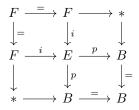
## Algebraic Topology I PS5

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- 1. Consider the fibration on the two-sheeted cover  $S^0 \to S^1 \xrightarrow{p} S^1$
- 2. Let  $F \xrightarrow{i} E \xrightarrow{p} B$  be a fibration with B path connected. Let  $\{c_j\} \in H^*(E;\mathbb{Z})$  with only finitely many in any degree, such that  $\{i^*c_j\}$  form a  $\mathbb{Z}$  basis for the cohomology of  $H^*(F;\mathbb{Z})$ . Note first that this condition implies that the induced map is  $i^*: H^q(E;\mathbb{Z}) \to H^q(F;\mathbb{Z})$  is a surjection. We have the natural Serre spectral sequence  $E_2^{p,q} = H^p(B; H^q(F,\mathbb{Z})) \Longrightarrow H^{p+q}(E;\mathbb{Z})$ , and also a Serre sequence arising from the fibration  $F \to F \to *, \tilde{E}_2^{p,q} = H^p(*; H^q(F;\mathbb{Z})) \Longrightarrow H^{p+q}(F;\mathbb{Z})$ . The following diagram commutes:



Then i induces a map on spectral sequences  $E_2^{p,q} \xrightarrow{i^*} \tilde{E}_2^{p,q}$  which collapses on the  $E_{\infty}$  page so the map

is just  $i^*$ . Hence as  $i^*$  is surjective, each inclusion map must also be a bijection, so the  $d_r: E_r^{0,q} \to E_r^{r,q-r+1}$  vanish for all r. Also on the  $E_2$ -page,  $H^n(F;\mathbb{Z})$  is a finitely generated free  $\mathbb{Z}$ -module. So by UCT  $H^p(B;H^q(F;\mathbb{Z})) = H^p(B;\mathbb{Z}) \otimes H^q(F;\mathbb{Z})$ . Since  $d_r$  is zero on both the p and q axes, by multiplicative structure (ADD DETAIL) it is zero everywhere to the sequence collapses on the  $E_2$  page. Then  $H^p(B;\mathbb{Z}) \otimes H^q(B;\mathbb{Z}) \to H^{p+q}(E;\mathbb{Z})$ ,  $x \otimes y \mapsto s(x) \cap \pi^*(y)$  is an isomorphism.