Algebraic Topology I PS 10

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1. (a) Let $\gamma_{\mathbb{R}}^{1,n+1}$ be the tautological bundle over $\mathbb{R}P^n$

2. (a) Let ξ be a n-vector bundle $p : E \to B$ such that B is compact. We first show E is Hausdorff. Pick $x, y \in E$ such that $x \neq y$. If $p(x) = p(y) = b$, then the trivialising neighbourhood $\mathcal{U} \subset B$ of b is such that $x, y \in p^{-1}(\mathcal{U}) \cong \mathcal{U} \times \mathbb{F}^n$. Since $\mathcal{U} \times \mathbb{F}^n$ is also Hausdorff the result follows. For local compactness, note that for $\mathcal{U} \subset B$ an open subset, $\mathcal{U}$ is locally compact as B compact (so locally compact). Then any point $x \in E$ is contained in $p^{-1}(\mathcal{U}) \cong \mathcal{U} \times \mathbb{F}^n$ which is also locally compact, thus we have open $\mathcal{V}$ and compact K of $p^{-1}(\mathcal{U})$ such that $x \in \mathcal{V} \subset K$. Moreover, K is compact in E hence the result follows. Denote one-point compactification of E as E^+.

   (b) Consider the map $D(E) \to E^+ = E \cup \{\infty\}$ defined fibrewise as $v \mapsto v/\sqrt{1-|v|^2}$ on $D(E)\backslash S(E)$ and maps $S(E) \subset D(E)$ to ∞. This is evidently continuous as a map from $S(E) \subset D(E)$ to E. Also on fibres it sends $D(E)_b \overset{1:1}{\longleftrightarrow} E_b$ and so is a bijection from $D(E)/S(E)$ to E^+.