Importing required libraries

In [80]:
```
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.arima.model import ARIMA
from itertools import product
from tqdm import tqdm_notebook
import warnings
import numpy as np
import statsmodels as sm

warnings.filterwarnings('ignore')

plt.rcParams["figure.figsize"] = (25,5)
```

Problem 1

The data for this project (Problem1_DataSet.csv) represents 7 years of monthly data on airline miles flown in the United Kingdom. You are tasked with the goal of developing a forecasting model that can accurately predict the trend for future years. To achieve the final goal, answer each of the questions below.

In [81]:
```
df = pd.read_csv('Problem1_DataSet.csv', index_col = 'Month')
df
```

Out[81]:

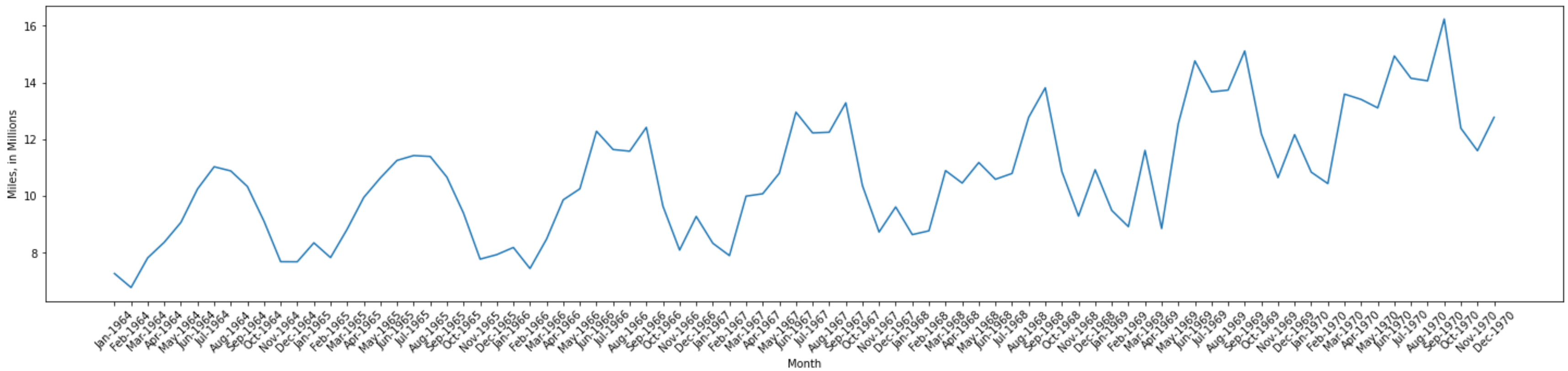
	Miles, in Millions
Month	
Jan-1964	7.269
Feb-1964	6.775
Mar-1964	7.819
Apr-1964	8.371
May-1964	9.069
...	...
Aug-1970	14.057
Sep-1970	16.234
Oct-1970	12.389
Nov-1970	11.594
Dec-1970	12.772

84 rows × 1 columns

Part 1

Create a time series of the plot of the data provided. (5 pts)

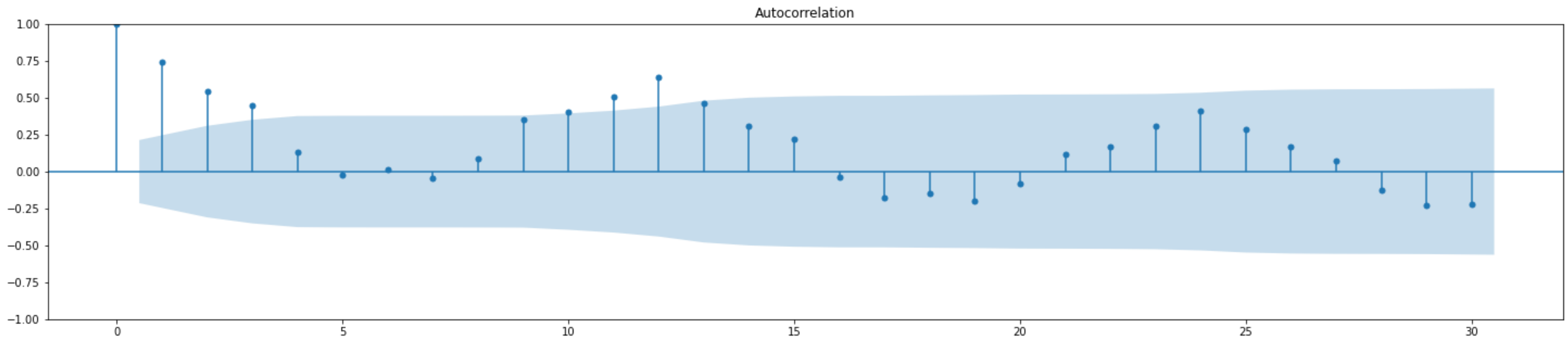
In [82]:
```
plt.xlabel('Month')
plt.ylabel('Miles, in Millions')
plt.xticks(rotation = 45)
plt.plot(df)
plt.show()
```



Part 2

Plot the autocorrelation function (ACF). From the ACF, what is the seasonal period? (5 pts)

In [83]:
```
plot_acf(df['Miles, in Millions'], lags = 30);
```



The seasonal period is about 1 year.

Part 3

Compute a moving average for the data to determine the trend in the data and overlay on the original time-series plot. What is a suitable choice for the moving average window length? (5 pts)

In [84]:
```
def sma(data, time_span):
    start_index = time_span-1
    periods = []
    new_data = []

    for i in range(start_index, len(data)):
        point = data.iloc[i-time_span+1:i+1, 0].mean()
        new_data.append(point)
        periods.append(data.index[i])

    return pd.DataFrame(data={'Miles, in Millions':new_data, 'Month':periods}).set_index('Month')
```

