**Programming Languages (COP 4020/CIS 6930) [Fall 2012]**

Assignment I

**Objectives**

1. To become acquainted with the SML/NJ compiler.
2. To understand basic ML constructs such as lists, functions, pattern matching, anonymous variables, and let-environments.
3. To gain experience defining recursive functions in a functional programming language.

**Due Date:** Sunday, September 9, 2012 (at 11:59pm).

**Machine Details**

Complete this assignment by yourself on the following CSEE network computers: c4lab01, c4lab02, ..., c4lab20. These machines are physically located in the Center 4 lab (ENB 220). Do not use any server machines like grad, babbage, sunblast, etc. You can connect to the C4 machines from home using SSH. (Example: Host name: *c4lab01.csee.usf.edu* Login ID and Password: <your NetID username and password>) You are responsible for ensuring that your programs compile and execute properly on these machines.

**Assignment Description**

1. Read Sections 1-3.6.3 and 4.1-4.2 of the *Elements of ML Programming* textbook.
2. Let’s represent (finite) sets of integers as pairs in ML. The first part of the pair is an int indicating the set’s size; the second part of the pair is an int list containing all the set elements. Hence, our sets have type int \* int list. For example, the value   
   (5, [1,3,5,7,9]) represents the set {1, 3, 5, 7, 9}. The order of integers in the int list is irrelevant, so (5, [3,7,1,9,5]) also represents the set {1, 3, 5, 7, 9}.

For this assignment, two data-structure invariants must be maintained. First, the size of a list must be correct; for example, (4, [1,3,5,7,9]) and (6, [1,3,5,7,9]) are invalid sets. Second, no element can appear more than once in the int list; for example, (5, [3,7,1,9,5,3]) and (6, [3,7,1,9,5,3]) are invalid sets.

Using this representation of sets, implement the following functions in a file named *sets.sml*:

**(a)** **cardinality**

This function returns the size of a finite set of integers. For example, cardinality((3,[2,7,4])) returns 3.

**(b) elementOf**

This function tests whether an integer is an element of a given set of integers. For example, elementOf(4, (3,[2,7,4])) returns true because 4 is an element of {2, 7, 4}, while elementOf(3, (3,[2,7,4])) returns false.

**(c) subset**

This function tests whether one set is a subset of another. For example, subset((3,[2,7,4]),(4,[7,9,4,2])) returns true because {2, 7, 4} is a subset of {7, 9, 4, 2}. Note that the empty set (0,[]) is a subset of every set.

**(d) equals**

This function tests whether two sets are equal. For example, equals((3,[2,7,4]), (3,[4,2,7])) returns true because {2, 7, 4} is the same as {4, 2, 7}.

**(e) union**

This function returns the union of two sets. For example, union((3,[2,7,4]), (2,[7,3])) may return (4,[4,2,3,7]) because {2, 7, 4} ∪ {7, 3} = {4, 2, 3, 7}.

**(f) funion**

This function returns the union of every set in a list (family) of sets. When passed an empty list, funion returns the empty set. For example, funion([(3,[2,7,4]), (2,[7,3]), (0,[])]) may return (4,[4,2,3,7]) because {2, 7, 4} ∪ {7, 3} ∪ ∅ = {4, 2, 3, 7}.

**(g) intersection**

This function returns the intersection of two sets. For example, intersection( (3,[2,7,4]), (2,[7,3])) returns (1,[7]) because {2, 7, 4} ∩ {7, 3} = {7}.

**(h) fintersection**

This function returns the intersection of every set in a list (family) of sets. When passed an empty list, fintersection returns the empty set. For example, fintersection( [(3,[2,7,4]), (2,[7,3]), (0,[])]) returns (0,[]) because {2, 7, 4} ∩   
{7, 3} ∩ ∅ = ∅.

**(i) powerset**

This function takes a set of integers *S* and returns a list of the subsets of *S*. For example, powerset((3,[2,7,4])) may return [(0,[]),(1,[2]),(1,[7]),(1,[4]), (2,[2,7]),(2,[2,4]),(2,[7,4]),(3,[2,7,4])] because 2{2,7,4} = {∅, {2}, {7}, {4}, {2,7}, {2,4}, {7,4}, {2,7,4}}. The order of elements in lists returned by powerset is irrelevant.

**(j) invpowerset**

This function takes a list *L* of sets and tests whether there exists a set *S* such that powerset(*S*)=*L*. If such an *S* exists then invpowerset returns SOME *S*; otherwise invpowerset returns NONE. For example, invpowerset([(1,[3]),(1,[7]), (0,[]),(2,[7,3])]) may return SOME (2,[7,3]) because 2{7,3} = {{3}, {7}, ∅, {7,3}}, while invpowerset([(1,[3]),(1,[7]),(2,[7,3])]) returns NONE.

**(k) cartesian** [Note: For undergraduates, this function is extra credit, worth up to +10%.]

This function takes a list of sets *S1, S2, ..., Sn* and returns a list of all the lists that contain an element of *S1* in their first position, an element of *S2* in their second position, etc. If one of *S1..Sn* is the empty set, then cartesian simply returns [], and if cartesian is called on an empty-list argument, then it returns [[]]. For example, cartesian([ (3,[2,7,4]),(2,[7,3])]) may return [[2,7],[2,3],[7,7],[7,3], [4,7],[4,3]] because {2, 7, 4} x {7, 3} = {(2,7), (2,3), (7,7), (7,3), (4,7), (4,3)}. The order of lists returned by cartesian is irrelevant, so cartesian([(3,[2,7,4]), (2,[7,3])]) could alternatively return [[4,3],[4,7],[7,3],[7,7],[2,3], [2,7]], etc.

