# Question 1

Write a program using Python that does the following:

- Takes two matrices of any size as the input
- Returns their dot product as the output

Note: You cannot use pre-packaged algorithms for matrix operations for this question. You can use numpy or pandas to store your data (not for calculations). Please do the following:

a. Please test the following matrix multiplications using your hand-written code and report the result:

$$\begin{bmatrix} -4 & -3 & -2 \\ 6 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 6 & 7 \\ -4 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

b. Compare the result to the packaged dot product numpy.dot. Are they same?

c. Please add your code to your .pdf file and also save it as an .ipynb file

In [15]:

```
import numpy as np

def create_matrix():
    num_rows_A = int(input("Enter number of rows for matrix A: "))
    num_cols_A = int(input("Enter numer of columns for matrix A: "))

    matrix_A = []

    for i in range(0, num_rows_A):
        matrix_A_row = []
        for j in range(0, num_cols_A):
            ele = int(input("Enter value: "))
            matrix_A_row.append(ele)
        matrix_A.append(matrix_A_row)

    print("MATRIX A")
    print(np.array(np.mat(matrix_A)))

    num_rows_B = int(input("Enter number of rows for matrix B: "))
    num_cols_B = int(input("Enter numer of columns for matrix B: " ))

    matrix_B = []

    for i in range(0, num_rows_B):
        matrix_B_row = []
        for j in range(0, num_cols_B):
            ele = int(input("Enter value: "))
            matrix_B_row.append(ele)
```

```
        matrix_B.append(matrix_B_row)

    print("MATRIX B")
    print(np.array(np.mat(matrix_B)))

    return matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols_B
```

In [16]:

```
def dot_product(matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols
    if num_rows_B == num_cols_A:
            matrix_dotprodAB = np.zeros((num_rows_A, num_cols_B))

            for i in range(len(matrix_A)):
                for j in range(len(matrix_B[0])):
                    for k in range(len(matrix_B)):
                        matrix_dotprodAB[i][j] += matrix_A[i][k] * matrix_B[
            return matrix_dotprodAB
    else:
        print("The matrices are not compatible")
        return None
```

## Part a:

In [17]:

```
matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols_B = create_matr
```

```
Enter number of rows for matrix A: 3
Enter numer of columns for matrix A: 3
Enter value: -4
Enter value: -3
Enter value: -2
Enter value: 6
Enter value: 0
Enter value: -1
Enter value: 2
Enter value: 1
Enter value: 3
MATRIX A
[[-4 -3 -2]
 [ 6  0 -1]
 [ 2  1  3]]
Enter number of rows for matrix B: 3
Enter numer of columns for matrix B: 2
Enter value: 5
Enter value: 4
Enter value: 6
Enter value: 7
Enter value: -4
Enter value: -3
MATRIX B
[[ 5  4]
 [ 6  7]
 [-4 -3]]
```

In [18]:

```
dot_product(matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols_B)
```

Out[18]:

```
array([[-30., -31.],
       [ 34.,  27.],
       [  4.,   6.]])
```

In [19]:
```
dotProd_AB = np.dot(matrix_A, matrix_B)
print(dotProd_AB)
```

```
[[-30 -31]
 [ 34  27]
 [  4   6]]
```

In [20]:
```
matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols_B = create_matr
```

```
Enter number of rows for matrix A: 2
Enter numer of columns for matrix A: 2
Enter value: 1
Enter value: 0
Enter value: 0
Enter value: 1
MATRIX A
[[1 0]
 [0 1]]
Enter number of rows for matrix B: 2
Enter numer of columns for matrix B: 4
Enter value: 1
Enter value: 2
Enter value: 3
Enter value: 4
Enter value: 5
Enter value: 6
Enter value: 7
Enter value: 8
MATRIX B
[[1 2 3 4]
 [5 6 7 8]]
```

In [21]:
```
dot_product(matrix_A, matrix_B, num_rows_B, num_cols_A, num_rows_A, num_cols_B)
```

Out[21]:
```
array([[1., 2., 3., 4.],
       [5., 6., 7., 8.]])
```

In [22]:
```
dotProd_AB = np.dot(matrix_A, matrix_B)
print(dotProd_AB)
```

```
[[1 2 3 4]
 [5 6 7 8]]
```

### Part b:

Yes, the packaged dot product numpy.dot and the handwritten code for matrix multiplication give the same results.

