Question 1 : Classification of XOR data

At first sight, it may seem as if separating the XOR data is a simple task. However, due to the fact that the data is not linearly separable and that fitting the data requires non-trivial learning, the XOR problem has been a case study of interest on many topics related to training of feedforward networks. In the 1960s, Minsky and Papert's observations that perceptrons (neural network ancestors) were unable to fit the XOR data contributed to the rise of the first AI winter. XOR data makes for such an interesting case study that papers describing learning properties of networks trained on it are still being published to this day.

Create arrays containing the input data and the corresponding output labels for the XOR operator. Recall that XOR takes as input two binary variables, and outputs a 0/1 if they have the same/different value.

In other words, the XOR training includes four, two-dimensional data samples with labels. Create and train a network with at least three hidden layers that separates the XOR data, that is, a network that gets 100% performance on the four training samples above. Monitor the accuracy on the training set as the training progresses.

Installing required libraries

In [2]:
```
!pip install torch
```

```
Requirement already satisfied: torch in /Users/haileythanki/opt/anaconda
3/lib/python3.9/site-packages (1.13.0)
Requirement already satisfied: typing-extensions in /Users/haileythanki/o
pt/anaconda3/lib/python3.9/site-packages (from torch) (4.1.1)
```

In [3]:
```
import torch
import torch.nn as nn
import torch.nn.functional as F
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

Function Definitions

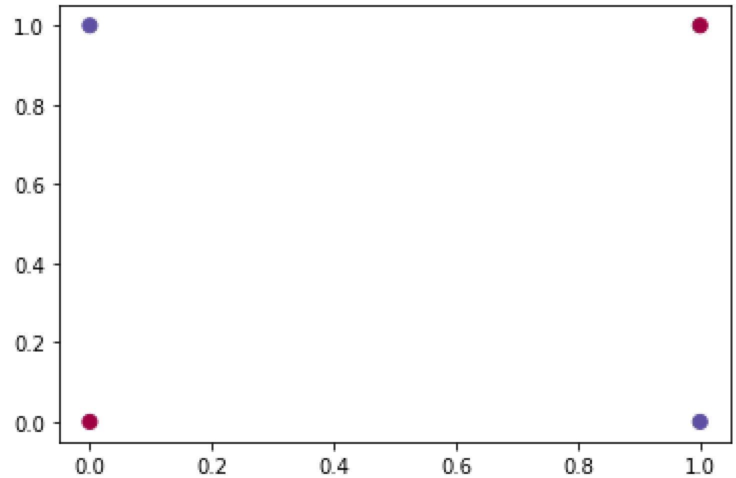
In [4]:
```
class Net(nn.Module):

    def __init__(self):
        super(Net, self).__init__()
        self.fc1 = nn.Linear(2, 3)
        torch.nn.init.uniform_(self.fc1.weight, a=0, b=1)
        self.fc2 = nn.Linear(3, 3)
        torch.nn.init.uniform_(self.fc2.weight, a=0, b=1)
        self.fc3 = nn.Linear(3, 3)
        torch.nn.init.uniform_(self.fc3.weight, a=0, b=1)
        self.fc4 = nn.Linear(3, 3)
        torch.nn.init.uniform_(self.fc4.weight, a=0, b=1)
        self.fc5 = nn.Linear(3, 2)
        torch.nn.init.uniform_(self.fc5.weight, a=0, b=1)

    def forward(self, x):
        x = F.relu(self.fc1(x))
        x = F.relu(self.fc2(x))
        x = F.relu(self.fc3(x))
        x = F.relu(self.fc4(x))
        x = self.fc5(x)
        return F.log_softmax(x)

# function for plotting
def plot_data(X, y, filename):
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral, s = 1)
    plt.savefig(filename)
    plt.close()

# function for plotting decision boundary
def plot_decision_boundary(clf, X, y, filename):
    x_min, x_max = -0.5, 1.5
    y_min, y_max = -0.5, 1.5
    h = 0.01
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max
    X_out = net(torch.tensor(np.c_[xx.ravel(), yy.ravel()], dtype = torch.f
    Z = X_out.data.max(1)[1]
    Z = Z.reshape(xx.shape)
    plt.contourf(xx, yy, Z, cmap=plt.cm.Spectral)
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral, s = 1)
    plt.savefig(filename)
    plt.close()
```

The data

In [5]:
```
xor_data = pd.read_csv('XOR.csv')
```

In [6]:
```
X = xor_data.values[:, 0:2]  # Take only the first two features.
X = torch.tensor(X, dtype = torch.float)
y = xor_data.values[:, 2]
y = torch.tensor(y, dtype = torch.long)
```

In [7]:
```
plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral, s = 50)
plt.show()
```



Part a

Plot the decision boundaries of the earliest network in the training process that achieves 100% accuracy by plotting the network outputs in a densely sampled region around [-0.5,1.5] × [-0.5,1.5]. (5 points)

In [8]:
```
net = Net()

# create a stochastic gradient descent optimizer
learning_rate = .01
optimizer = torch.optim.SGD(net.parameters(), lr=learning_rate, momentum=0.


