches have been able to formulate a hierarchy of importance [42].

Hence, given a sketch, the first step is to identify the features contained in the sketch. To identify the features in a sketch, the service characteristics are identified first. Then, the features are extracted based on the service characteristics. In this paper, a hand-drawn sketch of a bicycle was implemented as a case study. To this end, the service characteristics considered were energy, control, and interface [41,43]. In this paper, the adaptation of the service characteristics did not depend on the order of evaluation. The energy characteristic is the service characteristic responsible for supplying the energy required for the system to achieve its objective function. On the other hand, the control characteristic is responsible for governing the performance of the system. The interface characteristic is an interaction between the system and the environment as well as among the components of the system.

After the features of a sketch have been identified, they will be ranked. The process of ranking the identified features of a sketch starts with the refinement of the service characteristics based on the completion of the objective function of the system in the sketch. For example, to refine the energy characteristic of the bicycle, the types of movement the bicycle employs to achieve its objective function were identified.

The main movement types, for example, steering and propulsion, will be the refined energy service characteristics. The second step in ranking the features is construction of the service characteristics assessment matrix. This is performed by listing all the refined service characteristics in one column and each of the identified features in other columns (see Table 1). The matrix can then be used to assess if the identified features in the system have an impact on each refined service characteristic. If the feature is determined to have an impact on the service characteristic, the corresponding cell intersecting the selected "Feature" column and the refined service characteristics row shall be marked "1." On the other hand, if a feature does not have an impact, the corresponding cell in the table will be marked "0." The 1's of each feature in the table are finally added to provide the pleiotropy score of each feature, q_k. The pleiotropy score is a parameter that is used for ranking the features. That is, the feature with the highest pleiotropy score will be ranked #1 and so on. The ranking of the features was designated by the variable i.

Determining the Probability of Importance. The probability of importance is the fundamental factor driving the level of uncertainty in a sketch, as it reflects not only the level of importance of each of the composing features but also links its importance to its level. Thus, order to determine the probability of importance, two tasks should be addressed. Initially, it is necessary to determine the probability of likeliness of each feature in the sketch as compared to the images of the corresponding actual parts. Then, the probability of likeliness space is transformed into the probability of importance space using the ranking of features presented in the previous section. Next, the principles on which both tasks are AQ6 based, and the process used to determine each one of them are described.

Probability of Likeliness. The probability of likeliness of features in a sketch is determined by comparing the "likeness" (i.e.,

Table 1 Functional service characteristics assessment matrix

Service characteristic						
Main	Refined	Sketch	Feature #1	Feature #2	…	Feature #N
Energy characteristics	Characteristic #1	1	0	1	…	0
	Characteristic #2	1	0	0	⋰	0
	⋮	⋮	⋮	⋮	⋱	⋮
	Characteristic #n	1	0	0	…	0
Control characteristics	Characteristic #1	1	1	0	…	0
	Characteristic #2	1	1	0	⋰	0
	⋮	⋮	⋮	⋮	⋱	⋮
	Characteristic #n	1	1	0	…	0
Interface characteristics	Characteristic #1	1	1	0	…	1
	Characteristic #2	1	0	0	⋰	1
	⋮	⋮	⋮	⋮	⋱	⋮
	Characteristic #n	1	0	1	…	1
Pleiotropy		q	q_1	q_2	…	q_N
Feature rank		—	i_1	i_2	…	i_N

similarity) of the features in a sketch with respect to images of an actual mechanical component [44]. The concept of similarity is complex and can be used to describe several characteristics of different images. Therefore, this paper focuses only on the shape similarity measurement, which is based on the comparison between the geometrical attributes of the shape (contour) of the sketch and the actual component [45].

Among the several shape matching methodologies available to determine the similarity between two images, the inner-distance shape context (IDSC) methodology proposed by Ling and Jacobs [46] was selected. IDSC is a shape matching approach based on shape descriptors that use the concept of inner-distance, i.e., the length of the shortest path between landmark points within a shape's profile, in order to create shape descriptors of an object. Similar to the method originally proposed in Refs. [24,46], the methodology employs n sample points from a histogram h_l to calculate the relative distances between a landmark point x_l and the remaining $n-1$ points

$$h_l(r) = \#\{x_j : j \neq 1,\ (x_j - x_l) \in \mathrm{bin}(r)\} \tag{1}$$

where r is a bin which is uniformly divided into log-polar spatial coordinates [20]. Given a point, x_l, on the contour of the first shape and a point, y_j, on the contour of the second shape, Belongie et al. [24] also defined the cost of matching of these two points $(x_l, y_j) \equiv c_{lj}$. The definition uses chi-square test statistic and is given in terms of the frequencies represented by the histograms and is given by

$$c_{lj} = \frac{1}{2}\sum_{r=1}^{R}\frac{\left[h_l(r) - h_j(r)\right]^2}{h_l(r) + h_j(r)} \tag{2}$$

where $h_l(r)$ and $h_j(r)$ denote the Rth-bin normalized histogram at x_l and y_j, respectively.

In other words, Eq. (2) represents the error of matching any point of shape x to any point of shape y. In order to solve the matching problem and determine the pairwise correspondence of points x_l and y_j, a dynamic programming (DP) optimization procedure was used [47]. DP finds the pairwise correspondence of corresponding points x_l, y_j that minimizes the total cost of matching [24] given by

$$C(\pi) = \sum_{l=1}^{N_{cp}} c(x_l, y_{\pi_l}) \tag{3}$$

AQ7 where $C(\pi)$ is the total cost of matching of the corresponding points of the two shapes, π is a vector whose elements (lth) are the indexes of the points of the shape y_j which correspond to the points of the shape x_i, and N_{cp} is the total number of corresponding points.

Since Eq. (3) is a summation of costs (or errors), which measures the overall cost of matching between the two shapes, it is reasonable to define an average cost between the shapes by dividing the total cost of matching by the total number of corresponding points. This results in

$$\bar{C}(\pi) \equiv \frac{C(\pi)}{N_{cp}} = \frac{\sum_{l=1}^{N_{cp}} c(x_l, y_{\pi_l})}{N_{cp}} \tag{4}$$

The average cost between the corresponding points of the two shapes given by Eq. (4) quantifies the dissimilarity between two shapes. It is a probability number with limits ranging from 0 (for identical shapes) to 1 (for extremely different shapes). Feng et al. [25] used a similar concept to define a discrepancy factor to compute the cost of matching between 3D shapes. Then, the complement of the average cost of matching, $1-\bar{C}(\pi)$, is a probability number which accounts for the similarity, or the probability of likeliness, L_i, between the two shapes and is given by

$$L_i \equiv 1 - \bar{C}(\pi) = 1 - \frac{\sum_{i=1}^{N_{cp}} c(x_i, y_{\pi_i})}{N_{cp}} \tag{5}$$

The value of the probability of likeliness is also contained in the range $0 < L_i < 1$. This range was referred to as "probability of likeliness space." In order to calculate the probability of importance, p_i, the probability of likeliness space was mapped to the "probability of importance space." This mapping follows a set of rules based on the ranking of features and the principles of Shannon's information theory.