# Question 2

Assume that we have two (2) d-dimensional real vectors x and y. And denote by xi (or yi) the value in the i-th coordinate of x (or y). Prove or disprove the following statements by checking non-negativity, definiteness, homogeneity, and triangle inequality.

a. The following distance function is a metric. (5 points)

$$L_1(x,y) = \sum_{i=1}^{d} |x_i - y_i|$$

b. The following distance function is a metric. (5 points)

$$L_2(x,y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$$

c. The following distance function is a metric. (10 points)

$$L_2^2(x,y) = \sum_{i=1}^{d} (x_i - y_i)^2$$

## In order to check whether a distance function is a metric, we need to check for the following properties:

Non-negativity: For all $x \in R^n$, $f(x) \geq 0$

Definiteness: f(x) = 0 if x = 0

Homogeneity: For all $x \in R^n, t \in R \rightarrow f(tx) = |t| f(x)$

Triangle inequality: $d(x,y) \leq d(x,z) + d(z,y)$ for all points $x, y, z \in X$

### Part a:

$$L_1(x,y) = \sum_{i=1}^{d} |x_i - y_i|$$

$|x_i - y_i| \geq 0$ for all 1 ≤ i ≤ d

Hence,

$$\sum_{i=1}^{d} |x_i - y_i| \geq 0$$

The distance function is non-negative.

if $x_i - y_i = 0$

then, $|x_i - y_i| = 0$ for all 1 ≤ i ≤ d

Hence,

$$\sum_{i=1}^{d} |x_i - y_i| = 0$$

The distance function is definite.

$|t.\, x_i - t.\, y_i| = |t|.\, |x_i - y_i|$ for all 1 ≤ i ≤ d

Hence,

$$\sum_{i=1}^{d} |t.\, x_i - t.\, y_i| = \sum_{i=1}^{d} |t|.\, |x_i - y_i| = |t|.\, \sum_{i=1}^{d} |x_i - y_i|$$

The distance function is homogenous.

Consider,

$$L_1(x, y) = \sum_{i=1}^{d} |x_i - y_i|$$

$$L_1(x, z) = \sum_{i=1}^{d} |x_i - z_i|$$

$$L_1(z, y) = \sum_{i=1}^{d} |z_i - y_i|$$

Hence,

$$L_1(x, y) = \sum_{i=1}^{d} |x_i - y_i| = \sum_{i=1}^{d} |x_i - z_i + z_i - y_i| \le \sum_{i=1}^{d} |x_i - z_i| + \sum_{i=1}^{d} |z_i - y_i|$$

$$L_1(x, y) \le L_1(x, z) + L_1(z, y)$$

The distance function satisfies the triangle inequality condition.

The distance function $L_1(x, y) = \sum_{i=1}^{d} |x_i - y_i|$ is a metric.

## Part b:

$$L_2(x, y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$$

$(x_i - y_i)^2 \ge 0$ for all 1 ≤ i ≤ d

$\sum_{i=1}^{d} (x_i - y_i)^2 \ge 0$ is a non-negative real function since it is a metric and the square root of any non-negative real number is also a non-negative real number.

Hence,

$\sqrt{\sum_{i=1}^{d} (x_i - y_i)^2} \ge 0$ is a non-negative real function.