# create a loss function
criterion = nn.NLLLoss()

nepochs = 20000
data, target = X, y

# run the main training loop
for epoch in range(nepochs):
    # forward propagate
    net_out = net(data)
    # compute loss
    loss = criterion(net_out, target)
    # backpropagate
    loss.backward()
    # update parameters
    optimizer.step()
    # print out report
    if epoch % 10 == 0:
        print('Epoch =', epoch, ', Loss =', loss.item())
        net_out = net(data)
        pred = net_out.data.max(1)[1]  # get the index of the max log-proba
        correctidx = pred.eq(target.data)
        ncorrect = correctidx.sum()
        accuracy = ncorrect.item()/len(data)
        if accuracy == 1:
            print('Epoch =', epoch, ', Loss = ', loss.item())
            print('Training accuracy = ', accuracy)
            break

# compute accuracy on training data
net_out = net(data)
pred = net_out.data.max(1)[1]  # get the index of the max log-probability
correctidx = pred.eq(target.data)
ncorrect = correctidx.sum()
accuracy = ncorrect.item()/len(data)
print('Training accuracy =', accuracy)
```

```
Epoch = 740 , Loss = 85.35719299316406
Epoch = 750 , Loss = 127.64102935791016
Epoch = 760 , Loss = 165.4411163330078
Epoch = 770 , Loss = 198.42117309570312
Epoch = 780 , Loss = 226.4639892578125
Epoch = 790 , Loss = 249.5286865234375
Epoch = 800 , Loss = 267.6009826660156
Epoch = 810 , Loss = 280.6759033203125
Epoch = 820 , Loss = 288.7518310546875
Epoch = 830 , Loss = 291.8280639648437 5
Epoch = 840 , Loss = 289.9044189453125
Epoch = 850 , Loss = 282.9808044433594
```

```
Epoch = 860 , Loss = 271.0572204589844
Epoch = 870 , Loss = 254.13363647460938
Epoch = 880 , Loss = 232.21005249023438
Epoch = 890 , Loss = 205.28646850585938
Epoch = 900 , Loss = 173.36288452148438
Epoch = 910 , Loss = 136.43930053710938
Epoch = 920 , Loss = 94.51571655273438
```

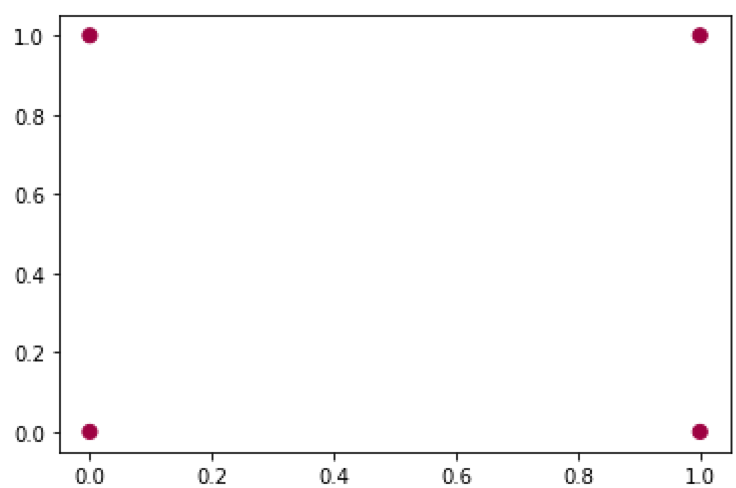
In [9]:

```
print(net.fc1.weight)
print(net.fc2.weight)
print(net.fc3.weight)
print(net.fc4.weight)
print(net.fc5.weight)
```

```
Parameter containing:
tensor([[-1528.8915, -5523.7012],
        [  530.2429,   208.7794],
        [-3193.6394, -7617.1128]], requires_grad=True)
Parameter containing:
tensor([[-9772.2744,   578.9077, -6740.3535],
        [-5649.1328,   232.2209, -4062.5300],
        [-5516.3081,   594.4395, -4140.3818]], requires_grad=True)
Parameter containing:
tensor([[ -485.6030,  -731.4153,    99.3140],
        [-3521.9634, -6071.2646,  -573.5346],
        [-2158.3091, -3701.6282,  -527.1506]], requires_grad=True)
Parameter containing:
tensor([[  -204.4523,  -3000.9285,  -2001.5150],
        [  -539.2286,  -9698.4912, -10752.3486],
        [  -138.9335,  -3525.7830,  -3522.7957]], requires_grad=True)
Parameter containing:
tensor([[  999.0788, -8610.5957, -3965.6099],
        [ -997.6282,  8611.7285,  3967.4583]], requires_grad=True)
```

In [10]:

```
plt.scatter(X[:, 0], X[:, 1], c=pred, cmap=plt.cm.Spectral, s = 50)
plt.show()
```



In [12]:
```
plot_decision_boundary(net, X, y, 'Q1_part_a.pdf')
```

```
/var/folders/_y/ch74wgzn7s1dxtq4ysb993sr0000gn/T/ipykernel_42258/39823238
33.py:22: UserWarning: Implicit dimension choice for log_softmax has been
deprecated. Change the call to include dim=X as an argument.
  return F.log_softmax(x)
```

Part b

Plot the decision boundaries of a network after the loss falls below 1×10-4. (5 points)

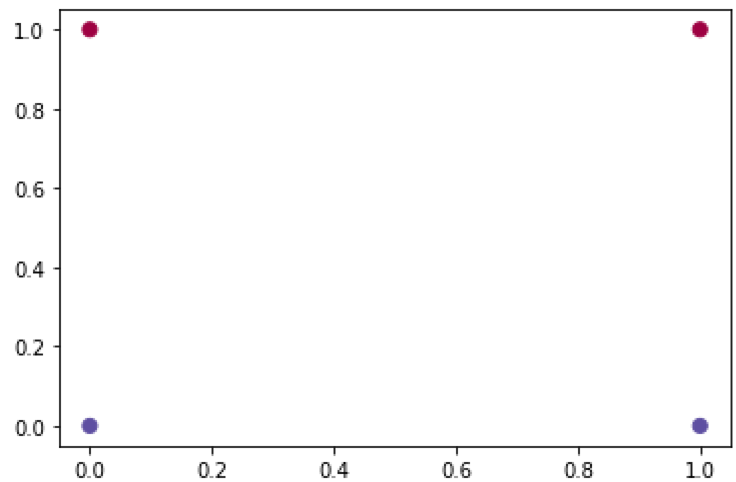
In [13]:
```
#train
net = Net()

# create a stochastic gradient descent optimizer
learning_rate = .01
optimizer = torch.optim.SGD(net.parameters(), lr=learning_rate, momentum=0.

# create a loss function
criterion = nn.NLLLoss()

nepochs = 40000
data, target = X, y
# run the main training loop
for epoch in range(nepochs):
    optimizer.zero_grad()
    # forward propagate
    net_out = net(data)
    # compute loss
    loss = criterion(net_out, target)
    # backpropagate
    loss.backward()
    # update parameters
    optimizer.step()
    # print out report

    if epoch % 10 == 0:
        print('Epoch =', epoch, ', Loss =', loss.item())
        net_out = net(data)
        pred = net_out.data.max(1)[1]  # get the index of the max log-proba
        correctidx = pred.eq(target.data)
        ncorrect = correctidx.sum()
        accuracy = ncorrect.item()/len(data)

    if loss.item() < 0.0001:
        print('Epoch =', epoch, ', Loss =', loss.item())
        print('Training accuracy is ', accuracy)
        break

# compute accuracy on training data

net_out = net(data)
pred = net_out.data.max(1)[1]  # get the index of the max log-probability
correctidx = pred.eq(target.data)
ncorrect = correctidx.sum()
accuracy = ncorrect.item()/len(data)
print('Training accuracy is ', accuracy)
```

```
Epoch = 210 , Loss = 0.6932947635650635
Epoch = 220 , Loss = 0.6932694911956787
Epoch = 230 , Loss = 0.6932486295700073
Epoch = 240 , Loss = 0.693231344223022 5
Epoch = 250 , Loss = 0.6932169198989868
Epoch = 260 , Loss = 0.6932049989700317
Epoch = 270 , Loss = 0.693195104598999
Epoch = 280 , Loss = 0.6931869983673096
Epoch = 290 , Loss = 0.6931802034378052
Epoch = 300 , Loss = 0.6931745409965515
```

```
Epoch = 300 , Loss = 0.6931745409965515
Epoch = 310 , Loss = 0.6931699514389038
Epoch = 320 , Loss = 0.6931660175323486
Epoch = 330 , Loss = 0.6931627988815308
Epoch = 340 , Loss = 0.6931600570678711
Epoch = 350 , Loss = 0.6931579113006592
Epoch = 360 , Loss = 0.6931561231613159
Epoch = 370 , Loss = 0.6931544542312622
Epoch = 380 , Loss = 0.6931532621383667
Epoch = 390 , Loss = 0.6931523084640503
Epoch = 400 , Loss = 0.6931513547897339
```

In [14]:
```python
plt.scatter(X[:, 0], X[:, 1], c=pred, cmap=plt.cm.Spectral, s = 50)
```

Out[14]:
```
<matplotlib.collections.PathCollection at 0x7f84c8ee8190>
```



In [15]:
```python
plot_decision_boundary(net, X, y, 'Q1_part_b.pdf')
```

```
/var/folders/_y/ch74wgzn7s1dxtq4ysb993sr0000gn/T/ipykernel_42258/39823238
33.py:22: UserWarning: Implicit dimension choice for log_softmax has been
deprecated. Change the call to include dim=X as an argument.
  return F.log_softmax(x)
```

In [16]:
```
print(net.fc1.weight)
print(net.fc2.weight)
print(net.fc3.weight)
print(net.fc4.weight)
print(net.fc5.weight)
```

```
Parameter containing:
tensor([[0.6786, 0.5318],
        [0.4859, 0.7700],
        [0.6909, 0.2302]], requires_grad=True)
Parameter containing:
tensor([[ 0.4240,  0.8782,  0.4689],
        [ 0.1468,  0.5095,  0.2632],
        [-0.0549,  0.8159,  0.8000]], requires_grad=True)
Parameter containing:
tensor([[ 0.4404,  0.2160,  0.5122],
        [ 0.5325,  0.0720,  0.3663],
        [-0.1219,  0.3828,  0.5496]], requires_grad=True)
Parameter containing:
tensor([[-0.0278,  0.8549,  0.0206],
        [ 0.7540,  0.1916,  0.8293],
        [ 0.3344,  0.7597,  0.5536]], requires_grad=True)
Parameter containing:
tensor([[0.4891, 0.2502, 1.0537],
        [0.6476, 0.4687, 0.7347]], requires_grad=True)
```

Part c

Gradually decrease the capacity of the network above. Find the smallest network that can still separate the data, i.e., find the least number of hidden layers and neurons that produces an accuracy of 1 on the training set? (5 points) [A portion of the total points is allocated to your rank amongst your peers in achieving the smallest network]

In [18]:
```
class Net_smallest(nn.Module):

    def __init__(self):
        super(Net_smallest, self).__init__()
        self.fc1 = nn.Linear(2, 2)
        torch.nn.init.uniform_(self.fc1.weight, a=0, b=1)

        self.fc2 = nn.Linear(2, 2)
        torch.nn.init.uniform_(self.fc2.weight, a=0, b=1)

    def forward(self, x):
        x = F.relu(self.fc1(x))
        x = F.relu(self.fc2(x))
        return F.log_softmax(x)
        #return F.softmax(x)
```

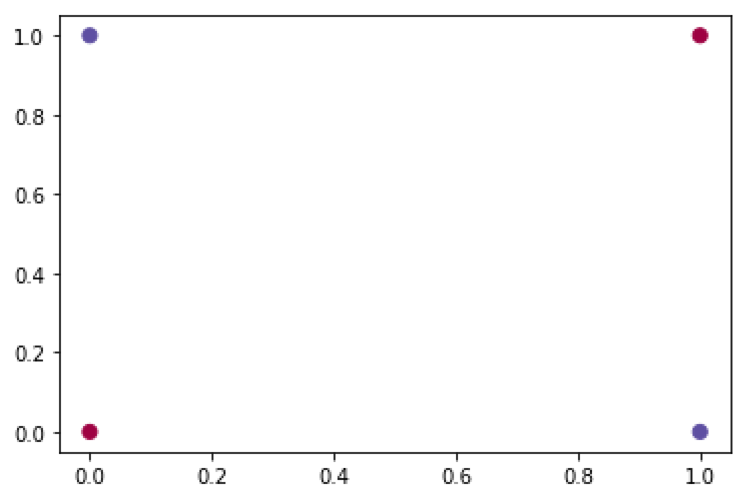
In [19]:
```
# train
net = Net_smallest()

# create a stochastic gradient descent optimizer
learning_rate = .01
optimizer = torch.optim.SGD(net.parameters(), lr=learning_rate, momentum=0.
# optimizer = torch.optim.Adam(net.parameters(), lr=learning_rate)


# create a loss function
criterion = nn.NLLLoss()

nepochs = 40000
data, target = X, y
# run the main training loop
for epoch in range(nepochs):
    optimizer.zero_grad()
    # forward propagate
    net_out = net(data)
    # compute loss
    loss = criterion(net_out, target)
    # backpropagate
    loss.backward()
    # update parameters
    optimizer.step()
    # print out report

    if epoch % 10 == 0:
        print('Epoch =', epoch, ', Loss =', loss.item())
        net_out = net(data)
        pred = net_out.data.max(1)[1]  # get the index of the max log-proba
        correctidx = pred.eq(target.data)
        ncorrect = correctidx.sum()
        accuracy = ncorrect.item()/len(data)
        if accuracy == 1:
            print('Epoch =', epoch, ', Loss =', loss.item())
            print('Training accuracy is ', accuracy)
            break

# compute accuracy on training data
net_out = net(data)
pred = net_out.data.max(1)[1]  # get the index of the max log-probability
correctidx = pred.eq(target.data)
ncorrect = correctidx.sum()
accuracy = ncorrect.item()/len(data)
print('Training accuracy is ', accuracy)
```

```
Epoch = 0 , Loss = 0.7040287852287292
Epoch = 10 , Loss = 0.6947921514511108
Epoch = 20 , Loss = 0.686323450088501
Epoch = 30 , Loss = 0.6813880205154419
Epoch = 40 , Loss = 0.674472451210022
Epoch = 50 , Loss = 0.6664308309555054
Epoch = 60 , Loss = 0.6570623517036438
Epoch = 70 , Loss = 0.6459668874740601
```

```
Epoch = 80 , Loss = 0.635154664516449
Epoch = 90 , Loss = 0.6239791512489319
Epoch = 100 , Loss = 0.6112431287765503
Epoch = 110 , Loss = 0.5980989933013916
Epoch = 120 , Loss = 0.5849587917327881
Epoch = 130 , Loss = 0.5706372261047363
Epoch = 140 , Loss = 0.5562910437583923
Epoch = 150 , Loss = 0.5419378280639648
Epoch = 160 , Loss = 0.5278540253639221
Epoch = 170 , Loss = 0.5128071904182434
Epoch = 180 , Loss = 0.49756360054016113
Epoch = 190 , Loss = 0.4824790060520172
Epoch = 200 , Loss = 0.46755239367485046
Epoch = 210 , Loss = 0.4526820778846741
Epoch = 220 , Loss = 0.43765148520469666
Epoch = 230 , Loss = 0.42197883129119873
Epoch = 240 , Loss = 0.4059251546859741
Epoch = 250 , Loss = 0.3898656368255615
Epoch = 260 , Loss = 0.3734908699989319
Epoch = 270 , Loss = 0.3557816743850708
Epoch = 270 , Loss = 0.3557816743850708
Training accuracy is  1.0
Training accuracy is  1.0

/var/folders/_y/ch74wgzn7s1dxtq4ysb993sr0000gn/T/ipykernel_42258/23387202
71.py:14: UserWarning: Implicit dimension choice for log_softmax has been
deprecated. Change the call to include dim=X as an argument.
  return F.log_softmax(x)
```

In [20]:

```
plt.scatter(X[:, 0], X[:, 1], c=pred, cmap=plt.cm.Spectral, s = 50)
plt.show()
```



In [22]:

```
plot_decision_boundary(net, X, y, 'Q1_part_c.pdf')
```

```
/var/folders/_y/ch74wgzn7s1dxtq4ysb993sr0000gn/T/ipykernel_42258/23387202
71.py:14: UserWarning: Implicit dimension choice for log_softmax has been
deprecated. Change the call to include dim=X as an argument.
  return F.log_softmax(x)
```

In [23]:
```
print(net.fc1.weight)
print(net.fc2.weight)
```

```
Parameter containing:
tensor([[0.7036, 1.0219],
        [1.3338, 1.3401]], requires_grad=True)
Parameter containing:
tensor([[ 0.0086,  2.3259],
        [ 1.3055, -0.6812]], requires_grad=True)
```

In [ ]: