1) show $m(a+bX) = a+b \times m(X)$

$$M(Y) = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$M(y) = \frac{1}{N}\sum_{i=1}^{N} a+bx_i$$

$$N(M(y)) = \sum_{i=1}^{N} a + \sum_{i=1}^{N} bxi$$

$$N(M(y)) = Na + b\sum_{i=1}^{N} X_i$$

$$M(Y) = \frac{N}{N} a + b\frac{1}{N}\sum_{i=1}^{N} X_i$$

$$M(y) = a + bM(X)$$

$$M(a+bx) = a+b \times M(x)$$

2) show that $\operatorname{cov}(X,X) = s^2$

$$\operatorname{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(X_i - M(X))(X_i - M(X))$$

$$\operatorname{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(X_i - M(x))^2$$

$$\operatorname{cov}(X,X) = s^2$$

3) show that $cov(X, a+bY) = b \times cov(X,Y)$

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(X_i - M(X))((a+by_i) - M(a+by))$$

$$cov(X, a+by) = \frac{1}{N}\sum_{i=1}^{N}(X_i - M(X))(a+by_i - a - bM(Y))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))(b(y_i - M(Y))$$

$$= b(\frac{1}{N}\sum_{i=1}^{N}(X_i - M(X))(y_i - M(Y))$$

$$= b\, cov(X, Y)$$

4) show that $cov(a+bX, a+bY) = b^2 cov(X,Y)$

From part 3 we know that

$$b\, cov(a+bX, \quad Y \quad) = b(b\, cov(X,Y))$$

$$cov(a+bX, a+bY) = b^2\, cov(X,Y)$$

5) Suppose $b>0$ and median of X be med(X). Is it true the median of $a+bX$ is equal to $a+b \times med(X)$? Is the IQR of $a+bX$ equal to $a+b \times IQR(X)$?

As $b>0$, order is maintained in the $X \to a+bX$ transformation. Thus, the "middle" maintains its position as middle after transformation.

$$med(a+bX) = a + b\,med(X)$$

all values are scaled by b and shifted by a then the median is taken

The median is taken, scaled by b then shifted by a

$y = a + bx$ where $b > 0$

$Q_1(Y) = a + bQ_1(X)$, $Q_3(Y) = a + bQ_3(X)$

$IQR(Y) = a + bQ_3(X) - a - bQ_1(X)$

$IQR(Y) = b(Q_3(X) - Q_1(X))$

$IQR(a+bx) = b(IQR(X))$

Thus $IQR(a+bx) = a + b(IQR(x))$

when $a = 0$

6) Show by example that the means of x^2 and $\sqrt{x}$ are generally not $(m(x))^2$ and $\sqrt{m(x)}$.

x^2

$x = \{0,1,2\}$

mean of $x^2 = \frac{0^2+1^2+2^2}{3} = \frac{5}{3}$

mean of $x = \frac{0+1+2}{3} = 1$

$(m(x))^2 = 1^2 = 1$

$1 \neq \frac{5}{3}$ by counterexample, $m(x^2) \neq (m(x))^2$

$\sqrt{x} \quad x = \{0, 1, 4\}$

mean of $\sqrt{x} = \frac{\sqrt{0}+\sqrt{1}+\sqrt{4}}{3} = \frac{3}{3} = 1$

mean of $x = \frac{0+1+4}{3} = \frac{5}{3}$

$\sqrt{(m(x))} = \sqrt{5/3}$ by counterexample

$m(\sqrt{x}) \neq \sqrt{m(x)}$