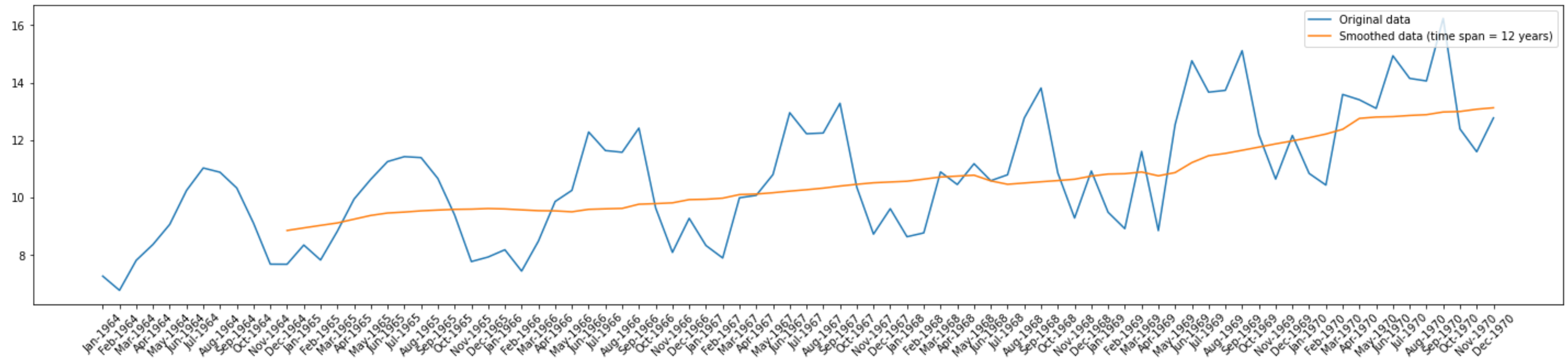
In [85]:
```
sma_df = sma(df, 12)
sma_df
```

Out[85]:

	Miles, in Millions
Month	
Dec-1964	8.856000
Jan-1965	8.946083
Feb-1965	9.033917
Mar-1965	9.118083
Apr-1965	9.249500
...	...
Aug-1970	12.884000
Sep-1970	12.977667
Oct-1970	12.994667
Nov-1970	13.073750
Dec-1970	13.124667

73 rows × 1 columns

In [86]:
```
plt.plot(df.index, df['Miles, in Millions'])
plt.plot(sma_df.index, sma_df['Miles, in Millions'])
plt.legend(["Original data", "Smoothed data (time span = 12 years)"], loc ="upper right")
plt.xticks(rotation = 45)
plt.show()
```



As can be seen from the above figure, a suitable choice for the moving average window length would by 12 months.

Part 4

Observing the moving average plot in Q3, is the trend line increasing or decreasing? (5 pts)

The trend is increasing.

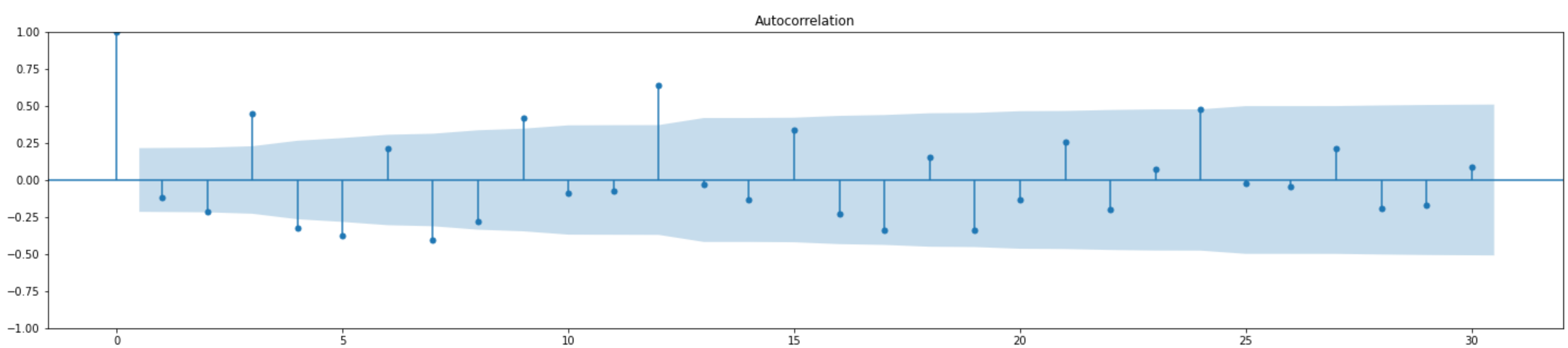
Part 5

Compute the first difference of the data and plot the ACF and PACF for the differenced data. What are the significant lags based on the ACF and PACF? (5 pts)

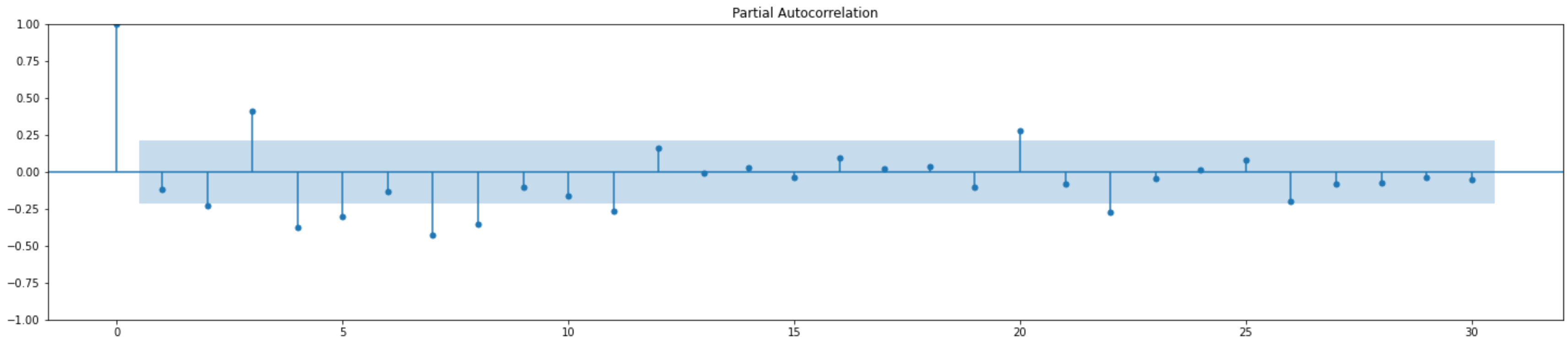
In [8]:
```
df_first_diff = df['Miles, in Millions'].diff()
df_first_diff
```

Out[8]:
```
Month
Jan-1964       NaN
Feb-1964    -0.494
Mar-1964     1.044
Apr-1964     0.552
May-1964     0.698
             ...
Aug-1970    -0.090
Sep-1970     2.177
Oct-1970    -3.845
Nov-1970    -0.795
Dec-1970     1.178
Name: Miles, in Millions, Length: 84, dtype: float64
```

In [9]:
```
plot_acf(df_first_diff.dropna(), lags = 30);
```



In [10]:
```
plot_pacf(df_first_diff.dropna(), lags = 30, method='ywm');
```



The significant lag based on the acf plot is 12 and based on the pacf plot is 3.

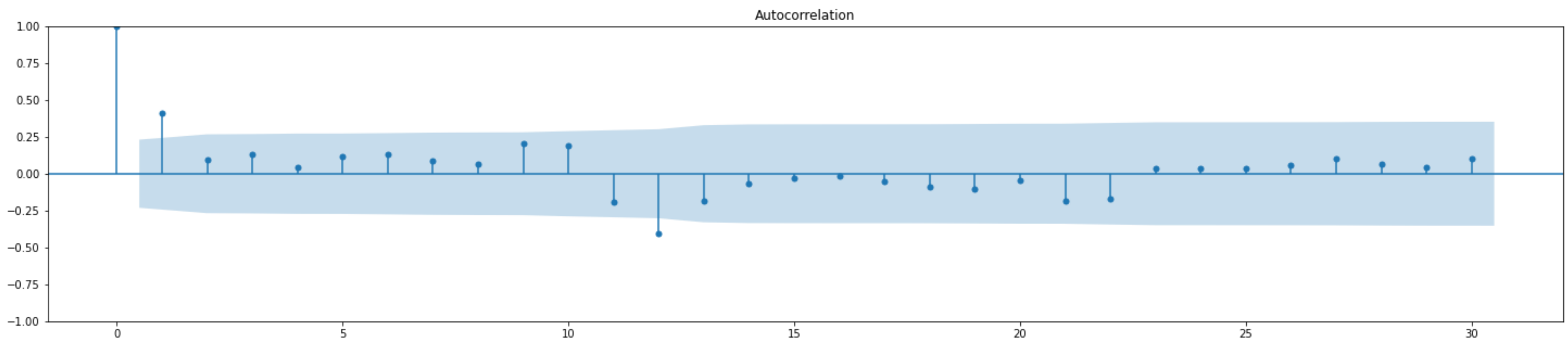
Part 6

Using the output from Q5 above, perform a first seasonal difference with the seasonal period you identified in Q2, and plot the ACF and PACF again. What are the significant lags based on the ACF and PACF? (5 pts)

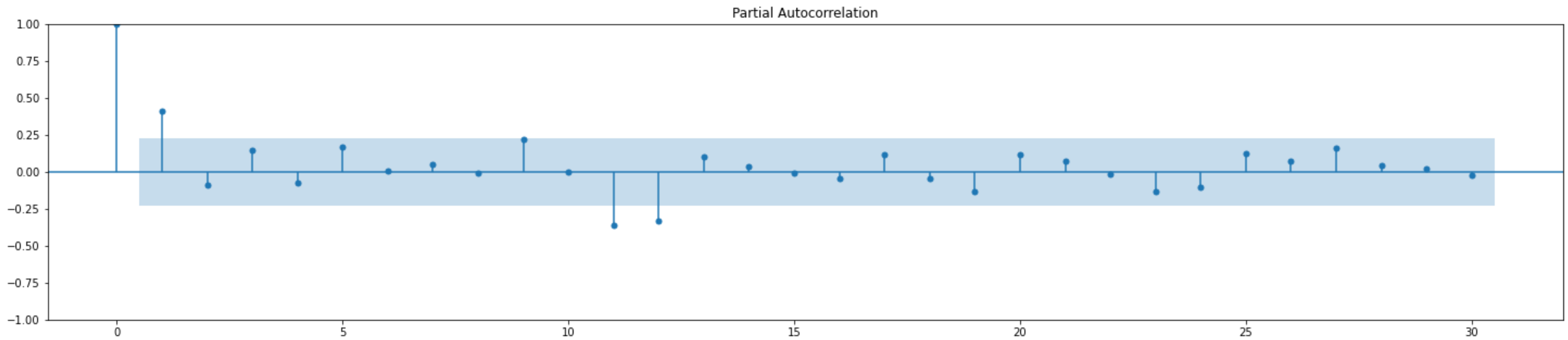
In [11]:
```
df_seasonal_diff = df['Miles, in Millions'] - df['Miles, in Millions'].shift(12)
df_seasonal_diff
```

Out[11]:
```
Month
Jan-1964      NaN
Feb-1964      NaN
Mar-1964      NaN
Apr-1964      NaN
May-1964      NaN
            ...
Aug-1970    0.326
Sep-1970    1.124
Oct-1970    0.204
Nov-1970    0.949
Dec-1970    0.611
Name: Miles, in Millions, Length: 84, dtype: float64
```

In [12]:
```
plot_acf(df_seasonal_diff.dropna(), lags = 30);
```



In [13]:
```
plot_pacf(df_seasonal_diff.dropna(), lags = 30, method='ywm');
```



Part 7

Develop a suitable SARIMA model that can be applied on the time series. Use the first 6 years of data only to develop the model. (20 pts)

a. To develop the model, vary the model parameters for the non-seasonal (p,d,q) and seasonal components (P,D,Q) and calculate the output for each combination of parameters.

b. Use an evaluation criteria such as AIC, BIC or sum squared error or mean squared error to determine the best choice of parameters (p,d,q,P,D,Q). Note: AIC and BIC are metrics that is readily output by the ARIMA model.

In [14]:
```
df_train = df.iloc[:72,:]
df_train
```

Out[14]:

	Miles, in Millions
Month	
Jan-1964	7.269
Feb-1964	6.775
Mar-1964	7.819
Apr-1964	8.371
May-1964	9.069
...	...
Aug-1969	13.731
Sep-1969	15.110
Oct-1969	12.185
Nov-1969	10.645
Dec-1969	12.161

72 rows × 1 columns

In [15]:
```
def optimize_SARIMA(parameters_list, d, D, s, exog):
    """
        Return dataframe with parameters, corresponding AIC and SSE

        parameters_list - list with (p, q, P, Q) tuples
        d - integration order
        D - seasonal integration order
        s - length of season
        exog - the exogenous variable
    """

    results = []

    for param in tqdm_notebook(parameters_list):
        try:
            model = SARIMAX(exog, order=(param[0], d, param[1]), seasonal_order=(param[2], D, param[3],
        except:
            continue

        aic = model.aic
        results.append([param, aic])

    result_df = pd.DataFrame(results)
    result_df.columns = ['(p,q)x(P,Q)', 'AIC']
    #Sort in ascending order, lower AIC is better
    result_df = result_df.sort_values(by='AIC', ascending=True).reset_index(drop=True)

    return result_df
```

In [16]:
```
p = range(0, 4, 1)
d = 1
q = range(0, 4, 1)
P = range(0, 4, 1)
D = 1
Q = range(0, 4, 1)
s = 12

parameters = product(p, q, P, Q)
parameters_list = list(parameters)
print(len(parameters_list))
```

```
256
```

In [17]:
```
result_df = optimize_SARIMA(parameters_list, 1, 1, 4, df_train['Miles, in Millions'])
result_df
```

```
100%                                        256/256 [01:07<00:00, 1.72it/s]
```

In [18]:
```
best_model = SARIMAX(df_train['Miles, in Millions'], order=(1, 1, 1), seasonal_order=(2, 1, 1, 12)).fit(
best_model.summary()
```

```
 This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            6     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.23780D+00    |proj g|=  3.88390D-01

At iterate    5    f=  1.00744D+00    |proj g|=  3.73641D-02

At iterate   10    f=  9.93131D-01    |proj g|=  3.05237D-02

At iterate   15    f=  9.91912D-01    |proj g|=  4.36098D-03

At iterate   20    f=  9.91383D-01    |proj g|=  1.89215D-03

At iterate   25    f=  9.91253D-01    |proj g|=  1.46766D-03

At iterate   30    f=  9.91169D-01    |proj g|=  2.60758D-03

At iterate   35    f=  9.91137D-01    |proj g|=  4.71780D-04

At iterate   40    f=  9.91129D-01    |proj g|=  2.16290D-04

At iterate   45    f=  9.91127D-01    |proj g|=  7.62136D-04

At iterate   50    f=  9.91127D-01    |proj g|=  1.85242D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
```

```
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    6     50     61      1     0     0   1.852D-04   9.911D-01
  F =   0.99112687604383687

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
```

Out[18]:

SARIMAX Results

Dep. Variable:	Miles, in Millions	**No. Observations:**	72
Model:	SARIMAX(1, 1, 1)x(2, 1, 1, 12)	**Log Likelihood**	-71.361
Date:	Wed, 26 Oct 2022	**AIC**	154.722
Time:	21:18:28	**BIC**	167.187
Sample:	01-01-1964	**HQIC**	159.588
	- 12-01-1969		
Covariance Type:	opg		

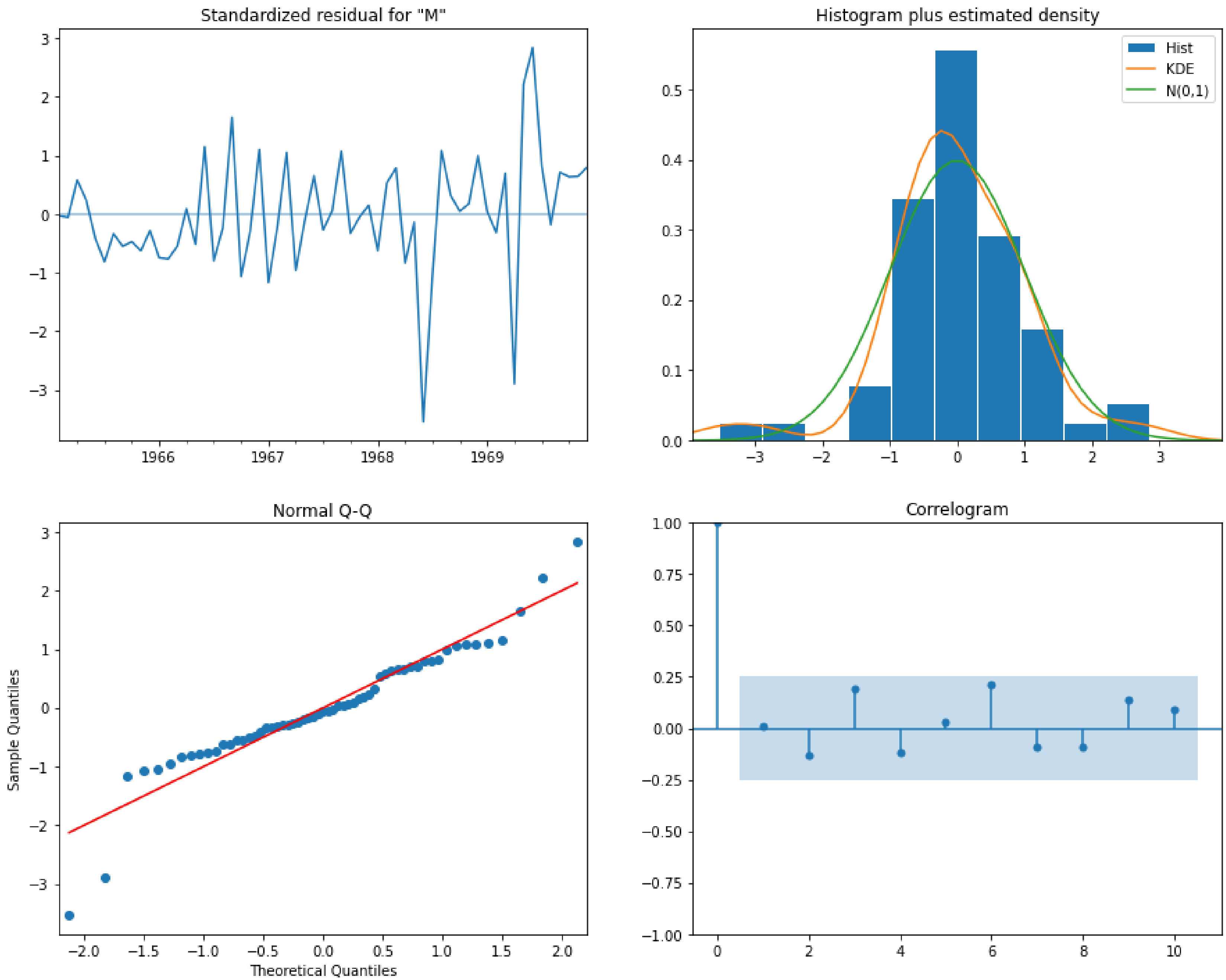
	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
ar.L1	0.3679	0.211	1.741	0.082	-0.046	0.782
ma.L1	-0.9989	6.680	-0.150	0.881	-14.091	12.093
ar.S.L12	-1.3529	2.758	-0.490	0.624	-6.759	4.054
ar.S.L24	-0.4261	1.061	-0.401	0.688	-2.507	1.654
ma.S.L12	0.9833	10.369	0.095	0.924	-19.340	21.307
sigma2	0.5707	7.108	0.080	0.936	-13.361	14.502

Ljung-Box (L1) (Q):	0.01	**Jarque-Bera (JB):**	21.43
Prob(Q):	0.93	**Prob(JB):**	0.00
Heteroskedasticity (H):	4.63	**Skew:**	-0.47

Prob(H) (two-sided):	0.00	**Kurtosis:**	5.80

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [19]:
```
best_model.plot_diagnostics(figsize=(15,12));
```



In [20]:
```
df_train['arima_model'] = best_model.fittedvalues
plt.figure(figsize=(25, 7.5))
plt.plot(df_train['arima_model'], color='r', label='model')
plt.plot(df_train['Miles, in Millions'], label='actual')
plt.xticks(rotation = 45)
plt.legend()
plt.show()
```

Part 8

Use the model parameters determined in Q7 above to forecast for the 7th year. Compare the forecast with actual values. Comment on your observations. (10 pts)

In [21]:
```
forecast = best_model.forecast(steps = 12)
forecast = df_train['arima_model'].append(forecast)
forecast = forecast.to_frame().reset_index().iloc[:,1]
forecast
```

Out[21]:
```
0      0.000000
1      7.268999
2      6.775000
3      7.819000
4      8.371000
         ...
79    14.182875
80    15.372481
81    12.478162
82    10.908102
83    12.404746
Name: 0, Length: 84, dtype: float64
```

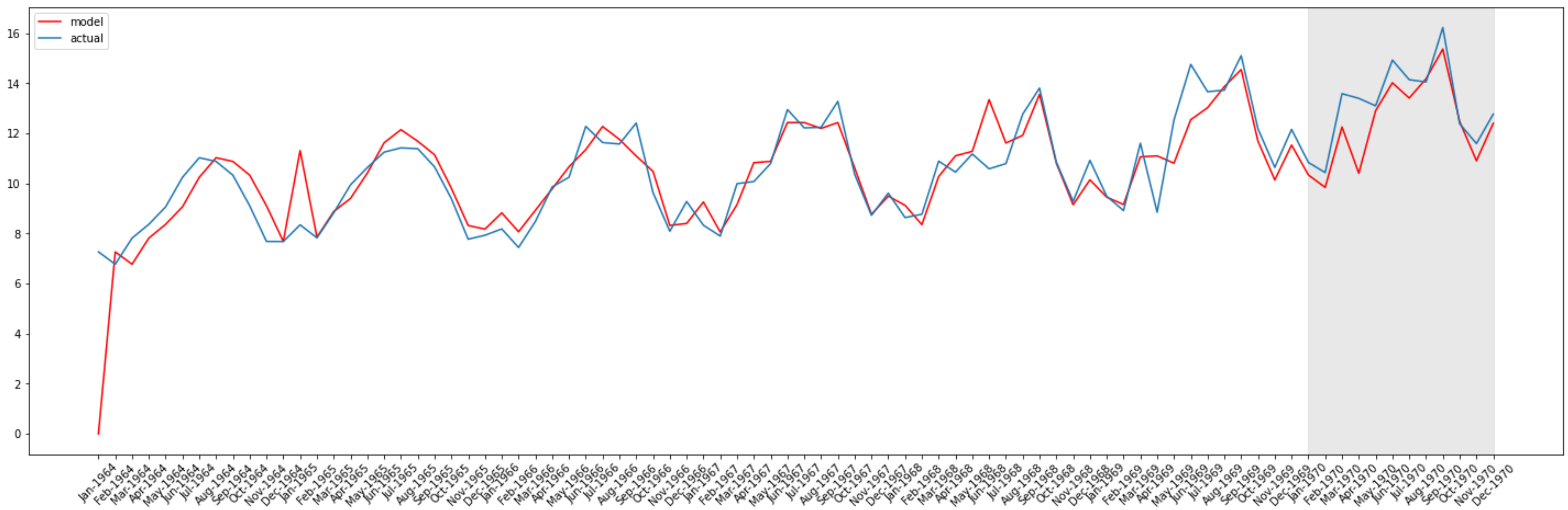
In [22]:
```
df_plot = df.reset_index()
df_plot
```

Out[22]:

	Month	Miles, in Millions
0	Jan-1964	7.269
1	Feb-1964	6.775
2	Mar-1964	7.819
3	Apr-1964	8.371
4	May-1964	9.069
...	...	...
79	Aug-1970	14.057
80	Sep-1970	16.234
81	Oct-1970	12.389
82	Nov-1970	11.594
83	Dec-1970	12.772

84 rows × 2 columns

In [23]:
```
plt.figure(figsize=(25, 7.5))
plt.plot(forecast, color='r', label='model')
plt.plot(df['Miles, in Millions'], label='actual')
plt.axvspan(df.index[-12], df.index[-1], alpha=0.5, color='lightgrey')
plt.xticks(rotation = 45)
plt.legend()
plt.show()
```



The forecasted values are in red inside the grey strip. They very closely capture the trend of the actual data for the most part except for about 2 months.

Problem 2

In this problem, you will develop a time-series model to analyze Wine consumption from the data file “TotalWine.csv”.

In [24]:
```
df = pd.read_csv('TotalWine.csv', index_col = 'Time (Quarter)')
df
```

Out[24]:

Time (Quarter)	TotalWine
1	1.486
2	1.915
3	1.844
4	2.808
5	1.287
6	1.861
7	2.034
8	2.739
9	1.656
10	1.918
11	2.265
12	2.902
13	1.691
14	2.033
15	2.141
16	2.932
17	1.847
18	2.157
19	2.318
20	2.974
21	1.977
22	2.328

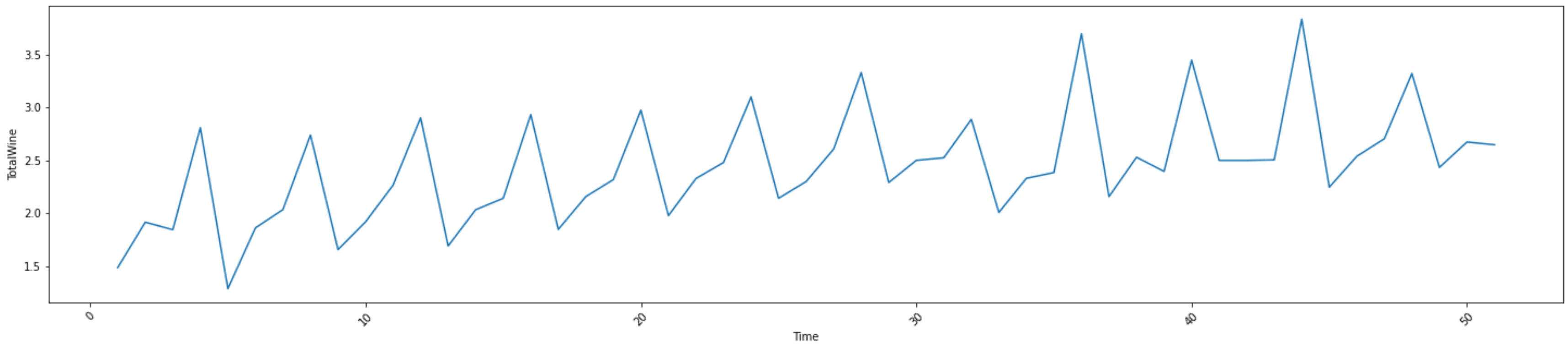
	TotalWine
Time (Quarter)	
23	2.479
24	3.099
25	2.141
26	2.299
27	2.606
28	3.330
29	2.290
30	2.499
31	2.524
32	2.887
33	2.007
34	2.330
35	2.384
36	3.696
37	2.157
38	2.529
39	2.395
40	3.447
41	2.499
42	2.499
43	2.504
44	3.834
45	2.246
46	2.538
47	2.704

	TotalWine
Time (Quarter)	
48	3.321
49	2.433
50	2.673
51	2.647

Part a

Plot the time series for TotalWine. What is the seasonal period for this time-series? (1 pt)

In [25]:
```
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.xticks(rotation = 45)
plt.plot(df)
plt.show()
```

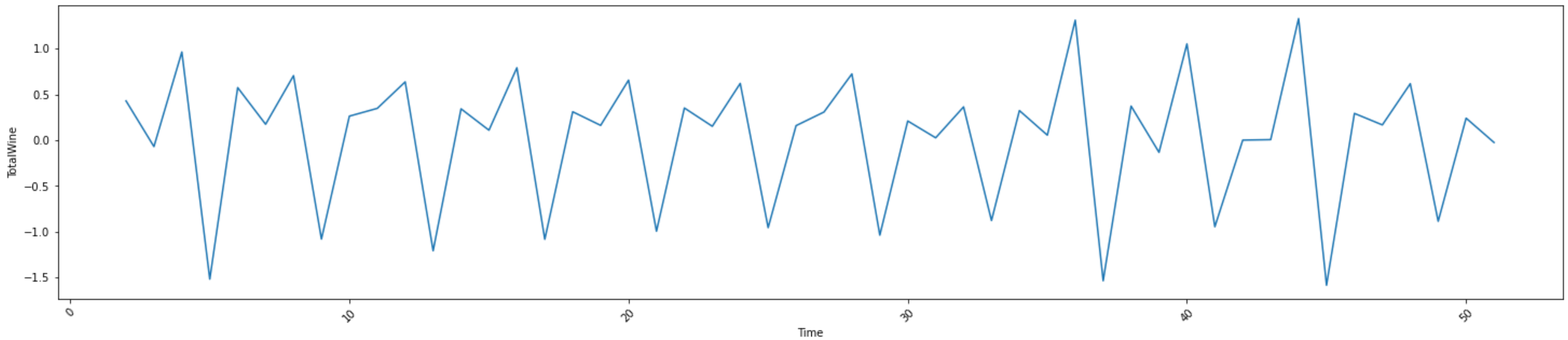


The seasonal period is 4.

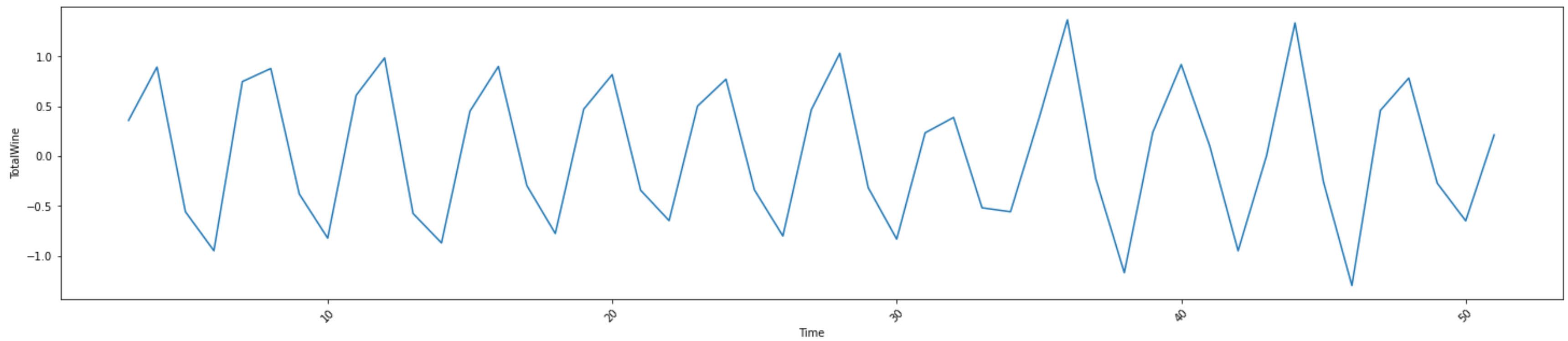
Part b

Apply seasonal differencing to the original time-series. Vary the difference lag from 1, 2, 4, 6. Plot the result for each of these lags. Which of these differences is most suitable to remove the seasonality? (2 pts)

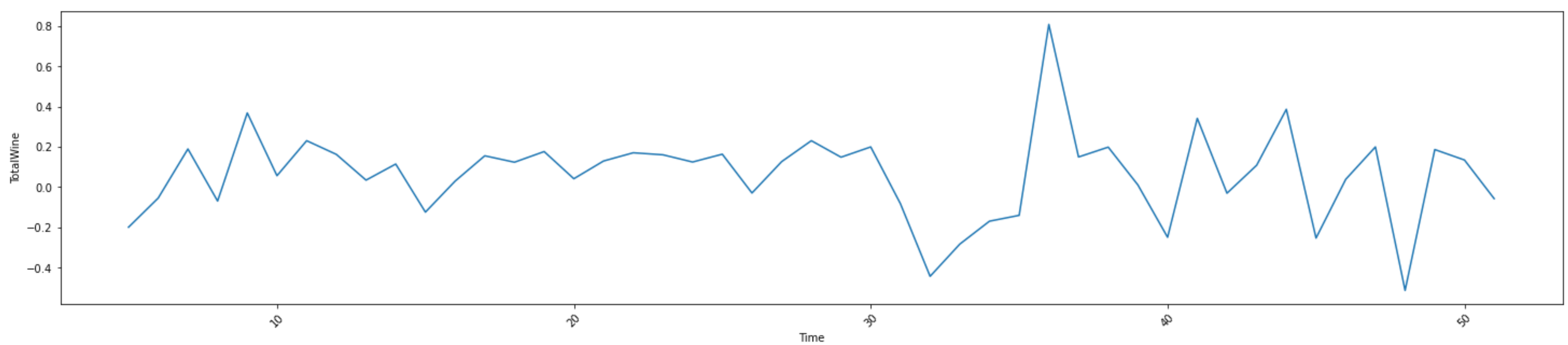
In [34]:
```
df_diff_1 = df['TotalWine'] - df['TotalWine'].shift(1)
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.xticks(rotation = 45)
plt.plot(df_diff_1)
plt.show()
```



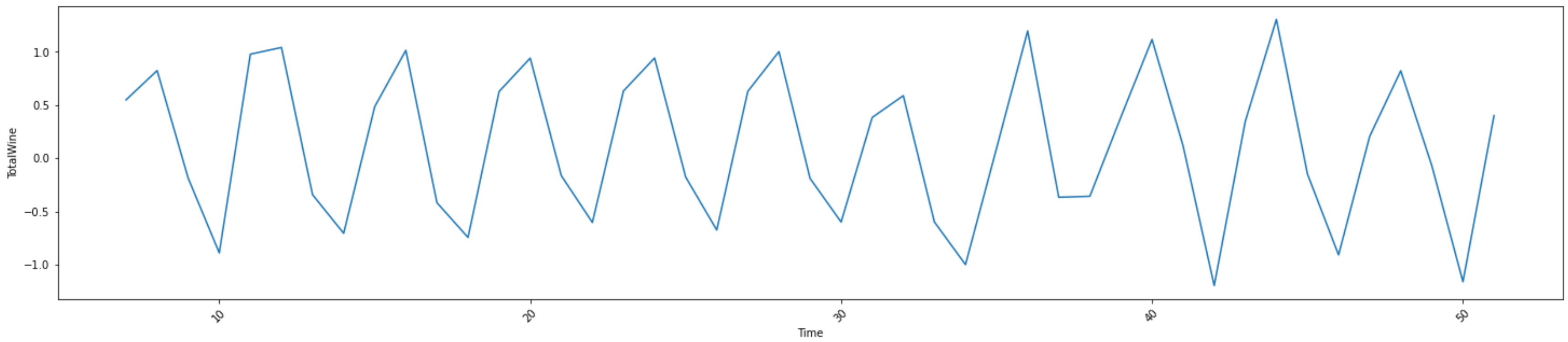
In [35]:
```
df_diff_2 = df['TotalWine'] - df['TotalWine'].shift(2)
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.xticks(rotation = 45)
plt.plot(df_diff_2)
plt.show()
```



In [36]:
```
df_diff_4 = df['TotalWine'] - df['TotalWine'].shift(4)
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.xticks(rotation = 45)
plt.plot(df_diff_4)
plt.show()
```



In [37]:
```
df_diff_6 = df['TotalWine'] - df['TotalWine'].shift(6)
plt.xlabel('Time')
plt.ylabel('TotalWine')
plt.xticks(rotation = 45)
plt.plot(df_diff_6)
plt.show()
```

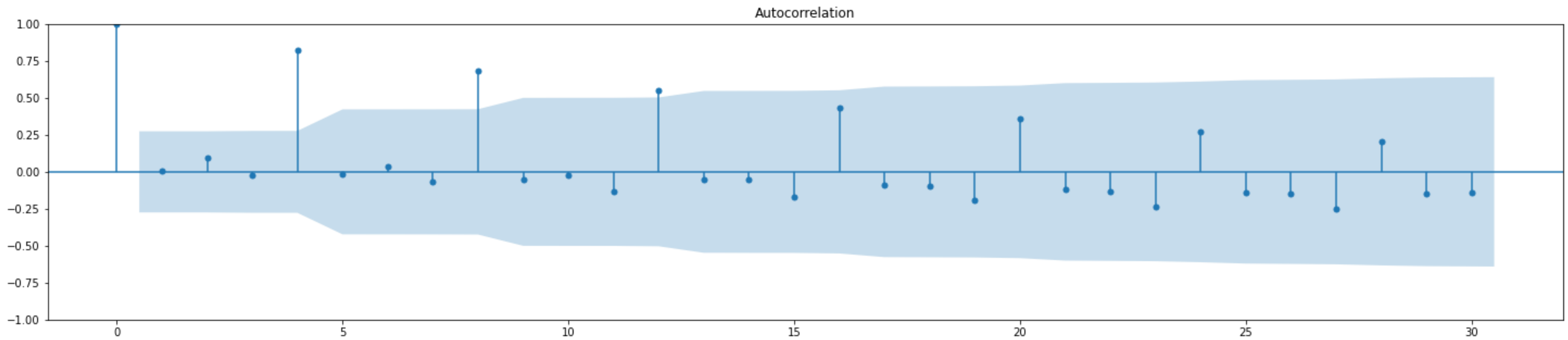


A lag of 4 is the most suitable to remove seasonality.

Part c

Compute and plot the Auto-correlation (ACF) function for the original time-series. What is the seasonal period you estimate from the ACF? (1 pt)

In [38]:
```
plot_acf(df, lags = 30);
```



The seasonal period is 4 quarters.

Part d

Define an AR model using tsa.AR available in statsmodels.api. Determine the optimal order using the “select_order” function. You will need to specify a maximum order p (recommend p=10) to consider and a criterion for deciding which model order is "best". [e.g. You can use AIC as the model selection criteria] (2 pts)

In [40]:
```
df_seasonal_diff = df['TotalWine'].diff(periods=4).dropna().reset_index(drop=True)
```

In [71]:
```
# AR model using tsa.AR available in statsmodels.api was unavailable

opt_order = sm.tsa.ar_model.AutoReg(df_seasonal_diff, lags=10).ar_lags
print('optimal lag = ', 5)
```

```
optimal lag =  5
```

Part e

Now, evaluate an AR(p) model for the time-series generated after seasonal differencing (using the best lag you found in part b above) (4 pts)

i. use the fit method specifying the optimal lag found above

ii. use the predict method to generate values starting at the optimal lag

iii. plot the predicted results and the corresponding seasonally differenced time-series

iv. Calculate the Mean Absolute Error (MAE) by comparing the predicted results with the seasonally differenced data.

In [69]:
```
model = ARIMA(df_seasonal_diff, order=(5,0,0))
model_fit = model.fit()
model_fit.summary()
```

Out[69]:

SARIMAX Results

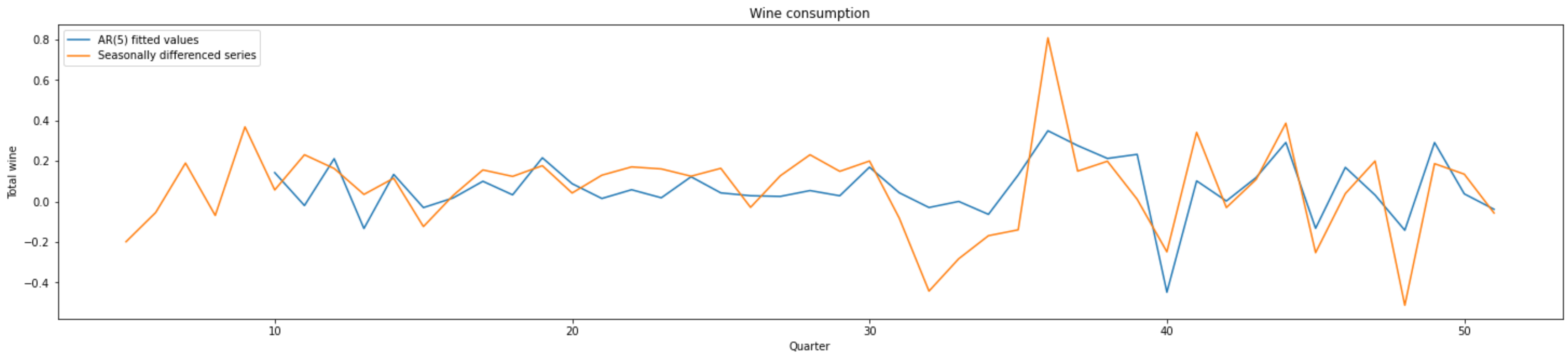
Dep. Variable:	TotalWine	**No. Observations:**	47
Model:	ARIMA(5, 0, 0)	**Log Likelihood**	16.795
Date:	Wed, 26 Oct 2022	**AIC**	-19.590
Time:	22:15:05	**BIC**	-6.639
Sample:	0	**HQIC**	-14.717
	- 47		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0722	0.021	3.389	0.001	0.030	0.114
ar.L1	0.0824	0.144	0.571	0.568	-0.200	0.365
ar.L2	0.0455	0.161	0.282	0.778	-0.270	0.361
ar.L3	0.0521	0.098	0.533	0.594	-0.139	0.244
ar.L4	-0.6723	0.119	-5.645	0.000	-0.906	-0.439
ar.L5	0.1421	0.145	0.983	0.325	-0.141	0.425
sigma2	0.0271	0.006	4.187	0.000	0.014	0.040

Ljung-Box (L1) (Q):	0.00	**Jarque-Bera (JB):**	0.67
Prob(Q):	0.97	**Prob(JB):**	0.71
Heteroskedasticity (H):	3.02	**Skew:**	-0.09
Prob(H) (two-sided):	0.03	**Kurtosis:**	3.56

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [78]:
```
model_predict = model_fit.predict(start=5,end=len(df_seasonal_diff) - 1, typ='levels').rename("Predictic
plt.plot(df.iloc[4+5:].index, model_predict, label='AR(5) fitted values')
plt.plot(df.iloc[4:].index, df_seasonal_diff, label='Seasonally differenced series')
plt.xlabel('Quarter')
plt.ylabel('Total wine')
plt.title('Wine consumption')
plt.legend(loc="upper left")
plt.show()
```



In [79]:
```
mae = (model_predict - df_seasonal_diff.loc[5:]).abs().sum() / len(model_predict)
mae
```

Out[79]: 0.13094130994233436

References

1. https://towardsdatascience.com/time-series-forecasting-with-sarima-in-python-cda5b793977b (https://towardsdatascience.com/time-series-forecasting-with-sarima-in-python-cda5b793977b)

In [ ]: