Question 1:

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Note: Please show your solution step-by-step by using what you know about marginal probability, conditional probability, joint probability, and the Bayes' theorem)

The probability of the car being behind the door the contestant picks inititally (door no. 1) is 1/3.

This means that the car being behind the doors not chosen by the contestant (door no. 2 or door no. 3) is 2/3.

Once the host opens one of the doors not chosen by the contestant (door no. 3) the probability of the car being behind the door of 2/3 shifts entirely to the door not chosen by the contestant and not opened by the host (door no. 2).

So, now the probability of teh car being behind the door initially chosen by the contestant (door no. 1) is 1/3. The probability of the car not chosen by the contestant and not opened by the host is 2/3.

Hence, it would be to the contestant's adavantage to switch.

Question 2:

Suppose we have two NBA teams – for simplicity team A and team B – who have made it to NBA Playoffs. In each game between these two teams, team A has a winning probability of 0.55, and team B has a winning probability of 0.45. What is the probability that these two teams will play the 7^{th} game in NBA Playoffs?

Notes:

i. There cannot be a tie in any game.

ii. Please check this link for more information about NBA Playoffs: https://en.wikipedia.org/wiki/NBA_playoffs (https://en.wikipedia.org/wiki/NBA_playoffs) and to think about possible combinations.

iii. Also, please show your solution step-by-step by using what you know about marginal probability, conditional probability, joint probability, and the Bayes' theorem.

The playoffs use a best-of-seven elimination format. This means two teams play each other up to seven times, with the team that wins four games progressing to the next round.

That is, the team which is first to win 4 rounds against the other team progresses to the next round.

Now, in order for 2 teams to play the 7th round against one another, they must win 3 games each in the first 6 games.

Let P(A) be probability of team A winning a game against team B and P(B) be probability of team B winning A game against team A.

P(A) = 0.55

P(B) = 0.45

If we think in terms of a binomial probability distribution, we can consider that for 6 trials, the probabiliity of success is probability of team A winning 3 times. This also means that team A also loses 3 times or team B wins 3 times.

Probability that team A and B play the 7th game in the NBA playoffs = ${}_6C_3 . P(A)^3 . P(B)^{6-3}$ = $\frac{6!}{3!(6-3)!} . (0.55)^3 . (0.45)^3$ = 20 x 0.166375 x 0.091125 = 0.3032184375

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