The distance function is non-negative.

if $x_i - y_i = 0$

⇒ $(x_i - y_i)^2 = 0$ for all 1 ≤ i ≤ d

⇒ $\sum_{i=1}^{d} (x_i - y_i)^2 = 0$

Hence,

$$\sqrt{\sum_{i=1}^{d} (x_i - y_i)^2} = 0$$

The distance function is definite.

$$f(x) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$$

$$f(t.x) = \sqrt{\sum_{i=1}^{d} (t.x_i - t.y_i)^2} = \sqrt{\sum_{i=1}^{d} t^2(x_i - y_i)^2} = |t|\sqrt{\sum_{i=1}^{d} (x_i - y_i)^2} = |t| f(x)$$

The distance function is homogenous.

$$L_2(x, y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$$

$$L_2(x, z) = \sqrt{\sum_{i=1}^{d} (x_i - z_i)^2}$$

$$L_2(z, y) = \sqrt{\sum_{i=1}^{d} (z_i - y_i)^2}$$

Hence,

$$L_2(x, y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2} \leq \sqrt{\sum_{i=1}^{d} (x_i - z_i)^2} + \sqrt{\sum_{i=1}^{d} (z_i - y_i)^2}$$

$$L_2(x, y) \leq L_2(x, z) + L_2(z, y)$$

The distance function satisfies the triangle inequality condition.

The distance function $L_2(x, y) = \sqrt{\sum_{i=1}^{d} (x_i - y_i)^2}$ is a metric.

## Part c:

$$L_2^2(x,y) = \sum_{i=1}^{d} (x_i - y_i)^2$$

$(x_i - y_i)^2 \geq 0$ for all 1 ≤ i ≤ d

⇒ $\sum_{i=1}^{d} (x_i - y_i)^2 \geq 0$

⇒ $L_2^2(x,y) \geq 0$

The distance function is non-negative.

if $x_i - y_i = 0$

⇒ $(x_i - y_i)^2 = 0$ for all 1 ≤ i ≤ d

⇒ $\sum_{i=1}^{d} (x_i - y_i)^2 = 0$

Hence,

$L_2^2(x,y) = 0$

The distance function is definite.

$$f(x) = \sum_{i=1}^{d} (x_i - y_i)^2$$

$$f(t.x) = \sum_{i=1}^{d} (t.x_i - t.y_i)^2 = \sum_{i=1}^{d} t^2(x_i - y_i)^2 = t^2\sum_{i=1}^{d} (x_i - y_i)^2$$

The distance function is homogenous.

Consider,

$$L_2^2(x,y) = \sum_{i=1}^{d} (x_i - y_i)^2$$

$$L_2^2(x,z) = \sum_{i=1}^{d} (x_i - z_i)^2$$

$$L_2^2(z,y) = \sum_{i=1}^{d} (z_i - y_i)^2$$

Hence,

$$L_2^2(x,z) + L_2^2(z,y)$$

$\Rightarrow \sum_{i=1}^{d} (x_i - z_i)^2 + \sum_{i=1}^{d} (z_i - y_i)^2$

$\Rightarrow \sum_{i=1}^{d} (x_i^2 - 2x_i . z_i + z_i^2 + z_i^2 - 2z_i . y_i + y_i^2)$

$\Rightarrow \sum_{i=1}^{d} (x_i^2 + y_i^2 + 2z_i^2 - 2x_i . z_i - 2z_i . y_i)$

$\Rightarrow \sum_{i=1}^{d} (x_i^2 + y_i^2 - 2x_i . y_i + 2x_i . y_i + 2z_i^2 - 2x_i . z_i - 2z_i . y_i)$

$\Rightarrow \sum_{i=1}^{d} ((x_i - y_i)^2 + 2x_i . y_i + 2z_i^2 - 2x_i . z_i - 2z_i . y_i)$

$\Rightarrow \sum_{i=1}^{d} ((x_i - y_i)^2 + 2(z_i^2 + x_i . y_i - x_i . z_i - z_i . y_i))$

$\Rightarrow \sum_{i=1}^{d} (x_i - y_i)^2 + 2\sum_{i=1}^{d} (z_i^2 + x_i . y_i - x_i . z_i - z_i . y_i)$

Now, consider $t = 2\sum_{i=1}^{d} (z_i^2 + x_i . y_i - x_i . z_i - z_i . y_i)$

LHS: $\sum_{i=1}^{d} (x_i - y_i)^2$

RHS: $\sum_{i=1}^{d} (x_i - y_i)^2$ + t

if t = 0, then LHS = RHS

if t > 0, then LHS < RHS

But, if t < 0, then LHS > RHS

i.e. if $2\sum_{i=1}^{d} (z_i^2 + x_i . y_i - x_i . z_i - z_i . y_i) < 0$ then $L_2^2(x, y) > L_2^2(x, z) + L_2^2(z, y)$

The distance function does not satisfy the triangle inequality condition for all cases.

The distance function $L_2^2(x, y) = \sum_{i=1}^{d} (x_i - y_i)^2$ is not a metric.

# Question 3

Calculating by hand, find the characteristic polynomial, eigenvalues and the eigenvectors of the following matrix:

$$\begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix}$$

Let's assume,

$$A = \begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix} \tag{1}$$

## characteristic polynomial of matrix:

step 1:

$$[A] - \lambda . [I] = \begin{bmatrix} 4 & 4 & 4 \\ -2 & -3 & -6 \\ 1 & 3 & 6 \end{bmatrix} - \lambda . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4-\lambda & 4 & 4 \\ -2 & -3-\lambda & -6 \\ 1 & 3 & 6-\lambda \end{bmatrix}$$

step 2:

$det([A] - \lambda * [I]) = \{(4-\lambda) * [(-3-\lambda) * (6-\lambda) - (-6 * 3)]\} - \{4 * [(-2 * (6-\lambda)) - (-$

$\{4 * [\lambda^2 - 3\lambda] - \lambda[\lambda^2 - 3\lambda]\} - \{-24 + 8\lambda\} + \{-12 + 4\lambda\}$

Answer:

$-\lambda^3 + 7\lambda^2 - 16\lambda + 12$

## eigenvalues of matrix:

### step 1:

The characteristic equation is:

$-\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$

### step 2:

factorizing the above equation:

$(\lambda - 2)(-\lambda^2 + 5\lambda - 6) = 0$

$(\lambda - 2)(-\lambda^2 + 2\lambda + 3\lambda - 6) = 0$

$(\lambda - 2)[-\lambda(\lambda - 2) + 3(\lambda - 2)] = 0$

$(\lambda - 2)(\lambda - 2)(\lambda - 3) = 0$

$\lambda = 2, \lambda = 3$

### Answer:

The eigenvalues are 2 & 3.

## eigenvectors of matrix:

### When $\lambda = 2$,

### step 1:

⇒ $\begin{bmatrix} 4-\lambda & 4 & 4 \\ -2 & -3-\lambda & -6 \\ 1 & 3 & 6-\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

⇒ $\begin{bmatrix} 2 & 4 & 4 \\ -2 & -5 & -6 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

### step 2:

Setting X = 1, we get the following equations:

$2(1) + 4Y + 4Z = 0$

$-2(1) - 5Y - 6Z = 0$

### step 3:

⇒ $[2 + 4Y + 4Z = 0] - - - - - - > X3$ (eq1)

$[-2 - 5Y - 6Z = 0] - - - - - - > X2$ (eq2)

---

⇒ $12Y + 12Z + 6 = 0$

$-10Y - 12Z - 4 = 0$

---

⇒ $2Y + 2 = 0$

⇒ $Y = -1$

---

Substituting Y = -1 in eq1

⇒ $2 + 4(-1) + 4Z = 0$

⇒ $4Z - 2 = 0$

⇒ $Z = 1/2$

### step 4:

The eigenvector when $\lambda = 2$ is (1, -1, 1/2)

For convenience, we can scale up by a factor of 2, to get:

(2, -2, 1)

### When $\lambda = 3$,

### step 1:

$$\Rightarrow \begin{bmatrix} 4-\lambda & 4 & 4 \\ -2 & -3-\lambda & -6 \\ 1 & 3 & 6-\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 4 \\ -2 & -6 & -6 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

### step 2:

Setting X = 1, we get the following equations:

$1(1) + 4Y + 4Z = 0$

$-2(1) - 6Y - 6Z = 0$

### step 3:

⇒ $[1 + 4Y + 4Z = 0] - - - - - - - > X3$ (eq1)

$[-2 - 6Y - 6Z = 0] - - - - - - - > X2$ (eq2)

---

⇒ $12Y + 12Z + 3 = 0$

$-12Y - 12Z - 4 = 0$

---

⇒ $X \neq 1$

### step 4:

Setting Y = 1, we get the following equations:

$X + 4(1) + 4Z = 0$

$-2X - 6(1) - 6Z = 0$

### step 5:

⇒ $X + 4Z + 4 = 0 - - - - - - - - X2$ (eq1)

$-2X - 6Z - 6 = 0$ (eq2)

---

⇒ $2X + 8Z + 8 = 0$

$-2X - 6Z - 6 = 0$

---

⇒ $2Z + 2 = 0$

---

⇒ $Z = -1$

### step 6:

Substituting $Z = -1$ in eq1

$X = 0$

### step 7:

The eigenvector when $\lambda = 2$ is (1, -1, 1/2)

For convenience, we can scale up by a factor of 2, to get:

(0, 1, -1)

### Answer:

The Eigenvector corresponding to $\lambda = 2$ is $[2, -2, 1]$

The Eigenvector corresponding to $\lambda = 3$ is $[0, 1, -1]$

# Question 4

Provide a proof for the following: Let A, B, and C be any n x n matrices:

a. Show that trace(ABC) = trace(CAB) = trace(BCA) (10 points)

b. trace(ABC) = trace(BAC). Provide a proof or a counterexample (10 points)

### Part a:

Consider the following two matrices,

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \dots & \dots & \dots & \dots & \dots\dots \\ x_{n1} & x_{n2} & x_{n3} & \dots & x_{nn} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_{11} & y_{12} & y_{13} & \cdots & y_{1n} \\ y_{21} & y_{22} & y_{23} & \cdots & y_{2n} \\ y_{31} & y_{32} & y_{33} & \cdots & y_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & y_{n3} & \cdots & y_{nn} \end{bmatrix}$$

---

Then,

$$X.Y = \begin{bmatrix} x_{11}.y_{11} + x_{12}.y_{21} + \ldots + x_{1n}.y_{n1} & \cdots & \cdots & \cdots \\ \cdots & x_{21}.y_{12} + x_{22}.y_{22} + \ldots + x_{2n}.y_{n2} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & x_{n} \end{bmatrix}$$

---

This means that,

$$Trace(AB) = [x_{11}.y_{11} + x_{12}.y_{21} + \ldots + x_{1n}.y_{n1}] + [x_{21}.y_{12} + x_{22}.y_{22} + \ldots + x_{2n}.y_{n2}] + \ldots + [$$

In simpler terms it can be expressed as follows,

$$Trace(XY) = \sum x_{ij}.y_{ji}$$

$$\Rightarrow Trace(XY) = \sum_i \sum_j x_{ij}.y_{ji}$$

Since $x_{ij}, y_{ij}$ are scalar,

$$x_{ij}.y_{ij} = y_{ij}.x_{ij}$$

Hence,

$$\Rightarrow Trace(XY) = \sum_i \sum_j y_{ij}.x_{ji}$$

$$\Rightarrow Trace(XY) = Trace(YX)$$

---

Now consider,

$$X = AB$$

$$Y = C$$

$$Trace(XY) = Trace(YX)$$

$$\Rightarrow Trace((AB)C) = Trace(C(AB))$$

$$\Rightarrow Trace(ABC) = Trace(CAB)$$

---

Also consider,

$X = CA$

$Y = B$

$Trace(XY) = Trace(YX)$

$\Rightarrow Trace((CA)B) = Trace(B(AC))$

$\Rightarrow Trace(CAB) = Trace(BAC)$

---

Therefore,

$Trace(ABC) = Trace(CAB) = Trace(BAC)$

### Part b:

Let

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$ABC = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 31 & 15 \\ 21 & 18 \end{bmatrix}$$

$$BAC = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 27 & 9 \\ 27 & 18 \end{bmatrix}$$

$Trace(ABC) = 31 + 18 = 49$

$Trace(BAC) = 27 + 18 = 45$

Thus,

$Trace(ABC) \neq Trace(BAC)$

# Question 5

Let A and B be n x n matrices with AB = 0.

Each question below is 5 points. Provide a proof or counterexample for each of the following:

a. BA = 0

b. Either A = 0 or B = 0 (or both)

c. If det(A) = -3, then B = 0

d. There is a vector v ≠ 0 such that BAv = 0

## Part a:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

Hence, the statement if AB = 0 then BA = 0 is False.

## Part b:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

But,

$A \neq 0$ and $B \neq 0$

Hence, the statement Either A = 0 or B = 0 (or both) is False.

## Part c:

Method 1:

$AB = 0$

⇒ $det(AB) = 0$

$det(AB) = det(A).\,det(B)$

Consider,

$$A = \begin{bmatrix} n_1 & n_2 \\ n_3 & n_4 \end{bmatrix}$$

And, $det(A) = n_1.n_4 - n_2.n_3 = -3$

$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$A.B = \begin{bmatrix} n_1 b_{11} + n_2 b_{21} & b_1 b_{12} + n_2 b_{22} \\ n_3 b_{11} + n_4 b_{21} & n_3 b_{12} + n_4 b_{22} \end{bmatrix}$

And, we know that AB=0

So we get the following equations:

$n_1 b_{11} + n_2 b_{21} = 0$ (eq 1)

$n_3 b_{11} + n_4 b_{21} = 0$ (eq 2)

$b_1 b_{12} + n_2 b_{22} = 0$ (eq 3)

$n_3 b_{12} + n_4 b_{22} = 0$ (eq 4)

Considering eq1 and eq2;

$n_1 b_{11} + n_2 b_{21} = 0 \ldots [eq1xn_3]$

$n_3 b_{11} + n_4 b_{21} = 0 \ldots [eq2x - n_1]$

⇒ $n_1 n_3 b_{11} + n_2 n_3 b_{21} = 0$

$-n_1 n_3 b_{11} - n_1 n_4 b_{21} = 0$ (eq 2)

Solving the above 2 equations, we get

$[n_1 n_4 - n_2 n_3] b_{21} = 0$

So, either $n_1 n_4 - n_2 n_3 = 0$ or $b_{21} = 0$

But, we know that $n_1 n_4 - n_2 n_3 = det(A) = -3$

Hence, $b_{21} = 0$

Similarly, $b_{11} = 0, b_{21} = 0, b_{22} = 0$

So if det(A) = -3 then det(B) should be equal to 0.

Method 2:

$AB = 0$

We can assume that $A^{-1}$ exists since $det(A) \neq 0$

Multiplying both sides by $A^{-1}$

⇒ $A.B.A^{-1} = 0.A^{-1}$

⇒ $B = 0$

⇒ $det(B) = 0$

So if det(A) = -3 then det(B) should be equal to 0.

### Part d:

There is a vector v ≠ 0 such that BAV = 0

AB = 0

⇒ det(AB) = det(BA) = 0

⇒ BA is not invertible (Since, a matrix is invertible only when it's product with non-zero vector is 0).

Hence, there is a vector v ≠ 0 such that BAv = 0