

Simulation Challenge

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Investment Game Simulation Analysis

This report analyzes two different investment strategies using Monte Carlo simulation to understand their risk and return characteristics. We'll examine how small differences in strategy rules can lead to dramatically different outcomes over time.

Question 1: Expected Value Analysis

Let's start by calculating the expected value of your account balance after just one coin flip for the original game. This tells us what we should expect to happen on average.

Original Game Rules: - Start with \$1,000 - Heads: increase balance by 50% (multiply by 1.5) - Tails: reduce balance by 40% (multiply by 0.6) - Probability of heads = 0.5, tails = 0.5

What we're calculating: Expected value is the average outcome if we repeated this game many times. It's calculated by multiplying each possible outcome by its probability and adding them up.

Initial balance: \$1,000

Balance after heads (50% gain): \$1,500

Balance after tails (40% loss): \$600

Expected value after 1 flip: \$1,050

Key Insight: The expected value of \$1,050 means that on average, we expect to gain \$50 per flip. This is positive, suggesting the game is favorable in the long run.

Question 2: Expectation vs. Reality Analysis

Now let's understand what this expected value actually means for our investment strategy.

Expected value: \$1,050

Change from initial: \$50

Percentage change: 5%

The expected value is POSITIVE - we expect to gain money on average

Based on this result, we expect our account to be worth \$1,050

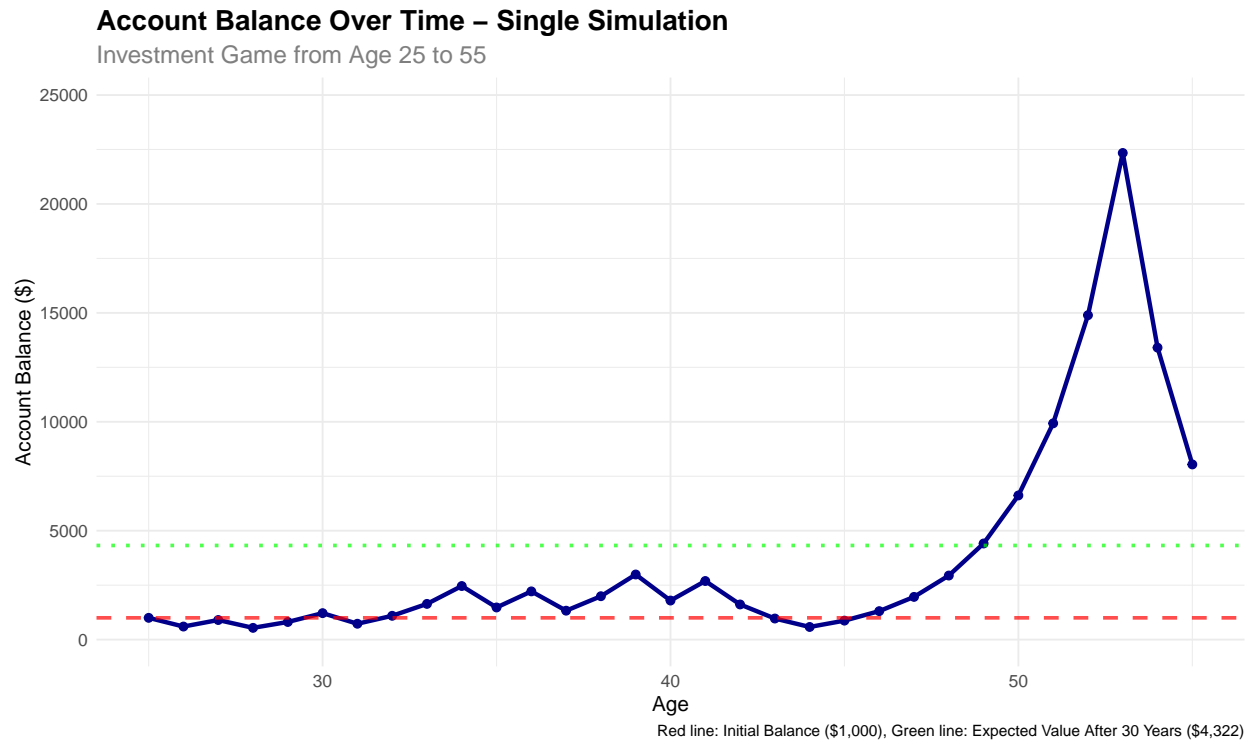
This is \$50 MORE than our initial \$1,000

