# Algebraic Topology I PS 9

December 15, 2023

2 Let $p : E \to B$ a n-vector bundle where $B$ is paracompact. Take trivialisations at $x$, $\varphi : \mathcal{U}_x \times \mathbb{F}^n \to p^{-1}(\mathcal{U}_x)$, which form a cover $\bigcup_{x \in B} \mathcal{U}_x \supset B$. Take a locally finite refinement $\{\mathcal{V}_x\}_{x \in B}$, and restrict $\varphi$ to $\varphi|_{\mathcal{V}_x} : \mathcal{V}_x \times \mathbb{F}^n \to p^{-1}(\mathcal{V}_x)$ Since the sub-bundle $E|_{\mathcal{V}_x}$ is trivial, it admits a Euclidean metric $\mu_x$ from the standard euclidean metric on $\mathbb{F}^n$. Locally finiteness condition means we can take a partition of unity $\{\phi_x\}$ subordinate to $\{\mathcal{V}_x\}$. Define

$$\mu = \sum_{x \in B} \phi_x \mu_x$$

where $\phi_x \mu_x$ is well defined on all $E$ as we can define it to vanish outside of $E|_{\mathcal{V}_x}$ and sum is finite. This is a Euclidean metric on $E$ as on each fibre, $\mu|_{E_p} : v \to \sum_{x \in B} \phi_x(p) \mu_x|_{E_p}(v)$. Quadratic property is obvious and positive definiteness holds as at least one $\phi_x(p) > 0$.