Probability of Importance. In order to continue with the process of determining the probability of importance of the features in a sketch, the topic of transformation in probability space is introduced. The transformation in probability space is presented as a set of rules established based on the ranking of features and the principles of Shannon's information theory. The rules are used to transform the probability of likeliness space into the probability of importance space.

Consider a sketch that has N total number of features ranked based on their importance from $i=1$ to n, having k_i number of features in each rank and where rank $i=1$ is given to the most important feature. The rules of transformation from the probability of likeliness space to the probability of importance space are as follows:

(1) There will only be one most important feature in a sketch, i.e.,

$$k_1 = 1 \tag{6}$$

(2) The summation of the probability of importance, p_i, of all features is given by [48]

$$\sum_{i=1}^{n} k_i p_i = 1 \tag{7}$$

(3) The probability of importance of the most important feature in a sketch, p_1, cannot be less than $1/N$ and will have a maximum value of 1, i.e.,

$$\frac{1}{N} \le p_1 \le 1 \tag{8}$$

(4) The probability of importance, p_i, of each of the ith ranked feature(s), where $2 \le i \le n-1$, is given by

$$\left(\frac{1 - \sum_{j=1}^{i-1} k_j p_j}{N - \sum_{j=1}^{i-1} k_j} \right) \le p_i \le \left(1 - \sum_{j=1}^{i-1} k_j p_j \right) \tag{9}$$

(5) The probability of importance, p_n, of each of the nth ranked feature(s) is given by

$$p_n = \frac{1 - \sum_{i=1}^{n-1} \sum k_i p_i}{k_n} \tag{10}$$

(6) Probability of likeliness space of the ith ranked feature $(i = 1,\ 2, \ldots, n-1)$, $0 < L_i < 1$, determined using IDSC, shall be mapped to the probability of importance space, $p_{L_i} < p_i < p_{U_i}$, defined in rule 3 and rule 4, of the same feature according to a linear transformation given by

$$p_i = (p_{U_i} - p_{L_i}) L_i + p_{L_i} \tag{11}$$

For example, the mapping function for the most important feature $(i = 1)$ will be

$$p_1 = \left(\frac{N-1}{N} \right) L_1 + \frac{1}{N} \tag{12}$$

Note that the constrained probability space gets narrower as one progresses through the ranking.

Quantification of Uncertainty. Finally, the quantification of uncertainty formalizes the role of all the features in a sketch and gives an integrated quantitative value of the uncertainty in a sketch. The quantification of uncertainty is founded on the application of the principles that guide the use of Shannon's information theory [48] and the application of the normalized entropy [49]. Thus, in order to quantify the uncertainty in a sketch, the following quantities need to be calculated:

(1) The total entropy of a sketch given by

$$H = -\sum_{i=1}^{n} k_i p_i \log_2 p_i \tag{13}$$

(2) The uncertainty of a sketch, based on the normalized value of the entropy, where $0 \le u \le 1$, is given by

$$u = \frac{H}{\log_2(N)} \tag{14}$$

Hence, through the application of Eq. (13) whose inputs are the probabilities of importance p_i's of the features of the sketch, followed by Eq. (14) whose inputs are H and N, the quantification of the uncertainty of a sketch is accomplished.

Figure 1 shows the description of the improved framework for the quantification of uncertainty in sketches. The application of the improved framework in uncertainty quantification of sketches is illustrated in the following case study.

The proposed framework targets shared sketches and persuasive sketches [3]. It was noted in "Introduction" that the shared sketches have application in the conceptual design stage, and the persuasive sketches have applications in eliciting user preferences, sketch-based shape retrieval, and sketch-based modeling. Thus, this framework has application (1) at the conceptual design stage of the design process, (2) in eliciting user preferences during the later stages of the design process, (3) sketch-based shape retrieval, and (4) sketch-based modeling. The framework developed is applicable for both hand-drawn (pen and paper) or digital sketches. In this paper, hand-drawn sketches (pen and paper) were used.

Case Study

Formulation. The case study section is focused on demonstrating the use of the improved framework for determining probability of importance of features in a sketch and consequently quantifying its uncertainty (see Fig. 1). This has additional purposes: (a) validating the research work presented in this article and (b) showing the applicability of uncertainty quantification tools toward in application areas where sketches are central. A bicycle sketch will be investigated as a case study by following the approaches designed to aid in response to each research question as presented in the section "Methodology."

A bicycle (see Fig. 2) is a two wheeled steerable machine that is pedaled by a person, the bicycle rider, in order to propel it and thus achieve movement. Bicycles generally have four components and configurations. The mounting system is composed of the saddle and the bicycle frame. Its main function is to hold the components of the bicycle, as well as to allow the rider to maintain a position that allows her or him to operate the bicycle. The steering system is composed of the handlebar, the fork, and the front wheel. Its main function is to allow the rider to control and guide the bicycle, through the interaction of the rider with the handlebar

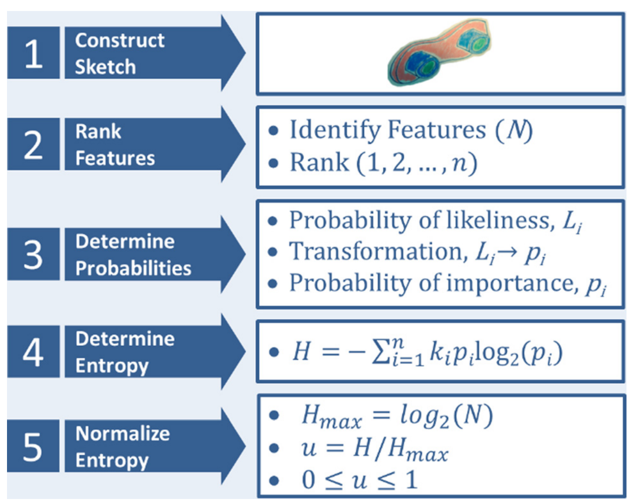


Fig. 1 Improved framework for the quantification of uncertainty in sketches

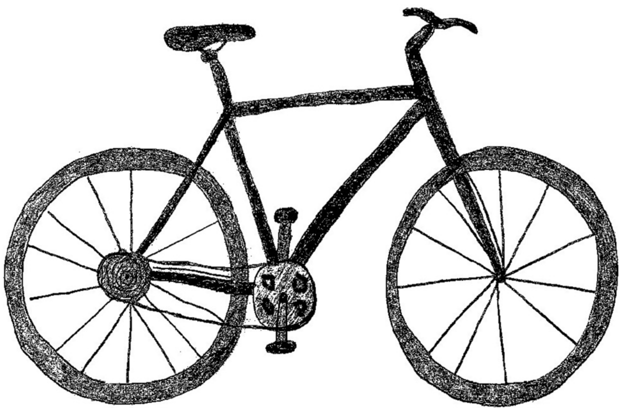


Fig. 2 Sketch of a bicycle

and the interface of the front wheel with the terrain. Another component is the power system, which is composed of the drivetrain. Its main function is to supply the power necessary to propel the bicycle. Finally, the rear wheel system is main function is to make effective use of the power supplied by the drivetrain and to enable the displacement of the bicycle. The bicycle functions by the rider sitting on the saddle of the mounting system, steering by leaning and turning the handlebar of the steering system, and by turning the pedals attached to the cranks and the chain-wheel in order to generate the power needed by the power system, which is then transmitted by the chain-wheel to a sprocket on the rear wheel of the rear wheel system in order to enable the displacement of the bicycle [50]. Nowadays, bicycles are generally grouped into six main categories based on their specific purpose/application: utility, touring, racing, mountain, hybrid, and BMX. As a result of these differences in purpose/application, bicycles come in different sizes and shapes, and therefore the sketches that are used to depict and communicate their designs are diverse as well. The application of the improved framework for the quantification of uncertainty presented in Fig. 1 is now applied on the sketch of a bicycle shown in Fig. 2.

Ranking of Features. The first step in order to rank its features is to identify them. This means to distinguish the features that embody the sketch of a bicycle based on its service characteristics, namely [40]: energy, control, and interface. Thus, using the aforementioned service characteristics, the features of the sketch of a bicycle were identified as: (a) mounting system = frame + saddle, (b) steering system = handlebar + fork + front wheel, (c) power system = pedals + crank(s) + chain-wheel + sprocket(s), and (d) rear wheel system = rear wheel, shown in Fig. 3. The identified features were then assessed, using the service characteristics in conjunction with the refined service characteristics: steering, propulsion, interface terrain, interface connector, and interface rider. Thus, based on the service characteristics, the features of the sketch of a bicycle were assessed, and the pleiotropy score for each of the identified features was determined through the use of the functional service characteristics assessment matrix (FSCA) [41] shown in Table 2. Finally, using the pleiotropy scores obtained using the FSCA, the features of the sketch were ranked in descending order from highest to lowest based on their pleiotropy scores. This yielded the following ranking for the features of the sketch of a bicycle: (d) rear wheel system (rank 1), (c) power system (rank 2), (b) steering system (rank 3), and (a) mounting system (rank 4) shown in Table 3. Therefore, in the sketch of a bicycle (see Fig. 3), the (d) rear wheel system is the highest ranking feature of the sketch of a bicycle, this implies that the rear wheel system may play the most important role as the identifier of a sketch of a bicycle.

Determining the Probability of Importance. After the ranking of the features of the sketch was established, the probability of importance, p_i, for each of the features of the sketch of a bicycle was determined. In order to achieve this goal, standard images of bicycle components were determined (see Fig. 4) and then compared to the features of the sketch of a bicycle in order to calculate

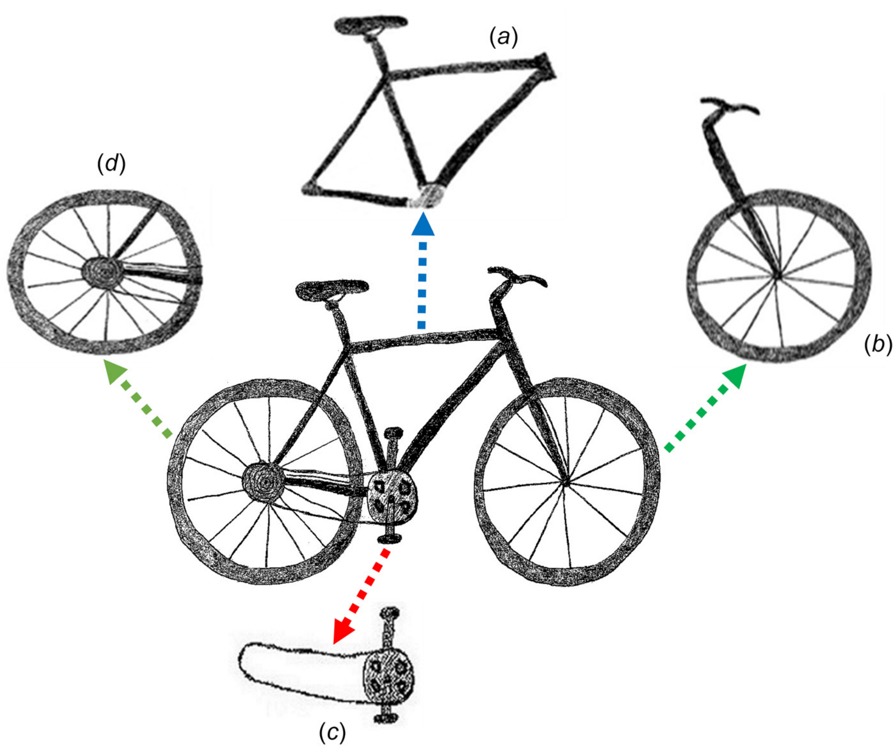


Fig. 3 Identified features in the sketch of a bicycle: (*a*) mounting system, (*b*) steering system, (*c*) power system, and (*d*) rear wheel system

Table 2 FSCA for the sketch of a bicycle

Function: Transport person						
Service characteristic		Sketch	Features			
Main	Refined	Bicycle	Mounting system (a)	Steering system (b)	Power system (c)	Rear wheel system (d)
Energy characteristics	Steering	1	0	1	0	0
	Propulsion	1	0	0	1	1
Control characteristics	Steering	1	0	1	0	0
	Propulsion	1	0	0	1	1
Interface characteristics	Terrain	1	0	1	0	1
	Two features	1	1	0	1	1
	Person	1	1	1	1	0
Pleiotropy		7	2	4	4	4
Tie priority factor		—	—	3.5	—	4.5
Feature rank		—	4	3	2	1

Table 3 Ranking of features of sketch of a bicycle based on their pleiotropy scores

Feature of sketch assessed	Pleiotropy score of feature	Rank of feature
(d) Rear wheel system	4.5	1
(c) Power system	4	2
(b) Steering system	3.5	3
(a) Mounting system	2	4

Table 4 Values for the probability of likeliness (L_i) of the ranked features of the sketch of a bicycle using the IDSC algorithm

Feature of sketch assessed	Probability of likeliness (L_i) (%)
(d) Rear wheel system	95.98
(c) Power system	71.55
(b) Steering system	81.16
(a) Mounting system	79.72

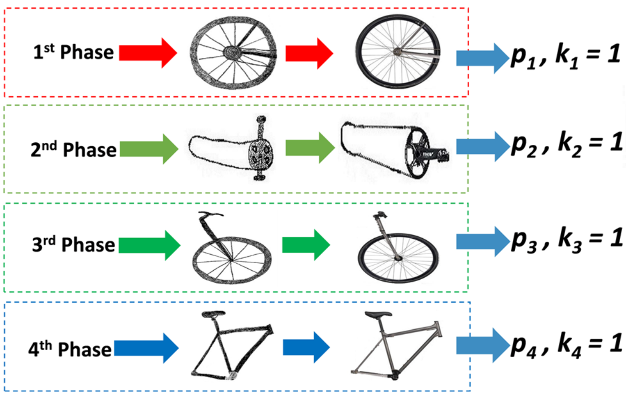


Fig. 4 Determining standard images of bicycle components

the probability of likeliness (L_i) of each of the features of the sketch, using the inner-distance shape context (IDSC) [46]. The values obtained from the comparisons between the features of the sketch and the standard components are then used to determine the probability of likeliness, L_i, based on the overall matching cost $C(\pi)$ of the IDSC algorithm. The $C(\pi)$ value as it has been detailed before is a probabilistic value that allows to determine the level of similarity between two shapes based on the degree of agreement between the landmark points of their contours. As the $C(\pi)$ value is based on the entire range of values between two compared shapes, its measuring results are more reliable as they are not biased. Thus, the results obtained for the L_i's in the case of the results of the sketch of a bicycle (see Table 4) are more accurate estimates of the actual probability of likeliness of the features of the sketch based on their shape (see Fig. 5) than the results obtained with other approaches. Hence, this enables more reliable estimates of the probability of importance p_i for the features of a sketch.

Once the values for each of the features of the sketch of a bicycle were determined, their p_i's were determined. Thus, using the transformation in probability space, the calculation begins with the most important feature of the sketch of a bicycle; the mounting system $k_1 = 1$. This process begins by verifying Eq. (11) of the transformation in probability space, which states that there can only be a single most important feature in the sketch being analyzed. Next, the verification of Eq. (11) is presented. As it can be seen, the system complies with Eq. (11). Thus, the process of determining p_1 for the rear wheel system can continue. The next step in order to determine p_1 of the rear wheel system is transforming the probability of likeliness $L_i = L_1 = 95.98\%$ of the rear wheel system into p_1. Thus, in order to transform this probability of likeliness L_1 into p_1. The following process needs to be followed. The process starts by applying Eq. (8), which allows establishing the probability of importance limit values: Lower limit $= p_{L_i} = p_{L_1}$ and Upper limit $= p_{U_i} = p_{U_1}$ for the most important feature, i.e., the rear wheel system. Next, the application of Eq. (8) is presented. Where N is equal to the number of identified features in a sketch, which for the case of the sketch of a bicycle is $N = 4$

$$0.25 \le p_1 \le 1 \tag{15}$$

As it can be seen from Eq. (8), the probability limit values of $p_1, p_{L_1} = 0.25$ and $p_{U_1} = 1$, have been determined. Thus, the process of determining p_1 for the rear wheel system can be continued. This is done by applying Eq. (11), which allows mapping L_1 into p_1 through a linear transformation. Next, the application of Eq. (11) is presented as

$$p_1 = (p_{U_1} - p_{L_1})L_1 + p_{L_1} = (1 - 0.25)0.9598 + 0.25 = 0.9699 = 96.99\% \tag{16}$$

By applying Eq. (11), the probability of importance $p_1 = 96.99\%$ for the rear wheel system has been obtained. Thus, after p_1 has been determined, the next step is to determine the limit values p_{L_2} and p_{U_2} for p_2 of the power system. This is done through the application of Eq. (9), which allows establishing the

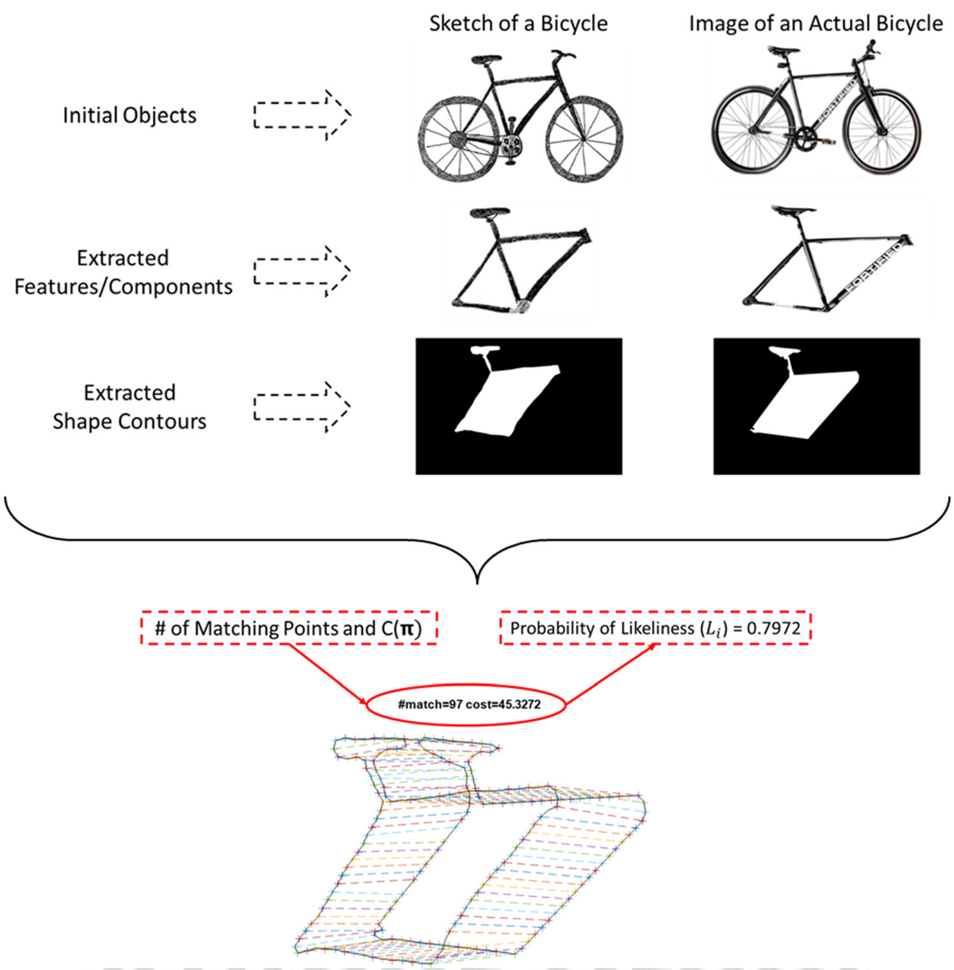


Fig. 5 Determining L_i for the features of the sketch of a bicycle

limit probability values for all the features that come after the most important feature in the sketch. Where $\sum_{j=1}^{i-1} k_j p_j$ is the total probability of importance value of the features whose values have been determined previously, and $\sum_{j=1}^{i-1} k_j$ is the total number of those same features whose probability of importance values has been determined previously. While considering the following as well:

$$p_{L_2} = \left(\frac{1 - \sum_{j=1}^{i-1} k_j p_j}{N - \sum_{j=1}^{i-1} k_j} \right) \quad \text{and} \quad p_{U_2} = \left(1 - \sum_{j=1}^{i-1} k_j p_j \right) \tag{17}$$

Next, the application of Eq. (9) to obtain the limit values for p_2 of the power system is presented

$$\sum_{j=1}^{i-1} k_j p_j = \sum_{j=1}^{2-1} k_1 p_1 = \sum_{j=1}^{1} 1 * 0.9699 = 96.99\% \tag{18}$$

$$\sum_{j=1}^{i-1} k_j = \sum_{j=1}^{2-1} k_1 = \sum_{j=1}^{1} 1 = 1 \tag{19}$$

$$\left(\frac{1 - 0.9699}{4 - 1} \right) \le p_2 \le (1 - 0.9699)$$
$$0.0100 \le p_2 \le 0.0301 \tag{20}$$

Thus, the development of Eq. (9) allows to determine that $p_{L_2} = 0.0100$ and $p_{U_2} = 0.0301$. After the limit values for p_2 of the power system have been established, Eq. (10) is applied. The purpose of Eq. (10) is to determine the maximum value of the probability of importance in the case where there are repeated features with the same ranking, e.g., $k_3 = 2$. It is important to notice that in the present case study there is only a single feature at each ranking level, i.e., the k_nth value for the sketch of a bicycle is always equal to 1. Therefore, at this point, all the maximum values of p_i's will be the same as those of p_{U_i}'s determined using Eq. (9) for each feature in the sketch of a bicycle. Thus, despite the present fact that the use of Eq. (10) does not underlie any effect on the present result of the case study, next we present its application to clarify its usage on the determination of p_2 for the power system feature

$$\sum_{i=1}^{n-1} k_i p_i = \sum_{i=1}^{2-1} k_1 p_1 = \sum_{i=1}^{1} 1 * 0.9699 = 0.9699 \tag{21}$$

$$p_2 = \frac{1 - \sum_{i=1}^{n-1} k_i p_i}{k_2} = \frac{1 - 0.9699}{1} = 0.0301 \tag{22}$$

Therefore, we will have a value $p_2 = 0.0301$.

Now, that the usage of Eq. (10) has been clarified; the process of determining p_2 of the rear wheel feature can continue. Next, the application of Eq. (11) is presented in order to determine p_2 of the rear wheel feature

$$p_2 = (p_{U_2} - p_{L_2})L_2 + p_{L_2} = (0.0301 - 0.0100)0.7155 + 0.01 = 0.0243 = 2.43\% \quad (23)$$

At this point, the probability of importance for the rear wheel system $p_1 = 96.99\%$, and the power system $p_2 = 2.43\%$ have been determined. Therefore, the case study can move forward to determine the probability of importance p_3 of the steering system. In order to determine p_3, the process is the same as for p_2. Equation (9) is applied to determine the limit values p_{L_3} and p_{U_3} for p_3 of the steering system

$$\sum_{j=1}^{i-1} k_j p_j = \sum_{j=1}^{3-1} k_1 p_1 + k_2 p_2 = \sum_{j=1}^{1} (1 * 96.99\%) + (1 * 2.43\%) = 99.42\% \quad (24)$$

$$\sum_{j=1}^{i-1} k_j = \sum_{j=1}^{3-1} k_1 + k_2 = \sum_{j=1}^{2} 1 + 1 = 2 \quad (25)$$

$$\left(\frac{1 - 0.9942}{4 - 2}\right) \leq p_3 \leq (1 - 0.9942)$$
$$0.0029 \leq p_3 \leq 0.0058 \quad (26)$$

The result of the development of Eq. (9) allows to determine that $p_{L_3} = 0.0029$ and $p_{U_3} = 0.0058$.

Now, in the case of determining p_3 for the steering system, Eq. (10) will not be applied due to the condition of singular features in each ranking as it was explained before, and so we directly proceed to the application of Eq. (11) for the determination of p_3 for the steering system

$$p_3 = (p_{U_3} - p_{L_3})L_3 + p_{L_3} = (0.0058 - 0.0029)0.8116 + 0.0029 = 0.0052 = 0.52\% \quad (27)$$

As it can be seen the probability of importance for the steering system $p_3 = 0.52\%$ has been calculated.

At this point, the probabilities of importance for three (p_1, p_2 and p_3) out of the four features in the sketch of a bicycle have been determined. This fact allows determining the final probability of importance p_4 of the mounting system by applying Eq. (7), which is the summation of the total probability of importance space

$$\sum_{i=1}^{n} k_i p_i = k_1 p_1 + k_2 p_2 + k_3 p_3 + k_4 p_4 = 1$$
$$= (1 * 0.9698) + (1 * 0.0243) + (1 * 0.0052) + (1 * p_4) = 1$$
$$= 0.99993 + p_4 = 1 \quad (28)$$

$$\therefore p_4 = 1 - 0.9993 = 0.0007 = 0.07\% \quad (29)$$

As it can be seen the probability of importance for the mounting system feature is $p_4 = 0.07\%$. In this case, it is essential to mention that for the determination of the probability of importance p_4 for the mounting system feature; Eq. (7) was used given that it was the last feature left in the determination of the probability of importance for the features in the sketch of a bicycle. Thus, its probability of importance value is driven by the residual probability remaining from the summation of the probability of importance values of the features with a higher ranking. This aspect is emphasized, given that if the mounting system feature was not the last feature, the process of determining the probability of importance p_4 for the mounting system feature would follow the same path, i.e., applying Eqs. (9) and (11), like in the determination of p_2 and p_3.

Thus, as it can be seen in Table 5, the probabilities of importance for all the features of the sketch of a bicycle have been determined. Therefore, the aim of determination of the probability of importance of the features of a sketch has been accomplished for the case study.

Table 5 Probabilities of importance for the features of the sketch of a bicycle

Feature of sketch assessed	Probability of likeliness (L_i) (%)	Probability of importance (p_i) (%)
(d) Rear wheel system	95.98	96.99
(c) Power system	71.55	2.43
(b) Steering system	81.16	0.52
(a) Mounting system	79.72	0.07

The same approach as in the previous section of the case study is followed. Therefore, next the benefits that the development of determining the probability of importance brings to the improved framework for the quantification of uncertainty in sketches are presented. The determination of the probability of importance plays a key role in the improvement of the uncertainty quantification framework. First, since it enables the assessment of sketches versus real objects through the use of a computational tool, i.e., IDSC, it eliminates the need of expert input in order to assess sketches. Second, it establishes a well-defined systematic stepwise procedure to calculate the probabilities of features in a sketch. Jointly, these two benefits result in a significant improvement to the framework for the quantification of uncertainty in sketches. Due to its more structured setup and application, the framework is now easier to be used by members of the design community.

AQ8

Quantification of Uncertainty. Quantification of uncertainty is the final chapter of the case study section. As its name implies, it allows quantifying the uncertainty of the sketch of a bicycle. To determine the uncertainty, the total entropy of the sketch of a bicycle will be calculated using Eq. (13), followed by the determination of the uncertainty of the sketch of a bicycle using Eq. (14). Next, the calculation of the uncertainty of the sketch of a bicycle is presented.

The process starts with the application of Eq. (13), which uses as inputs the probability of importance values of the features of the sketch of a bicycle, i.e., p_1, p_2, p_3, p_4, and is shown next

$$H = -\sum_{i=1}^{N} p_i \log_2(p_i) = -[p_1 \log_2(p_1) + p_2 \log_2(p_2) + p_3 \log_2(p_3) + p_4 \log_2(p_4)] = -[0.9699 \log_2(0.9699) + 0.0243 \log_2(0.0243) + 0.0052 \log_2(0.0052) + 0.0007 \log_2(0.0007)] = -[-0.0428 - 0.1303 - 0.0395 - 0.0064] = 0.219 \text{ bits} \quad (30)$$

After, the total entropy H has been calculated using Eq. (13), the quantification of uncertainty for the sketch of a bicycle continues with the application of Eq. (14)

$$u = \frac{H}{\log_2(N)} = \frac{0.219 \text{ bits}}{\log_2(4)} = \frac{0.219 \text{ bits}}{2 \text{ bits}} = 0.1095 \quad (31)$$

As it can be observed, the uncertainty for the sketch of a bicycle is $u = 0.1095$. This value indicates that the uncertainty in the sketch of a bicycle used in the present case study is very low, based on the range of the normalized entropy, i.e., $0 \leq u \leq 1$. Thus, in order to better understand the level of uncertainty, the meaning of the range of the uncertainty is explained next. The uncertainty, which is the normalized value of the total entropy of the features of the sketch of a bicycle, is interpreted in the following manner. The closer the value of the calculated uncertainty u is to 1, the higher the level of uncertainty of the assessed sketch. This means that if a sketch has an uncertainty value of 1, it has a maximum uncertainty level associated to it, and therefore the possibility of its design intent being understood by others will be very limited. On the other hand, if a sketch has zero uncertainty, it means that the sketch has a very high possibility of being correctly identified. Therefore, the sketch of a bicycle used in the present case study was determined to have a very low level of uncertainty.

At this point, the case study section is completed as the quantification of uncertainty for the sketch of a bicycle has been calculated. In the case of this last research question, the conclusion highlights that through the improvements made to the framework for the quantification of uncertainty in sketches, the results obtained with its use will be more accurate. As areas of the framework such as: the ranking of features and determination of the probabilities of the features, are structured and developed in such a way that their results are no longer dependent on the opinion or judgment of a design expert.

A general discussion is presented below regarding how the number of features in a sketch and the probability of importance values of those features impact the level of uncertainty of a sketch.

General Discussion

In this section, a general discussion is presented to illustrate the improvement of the framework for the quantification of uncertainty in sketches. The general discussion is established through the analysis of the uncertainty levels results for different sketch scenarios developed with respect to the following parameters: number of features of the sketch and probability of importance values of those features.

The first figure (Fig. 6) of this general discussion section presents four different scenarios, where the number of features that compose a sketch varies from 2 to 5 features in a sketch, each for different values of p_1, and where the residual probability is assigned equally among the remaining features. Next, the discussion on the important aspects is presented. The first important aspect is that as the number of features composing a sketch increases, the rate at which its uncertainty level increases with respect to the value of p_1 decreases. In this case, this means that as the number of features in a sketch increases; the significance of its most important feature, i.e., the need to have a most important feature with a high probability of importance value, decreases. Thus, sketches with fewer features need to have their most important feature drawn better than sketches with a greater number of features. The second important aspect is related to the fact that as $p_1 \rightarrow 1$, the significance of its value to the actual uncertainty of a sketch decreases. This means, for sketches with well-drawn most important features, e.g., $p_1 \geq 0.95$, the improvement of the drawing of their other features will not have a considerable effect in the reduction of their uncertainty, regardless of their total number of features. This second aspect further validates the fact of paying attention to the quality of the drawing of the most important feature in any sketch and also how this can avoid the need to improve other features in the sketch. The third aspect comes from the fact; that as the number of features N in a sketch increases, the range of $1/N \leq p_1 \leq 1$ also increases.

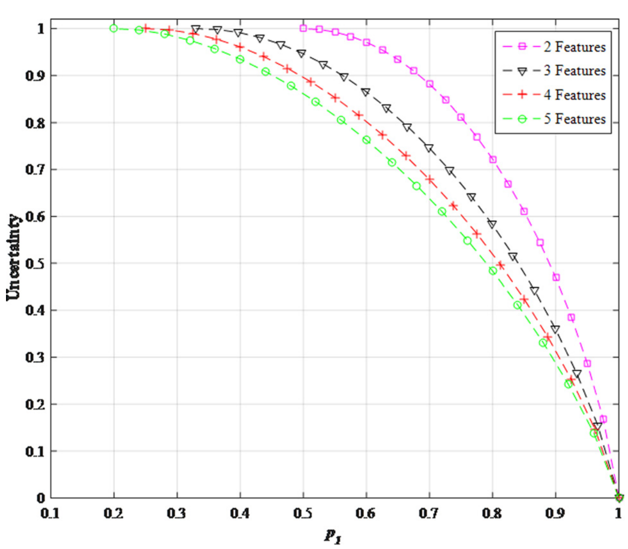


Fig. 6 Uncertainty as a function of the probability of importance value of the most important feature (p_1)

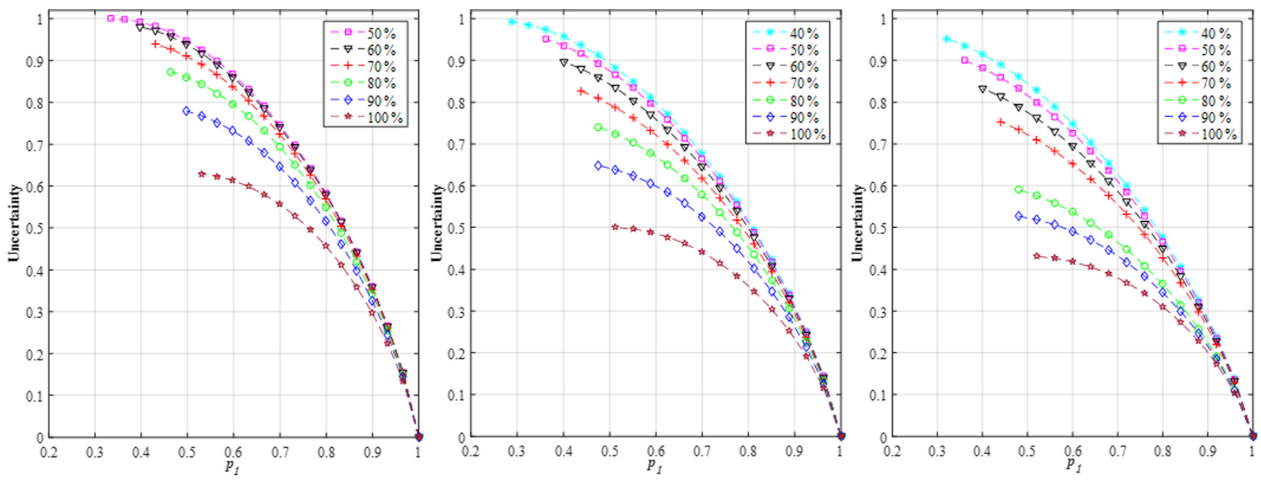
This aspect is very important, as it can be interpreted in different ways. In the case of the present research, the discussion is based on the idea of viewing a sketch as a visual representation of a design, with the purpose of only communicating its design intent. This means, a sketch needs to have only the necessary features to communicate its design intent and nothing more. So, sketches that need more features to be embodied inherently have from their start a greater chance to be misunderstood. Therefore, this implies that not only the most important feature in the sketch should be drawn in the best way possible but also the number of its embodying features should be kept to a minimum to avoid increasing the uncertainty of the sketch. Finally, the last aspect relates to the fact that for certain ranges of values of p_1, e.g., $0.7 \leq p_1 \leq 0.8$, the difference between the uncertainty values of the sketches regardless of the number of features is greater. Thus, one way to reduce the uncertainty of a sketch could be to increase its number of embodying features, instead of improving the quality of its most important feature. At this point, the present research is trying to establish a quantitative manner that could be incorporated as an addition to the framework that can allow establishing a decision in this sense.

Next, the discussion moves toward the presentation of the important aspects about the results shown in Fig. 7. In the case of Fig. 7, the main purpose of developing three different scenarios for sketches with different number of features is to highlight how the uncertainty of a sketch is related to the probability values of its lowered ranked features, after the probability of the most important feature, p_1, has been determined.

This is shown, by changing the value, i.e., percent value of the residual probability assigned to the second most important feature in a sketch, p_2, while the remaining residual probability is assigned equally to the remaining unranked features, i.e., $p_3 \ldots p_n$; to observe how this impacts the level of uncertainty of a sketch, using three different sketch scenarios, i.e., sketches with: (a) three features, (b) four features, and (c) five features, respectively.

Thus, in the analysis of Fig. 7, we would like to start with highlighting the common aspects with respect to Fig. 6. The first point in common is related to the uncertainty rate with respect to the number of features that embody a sketch. In the three scenarios, i.e., Figs. 7(*a*)–7(*c*), the same behavior as in Fig. 6 is observed, where the rate of change of the uncertainty increases as the number of features in the sketch decreases. The second point in common is that just as in Fig. 6 the difference in the value of the uncertainty for the different percentage values of p_2 for the three scenarios reduces significantly as the value of $p_1 \rightarrow 1$. This validates the importance and pertinence of defining the ranking of the features in a sketch and furthermore highlights the importance of determining which one of its features is its most important feature.

Next, important aspects specific to the scenarios presented in Fig. 7 are discussed. Initially, as it can be observed in all three cases of Fig. 7, the higher the percentage value of p_2, the greater its impact on the reduction of the uncertainty in a sketch. This fact allows to observe that the level of uncertainty of a sketch has a recursive nature and thus could be reduced if the value of p_2 is improved, i.e., increased with respect to the other features of the sketch after p_1 has been increased to its maximum possible value. The final aspect of Fig. 7 is related to the fact that when we have values of $p_1 \approx 0.5$, for all three scenarios, the uncertainty reaches its minimum when $p_2 = 100\%$. This is a very useful fact about the uncertainty of a sketch, as this can be used as an aiding principle to decrease the uncertainty in a sketch. Therefore, the decrease in the uncertainty of a sketch could be achieved through the improvement of the probability of importance of the second



(*a*) (*b*) (*c*)

Fig. 7 Uncertainty as a function of p_1 and the second feature ()% of residual probability for (*a*) three features, (*b*) four features, and (*c*) five features

feature, depending on the number of features in a sketch could decrease its uncertainty around 40%.

Finally, a discussion of Fig. 8, in the Appendix, shows the results on the level of uncertainty in the sketch of a bicycle, when the quality of its features is reduced. In this case, the example in Fig. 8 shows the sketch of a bicycle with incomplete features, i.e., an incomplete rear wheel system versus the sketch of a bicycle with complete features, where the uncertainty for the incomplete sketch is 60.08% and for the complete sketch is 10.95%. In this case, the results obtained in this example agree with the implication that sketches with reduced feature quality will have an increased uncertainty level. In addition, it agrees with the high impact that high ranking features have on the uncertainty level of a sketch. In this case, we expand on this issue by emphasizing the quality of a sketch as communication tool of design ideas. Thus, when in this case of sketch (a), the quality of its main feature is reduced, its uncertainty increases, but not only on one dimension, e.g., recognizing the bicycle, but also by making the idea of the bicycle ambiguous. For example, it may not be clear whether the sketch means to draw attention to a certain feature or whether something is missing.

This general discussion section emphasizes important aspects related to the quantification of uncertainty in sketches, and thus contributes to a better understanding of how uncertainty relates to the quantitative values of the features in a sketch. Furthermore, it allowed the expansion of aspects that in the future could contribute to the establishment of principles that could be implemented toward improving the capability of designers to manage the uncertainty associated to sketches.

Conclusion

The research work presented in this paper presents the development of the role of the probability of importance in determining the level of uncertainty in sketches and the development and application of an improved framework for the quantification of uncertainty in sketches. Hence, this allows to provide answers to the research questions: (1) How are the features in a sketch ranked? (2) What is the probability of importance of features in a sketch? (3) What is the level of uncertainty in a sketch?. This research contributed to its goal by accomplishing the following results. First, an improved, robust, user-friendly, and consistent ranking procedure was established. This was performed by the concept of design hierarchy and the assessment of the features of a sketch based on its systemic service characteristics, i.e., energy, control, and interface, as well as on its functional service characteristics, e.g., steering and propulsion, through the calculation of their pleiotropy scores using the functional service characteristics assessment matrix. Second, the probability of importance was determined, and the probabilities of features of a bicycle were calculated. Probabilities of features were calculated by means of pairwise assessment of the shape of the features of a sketch versus the shape of actual components using the matching cost $C(\pi)$ of the IDSC algorithm and the rules for transformation in probability space. This probability calculation eliminates the need of expert input in assessment of uncertainty in sketches. These attributes improve the accuracy of the uncertainty quantification framework and enhance its applicability among the members of the design community. Next, we would like to point out that the current efforts toward the further development of this research are focused on the improvement of the responses to each of the research questions formulated in the current body of work, which include the following: (i) enhancing the ranking scheme to make it applicable to more classes of sketches, i.e., not only sketches in conceptual design, (ii) improving the accuracy of the probability of importance estimation of the features of sketches, (iii) enhancing the assessment of the likeliness of features of sketches to actual components not only based on a single perceptual attribute, i.e., shape, but also others such as: aspect ratio and position, and (iv) optimizing the use of Shannon's normalized entropy to quantify the uncertainty in sketches in order to make the framework robust to cases where the uncertainty level is maximum. All of the aforementioned reasons have the ultimate goal of contributing to the development and enhancement of design communication.

Nomenclature

c = chi-square distribution
C = total matching cost
$\bar{C}$ = average matching cost
h = histogram
H = entropy
i = rank of features
k_i = number of features with a rank of i
L_i = probability of likeliness
n = total number of ranks
N = total number of features
N_{cp} = total number of corresponding points
p_i = probability of importance
p_{L_i} = lower limit of the probability of importance
p_{U_i} = upper limit of the probability of importance
q_k = pleiotropy score of a feature k
u = uncertainty based on normalized value
x_l, y_j = corresponding points on two shapes being compared
π = a vector containing the indexes of points x_l and y_j

Appendix

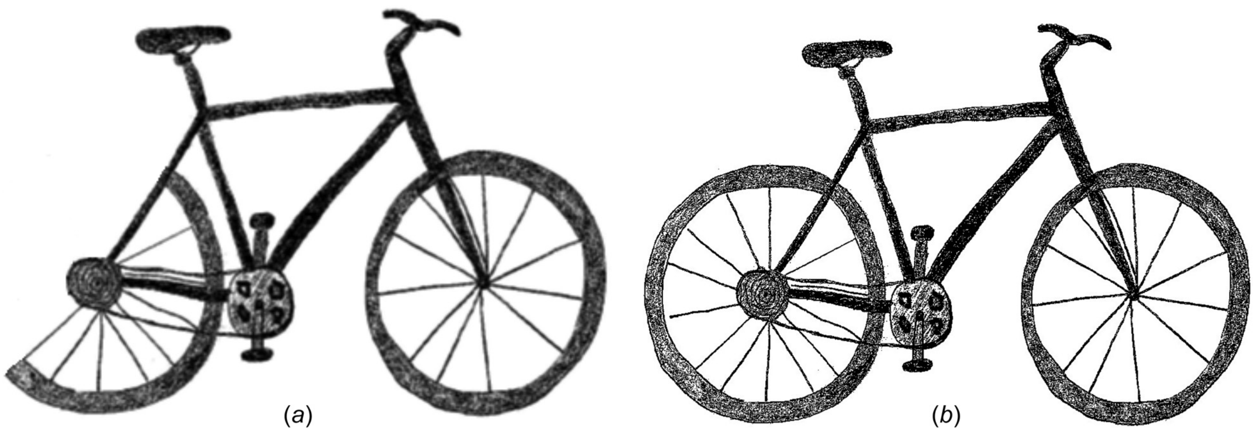


Fig. 8 Example of levels of uncertainty of sketches of a bicycle: (*a*) high uncertainty ($u = 60.08\%$) and (*b*) low uncertainty ($u = 10.95\%$)

A.1 Example of High and Low Uncertainty Sketches. Figure 8 shows an example of levels of uncertainty of sketches of a bicycle, Fig. 8(*a*) depicts the sketch of the bicycle with high level of uncertainty (i.e., $u = 60.08\%$), whereas Fig. 8(*b*) shows the sketch with low uncertainty (i.e., $u = 10.95\%$).

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