Key Insight: Even though we expect to gain money on average (+5%), this doesn't guarantee we'll make money in any single game. The expected value tells us about long-term trends, but individual outcomes can vary widely.

Question 3: Single Simulation with Time Series Plot

Now let's see what happens when we actually play this game over 30 years (from age 25 to 55). This simulation shows one possible path your investment could take.

What we're doing: We'll flip a coin 30 times, each time either gaining 50% or losing 40% of our current balance. This shows how compounding works - both gains and losses build on each other over time.



Single Simulation Results:

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Initial balance (age 25): \$1,000

Final balance (age 55): \$8,043

Total change: \$7,043

Total return: 704%

Coin flip results: 17 heads, 13 tails

Heads percentage: 57%

What this plot shows: The blue line shows how your account balance changes over time. The red dashed line shows your starting point (\$1,000), and the green dotted line shows the expected value after 30 years. Notice how the actual path can be very different from the expected value due to random chance.

Analysis of Single Simulation Results

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ANALYSIS OF SINGLE SIMULATION

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Key Results:

- Started with: \$1,000
- Ended with: \$8,043
- Net change: \$7,043
- This simulation was PROFITABLE!
- You would be happy with this outcome

Performance Analysis:

- Total return: 704%
- Annualized return: 7.2%

Luck Factor:

- Got 17 heads out of 30 flips
- Expected heads: 15
- You were LUCKY - got more heads than expected!

Key Insight:

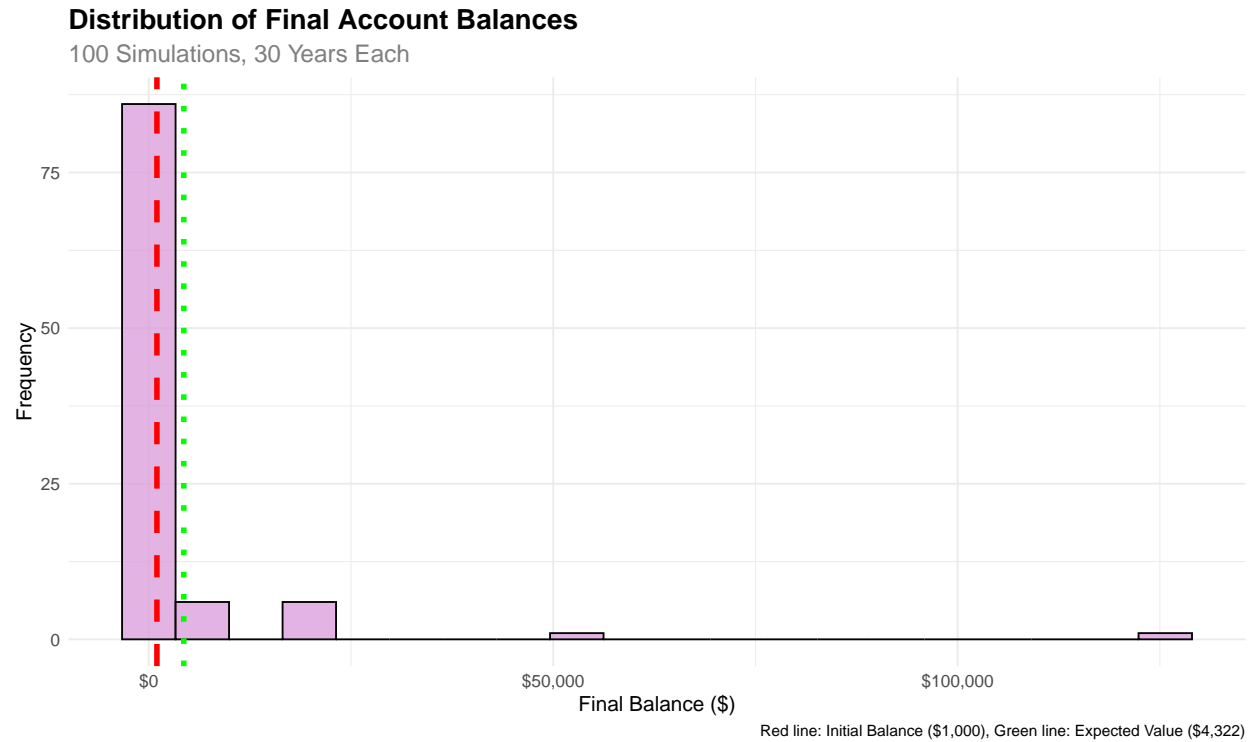
- Even though the expected value per flip is positive (\$1,050)
- Individual outcomes can vary dramatically due to compounding
- This single simulation shows the high variability possible
- Multiple simulations would give a better picture of the true risk

Key Insight: One simulation doesn't tell us the whole story. To understand the true risk and potential outcomes, we need to run many simulations and look at the distribution of results.

Question 4: Multiple Simulations (100 simulations)

Now let's run 100 different simulations to see the range of possible outcomes. This will show us the distribution of final balances and help us understand the true risk of this investment strategy.

What we're doing: We'll run the same 30-year simulation 100 times with different random coin flips. This gives us a probability distribution showing all possible outcomes and their likelihoods.



Summary Statistics for 100 Simulations:

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Mean final balance: \$3,872

Median final balance: \$206

Minimum final balance: \$0.054

Maximum final balance: \$125,666

Probability above initial (\$1,000): 0.3 (30%)

Probability above \$10,000: 0.08 (8%)

Analysis of 100 Simulations:

- You would NOT be happy with 70% of outcomes
- The median outcome results in a loss
- There's huge variability: from \$0.05 to \$1e+05

Question 5: Probability Analysis

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PROBABILITY ANALYSIS - ORIGINAL STRATEGY

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Key Probability Results:

- Probability that balance > \$1,000 at age 55: 0.3 (30%)
- Probability that balance ≤ \$1,000 at age 55: 0.7 (70%)

Interpretation:

- You have a LESS than 50% chance of ending up with more than you started!
- This suggests the strategy is unfavorable in the long run

Detailed Breakdown:

- 30 out of 100 simulations ended above \$1,000
- 70 out of 100 simulations ended at or below \$1,000

Key Insight: The histogram shows the distribution of final balances across 100 simulations. Most outcomes cluster around certain values, but there's a wide range of possible results. This is the reality of investment risk - even with a positive expected value, individual outcomes can vary dramatically.

Question 6: Strategy Comparison - Modified Game

Now let's compare the original strategy with a modified version to see how different rules affect outcomes.

Modified Game Rules: - Start with \$1,000 - Must bet exactly 50% of current balance each round - Heads: increase your bet by 50% (add 50% of bet to balance) - Tails: reduce your bet by 40% (subtract 40% of bet from balance) - Play for 30 years (age 25 to 55)

Key Difference: In the modified game, you can only bet 50% of your balance each round, rather than betting your entire balance. This should theoretically reduce risk, but let's see what the simulations show.

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MODIFIED GAME STRATEGY RESULTS

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Summary Statistics for Modified Game (100 Simulations):

Mean final balance: \$2,238

Median final balance: \$1,000

Minimum final balance: \$44

Maximum final balance: \$55,511

Probability above initial (\$1,000): 0.41 (41%)

Probability above \$10,000: 0.01 (1%)

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STRATEGY COMPARISON ANALYSIS

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Key Comparison Results:

- Original Strategy - Probability above \$10,000: 0.08 (8%)
- Modified Strategy - Probability above \$10,000: 0.01 (1%)

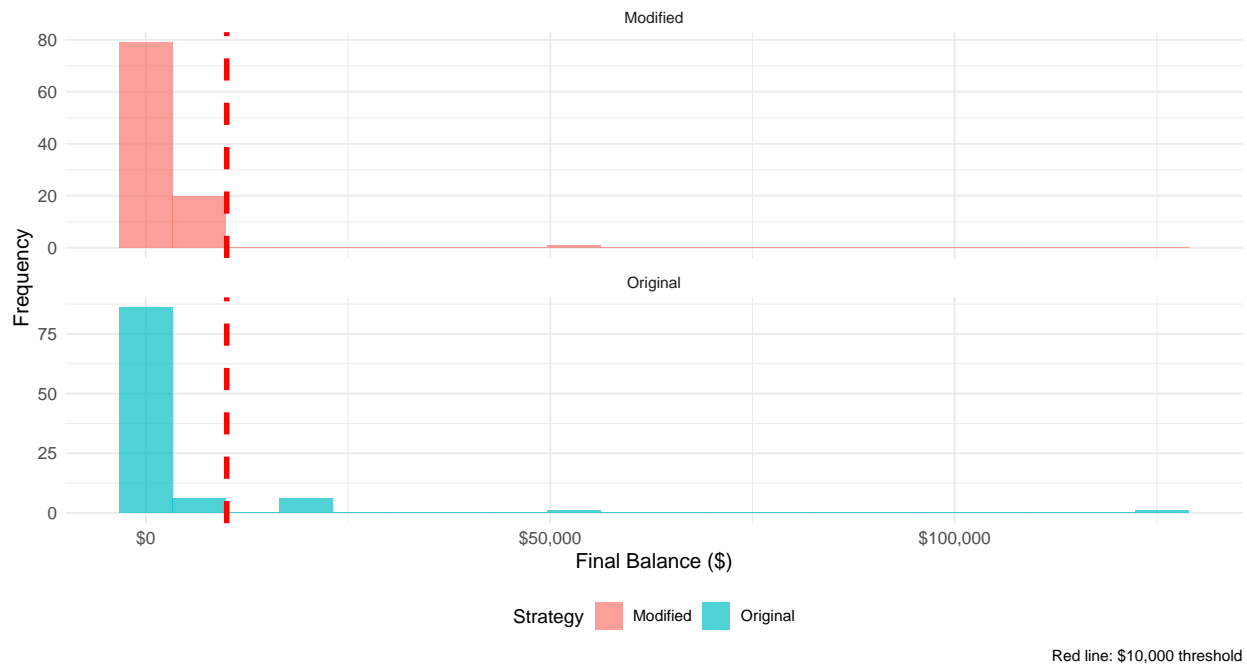
- The ORIGINAL strategy has a HIGHER probability of reaching \$10,000!
- Difference: -0.07 (-7%)

Strategic Insights:

- Original strategy has higher average returns
- Modified strategy has higher median returns
- Modified strategy range: \$44 to \$55,511
- Original strategy range: \$0.05 to \$1e+05

Strategy Comparison: Final Balance Distributions

100 Simulations Each – Original vs Modified Game



Final Analysis and Recommendations

FINAL ANALYSIS AND RECOMMENDATIONS

Key Findings:

1. Expected value per flip is positive (\$1,050)
2. Original strategy: 8% chance of reaching \$10,000
3. Modified strategy: 1% chance of reaching \$10,000
4. The ORIGINAL strategy is better for reaching \$10,000

Investment Implications:

- Both strategies show the power of compounding over time
- High variability means outcomes can be dramatically different
- The 'safer' modified strategy (betting only 50%) may actually be riskier
- Individual luck plays a huge role in short-term outcomes
- Multiple simulations reveal the true risk profile

Recommendation for a 25-year-old:

Consider the ORIGINAL strategy for higher probability of reaching \$10,000

However, remember that past performance doesn't guarantee future results!

Diversification and professional financial advice are always recommended.