**Sample Execution** (please remember that the order of elements in sets is irrelevant)

> **sml**

Standard ML of New Jersey v110.74 [built: Thu Aug 16 11:25:45 2012]

- **use "sets.sml";**

[opening sets.sml]

val cardinality = fn : int \* int list -> int

val elementOf = fn : int \* (int \* int list) -> bool

val subset = fn : (int \* int list) \* (int \* int list) -> bool

val equals = fn : (int \* int list) \* (int \* int list) -> bool

val union = fn : (int \* int list) \* (int \* int list) -> int \* int list

val funion = fn : (int \* int list) list -> int \* int list

val intersection = fn : (int \* int list) \* (int \* int list) -> int \* int list

val fintersection = fn : (int \* int list) list -> int \* int list

val powerset = fn : int \* int list -> (int \* int list) list

val invpowerset = fn : (int \* int list) list -> (int \* int list) option

val cartesian = fn : (int \* int list) list -> int list list

val it = () : unit

- **val E = (0,[]);**

val E = (0,[]) : int \* 'a list

- **val S = (3,[2,7,4]);**

val S = (3,[2,7,4]) : int \* int list

- **val T = (4,[7,9,4,2]);**

val T = (4,[7,9,4,2]) : int \* int list

- **val S' = (3,[4,2,7]);**

val S' = (3,[4,2,7]) : int \* int list

- **val U = (2,[7,3]);**

val U = (2,[7,3]) : int \* int list

- **cardinality(S);**

val it = 3 : int

- **elementOf(4,S);**

val it = true : bool

- **elementOf(0,E);**

val it = false : bool

- **subset(S,T);**

val it = true : bool

- **subset(T,S);**

val it = false : bool

- **equals(S,S');**

val it = true : bool

- **equals(S,E);**

val it = false : bool

- **union(S,U);**

val it = (4,[4,2,7,3]) : int \* int list

- **funion([S,T,S',U,E]);**

val it = (5,[9,2,4,3,7]) : int \* int list

- **funion([]);**

val it = (0,[]) : int \* int list

- **intersection(S,U);**

val it = (1,[7]) : int \* int list

- **intersection(S,E);**

val it = (0,[]) : int \* int list

- **fintersection([S,U]);**

val it = (1,[7]) : int \* int list

- **fintersection([]);**

val it = (0,[]) : int \* int list

- **powerset(E);**

val it = [(0,[])] : (int \* int list) list

- **val P = powerset(S);**

val P =

[(0,[]),(1,[2]),(1,[7]),(2,[2,7]),(1,[4]),(2,[2,4]),(2,[7,4]),(3,[2,7,4])]

: (int \* int list) list

- **invpowerset(P);**

val it = SOME (3,[2,7,4]) : (int \* int list) option

- **invpowerset([E]);**

val it = SOME (0,[]) : (int \* int list) option

- **invpowerset([(0,[]),(1,[2]),(1,[7])]);**

val it = NONE : (int \* int list) option

- **cartesian([S,U]);**

val it = [[2,7],[2,3],[7,7],[7,3],[4,7],[4,3]] : int list list

- **cartesian([]);**

val it = [[]] : int list list

- **cartesian([S,U,E,T]);**

val it = [] : int list list

**Grading**

For full credit, your implementation must:

* be commented and formatted appropriately (please use spaces instead of tabs for indentation).
* use anonymous variables, pattern matching, and let-environments when appropriate (e.g., define all helper functions in let-environments).
* compile on the C4 machines with no errors or warnings.
* not use any advanced ML features that cause *side effects* to occur (e.g., I/O or pointer use).
* not use any built-in (i.e., predefined or library) functions.
* not be significantly more complicated than is necessary.
* assume that incoming set arguments satisfy the data-structure invariants described on Page 1 (i.e., your functions should not try to enforce the validity of their set arguments).
* never pass as an argument a set that violates the data-structure invariants described on Page 1 (i.e., when invoking a function, your code may never pass an invalid set as an argument).
* never return from a function a set that violates the data-structure invariants described on   
  Page 1.

Please note that we will test submissions on inputs not shown in the sample execution above.

**Hints**

It took me 1-2 hours to implement and test my *sets.sml*, which is 46 lines of code (not counting whitespace/comments). If, after completely reading Sections 1-3.6.3 and 4.1-4.2 of the textbook, you find yourself spending a significant amount of time (e.g., more than 12 hours) on this assignment, please visit or email the teaching assistant to ask for help with whatever problems you are having.

**Submission Notes**

* Type the following pledge as an initial comment in your *sets.sml* file: “I pledge my Honor that I have not cheated, and will not cheat, on this assignment.” Type your name after the pledge. Not including this pledge will lower your grade 50%.
* Upload and submit your *sets.sml* file into the Blackboard folder for this assignment.
* You may submit your assignment in Blackboard as many times as you like; we will grade your latest submission.
* For every day that your assignment is late (up to 3 days), your grade reduces 10%.